Neutrosophic Soft Structures

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Abstract: In paper, neutrosophic soft points with the concept of one point greater than the other and their properties, generalized neutrosophic soft open set known as soft b-open set, neutrosophic soft separation axioms theoretically with support of suitable examples with respect to soft points, neutrosophic soft b0-space engagement with generalized neutrosophic soft closed set, neutrosophic soft b2-space engagement with generalized neutrosophic soft open set are addressed. In continuation, neutrosophic soft b0-space behave as neutrosophic soft b3-space with the plantation of some extra condition on soft b0-space, neutrosophic soft b3-space and related theorems, neutrosophic soft b4-space, monotonous behavior of neutrosophic soft function with connection of different neutrosophic soft separation axioms, monotonous behavior of neutrosophic soft function with connection of different neutrosophic soft close sets are reflected. Secondly, long touched has been given to neutrosophic soft countability connection with bases and sub-bases, neutrosophic soft product spaces and its engagement through different generalized neutrosophic soft open set and close sets, neutrosophic soft coordinate spaces and its engagement through different generalized neutrosophic soft open set and close sets, Finally, neutrosophic soft countability and its relationship with Bolzano Weirstrass Property through engagement of compactness, neutrosophic soft strongly spaces and its related theorems, neutrosophic soft sequences and its relation with neutrosophic soft compactness, neutrosophic soft Lindelof space and related theorems are supposed to address.

Keywords Neutrosophic soft set (NSS), neutrosophic soft point, neutrosophic soft b-open set and neutrosophic b-separation axioms.
1. Introduction

Cagman et al. [1] defined the concept of soft topology on a soft set, and presented its related properties. The authors also discussed the foundations of the theory of soft topological spaces. Shabir and Naz [2] addressed soft topological spaces with a fixed set of parameters over an initial universe. The notions of soft open sets, soft closed sets, soft closure, soft interior points, soft point neighborhood, soft separation axioms and their basic characteristics are addressed. The authors reflected that a soft topological space gives birth to a family of crisp topological spaces that are parameterized. The authors scanned a soft topological space’s subspaces, and explored open and soft closed sets of characterization w. r. t. soft open set. Finally, the authors tackled in depth the notion $T_i$ spaces, soft normal space and soft regular spaces.

Bayramov and Gunduz [3] investigated some basic notions of STS by using soft point concept. Later on the authors addressed $T_i$-soft spaces and the ties between them. Finally, the authors defined soft compactness and leaked out some of its important characteristics.

Khattak et al [4] introduced the concept of soft $\alpha$-open soft $\beta$-open, soft $\alpha$-separations axioms and soft $\beta$-separation axioms in soft single point topology. The authors have addressed soft $(\alpha, \beta)$ separation axioms with regard to ordinary points and soft points in soft topological spaces.

Zadeh [5] exposed the concept of fuzzy set. The author described that a fuzzy set is a class of objects with a continuum of grades of membership. The authors further defined the set through a membership feature, assigning membership grade to each group candidate. The notions of inclusion, union, intersection, complement, relation, convexity, etc. have been applied to such sets, different properties of these notions have been developed in the sense of fuzzy sets. In particular, it has been proven that a soft separation axiom theorem for convex fuzzy set ignored the prerequisites of mutually exclusive fuzzy sets.

Atanassov [6] developed the ‘intuitionistic fuzzy set’ (IFS) concept, which is an extension of the ‘fuzzy set’ definition. The authors explored different properties including operations and set-over relationships. Bayramov and Gunduz [7] introduced some important features of intuitive fuzzy soft topological spaces and established the intuitive soft closure and interior of an intuitive soft set. In addition, their research also addressed intuitionistic fuzzy continuous mapping and structural characteristics. Deli and Broumi [8] defined for the first a relation on neutrosophic soft sets. The new concept allows two neutrosophical soft sets to be composed. It is conceived to extract useful information by combing neutrosophical soft sets. Eventually a decision making approach is based on neutrosophic soft sets.

In a new approach, Bera and Mahapatra [9] introduced the concept of cartesian product and the neutrosophic soft sets in a new approach. Some properties of this principle were discussed and checked with relevant examples from real life. Smarandache [10] for the first time initiated the concept of neutrosophic set which is generalization of the intuitionistic fuzzy set (IFS), and intuitionistic set (NS). Some related examples are presented. Peculiarities between NS and IFS are underlined.

Maji [11] broadened the Smarandache analysis. The author used the idea of soft set neutrosophic set and incorporated neutrosophic soft set. On neutrosophic soft set those meaning and related operations were addressed.
Bera and Mahapatra [12] developed topology formulation on a neutrosophic soft set (NSS). This study studies the notion of neutrosophic soft interior, neutrosophic soft closure, neutrosophic soft neighborhood, neutrosophic soft boundary, normal NSS and their basic properties. Topology and topology for subspaces on the NSS are described with appropriate examples. It also developed some related properties. In addition to this, the concept of separation axioms on neutrosophic soft topological space was introduced along with investigation of several structural features.

Khattak et al. [13] for the first time leaked out the idea of neutrosophic soft b-open set, neutrosophic soft b-closed sets and their properties. Also the idea of neutrosophic soft b-neighborhood and neutrosophic soft b-separation axioms in neutrosophic soft topological structures are reflected. Later on the important results are discussed related to these newly defined concepts with respect to soft points. The concept of neutrosophic soft b-separation axioms of neutrosophic soft topological spaces is diffused in different results with respect to soft points. Furthermore, properties of neutrosophic soft b\(T_i\)-space \(i = 0, 1, 2, 3, 4\) and some associations between them are discussed.

C.G. Aras et al. [14] leaked out some basic notions of neutrosophic soft sets and redefined some neutrosophic soft point concept. Later on the authors addressed some neutrosophic soft \(T_i\)-space and the relationships among them.

T. Y. Ozturk et al. [15] re-defined some operations on neutrosophic soft sets differently as defined by others authors. The authors supported and defended their approach through interesting examples. The authors further beautifully addressed different results with this new approach.

M Al-Tahan, B Davvaz [16] discussed a relationship between SVNS and neutrosophic \(\kappa\)-structures and study it. Moreover, the authors apply results to algebraic structures (hyper structures) and prove that the results on neutrosophic \(\kappa\)-substructure (sub hyper structure) of a given algebraic structure (hyper structure) can be deduced from single valued neutrosophic algebraic structure (hyper structure) and vice versa.

Adeleke et al. [17] studied refined neutrosophic rings, Substructures of refined neutrosophic rings and their elementary properties and it is shown that every refined neutrosophic ring is a ring. Adeleke et al. [18] studied refined neutrosophic ideals and refined neutrosophic homomorphism along their elementary properties. Madeleine et al. [19] provided a connection between neutrosophic \(\kappa\)-structures and subtraction algebras. In this regard, the authors introduced the concept of neutrosophic \(\kappa\)-ideals in subtraction algebra. Moreover, the authors studied its properties and find out a necessary and sufficient condition for a neutrosophic \(\kappa\)-structure to be a neutrosophic \(\kappa\)-ideal. M. Parimala et al. [20] introduced the notion of neutrosophic \(\omega\)-closed sets and study some of the properties of neutrosophic \(\omega\)-closed sets. Further, the authors investigated neutrosophic \(\omega\)-continuity, neutrosophic \(\omega\)-irresoluteness, neutrosophic \(\omega\) connectedness and neutrosophic contra \(\omega\) continuity along with examples. Abdel-Basset et al. [21] proposed a powerful framework based on neutrosophic sets to aid with patients with cancer. Abdel-Basset et al. [22] developed a novel intelligent medical decision support model based on soft computing and IOT as the use of neutrosophical sets decision-making. Abdel-Basset et al. [23] concentrated on the evaluation of supply chain sustainability based on the two critical dimensions. The authors further added that the first is the importance of evolution metrics based on economic, environmental and social aspect, and
the second is the degree of difficulty of information gathering. The authors guaranteed that the aim of this paper increase the accuracy of the evacuation. Abdel-Basset et al. [24] suggested that this article proposed a hybrid combination between analytical hierarchical process (AHP) as an MCDM method and neutrosophic theory to successfully detect and handle the uncertainty and inconsistency challenges.

A. Mehmood et al. [26] introduced generalized neutrosophic separation axioms in neutrosophic soft topological spaces. A. Mehmood et al. [27] discussed soft α-connectedness, soft α-dis-connectedness and soft α-compact spaces in bi-polar soft topological spaces with respect to ordinary points. For better understanding the authors provided suitable examples.

2. Preliminaries

In this section we now state certain useful definitions, theorems, and several existing results for neutrosophic soft sets that we require in the next sections.

Definition 2.1 [13] NSS on a father set (X) is characterized as:

\[ \mathcal{A}_{\text{neutrosophic}} = \left\{ (x, T_{\text{neutrosophic}}(x), I_{\text{neutrosophic}}(x), F_{\text{neutrosophic}}(x) : x \in (X)) \right\} \]

\[ T: (X) \rightarrow [0, 1]^* \]

\[ I: (X) \rightarrow [0, 1]^* \]

\[ F: (X) \rightarrow [0, 1]^* \]

so that’s it

\[ 0^- \leq \{ T + I + F \} \leq 3^+ \]

Definition 2.2 [10] let (X) be a father set, \( d_{\text{parameter}} \) be a set of all conditions, and \( L((X)) \) denote the efficiency set of\((X)\). A pair \((f, d_{\text{parameter}})\) is referred to as a soft set over\((X)\), where\( f \) is a map given by: \( d_{\text{parameter}} \rightarrow L((X)) \).

For \( n \in d_{\text{parameter}} \), \( f(n) \) may be viewed as the set of soft set elements \((f, d_{\text{parameter}})\), or as a set of \( n \) -estimated the soft set components, i.e. \((f, d_{\text{parameter}}) = \{(n, f(n) : n \in d_{\text{parameter}}, f: d_{\text{parameter}} \rightarrow L((X))\})\).

Definition 2.3 [7] Let (X) be a father set, \( d_{\text{parameter}} \) be a set of all conditions, and \( L((X)) \) denote the efficiency Set of\((X)\). A pair \((f, d_{\text{parameter}})\) is referred to as a soft set over\((X)\), where\( f \) is a map given by: \( d_{\text{parameter}} \rightarrow L((X)) \).

Then a (NS)set\((f, d_{\text{parameter}})\)over\((X)\)is a set defined by a set of valued functions signifying a mapping \( d_{\text{parameter}} \rightarrow \)

\( L((X)) \), is referred to as the approximate (NS) function\((f, d_{\text{parameter}})\). In other words, the (NS) is a group of conditions of certain elements of the set \( L((X)) \) so it can be written as a set of ordered pairs:

\[(f, d_{\text{parameter}}) = \{(n, [x, T_{f(x)}(c), I_{f(x)}(c), F_{f(x)}(c) : x \in (X)) : n \in d_{\text{parameter}} \} \]

Obviously, \( T_{f(x)}(c), I_{f(x)}(c), F_{f(x)}(c) \in [0, 1] \) are membership of truth, membership of indeterminacy and membership of falsehood\((n)\). Since the supremum of each\( T, I, F \) is 1, the inequality that \( 0^- \leq T_{f(x)}(c) + I_{f(x)}(c) + F_{f(x)}(c) \leq 3^+ \) is obvious.
Definition 2.4 [5] Let \( f_\Delta \text{parameter} \) be a NSS over the father set \( X \). The complement of \( f_\Delta \text{parameter} \) is signified \( f_\Delta \text{parameter}^c \) and is defined as follows:

\[
( f_\Delta \text{parameter}^c)^c = \left\{ \left[ n, \left[ x, T_{f(x)(\delta)}, 1 - I_{f(x)(\delta)}, F_{f(x)(\delta)}; x \in (X) \right] : n \in \text{parameter} \right) \right\}
\]

It’s clear that

\[
( f_\Delta \text{parameter}^c)^c = ( f_\Delta \text{parameter}).
\]

Definition 2.5 [9] Let \( f_\Gamma, n \) and \( f_\Omega, n \) two NSS over a father set \( X \). \( f_\Gamma, n \) is supposed to be NSS of \( f_\Omega, n \) if

\[
T_{f(x)(\delta)} \leq T_{h(x)(\delta)}, I_{f(x)(\delta)} \leq I_{h(x)(\delta)}, F_{f(x)(\delta)} \geq F_{h(x)(\delta)}, \forall n \in \text{parameter} \text{ and } x \in (X).
\]

It is signifies as \( f_\Gamma, n \subseteq (f_\Omega, n) \). \( f_\Gamma, n \) is said to be (NS) equal to \( f_\Omega, n \) if \( f_\Gamma, n \) is (NSSS) of \( f_\Omega, n \) and \( f_\Omega, n \) is NSSS of if \( f_\Gamma, n \). It is symbolized as \( f_\Gamma, n \equiv (f_\Omega, n) \).

3. Neutrosophic Soft Points and Their Characteristics

Definition 3.1 Let \( f_\Gamma, n \& f_\Delta, n \) be two NSSS over a father set \( X \) s.t. \( f_\Gamma, n \neq f_\Delta, n \). Then their union is signifies as \( f_\Gamma, n \cup f_\Delta, n \) is defined as \( f_\Gamma, n \) = \( f_\Gamma, n \) & is defined as \( f_\Gamma, n \) = \( \left\{ \left[ n, x, T_{b(x)(\delta)}, I_{b(x)(\delta)}, F_{b(x)(\delta)}; x \right] : n \in \text{parameter} \right\} \)

\[
\begin{align*}
T_{b(x)(\delta)} &= \max \left\{ T_{A(x)(\delta)}, T_{B(x)(\delta)} \right\}, \\
I_{b(x)(\delta)} &= \max \left\{ I_{A(x)(\delta)}, I_{B(x)(\delta)} \right\}, \\
F_{b(x)(\delta)} &= \min \left\{ F_{A(x)(\delta)}, F_{B(x)(\delta)} \right\}.
\end{align*}
\]

Definition 3.2 Let \( f_\Gamma, n \& f_\Delta, n \) be two NSSS over the father set \( X \) s.t. \( f_\Gamma, n \neq f_\Delta, n \). Then their intersection is signifies as \( f_\Gamma, n \cap f_\Delta, n \) is defined as follows \( f_\Gamma, n \) = \( \left\{ \left[ n, x, T_{b(x)(\delta)}, I_{b(x)(\delta)}, F_{b(x)(\delta)}; x \right] : n \in \text{parameter} \right\} \)

\[
\begin{align*}
T_{b(x)(\delta)} &= \min \left\{ T_{A(x)(\delta)}, T_{B(x)(\delta)} \right\}, \\
I_{b(x)(\delta)} &= \min \left\{ I_{A(x)(\delta)}, I_{B(x)(\delta)} \right\}, \\
F_{b(x)(\delta)} &= \max \left\{ F_{A(x)(\delta)}, F_{B(x)(\delta)} \right\}.
\end{align*}
\]

Definition 3.3 NSSet \( f_\Gamma, n \) be a NSS over the father set \( X \) is said to be a null neutrosophic soft set

\[
\text{If } T_{_f(x)(\delta)} = 0, I_{_f(x)(\delta)} = 0, F_{_f(x)(\delta)} = 1; \forall e \in n \text{ and } x \in (X), \text{ it is signifies as } 0_{(X), n}.
\]

Definition 3.4 NSS \( f_\Gamma, n \) over the father set \( X \) is said to be an absolute neutrosophical softness i

\[
\text{If } T_{_f(x)(\delta)} = 1, I_{_f(x)(\delta)} = 1, F_{_f(x)(\delta)} = 0; \forall e \in n \text{ and } x \in (X).
\]

It is signifies as \( 1_{(X), n} \). Clearly, \( 0_{(X), n}^c = 1_{(X), n} \) and \( 1_{(X), n}^c = 0_{(X), n} \).
Definition 3.5 Let $\text{NSS}(\tilde{X}, \mathcal{A}_{\text{parameter}})$ be the family of all NS soft sets over the father set $(\tilde{X})$ and $\tau \in \text{NSS}(\tilde{X}, \mathcal{A}_{\text{parameter}})$, then $\tau$ is said to be an NS soft topology on $(\tilde{X})$ if:

1. $\{0, \{X_{a,b,c}, 1 \} \} \in \tau$,
2. The union of any number of NS soft sets in $\tau$ belongs to $\tau$,
3. The intersection of a finite number of (NS) soft sets in $\tau$ belongs to $\tau$. Then $(\tilde{X}, \tau, \mathcal{A}_{\text{parameter}})$ is said to be a (NSTS) over $(\tilde{X})$. Each member of $\tau$ is said to be a NS soft open set.

Definition 3.6 Let $(\tilde{X}, \tau, \mathcal{A}_{\text{parameter}})$ be a NSTS over $(\tilde{X})$ and $(\tilde{f}, \mathcal{A}_{\text{parameter}})$ be a NS set over $(\tilde{X})$. Then $(\tilde{f}, \mathcal{A}_{\text{parameter}})$ is supposed to be a NS closed set if its complement is a NS open set.

Definition 3.7 Let $\text{NS}$ be the family of all NS over father set $(\tilde{X})$ and $x \in (\tilde{X})$ then NS $x_{(a,b,c)}$ is supposed to be a N point, for $0 < a$, $b$, $c \leq 1$ and is defined as follows:

$x_{(a,b,c)} = \{x \mid y = x \}$

It is obvious that every (NS) is actually the union of its N points.

Example 3.8 Suppose that $(\tilde{X}) = \{x_1, x_2\}$ then N set $A = \{x_{3}, 0.1, 0.3, 0.5\}, (x_{2}, 0.5, 0.4, 0.7\})$ is the union of N points $x_{1(0.1,0.3,0.5)} \& x_{2(0.5,0.4,0.7)}$. Now we define the concept of NS points for NS sets.

Definition 3.9 Let $\text{NSS}(\tilde{X})$ be the family of all NS soft sets over the father set $(\tilde{X})$. Then $\text{NSS}(x_{(a,b,c)})^e$ is called a NS point, for every $x \in (\tilde{X}), 0 < a$, $b$, $c \leq 1$, $e \in \mathcal{A}_{\text{parameter}}$, and is defined as follows:

$x^{e}_{(a,b,c)(y)} = \{x \mid e = e \land y = x \}$

Definition 3.10 Suppose that the father set $(\tilde{X})$ is assumed to be $\tilde{X} = \{x_1, x_2\}$ & the set of conditions by $\mathcal{A}_{\text{parameter}} = \{e_1, e_2\}$. Let us consider $\text{NSS}(\tilde{f}, \mathcal{A}_{\text{parameter}})$ over the father set $(\tilde{X})$ as follows:

$(\tilde{f}, \mathcal{A}_{\text{parameter}}) = \{e_1 = \{(x_{1}, 0.3, 0.7, 0.6), (x_{2}, 0.4, 0.3, 0.8)\}, e_2 = \{(x_{1}, 0.4, 0.6, 0.8), (x_{2}, 0.3, 0.7, 0.2)\}\}$. It is clear that

$(\tilde{f}, \mathcal{A}_{\text{parameter}})$ is the union of its NS points.

\[
\begin{align*}
&\text{Where,} \\
&f_{e_1}^{e_1(0.3,0.7,0.6)} = \{ e_1 = \{(x_{1}, 0.3, 0.7, 0.6), (x_{2}, 0.0, 0.1)\}, \} \\
&f_{e_2}^{e_2(0.4,0.6,0.8)} = \{ e_2 = \{(x_{1}, 0.0, 0.1), (x_{2}, 0.0, 0.1)\}, \} \\
&f_{e_1}^{e_1(2,0.3,0.7,0.6)} = \{ e_1 = \{(x_{1}, 0.0, 0.1), (x_{2}, 0.0, 0.1)\}, e_2 = \{(x_{1}, 0.0, 0.1), (x_{2}, 0.0, 0.1)\} \} \\
&f_{e_2}^{e_2(2,0.3,0.7,0.2)} = \{ e_1 = \{(x_{1}, 0.0, 0.1), (x_{2}, 0.0, 0.1)\}\} \\
&f_{e_1}^{e_1(0.3,0.7,0.6)} = \{ e_1 = \{(x_{1}, 0.3, 0.7, 0.6), (x_{2}, 0.0, 0.1)\}, \} \\
&f_{e_2}^{e_2(0.4,0.6,0.8)} = \{ e_2 = \{(x_{1}, 0.0, 0.1), (x_{2}, 0.0, 0.1)\}, \} \\
&f_{e_1}^{e_1(2,0.3,0.7,0.6)} = \{ e_1 = \{(x_{1}, 0.0, 0.1), (x_{2}, 0.0, 0.1)\}, e_2 = \{(x_{1}, 0.0, 0.1), (x_{2}, 0.0, 0.1)\} \} \\
&f_{e_2}^{e_2(2,0.3,0.7,0.2)} = \{ e_1 = \{(x_{1}, 0.0, 0.1), (x_{2}, 0.0, 0.1)\}\} \\
\end{align*}
\]
Definition 3.11 Let $(\tilde{f}, d_{\text{parameter}})$ be a NSS over the father set $\tilde{X}$, we say that $x^e_{(a,b,c)} \in (\tilde{f}, d_{\text{parameter}})$ read as belonging to the NSS $(\tilde{f}, d_{\text{parameter}})$ whenever $a \leq T_{(\tilde{f})(x)}, b \leq L_{(\tilde{f})(x)}, c \geq F_{(\tilde{f})(x)}$.

Definition 3.12 Let $x^e_{(a,b,c)}$ and $x^{e'}_{(a',b',c')}$ be two NS points. For the NS points Over father set $(\tilde{X})$, we say that the NS points are distinct points $x^e_{(a,b,c)} \cap x^{e'}_{(a',b',c')} = 0_{(\tilde{X}),d_{\text{parameter}}}$. It is clear that $x^e_{(a,b,c)}$ and $x^{e'}_{(a',b',c')} \neq x^e_{(a,b,c)}$ are distinct NS points if and only if $x > y$ or $x < y$ or $e' > e$ or $e' < e$.

4. Neutrosophic Soft b-Separation Axioms

In this phase we define generalized neutrosophic soft separation axioms.

Definition 4.1 Let $((\tilde{X}), \tau, d_{\text{parameter}})$ be a NSTS over $(\tilde{X})$ & $(\tilde{f}, d_{\text{parameter}})$ be a neutrosophic soft set over $(\tilde{X})$. Then $(\tilde{f}, d_{\text{parameter}})$ is supposed to be a NS b-open if $(\tilde{f}, d_{\text{parameter}}) \subseteq \text{cl}(\text{int}(\tilde{f}, d_{\text{parameter}})) \cup \text{cl}(\text{cl}(\tilde{f}, d_{\text{parameter}}))$ and NS b-close if $(\tilde{f}, d_{\text{parameter}}) \supseteq \text{int}(\text{cl}(\tilde{f}, d_{\text{parameter}})) \cap \text{cl}(\text{in}(\tilde{f}, d_{\text{parameter}}))$.

Definition 4.2 Let $((\tilde{X}), \tau, d_{\text{parameter}})$ be a NSTS over $(\tilde{X})$, and $x^e_{(a,b,c)} > y^{e'}_{(a',b',c')}$ or $x^e_{(a,b,c)} < y^{e'}_{(a',b',c')}$ are NS points. If there exist NSb open sets $(\tilde{f}, d_{\text{parameter}}) \& (\tilde{g}, d_{\text{parameter}})$ such that

$$x^e_{(a,b,c)} \in (\tilde{f}, d_{\text{parameter}}), x^e_{(a,b,c)} \cap (\tilde{g}, d_{\text{parameter}}) = 0_{(\tilde{X}),d_{\text{parameter}}} \text{ or } y^{e'}_{(a',b',c')} \in (\tilde{g}, d_{\text{parameter}}),$$

$$y^{e'}_{(a',b',c')} \cap (\tilde{f}, d_{\text{parameter}}) = 0_{(\tilde{X}),d_{\text{parameter}}} \text{ Then } ((\tilde{X}), \tau, d_{\text{parameter}}) \text{ is called a NSb}_0$$

Definition 4.3 Let $((\tilde{X}), \tau, d_{\text{parameter}})$ be a NSTS over $(\tilde{X})$, and $x^e_{(a,b,c)} > y^{e'}_{(a',b',c')}$ or $x^e_{(a,b,c)} < y^{e'}_{(a',b',c')}$ are NS points. If there exist NSb open sets $(\tilde{f}, d_{\text{parameter}}) \& (\tilde{g}, d_{\text{parameter}})$:

$$x^e_{(a,b,c)} \in (\tilde{f}, d_{\text{parameter}}), x^e_{(a,b,c)} \cap (\tilde{g}, d_{\text{parameter}}) = 0_{(\tilde{X}),d_{\text{parameter}}} \text{ or } y^{e'}_{(a',b',c')} \in (\tilde{g}, d_{\text{parameter}}),$$

$$y^{e'}_{(a',b',c')} \cap (\tilde{f}, d_{\text{parameter}}) = 0_{(\tilde{X}),d_{\text{parameter}}} \text{ Then } ((\tilde{X}), \tau, d_{\text{parameter}}) \text{ is called a NSb}_1.$$

Definition 4.4 Let $((\tilde{X}), \tau, d_{\text{parameter}})$ be a NSTS over $(\tilde{X})$, and $x^e_{(a,b,c)} > y^{e'}_{(a',b',c')}$
\( x^e_{(a,b,c)} < y^e_{(a',b',d')} \) are NS points. if \( \exists \) NSb open sets \((\bar{f},d_{\text{parameter}}) \& (\bar{g},d_{\text{parameter}}) \) s.t.

\[ x^e_{(a,b,c)} \in (\bar{f},d_{\text{parameter}}) \& y^e_{(a',b',d')} \in (\bar{g},d_{\text{parameter}}) \cap (\bar{f},d_{\text{parameter}}) = 0 \]

Then \((\bar{X}, \tau, d_{\text{parameter}})\) is called a NSb2 space.

**Example 4.5** Suppose that the father set \((\bar{X})\) is assumed to be

\( (\bar{X}) = \{x_1, x_2\} \) & the set of conditions by \( d_{\text{parameter}} = \{e_1, e_2\} \). Let us consider NS set \((\bar{f},d_{\text{parameter}})\) over the father set \((\bar{X})\) & \( x^{e_1}_{(0,1,0,4,0,7)}, x^{e_2}_{(0,2,0,5,0,6)}, x^{e_3}_{(3,0,3,0,5)}, x^{e_4}_{(4,0,4,0,4)} \) be NS points. Then the family \( \tau = 0 \) (\((\bar{X}),d_{\text{parameter}}\)) \( \{ (\bar{f}_1,d_{\text{parameter}}), (\bar{f}_2,d_{\text{parameter}}), (\bar{f}_3,d_{\text{parameter}}), (\bar{f}_4,d_{\text{parameter}}), (\bar{f}_5,d_{\text{parameter}}), (\bar{f}_6,d_{\text{parameter}}) \} \),

where

\[ \left( \bar{f}_1, d_{\text{parameter}} \right) = x^{e_1}_{(0,1,0,4,0,7)} \left( \bar{f}_2, d_{\text{parameter}} \right) = x^{e_2}_{(0,2,0,5,0,6)} \left( \bar{f}_3, d_{\text{parameter}} \right) = x^{e_3}_{(3,0,3,0,5)} \left( \bar{f}_4, d_{\text{parameter}} \right) = x^{e_4}_{(4,0,4,0,4)} \left( \bar{f}_5, d_{\text{parameter}} \right) = \left( \bar{f}_6, d_{\text{parameter}} \right) \]

\[ \left( \bar{f}_{10}, d_{\text{parameter}} \right) = \{ x^{e_1}_{(0,1,0,4,0,7)}, x^{e_2}_{(0,2,0,5,0,6)}, x^{e_3}_{(3,0,3,0,5)}, x^{e_4}_{(4,0,4,0,4)} \} \]

is a NSTS over the father set \((\bar{X})\). Thus \((\bar{X}), \tau, d_{\text{parameter}}\) be a NSTS over the father set \((\bar{X})\). Also \((\bar{X}), \tau, d_{\text{parameter}}\) is NSb0 structure but it is not NSb1 because for NS points \( x^{e_1}_{(0,1,0,4,0,7)}, x^{e_2}_{(0,2,0,5,0,6)} \) \((\bar{X}), \tau, d_{\text{parameter}}\) not NSb1.

**Example 4.6** Suppose that the father set \((\bar{X})\) is assumed to be

\( (\bar{X}) = \{x_1, x_2\} \) & the set of conditions by \( d_{\text{parameter}} = \{e_1, e_2\} \). Let us consider NS set \((\bar{f},d_{\text{parameter}})\) over the father set \((\bar{X})\) & \( x^{e_1}_{(0,1,0,4,0,7)}, x^{e_2}_{(0,2,0,5,0,6)} x^{e_3}_{(3,0,3,0,5)}, x^{e_4}_{(4,0,4,0,4)} \) be NS points. Then the family \( \tau = 0 \) (\((\bar{X}),d_{\text{parameter}}\)) \( \{ (\bar{f}_1,d_{\text{parameter}}), (\bar{f}_2,d_{\text{parameter}}), (\bar{f}_3,d_{\text{parameter}}), (\bar{f}_4,d_{\text{parameter}}), (\bar{f}_5,d_{\text{parameter}}), (\bar{f}_6,d_{\text{parameter}}) \} \),

where

\[ \left( \bar{f}_1, d_{\text{parameter}} \right) = x^{e_1}_{(0,1,0,4,0,7)} \left( \bar{f}_2, d_{\text{parameter}} \right) = x^{e_2}_{(0,2,0,5,0,6)} \left( \bar{f}_3, d_{\text{parameter}} \right) = x^{e_3}_{(3,0,3,0,5)} \left( \bar{f}_4, d_{\text{parameter}} \right) = x^{e_4}_{(4,0,4,0,4)} \left( \bar{f}_5, d_{\text{parameter}} \right) = \left( \bar{f}_6, d_{\text{parameter}} \right) \]

\[ \left( \bar{f}_{10}, d_{\text{parameter}} \right) = \{ x^{e_1}_{(0,1,0,4,0,7)}, x^{e_2}_{(0,2,0,5,0,6)}, x^{e_3}_{(3,0,3,0,5)}, x^{e_4}_{(4,0,4,0,4)} \} \]
Let \((\tilde{X}, \tau, d_{\text{parameter}})\) be a \((\text{N}ST_{2}\) over the father set \((\tilde{X})\). Thus \((\tilde{X}, \tau, d_{\text{parameter}})\) be a \((\text{NSS}_{2}\) over the father set \((\tilde{X})\). Also \((\tilde{X}, \tau, d_{\text{parameter}})\) is NSS \((\text{N}ST)\) structure. \(x^{e_{1}(0,1,0,0,0,7)} \cup x^{e_{2}(0,2,0,5,0,6)} \cup x^{e_{3}(0,3,0,3,0,5)} \cup x^{e_{4}(0,4,4,4,4)}\) is a \(\text{N}ST\) over the father set \((\tilde{X})\). Thus \((\tilde{X}, \tau, d_{\text{parameter}})\) is NSS \((\text{N}ST)\) structure. \(x^{e_{1}(0,1,0,0,0,7)} \cup x^{e_{2}(0,2,0,5,0,6)} \cup x^{e_{3}(0,3,0,3,0,5)} \cup x^{e_{4}(0,4,4,4,4)}\) is a \(\text{NSS}_{1}\) structure iff each \(\text{NSS}\) point is a \(\text{NSS}\) closed set.
Proof Let \( (x^e_{(a,b,c)}, d_{parameter}) > (y^{e/}_{(a',b',c')}, d_{parameter}) \) be two NS points in NSb\( T_2 \) space. Then \( \exists \) disjoint NS open sets \( (\tilde{G}, d_{parameter}) \& (\tilde{G}', d_{parameter}) \) s.t. \( (x^e_{(a,b,c)}, d_{parameter}) \in (\tilde{G}, d_{parameter}) \& (y^{e/}_{(a',b',c')}, d_{parameter}) \in (\tilde{G}', d_{parameter}) \). Since \( (x^e_{(a,b,c)}, d_{parameter}) \cap (y^{e/}_{(a',b',c')}, d_{parameter}) = 0 \). Then \( (\tilde{G}, d_{parameter}) \cap (\tilde{G}', d_{parameter}) = 0 \). Next suppose that \( (x^e_{(a,b,c)}, d_{parameter}) > (y^{e/}_{(a',b',c')}, d_{parameter}) \), \( \exists a NS b \)-open set \( (\tilde{G}, d_{parameter}) \) containing \( (x^e_{(a,b,c)}, d_{parameter}) \) but not \( (y^{e/}_{(a',b',c')}, d_{parameter}) \) s.t. \( (x^e_{(a,b,c)}, d_{parameter}) \cap (y^{e/}_{(a',b',c')}, d_{parameter}) = 0 \). Then \( (\tilde{G}, d_{parameter}) \) is a NSb\( T_2 \) space.

Theorem 4.9 Let \( (\tilde{X}, \tau, d_{parameter}) \) be a NS\( T_2 \) space over the father set \( \tilde{X} \). Then \( (\tilde{X}, \tau, d_{parameter}) \) is NS b-T\(_1\) space if every NS point \( (x^e_{(a,b,c)}, d_{parameter}) \in (\tilde{G}, d_{parameter}) \subseteq (\tilde{X}, \tau, d_{parameter}) \). If there exists a NSb open set \( (\tilde{G}, d_{parameter}) \) s.t. \( (x^e_{(a,b,c)}, d_{parameter}) \in (\tilde{G}, d_{parameter}) \subseteq (\tilde{X}, \tau, d_{parameter}) \). Then \( (\tilde{X}, \tau, d_{parameter}) \) is a NSb\( T_2 \) space.

Proof. Suppose \( (x^e_{(a,b,c)}, d_{parameter}) \cap (y^{e/}_{(a',b',c')}, d_{parameter}) = 0 \). Since \( (\tilde{X}, \tau, d_{parameter}) \) is NS\( b \)-close sets in \( (\tilde{X}, \tau, d_{parameter}) \). Then \( (x^e_{(a,b,c)}, d_{parameter}) \cap (y^{e/}_{(a',b',c')}, d_{parameter}) = 0 \). Thus \( \exists a NS b \) open set \( (\tilde{G}, d_{parameter}) \) s.t. \( (x^e_{(a,b,c)}, d_{parameter}) \in (\tilde{G}, d_{parameter}) \subseteq (\tilde{X}, \tau, d_{parameter}) \). Then \( (\tilde{G}, d_{parameter}) \) is a NS\( b \) soft NSb\( T_2 \) space.

5. Characterization of other NS b-Separation Axioms

Definition 5.1. Let \( (\tilde{X}, \tau, d_{parameter}) \) be a NS\( T_2 \) over the father set \( \tilde{X} \). \( (\tilde{G}, d_{parameter}) \) be a NS\( b \)-open set and \( (x^e_{(a,b,c)}, d_{parameter}) \cap (\tilde{G}, d_{parameter}) = 0 \). If \( \exists NS \) b-open sets \( (\tilde{G}_1, d_{parameter}) \& (\tilde{G}_2, d_{parameter}) \) s.t. \( (x^e_{(a,b,c)}, d_{parameter}) \in (\tilde{G}_1, d_{parameter}) \subseteq (\tilde{X}, \tau, d_{parameter}) \). Then \( (\tilde{G}_1, d_{parameter}) \& (\tilde{G}_2, d_{parameter}) \) is called

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a NS b-regular space. \(((\tilde{X}), \tau, d_{\text{parameter}})\) is said to be \(NS_{b}3\) space, if is both a \(NS_{regular}\) and \(NS_{b}1\) space.

**Theorem 5.2.** Let \(((\tilde{X}), \tau, d_{\text{parameter}})\) be a \(NST_{Sober}\) the father set \((\tilde{X})\), \(((\tilde{X}), \tau, d_{\text{parameter}})\) is soft \(bT_{3}\) space if for every \((x^{e}_{(a,b,c)}, d_{\text{parameter}})\) \(\in (\tilde{I}, d_{\text{parameter}})\), \(e \in (\tilde{G}, d_{\text{parameter}})\) \(\in (\tilde{X}, \tau, d_{\text{parameter}})\) s.t. \((x^{e}_{(a,b,c)}, d_{\text{parameter}}) \in (\tilde{G}, d_{\text{parameter}}) \subset (\tilde{G}, d_{\text{parameter}}) \subset (\tilde{I}, d_{\text{parameter}}).

**Proof.** Let \(((\tilde{X}), \tau, d_{\text{parameter}})\) is \(NS_{b}3\) space & \((x^{e}_{(a,b,c)}, d_{\text{parameter}}) \in (\tilde{I}, d_{\text{parameter}}) \in (\tilde{X}, \tau, d_{\text{parameter}})\). Since \((\tilde{X}, \tau, d_{\text{parameter}})\) is \(NST_{3}\) space for the NS point \((x^{e}_{(a,b,c)}, d_{\text{parameter}})\) & b-closed set \((\tilde{I}, d_{\text{parameter}}) \cap (\tilde{G}, d_{\text{parameter}}) \cap (\tilde{G}, d_{\text{parameter}}) = 0((\tilde{X}, d_{\text{parameter}}))\). Then we have \((x^{e}_{(a,b,c)}, d_{\text{parameter}}) \in (\tilde{G}, d_{\text{parameter}}) \subset (\tilde{G}, d_{\text{parameter}}) \subset (\tilde{I}, d_{\text{parameter}})\).

Conversely, let \((x^{e}_{(a,b,c)}, d_{\text{parameter}}) \cap (\tilde{H}, d_{\text{parameter}}) = 0((\tilde{X}, d_{\text{parameter}})) \cap (\tilde{H}, d_{\text{parameter}})\) be a \(NS_{b}\) closed set. \((x^{e}_{(a,b,c)}, d_{\text{parameter}}) \cap (\tilde{H}, d_{\text{parameter}}) = 0((\tilde{X}, d_{\text{parameter}})) \cap (\tilde{H}, d_{\text{parameter}})\) & from the condition of the theorem, we have \((x^{e}_{(a,b,c)}, d_{\text{parameter}}) \in (\tilde{G}, d_{\text{parameter}}) \subset (\tilde{G}, d_{\text{parameter}}) \subset (\tilde{I}, d_{\text{parameter}})\).

\((\tilde{X}), \tau, d_{\text{parameter}})\) is said to be a \(NS_{b}4\) space if it is both a \(NS_{normal}\) and \(NS_{b}1\) space.

**Definition 5.3.** Let \(((\tilde{X}), \tau, d_{\text{parameter}})\) be a \(NST_{Sober}\) the father set \((\tilde{X})\). This space is a \(NS_{normal}\) space, if for every pair of disjoint \(NS_{b}\) closed sets \((\tilde{I}_{1}, d_{\text{parameter}}) \& (\tilde{I}_{2}, d_{\text{parameter}})\), \(\exists\) disjoint \(NS_{b}\) open sets \((\tilde{G}_{1}, d_{\text{parameter}}) \& (\tilde{G}_{2}, d_{\text{parameter}})\) s.t. \((\tilde{I}_{1}, d_{\text{parameter}}) \subset (\tilde{G}_{1}, d_{\text{parameter}}) \& (\tilde{I}_{2}, d_{\text{parameter}}) \subset (\tilde{G}_{2}, d_{\text{parameter}})\).

\((\tilde{X}), \tau, d_{\text{parameter}})\) is said to be a \(NS_{b}4\) space if it is both a \(NS_{normal}\) and \(NS_{b}1\) space.

**Theorem 5.4.** Let \(((\tilde{X}), \tau, d_{\text{parameter}})\) be a \(NST_{Sober}\) the father set \((\tilde{X})\). This space is a \(NS_{bT_{4}}\) space \(\iff\), for each \(NS_{b}\) closed set \((\tilde{I}, d_{\text{parameter}})\) and \(NS_{b}\) open set \((\tilde{G}, d_{\text{parameter}})\), \(\exists\) a \(NS_{b}\) open set \((\tilde{B}, d_{\text{parameter}})\).

**Proof.** Let \(((\tilde{X}), \tau, d_{\text{parameter}})\) be a \(NS_{b}\) over the father set \((\tilde{X})\). & let \((\tilde{I}, d_{\text{parameter}}) \subset (\tilde{G}, d_{\text{parameter}})\). Then \((\tilde{G}, d_{\text{parameter}})\) is is a \(NS_{b}\) closed set and \((\tilde{I}, d_{\text{parameter}}) \cap (\tilde{G}, d_{\text{parameter}}) = 0((\tilde{X}, d_{\text{parameter}}))\).

Since \((\tilde{X}), \tau, d_{\text{parameter}})\) be a \(NS_{b}4\) space, \(\exists\) \(NS_{b}\) open sets \((\tilde{B}_{1}, d_{\text{parameter}}) \& (\tilde{B}_{2}, d_{\text{parameter}})\) s.t. \((\tilde{I}, d_{\text{parameter}}) \subset (\tilde{B}_{1}, d_{\text{parameter}}) \& (\tilde{I}, d_{\text{parameter}}) \subset (\tilde{B}_{2}, d_{\text{parameter}})\).

Thus \((\tilde{B}_{1}, d_{\text{parameter}}) \subset (\tilde{B}_{1}, d_{\text{parameter}}) \subset (\tilde{B}_{2}, d_{\text{parameter}}) \subset (\tilde{G}, d_{\text{parameter}})\) is a \(NS_{b}\) closed set and \((\tilde{B}_{2}, d_{\text{parameter}}) \subset (\tilde{B}_{2}, d_{\text{parameter}}) \subset (\tilde{G}, d_{\text{parameter}})\). So \((\tilde{I}, d_{\text{parameter}}) \subset (\tilde{B}_{1}, d_{\text{parameter}}) \subset (\tilde{B}_{2}, d_{\text{parameter}}) \subset (\tilde{G}, d_{\text{parameter}})\).
Conversely, let \((f_1, d_{\text{parameter}})\) and \((f_2, d_{\text{parameter}})\) be two disjoint NSb closed sets. Then \((f_1, d_{\text{parameter}}) \not\subset (f_2, d_{\text{parameter}})^c\). From the condition of theorem, there exists a NS b open set \((B, d_{\text{parameter}})\) s.t. \((f_1, d_{\text{parameter}}) \not\subset (B, d_{\text{parameter}})^c\). Thus \((B, d_{\text{parameter}})\) and \((B, d_{\text{parameter}})^c\) are NS open sets and \((f_1, d_{\text{parameter}}) \not\subset (f_2, d_{\text{parameter}})^c\), \((f_2, d_{\text{parameter}}) \not\subset (B, d_{\text{parameter}})^c\) and \((B, d_{\text{parameter}}) \not\subset (B, d_{\text{parameter}})^c\). Hence, \((\mathcal{X}), \tau, d_{\text{parameter}}\) be a NS b space.

6. Monotonous behavior of NS b-Separation Axioms

Theorem 6.1. Let \((X^{\text{crs}}, \mathcal{X}, \partial)\) be NSST such that it is NSb Hausdorff space and \((Y^{\text{crs}}, \mathcal{Y}, \partial)\) be any NSST. Let \((\mathcal{Y}, \partial) : (X^{\text{crs}}, \mathcal{X}, \partial) \rightarrow (Y^{\text{crs}}, \mathcal{Y}, \partial)\) be a soft function such that it is soft monotone and continuous. Then \((Y^{\text{crs}}, \mathcal{Y}, \partial)\) is also of characteristics of NSb Hausdorffness.

Proof: Suppose \((x^{0}_{(a,b,c)}), d_{\text{parameter}}) \not\supset (x^{0}_{(a,b,c)}), d_{\text{parameter}} \not\subset (x^{0}_{(a,b,c)}), d_{\text{parameter}}\) such that either \((x^{0}_{(a,b,c)})_1 \not\supset (x^{0}_{(a,b,c)})_2\) or \((x^{0}_{(a,b,c)})_1 \not\subset (x^{0}_{(a,b,c)})_2\). Since \((\mathcal{Y}, \partial)\) is soft monotone. Let us suppose the monotonically increasing case. So, \((x^{0}_{(a,b,c)})_1 \not\supset (x^{0}_{(a,b,c)})_2\). Suppose \((g^{0}_{(a',b',c')}, d_{\text{parameter}}) \not\supset (g^{0}_{(a',b',c')}, d_{\text{parameter}}) \not\subset (g^{0}_{(a',b',c')}, d_{\text{parameter}})\) such that \((g^{0}_{(a',b',c')})_1 \not\supset (g^{0}_{(a',b',c')})_2\). Since, \((X^{\text{crs}}, \mathcal{X}, \partial)\) is NSb Hausdorff space so there exists mutually disjoint NS b-open sets \((k_1, \partial)\) and \((k_2, \partial)\) \((X^{\text{crs}}, \mathcal{X}, \partial) \supset (k_1, \partial) \supset (k_2, \partial) \not\subset (X^{\text{crs}}, \mathcal{X}, \partial)\). We claim that \((g^0_{(a',b',c')})(k_1, \partial) \cap (g^0_{(a',b',c')})(k_2, \partial) = 0\). Otherwise \((g^0_{(a',b',c')})(k_1, \partial) \neq 0\). Suppose \((x^{0}_{(a',b',c')}, d_{\text{parameter}}) \not\supset (x^{0}_{(a',b',c')}, d_{\text{parameter}}) \not\subset (x^{0}_{(a',b',c')}, d_{\text{parameter}})\) such that \((x^{0}_{(a',b',c')})_1 \not\supset (x^{0}_{(a',b',c')})_2\). Then \((g^{0}_{(a',b',c')})_1 \not\supset (g^{0}_{(a',b',c')})_2\).
Finally,

\[(e^{eff}_{(a,b',c')}, d_{\text{parameter}})_{1} = \# \left( (e^{eff}_{(a,b',c')}, d_{\text{parameter}})_{2} \right) \forall (e^{eff}_{(a,b',c')}, d_{\text{parameter}}) \in \#((k_{2}, \partial)) \Rightarrow \exists \left( e^{eff}_{(a,b',c')}, d_{\text{parameter}} \right) \in (k_{2}, \partial) \text{ s.t. } (e^{eff}_{(a,b',c')}, d_{\text{parameter}})_{3} = \# \left( (e^{eff}_{(a,b',c')}, d_{\text{parameter}})_{2} \right) \Rightarrow \# \left( (e^{eff}_{(a,b',c')}, d_{\text{parameter}})_{2} \right) > \# \left( (e^{eff}_{(a,b',c')}, d_{\text{parameter}})_{3} \right) \text{ since } f \text{ is soft one-to-one} \Rightarrow \left( e^{eff}_{(a,b',c')}, d_{\text{parameter}} \right)_{2} = \# \left( (e^{eff}_{(a,b',c')}, d_{\text{parameter}})_{3} \right) \Rightarrow \left( e^{eff}_{(a,b',c')}, d_{\text{parameter}} \right)_{2} \in \#((k_{2}, \partial)) \Rightarrow (e^{eff}_{(a,b',c')}, d_{\text{parameter}})_{2} \in \#((k_{1}, \partial) \cap \#((k_{2}, \partial)). \text{ This is contradiction because } (k_{1}, \partial) \cap (k_{2}, \partial) = 0_{(\Xi_{d_{\text{parameter}}})}. \text{ So, } f((k_{1}, \partial) \cap f((k_{2}, \partial)) = 0_{(\Xi_{d_{\text{parameter}}})}. \text{ Finally,} \]

\[
0_{(\Xi_{d_{\text{parameter}}})} = \begin{cases} 
(x^{e}_{(a,b,c)}, d_{\text{parameter}})_{1} > (x^{e}_{(a,b,c)}, d_{\text{parameter}})_{2} \text{ or } \\
(x^{e}_{(a,b,c)}, d_{\text{parameter}})_{1} < (x^{e}_{(a,b,c)}, d_{\text{parameter}})_{2} \Rightarrow \\
(x^{e}_{(a,b,c)}, d_{\text{parameter}})_{1} \neq \#(x^{e}_{(a,b,c)}, d_{\text{parameter}})_{2} \Rightarrow \#(x^{e}_{(a,b,c)}, d_{\text{parameter}})_{1} > \#(x^{e}_{(a,b,c)}, d_{\text{parameter}})_{2} \Rightarrow \#(x^{e}_{(a,b,c)}, d_{\text{parameter}})_{1} \neq \#(x^{e}_{(a,b,c)}, d_{\text{parameter}})_{2} \end{cases}
\]

Given a pair of points

\[
(y^{e}_{(a,b',c')}, d_{\text{parameter}})_{1}, (y^{e}_{(a,b',c')}, d_{\text{parameter}})_{2} \in Y^{crisp} \exists \#(y^{e}_{(a,b',c')}, d_{\text{parameter}})_{1} \neq (y^{e}_{(a,b',c')}, d_{\text{parameter}})_{2} \neq (y^{e}_{(a,b',c')}, d_{\text{parameter}})_{2} \]

\[
(y^{e}_{(a,b',c')}, d_{\text{parameter}})_{2} \text{ We are able to find out mutually exclusive NSb open sets }\#((k_{1}, \partial)), \#((k_{2}, \partial)) \in \#(Y^{crisp}, \Xi_{d_{\text{parameter}}}) \text{ s.t. } (y^{e}_{(a,b',c')}, d_{\text{parameter}})_{1} \in \#((k_{1}, \partial)) \cap \#((k_{2}, \partial)). \text{ This proves that} \]

\[
(Y^{crisp}, \Xi_{d_{\text{parameter}}}) \text{ is NSb Hausdorff space.}
\]

**Theorem 6.2.** Let \((X^{crisp}, \Xi_{d_{\text{parameter}}})\) be NSST and \((Y^{crisp}, \Xi_{d_{\text{parameter}}})\) be another NSST which satisfies one more condition of NSb Hausdorffness. Let \((f, \partial): (X^{crisp}, \Xi_{d_{\text{parameter}}}) \rightarrow (Y^{crisp}, \Xi_{d_{\text{parameter}}})\) be a soft function s.t. it is soft monotone and continuous. Then \((X^{crisp}, \Xi_{d_{\text{parameter}}})\) is also of characteristics of NSb Hausdorffness.

**Proof:** Suppose \((x^{e}_{(a,b,c)}, d_{\text{parameter}})_{1}, (x^{e}_{(a,b,c)}, d_{\text{parameter}})_{2} \in X^{crisp} \text{ such that} \exists (x^{e}_{(a,b,c)}, d_{\text{parameter}})_{1} > \)

\[
(x^{e}_{(a,b,c)}, d_{\text{parameter}})_{2} \text{ or } (x^{e}_{(a,b,c)}, d_{\text{parameter}})_{1} < (x^{e}_{(a,b,c)}, d_{\text{parameter}})_{2} \text{ Let us suppose the NS monotonically increasing case. So, } (x^{e}_{(a,b,c)}, d_{\text{parameter}})_{1} > \)

\[
(x^{e}_{(a,b,c)}, d_{\text{parameter}})_{2} \text{ implies that } \#((x^{e}_{(a,b,c)}, d_{\text{parameter}})_{1} \Rightarrow \#((x^{e}_{(a,b,c)}, d_{\text{parameter}})_{2}) \Rightarrow \#((x^{e}_{(a,b,c)}, d_{\text{parameter}})_{2}) \Rightarrow \#((x^{e}_{(a,b,c)}, d_{\text{parameter}})_{1}) \Rightarrow \#((x^{e}_{(a,b,c)}, d_{\text{parameter}})_{2}) \Rightarrow \#((x^{e}_{(a,b,c)}, d_{\text{parameter}})_{1})
\]

\[
\#((x^{e}_{(a,b,c)}, d_{\text{parameter}})_{2}) \text{ respectively. Suppose } (y^{e}_{(a,b',c')}, d_{\text{parameter}})_{1}, (y^{e}_{(a,b',c')}, d_{\text{parameter}})_{2} \in Y^{crisp} \text{ such that} \}
\]

\[
(y^{e}_{(a,b',c')}, d_{\text{parameter}})_{1} > (y^{e}_{(a,b',c')}, d_{\text{parameter}})_{2} \text{ or } (y^{e}_{(a,b',c')}, d_{\text{parameter}})_{1} < (y^{e}_{(a,b',c')}, d_{\text{parameter}})_{2}
\]
Let $\mathcal{X}_{(a,b,c)}^{\alpha_1}$ and $\mathcal{Y}_{(a,b,c)}^{\beta_1}$ be soft mappings such that they are continuous mappings. Let $\mathcal{X}_{(a,b,c)}^{\alpha_1}$ be a soft mapping such that it is continuous mapping. Let

$$\mathcal{X}_{(a,b,c)}^{\alpha_1}$$

respectively such that $\mathcal{X}_{(a,b,c)}^{\alpha_1} \in Y_{\alpha_1}$ but $(Y_{\alpha_1}, X, \partial)$ is NSb Hausdorff space. So according to definition $\mathcal{X}_{(a,b,c)}^{\alpha_1} > \mathcal{X}_{(a,b,c)}^{\alpha_1} > \mathcal{X}_{(a,b,c)}^{\alpha_1}$. There definitely exists NS b-open sets $(k_1, \partial)$ and $(k_2, \partial) \in Y_{\alpha_1}$ such that $\mathcal{X}_{(a,b,c)}^{\alpha_1} \in (k_1, \partial)$ and $\mathcal{X}_{(a,b,c)}^{\alpha_1} \in (k_2, \partial)$ and these two NS b-open sets are guaranteed mutually exclusive because the possibility of one rules out the possibility of others. Since $\mathcal{X}_{(a,b,c)}^{\alpha_1}((k_1, \partial))$ and $\mathcal{X}_{(a,b,c)}^{\alpha_1}((k_2, \partial))$ are NS open in $(X_{\alpha_1}, X, \partial)$.

Now, $\mathcal{X}_{(a,b,c)}^{\alpha_1}((k_1, \partial)) \cap \mathcal{X}_{(a,b,c)}^{\alpha_1}((k_2, \partial)) = \mathcal{X}_{(a,b,c)}^{\alpha_1}((k_1, \partial)) \cap \mathcal{X}_{(a,b,c)}^{\alpha_1}((k_2, \partial)) = \mathcal{X}_{(a,b,c)}^{\alpha_1}((k_1, \partial))$  

$0((k_1, \partial), \partial)$ and $\mathcal{X}_{(a,b,c)}^{\alpha_1}((k_1, \partial), \partial) \cap \mathcal{X}_{(a,b,c)}^{\alpha_1}((k_2, \partial), \partial) = \mathcal{X}_{(a,b,c)}^{\alpha_1}((k_1, \partial)) \cap \mathcal{X}_{(a,b,c)}^{\alpha_1}((k_2, \partial)) = \mathcal{X}_{(a,b,c)}^{\alpha_1}((k_1, \partial), \partial) \Rightarrow $ 

$\mathcal{X}_{(a,b,c)}^{\alpha_1}((k_1, \partial), \partial) \cap \mathcal{X}_{(a,b,c)}^{\alpha_1}((k_2, \partial), \partial) = \mathcal{X}_{(a,b,c)}^{\alpha_1}((k_1, \partial), \partial)$  

$\mathcal{X}_{(a,b,c)}^{\alpha_1}((k_1, \partial), \partial) \cap \mathcal{X}_{(a,b,c)}^{\alpha_1}((k_2, \partial), \partial) \Rightarrow \mathcal{X}_{(a,b,c)}^{\alpha_1}((k_1, \partial), \partial) \cap \mathcal{X}_{(a,b,c)}^{\alpha_1}((k_2, \partial), \partial) = \mathcal{X}_{(a,b,c)}^{\alpha_1}((k_1, \partial), \partial)$ 

Accordingly, NSST is NS b Hausdorff space.

**Theorem 6.3.** Let $(X_{\alpha_1}, X, \partial)$ be NSST and $(X_{\alpha_1}, X, \partial)$ be another NSST. Let $(\mathcal{X}_{(a,b,c)}^{\alpha_1}, \gamma, \partial): (X_{\alpha_1}, X, \partial) \to (X_{\alpha_1}, X, \partial)$ be a soft mapping such that it is continuous mapping. Let $(X_{\alpha_1}, X, \partial)$ is NSb Hausdorff space. Then it is guaranteed that $\mathcal{X}_{(a,b,c)}^{\alpha_1}((X_{(a,b,c)}^{\alpha_1}, \partial), \partial) = \mathcal{X}_{(a,b,c)}^{\alpha_1}((X_{(a,b,c)}^{\alpha_1}, \partial), \partial)$ is a NSb closed sub-set of $(X_{\alpha_1}, X, \partial) \times (X_{\alpha_1}, X, \partial)$.

**Proof:** Given that $(X_{\alpha_1}, X, \partial)$ be NSST and $(Y_{\beta_1}, X, \partial)$ be another NSST. Let $(\mathcal{X}_{(a,b,c)}^{\alpha_1}, \gamma, \partial): (X_{\alpha_1}, X, \partial) \to (X_{\alpha_1}, X, \partial)$ be a soft mapping such that it is continuous mapping. $(Y_{\beta_1}, X, \partial)$ is NSb Hausdorff space. Then we will prove that $\mathcal{X}_{(a,b,c)}^{\alpha_1}((X_{(a,b,c)}^{\alpha_1}, \partial), \partial) = \mathcal{X}_{(a,b,c)}^{\alpha_1}((X_{(a,b,c)}^{\alpha_1}, \partial), \partial)$. 

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$\# \left( \left( y^e_{(a,b,c)} \right)^{\text{parameter}} \right)$ is a NSb closed sub-set of $(X^{\text{crip}}, \mathfrak{I}, \partial) \times (Y^{\text{criP}}, \mathfrak{I}, \partial)$. Equavilintly, we will prove that $\left( \left( x^e_{(a,b,c)} \right)^{\text{parameter}}, \left( y^e_{(a,b,c)} \right)^{\text{parameter}} \right)$ is NS b-open sub-set of $(X^{\text{crp}}, \mathfrak{I}, \partial) \times (X^{\text{crp}}, \mathfrak{I}, \partial)$. Let $(\left( x^e_{(a,b,c)} \right)^{\text{parameter}}, \left( y^e_{(a,b,c)} \right)^{\text{parameter}}) \in$ \\
$\left( \left( x^e_{(a,b,c)} \right)^{\text{parameter}}, \left( y^e_{(a,b,c)} \right)^{\text{parameter}} \right)$ with $(x^e_{(a,b,c)} \text{parameter}) >$ \\
$\left( y^e_{(a,b,c)} \right)^{\text{parameter}} : \# \left( \left( x^e_{(a,b,c)} \right)^{\text{parameter}} \right)$ \\
$\left( y^e_{(a,b,c)} \right)^{\text{parameter}} : \# \left( \left( x^e_{(a,b,c)} \right)^{\text{parameter}} \right)$ or $\left( x^e_{(a,b,c)} \right)^{\text{parameter}}, \left( y^e_{(a,b,c)} \right)^{\text{parameter}} \right)$ \\
$\left( y^e_{(a,b,c)} \right)^{\text{parameter}} : \# \left( \left( x^e_{(a,b,c)} \right)^{\text{parameter}} \right) < \left( y^e_{(a,b,c)} \right)^{\text{parameter}}$. Then, \\
$\# \left( \left( x^e_{(a,b,c)} \right)^{\text{parameter}} \right) > \# \left( y^e_{(a,b,c)} \right)^{\text{parameter}} \text{ or } \# \left( x^e_{(a,b,c)} \right)^{\text{parameter}} < \left( y^e_{(a,b,c)} \right)^{\text{parameter}} \left( y^e_{(a,b,c)} \right)^{\text{parameter}}$ accordingly. Since, $(X^{\text{crp}}, \mathfrak{I}, \partial)$ is NSb Hausdorff space. Definitely, \\
$\# \left( \left( x^e_{(a,b,c)} \right)^{\text{parameter}} \right), \# \left( y^e_{(a,b,c)} \right)^{\text{parameter}}$ are points of $(X^{\text{crp}}, \mathfrak{I}, \partial)$, there exists NS b-open sets $(G, \partial), (K, \partial) \in (X^{\text{crp}}, \mathfrak{I}, \partial)$ such that $\# \left( \left( x^e_{(a,b,c)} \right)^{\text{parameter}} \right) \in (G, \partial) \text{ and } \# \left( \left( x^e_{(a,b,c)} \right)^{\text{parameter}} \right) \in (K, \partial)$ provided $(G, \partial) \cap (K, \partial) = 0 (\overline{X})^{\text{parameter}}$. Since, $(\#, \partial)$ is soft continuous, $\#^{-1}((G, \partial) \text{ and } \#^{-1}((K, \partial)$ are NS b-open sets in $(X^{\text{crp}}, \mathfrak{I}, \partial)$ supposing $(x^e_{(a,b,c)} \text{parameter})$ and $(y^e_{(a,b,c)} \text{parameter})$ respectively and so $\#^{-1}((G, \partial) \times \#^{-1}((K, \partial)$ is basic NSb open set in $(X^{\text{crp}}, \mathfrak{I}, \partial) \times (X^{\text{crp}}, \mathfrak{I}, \partial)$ containing \\
$\left( \left( x^e_{(a,b,c)} \right)^{\text{parameter}}, \left( y^e_{(a,b,c)} \right)^{\text{parameter}} \right)$, since $(G, \partial) \cap (K, \partial) = 0 (\overline{X})^{\text{parameter}}$. it is clear by the definition of \\
$\left( \left( x^e_{(a,b,c)} \right)^{\text{parameter}}, \left( y^e_{(a,b,c)} \right)^{\text{parameter}} \right): \# \left( \left( x^e_{(a,b,c)} \right)^{\text{parameter}} \right) = \# \left( \left( y^e_{(a,b,c)} \right)^{\text{parameter}} \right)$ that \\
$\#^{-1}((G, \partial) \text{ and } \#^{-1}((K, \partial) \cap \left( \left( x^e_{(a,b,c)} \right)^{\text{parameter}}, \left( y^e_{(a,b,c)} \right)^{\text{parameter}} \right) : \#(x) = \# \left( \left( y^e_{(a,b,c)} \right)^{\text{parameter}} \right) = 0 (\overline{X})^{\text{parameter}},$ that is $\#^{-1}((G, \partial) \times \#^{-1}((K, \partial) \in$ \\
$\left( \left( x^e_{(a,b,c)} \right)^{\text{parameter}}, \left( y^e_{(a,b,c)} \right)^{\text{parameter}} \right): \# \left( \left( x^e_{(a,b,c)} \right)^{\text{parameter}} \right) = \# \left( \left( y^e_{(a,b,c)} \right)^{\text{parameter}} \right)$ \\
$\left( \left( x^e_{(a,b,c)} \right)^{\text{parameter}}, \left( y^e_{(a,b,c)} \right)^{\text{parameter}} \right): \# \left( \left( x^e_{(a,b,c)} \right)^{\text{parameter}} \right) = \# \left( \left( y^e_{(a,b,c)} \right)^{\text{parameter}} \right)$ \\
Hence, $\left( \left( x^e_{(a,b,c)} \right)^{\text{parameter}}, \left( y^e_{(a,b,c)} \right)^{\text{parameter}} \right): \# \left( \left( x^e_{(a,b,c)} \right)^{\text{parameter}} \right) = \# \left( \left( y^e_{(a,b,c)} \right)^{\text{parameter}} \right)$.
$\mathcal{F} \left( \left( y^{r/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right)^{c}$ implies that $\left\{ \left( x^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}}, \left( y^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right\}$; $\mathcal{F} \left( \left( x^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right) = $ $\mathcal{F} \left( \left( y^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right)$ is NS b-closed.

7. Mixed NS b-Separation Axioms

**Theorem 7.1.** Let $(X^{cr-iP}, \tau, \varnothing)$ be (NSSTS) and $(Y^{cr-iP}, \varnothing, \varnothing)$ be an-other (NSSTS). Let $(f, g): (X^{cr-iP}, \tau, \varnothing) \to (Y^{cr-iP}, \varnothing, \varnothing)$ be NSb open mapping such that it is onto. If the soft set $\left\{ \left( x^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}}, \left( y^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right\}$ is NS b-closed in $(X^{cr-iP}, \tau, \varnothing) \times (Y^{cr-iP}, \varnothing, \varnothing)$, then $(X^{cr-iP}, \tau, \varnothing)$ will behave as NSb Hausdorff space.

**Proof:** Suppose $\mathcal{F} \left( \left( x^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right), \mathcal{F} \left( y^{e/(a,b,c,d)} \right)$ be two points of $Y^{cr-iP}$ such that

either $\mathcal{F} \left( \left( x^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right) > \mathcal{F} \left( \left( y^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right)$ or $\mathcal{F} \left( \left( x^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right) \prec \mathcal{F} \left( \left( y^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right).

$\mathcal{F} \left( \left( y^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right). Then $\left( \left( x^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}}, \left( y^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right) \notin $ $\left\{ \left( x^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}}, \left( y^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right\}$

with $\left( x^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} > $ $\left( y^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}}.$

$\mathcal{F} \left( \left( y^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right) \notin \left\{ \left( x^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}}, \left( y^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right\}.$

with $\left( x^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} < $ $\left( y^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}}.$

that is $\left( x^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \notin \left\{ \left( x^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}}, \left( y^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right\}$

with $\left( x^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} > $ $\left( y^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}}.$

$\mathcal{F} \left( \left( y^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right) \notin \left\{ \left( x^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}}, \left( y^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right\}.$

with $\left( x^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \subset \left\{ \left( x^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}}, \left( y^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right\}$

$\mathcal{F} \left( \left( y^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right) \subset \left\{ \left( x^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}}, \left( y^{e/(a,b,c,d)} \right)^{d_{\text{parameter}}} \right\}.$
\[ \mathbb{f} \left( \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} \right) \]. Since, \( \left( x^e \left( a, b, c \right) \right)_{\text{parameter}}, \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} \in \] 
\[ \left\{ \left( x^e \left( a, b, c \right) \right)_{\text{parameter}}, \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} \right\} \]
\[ \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} : \mathbb{f} \left( \left( x^e \left( a, b, c \right) \right)_{\text{parameter}} \right) > \]
\[ \mathbb{f} \left( \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} \right) \] or \( \left( x^e \left( a, b, c \right) \right)_{\text{parameter}}, \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} \in \) 
\[ \left\{ \left( x^e \left( a, b, c \right) \right)_{\text{parameter}}, \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} \right\} \]
\[ \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} : \mathbb{f} \left( \left( x^e \left( a, b, c \right) \right)_{\text{parameter}} \right) < \mathbb{f} \left( \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} \right) \] is soft in \( \left( X^{cr, ip}, \mathfrak{I}, \mathfrak{A} \right) \times \left( Y^{cr, ip}, \mathfrak{F}, \mathfrak{B} \right) \), then \( \exists \mathfrak{B} \) b - open sets 
\( \left( g, \mathfrak{B} \right) \) and \( \left( k, \mathfrak{B} \right) \) in \( \left( X^{cr, ip}, \mathfrak{I}, \mathfrak{A} \right) \) s.t. \( \left( x^e \left( a, b, c \right) \right)_{\text{parameter}}, \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} \) \( \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} : \mathbb{f} \left( \left( x^e \left( a, b, c \right) \right)_{\text{parameter}} \right) > \]
\[ \mathbb{f} \left( \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} \right) \] or \( \left( x^e \left( a, b, c \right) \right)_{\text{parameter}}, \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} \in \) \( \left( g, \mathfrak{B} \right) \times \left( k, \mathfrak{B} \right) \) \( \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} : \mathbb{f} \left( \left( x^e \left( a, b, c \right) \right)_{\text{parameter}} \right) < \mathbb{f} \left( \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} \right) \] 
\[ \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} : \mathbb{f} \left( \left( x^e \left( a, b, c \right) \right)_{\text{parameter}} \right) < \mathbb{f} \left( \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} \right) \]. Then, since \( \mathbb{f} \) is NS b-open, 
\[ \mathbb{f} \left( \left( g, \mathfrak{B} \right) \right) \) and \( \mathbb{f} \left( \left( k, \mathfrak{B} \right) \right) \) are NS b-open sets in \( \left( Y^{cr, ip}, \mathfrak{G}, \mathfrak{D} \right) \) containing \( \mathbb{f} \left( \left( x^e \left( a, b, c \right) \right)_{\text{parameter}} \right) \) and 
\[ \mathbb{f} \left( \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} \right) \) respectively, and \( \mathbb{f} \left( \left( g, \mathfrak{B} \right) \right) \) \( \mathfrak{F} \) \( \mathbb{f} \left( \left( k, \mathfrak{B} \right) \right) = 0 \left( \left( \mathfrak{F}, \mathfrak{D} \right)_{\text{parameter}} \right) \) otherwise \( \mathbb{f} \left( \left( g, \mathfrak{B} \right) \right) \) 
\[ \mathbb{f} \left( \left( k, \mathfrak{B} \right) \right) \mathfrak{F} \{ \left( x^e \left( a, b, c \right) \right)_{\text{parameter}} \right\} \) with \( \left( x^e \left( a, b, c \right) \right)_{\text{parameter}} \) 
\[ \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} : \mathbb{f} \left( \left( x^e \left( a, b, c \right) \right)_{\text{parameter}} \right) > \]
\[ \mathbb{f} \left( \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} \right) \] or \( \left( x^e \left( a, b, c \right) \right)_{\text{parameter}}, \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} \in \) 
\[ \left\{ \left( x^e \left( a, b, c \right) \right)_{\text{parameter}}, \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} \right\} \]
\[ \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} : \mathbb{f} \left( \left( x^e \left( a, b, c \right) \right)_{\text{parameter}} \right) < \mathbb{f} \left( \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} \right) \] 
\( \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} : \mathbb{f} \left( \left( x^e \left( a, b, c \right) \right)_{\text{parameter}} \right) < \mathbb{f} \left( \left( y^e \left( w, b, c \right) \right)_{\text{parameter}} \right) \] is NSb Hausdorff space.
Theorem 7.2. Let $(\mathcal{X}^{cr\times p}, \mathcal{I}, \partial)$ be a NS second countable space then it is guaranteed that every family of non-empty dis-joint NS b-open subsets of a NS second countable space $(\mathcal{X}^{cr\times p}, \mathcal{I}, \partial)$ is NSb countable.

Proof: Given that $(\mathcal{X}^{cr\times p}, \mathcal{I}, \partial)$ be a NS second countable space. Then, $\exists$ a NS countable base $\mathcal{B} = (\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4, \ldots \mathcal{B}_n: n \in \mathbb{N})$ for $(\mathcal{X}^{cr\times p}, \mathcal{I}, \partial)$. Let $(\mathcal{C}, \partial)$ be a family of non-vacuous mutually exclusive NS b-open sub-sets of $(\mathcal{X}^{cr\times p}, \mathcal{I}, \partial)$. Then, for each $(\mathcal{F}, \partial)$ of in $(\mathcal{C}, \partial)$ $\exists$ a soft $\mathcal{B}^n \in \mathcal{B}$ in such a way that $\mathcal{B}^n \in (\mathcal{F}, \partial)$. Let us attach with $(\mathcal{F}, \partial)$, the smallest positive integer $n$ such that $\mathcal{B}^n \in (\mathcal{F}, \partial)$. Since the candidates of $(\mathcal{C}, \partial)$ are mutually exclusive because of this behavior distinct candidates will be associated with distinct positive integers. Now, if we put the elements of $(\mathcal{C}, \partial)$ in order so that the order is increasing relative to the positive integers associated with them, we obtain a sequence of candidates of $(\mathcal{C}, \partial)$. This verifies that $(\mathcal{C}, \partial)$ is NS countable.

Theorem 7.3. Let $(\mathcal{X}^{cr\times p}, \mathcal{I}, \partial)$ be a NS second countable space and let $(\mathcal{F}, \partial)$ be NS uncountable subset of $(\mathcal{X}^{cr\times p}, \mathcal{I}, \partial)$. Then, at least one point of $(\mathcal{F}, \partial)$ is a soft limit point of $(\mathcal{F}, \partial)$. Let, if possible, no point of $(\mathcal{F}, \partial)$ be a soft limit point of $(\mathcal{F}, \partial)$. Then, for each $(x^{e}_{(a,b)}, \partial)$, such that $(x^{e}_{(a,b)}, \partial) \in (\mathcal{F}, \partial)$, $\exists$ NS $\mathcal{B} = (\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4, \ldots \mathcal{B}_n: n \in \mathbb{N})$ for $(\mathcal{X}^{cr\times p}, \mathcal{I}, \partial)$. Let, if possible, no point of $(\mathcal{F}, \partial)$ be a soft limit point of $(\mathcal{F}, \partial)$. Then, for each $(x^{e}_{(a,b)}, \partial)$, such that $(x^{e}_{(a,b)}, \partial) \in (\mathcal{F}, \partial)$, $\exists$ NS $\mathcal{B} = (\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4, \ldots \mathcal{B}_n: n \in \mathbb{N})$ for $(\mathcal{X}^{cr\times p}, \mathcal{I}, \partial)$.
contradiction is taking birth that on point of \((\mathfrak{f}, \mathfrak{d})\) is a soft limit point of \((\mathfrak{f}, \mathfrak{d})\), so at least one point of \((\mathfrak{f}, \mathfrak{d})\) is a soft limit point of \((\mathfrak{f}, \mathfrak{d})\).

**Theorem 7.4.** Let \((X^{crisp}, \mathfrak{I}, \mathfrak{d})\) NSSTS such that is is NS countably compact then this space has the characteristics of Bolzano Weierstrass Property (BW P).

**Proof:** Let \((X^{crisp}, \mathfrak{I}, \mathfrak{d})\) be a NS countably compact space and suppose, if possible, that it negate the Bolzano Weierstrass Property (BW P). Then there must exists an infinite NS set \((\mathfrak{f}, \mathfrak{d})\) supposing no soft limit point. Further suppose \((\rho, \mathfrak{d})\) be soft countably infinite soft sub-set \((\mathfrak{f}, \mathfrak{d})\) that is \((\rho, \mathfrak{d}) \in (\mathfrak{f}, \mathfrak{d})\). Then this guarantees \((\rho, \mathfrak{d})\) has no soft limit poit. It follows that \((\rho, \mathfrak{d})\) is NS considered closed set. Also for each \((x^{\mathfrak{f}(a,b),}_{(a,b),d,parameter})_n \in (\rho, \mathfrak{d}), (x^{\mathfrak{f}(a,b),}_{(a,b),d,parameter})_n\) is not a soft limit point of \((\rho, \mathfrak{d})\). Hence there exists NS b-open set \((\mathfrak{G}_n, \mathfrak{d})\), such that for each \((\mathfrak{G}_n, \mathfrak{d})\) \(\cap (\rho, \mathfrak{d}) = (x^{\mathfrak{f}(a,b),}_{(a,b),d,parameter})_n\). The the collection \((\mathfrak{G}_n, \mathfrak{d})_n \in N\) \(\cap (\rho, \mathfrak{d})\) is countable NS-b-open cover of \((X^{crisp}, \mathfrak{I}, \mathfrak{d})\). This soft cover has no finite sub-cover. For this we exhaust a single \((\mathfrak{G}_n, \mathfrak{d})\), it would not be a soft cover of \((X^{crisp}, \mathfrak{I}, \mathfrak{d})\) since then \((x^{\mathfrak{f}(a,b),}_{(a,b),d,parameter})_n\) would be covered. Result in \((X^{crisp}, \mathfrak{I}, \mathfrak{d})\) is not NS countably compact. But this contradicts the given. Hence, we are compelled to accept \((X^{crisp}, \mathfrak{I}, \mathfrak{d})\) must have Bolzano Weierstrass Property.

**Theorem 7.5.** Let \((X^{crisp}, \mathfrak{I}, \mathfrak{A})\) and \((X^{crisp}, \mathfrak{I}, \mathfrak{A})\) be two NSSTS and suppose \((\mathfrak{f}, \mathfrak{d})\) be a NS continuous function such that \((\mathfrak{f}, \mathfrak{d}): (X^{crisp}, \mathfrak{I}, \mathfrak{A}) \rightarrow (X^{crisp}, \mathfrak{I}, \mathfrak{A})\) is NS continuous function and let \((\mathcal{L}, \mathfrak{d}) \in (X^{crisp}, \mathfrak{I}, \mathfrak{A})\) supposes the B. V. P. then safely \(f((\mathcal{L}, \mathfrak{d}))\) has the B. V. P.

**Proof:** Suppose \((\mathcal{L}, \mathfrak{d})\) be an infinite NS sub-set of \((\mathfrak{f}, \mathfrak{d})\), so that \((\mathcal{L}, \mathfrak{d})\) contains an enumerable NS set \((x^{\mathfrak{f}(a,b),}_{(a,b),d,parameter})_n : n \in N\) then there exists enumerable NS set \((x^{\mathfrak{f}(a,b,c),}_{(a,b,c),d,parameter})_n : n \in N\) in \((\mathcal{L}, \mathfrak{d})\) s.t. \(f((x^{\mathfrak{f}(a,b,c),}_{(a,b,c),d,parameter})_n) = (x^{\mathfrak{f}(a,b,c),}_{(a,b,c),d,parameter})_n\). \(\mathcal{L}, \mathfrak{d}\) has B.V.P. \(\Rightarrow\) every infinite soft subset of \((\mathcal{L}, \mathfrak{d})\) supposes soft accumulation point belonging to \((\mathcal{L}, \mathfrak{d})\), \(\Rightarrow\) \((x^{\mathfrak{f}(a,b,c),}_{(a,b,c),d,parameter})_n : n \in N\) has soft neutrosophic limit point, say, \((x^{\mathfrak{f}(a,b,c),}_{(a,b,c),d,parameter})_n \in (\mathcal{L}, \mathfrak{d})\). \(\Rightarrow\) the limit of soft sequence \((x^{\mathfrak{f}(a,b,c),}_{(a,b,c),d,parameter})_n : n \in N\) is \((y^{\mathfrak{f}(a,b,c),}_{(a,b,c),d,parameter})_0 \in (\mathcal{L}, \mathfrak{d})\) \(\Rightarrow\) \((y^{\mathfrak{f}(a,b,c),}_{(a,b,c),d,parameter})_n \rightarrow (y^{\mathfrak{f}(a,b,c),}_{(a,b,c),d,parameter})_n \in (\mathcal{L}, \mathfrak{d})\) if is soft continuous \(\Rightarrow\) it is soft continuous. Furthermore \((y^{\mathfrak{f}(a,b,c),}_{(a,b,c),d,parameter})_n \rightarrow (y^{\mathfrak{f}(a,b,c),}_{(a,b,c),d,parameter})_n \in (\mathcal{L}, \mathfrak{d})\) \(\Rightarrow\) \(f((y^{\mathfrak{f}(a,b,c),}_{(a,b,c),d,parameter})_n) \rightarrow \)
Let \((X^{crisp}, \mathcal{I}, \mathcal{d})\) be a \(NSSTS\) so that (i) \(\mathcal{F}, \mathcal{d}\) is \(NSb\) compact soft sub-set of \((X^{crisp}, \mathcal{I}, \mathcal{d})\) and \((x^e_{(a,b,c)}, d_{parameter})\) be a crisp point in \(X^{crisp}\) such that \((x^e_{(a,b,c)}, d_{parameter})\) can be strongly separated from every point \((y^e_{(a'/b'/c')}, d_{parameter})\) in \((\mathcal{F}, \mathcal{d})\), then it is guaranteed that \((x^e_{(a,b,c)}, d_{parameter})\) and \((\mathcal{F}, \mathcal{d})\) can also be soft strongly separated in \((X^{crisp}, \mathcal{I}, \mathcal{d})\). (ii) if \((y, \mathcal{d})\) and \((\mathcal{F}, \mathcal{d})\) are two \(NSb\) compact soft sub-sets of \((X^{crisp}, \mathcal{I}, \mathcal{d})\) such that every point \((x^e_{(a,b,c)}, d_{parameter})\) in \((y, \mathcal{d})\) can be strongly separated from every point \((y^e_{(a'/b'/c')}, d_{parameter})\) in \((\mathcal{F}, \mathcal{d})\), then it is guaranteed that \((y, \mathcal{d})\) and \((\mathcal{F}, \mathcal{d})\) can be strongly separated in \((X^{crisp}, \mathcal{I}, \mathcal{d})\).

**Proof i)** Let \((\mathcal{U}, \mathcal{d})\) \((y^e_{(a'/b'/c')}, d_{parameter})\) \((x^e_{(a,b,c)}, d_{parameter})\) and \((\mathcal{L}, \mathcal{d})(x^e_{(a,b,c)}, d_{parameter})\) \((y^e_{(a'/b'/c')}, d_{parameter})\) separate strongly the point \(x\) from a point \((y^e_{(a'/b'/c')}, d_{parameter})\) \(\in (\mathcal{F}, \mathcal{d})\). As \((y^e_{(a'/b'/c')}, d_{parameter})\) runs over \((\mathcal{F}, \mathcal{d})\), the corresponding \(NS\) sets \((\mathcal{L}, \mathcal{d})(x^e_{(a,b,c)}, d_{parameter})\) \((y^e_{(a'/b'/c')}, d_{parameter})\) form \(NS\) \(b\)-open covering of \((\mathcal{F}, \mathcal{d})\), for which there exists a finite soft sub-covering, \((\mathcal{L}, \mathcal{d})(x^e_{(a,b,c)}, d_{parameter})\) \((y^e_{(a'/b'/c')}, d_{parameter})\)_1, ..., \((\mathcal{L}, \mathcal{d})(x^e_{(a,b,c)}, d_{parameter})\) \((y^e_{(a'/b'/c')}, d_{parameter})\)_n, say, since \((\mathcal{F}, \mathcal{d})\) is \(NSb\) compact. Let

\[
\begin{align*}
(\mathcal{U}, \mathcal{d})(y^e_{(a'/b'/c')}, d_{parameter})_1, \ldots, (\mathcal{U}, \mathcal{d})(y^e_{(a'/b'/c')}, d_{parameter})_n, (\mathcal{U}, \mathcal{d})_1, \ldots, (\mathcal{U}, \mathcal{d})_n.
\end{align*}
\]

be the corresponding \(NSb\) open sets supposing the point \((x^e_{(a,b,c)}, d_{parameter})\).

Let \((\mathcal{U}, \mathcal{d})(x^e_{(a,b,c)}, d_{parameter}) = (\mathcal{U}, \mathcal{d})(y^e_{(a'/b'/c')}, d_{parameter})_1, \ldots, (\mathcal{U}, \mathcal{d})(y^e_{(a'/b'/c')}, d_{parameter})_n, (\mathcal{U}, \mathcal{d})_1, \ldots, (\mathcal{U}, \mathcal{d})_n.

\[f((y^e_{(a'/b'/c')}, d_{parameter})_0) \in f((\mathcal{L}, \mathcal{d})) \Rightarrow (x^e_{(a,b,c)}, d_{parameter})_n \rightarrow f((y^e_{(a'/b'/c')}, d_{parameter})_0) \in f((\mathcal{L}, \mathcal{d})) \Rightarrow \text{limit of a soft sequence } (f((x^e_{(a,b,c)}, d_{parameter})_n))_{n \in N} \Rightarrow f((y^e_{(a'/b'/c')}, d_{parameter})_0) \in f((\mathcal{L}, \mathcal{d})) \Rightarrow \text{limit of a soft sequence } (f((x^e_{(a,b,c)}, d_{parameter})_n))_{n \in N} \Rightarrow f((y^e_{(a'/b'/c')}, d_{parameter})_0) \in f((\mathcal{L}, \mathcal{d}))
\] containing a limit point \(f((y^e_{(a'/b'/c')}, d_{parameter})_0) \in f((\mathcal{L}, \mathcal{d})).\) This guarantees that \(f((\mathcal{L}, \mathcal{d}))\) has \(B.V.P.\)
\( \tilde{\mathcal{L}}(\mathcal{F}, \partial) \left( y^{(a,b,c)}_{\partial} \right) \) and (L, \partial)_1(x^{(a,b,c)}_{\partial}) (F, \partial) = \emptyset (x^{(a,b,c)}_{\partial}) \)

= \( (\mathcal{L}, \partial)_1(x^{(a,b,c)}_{\partial}) \left( y^{(a,b,c)}_{\partial} \right) \cup \)

(\mathcal{V}, \partial)_1(x^{(a,b,c)}_{\partial}) \left( y^{(a,b,c)}_{\partial} \right) \cup (\mathcal{L}, \partial)_2(x^{(a,b,c)}_{\partial}) \left( y^{(a,b,c)}_{\partial} \right) \cup (\mathcal{V}, \partial)_2(x^{(a,b,c)}_{\partial}) \left( y^{(a,b,c)}_{\partial} \right) \cup \ldots

Then (x^{(a,b,c)}_{\partial}) \in

(\mathcal{U}, \partial)_1(F, \partial) \left( x^{(a,b,c)}_{\partial} \right) \text{ and } (F, \partial) \in (L, \partial)_1(x^{(a,b,c)}_{\partial}) (F, \partial).

Also,

since (\mathcal{U}, \partial)_1(F, \partial) \left( x^{(a,b,c)}_{\partial} \right) \mathcal{H}(L, \partial)_1(x^{(a,b,c)}_{\partial}) \left( y^{(a,b,c)}_{\partial} \right) \mathcal{H}(L, \partial)_2(x^{(a,b,c)}_{\partial}) \left( y^{(a,b,c)}_{\partial} \right) =

0(\mathcal{Y}, \partial) \text{ and } (\mathcal{U}, \partial)_1(F, \partial) \left( x^{(a,b,c)}_{\partial} \right) \in (U, \partial) \left( y^{(a,b,c)}_{\partial} \right) \text{ for } i = 1, 2, 3, 4, \ldots, n.

Thus (x^{(a,b,c)}_{\partial}) \text{ and } (F, \partial) \text{ are separated strongly by the pair of disjoint NSb open sets}

(\mathcal{U}, \partial)_1(F, \partial) \left( x^{(a,b,c)}_{\partial} \right) \text{ and } (L, \partial)_1(x^{(a,b,c)}_{\partial}) (F, \partial) \text{ in } (X^{cr,p}, \mathcal{I}, \partial).

(H) Suppose (x^{(a,b,c)}_{\partial}) \text{ runs over } (\partial, \partial) \text{ corresponding soft open sets } (U, \partial)_1(F, \partial) \left( x^{(a,b,c)}_{\partial} \right)

generate soft covering of (\partial, \partial), for which there exists a finite soft sub-covering

\[ \left\{ (U, \partial)_1(F, \partial) \left( x^{(a,b,c)}_{\partial} \right) \right\}, \]

\[ \left\{ (U, \partial)_2(F, \partial) \left( x^{(a,b,c)}_{\partial} \right) \right\}, \]

\[ \ldots, (U, \partial)_m(F, \partial) \left( x^{(a,b,c)}_{\partial} \right) \]

say, for (\partial, \partial) \text{ (since (\partial, \partial) is soft NSb compact). Let}

\( (L, \partial)_1(x^{(a,b,c)}_{\partial}) (F, \partial) \)

\( (L, \partial)_2(x^{(a,b,c)}_{\partial}) (F, \partial) \)

\( (L, \partial)_3(x^{(a,b,c)}_{\partial}) (F, \partial) \)

\( \ldots, (L, \partial)_m(F, \partial) \left( x^{(a,b,c)}_{\partial} \right) \)

(\mathcal{L}, \partial)_1(x^{(a,b,c)}_{\partial}) (F, \partial) \text{ be the corresponding NSb open sets containing } (F, \partial). \text{ Then } (U, \partial)((\partial, \partial)) =

\[ \left\{ (U, \partial)_1(F, \partial) \left( x^{(a,b,c)}_{\partial} \right) \right\} \cup \left\{ (U, \partial)_2(F, \partial) \left( x^{(a,b,c)}_{\partial} \right) \right\} \cup \left\{ (U, \partial)_3(F, \partial) \left( x^{(a,b,c)}_{\partial} \right) \right\} \cup \ldots, (L, \partial)_1(x^{(a,b,c)}_{\partial}) (F, \partial) \)

\[ \text{and } (L, \partial)((F, \partial)) = (L, \partial)_1(x^{(a,b,c)}_{\partial}) (F, \partial) \]

\[ (L, \partial)_2(x^{(a,b,c)}_{\partial}) (F, \partial) \]

\[ (L, \partial)_3(x^{(a,b,c)}_{\partial}) (F, \partial) \]

\[ \ldots, (L, \partial)_m(F, \partial) \left( x^{(a,b,c)}_{\partial} \right) \]

(\mathcal{H}, \partial)_1(F, \partial) \left( x^{(a,b,c)}_{\partial} \right) \text{ are two disjoint NS open sets, (as in (i)), which separate } (\partial, \partial) \text{ and } (F, \partial) \text{ strongly.}

**Theorem 7.7.** Let \( X^{cr,p}, \mathcal{I}, \partial \) be a NSSTS and \( \{F^{cr,p}, \mathcal{I}, \partial\} \) be NS sub-space of \( (X^{cr,p}, \mathcal{I}, \partial) \). The necessary and sufficient condition for \( \{F^{cr,p}, \partial\} \) to be NS compact relative to \( \{F^{cr,p}, \mathcal{I}, \partial\} \) is that \( \{F^{cr,p}, \partial\} \) is NS compact relative to \( (X^{cr,p}, \mathcal{I}, \partial) \).

**Proof:** First we prove that \( \{F^{cr,p}, \partial\} \) relative to \( (X^{cr,p}, \mathcal{I}, \partial) \). Let \( \{t, \partial\} : i \in I \) that is

\[ \{(t, \partial, t, \partial, t, \partial, t, \partial) \} \text{ be } (X^{cr,p}, \mathcal{I}, \partial) - \text{NSb open cover of } (F^{cr,p}, \partial), \text{then } (F^{cr,p}, \partial) \in \mathcal{U},\{t, \partial\} : (t, \partial) \in I \in \mathcal{V} \]
\((X^{crp}, \mathcal{I}, \partial) \Rightarrow \exists (g, \partial)_1 \in (X^{crp}, \mathcal{I}, \partial) \text{ s.t. } (t, \partial)_1 = (g, \partial)_1 \cap (t^{crp}, \partial) \subset (g, \partial)_1 \Rightarrow \exists (g, \partial)_1 \in \\
(X^{crp}, \mathcal{I}, \partial) \text{ s.t. } (t, \partial)_1 \in (g, \partial)_1 \Rightarrow \overline{U_{t} (t, \partial)} \cup (t, \partial)_1 \text{ but } (t^{crp}, \partial) \not\subset (t, \partial)_1. \text{ So that } (t^{crp}, \partial) \not\subset \overline{U_{t} (t, \partial)}_1. \text{ This guarantees that } \{(g, \partial)_i; i \in I\} \text { is a } (X^{crp}, \mathcal{I}, \partial) - NSb \text{ open cover of } (t^{crp}, \partial) \text{ which is known to be NSb compact relative }
\langle X^{crp}, \mathcal{I}, \partial \rangle \text{ and hence the soft cover } \{|(g, \partial)_i; (x^{crp}_{(ab)}, a_{\text{parameter}})|\} \text{ must be freezable to a finite soft sub cover, say,}
\{(g, \partial)_r; r = 1, 2, 3, 4, \ldots, n\}. \text{ Then } (t^{crp}, \partial) \subset \bigcup_{t=1}^{n} (g, \partial)_r \Rightarrow \\
(t^{crp}, \partial) \cap (t^{crp}, \partial) \subset (t^{crp}, \partial) \bigcap \bigcup_{t=1}^{n} (g, \partial)_r \\
= \bigcup_{t=1}^{n} ((t^{crp}, \partial \cap (t^{crp}, \partial) \subset \bigcup_{t=1}^{n} (g, \partial)_r \text{ or } (t^{crp}, \partial) \subset \bigcup_{t=1}^{n} (g, \partial)_r \Rightarrow \{t, \partial\}_r: 1 \leq r \leq n\} \text{ is a } (X^{crp}, \mathcal{I}, \partial) - NS \text{ open cover of } (t^{crp}, \partial). \text{ Steping from an arbitrary } (X^{crp}, \mathcal{I}, \partial) - \text{open cover of } (t^{crp}, \partial), \text{ we are able to show that the NS cover is freezable to a finite soft subcover } \{|(t, \partial)_r; 1 \leq r \leq n\} \text{ of } (t^{crp}, \partial), \text{ meaning there by } (t^{crp}, \partial) \text{ is }
(X^{crp}, \mathcal{I}, \partial) - NSb \text{ compact. The condition is sufficient: Suppose } (t^{crp}, \mathcal{I}, \partial) \text{ be soft sub-space of } (X^{crp}, \mathcal{I}, \partial)
\text{ and also } (t^{crp}, \partial) \text{ is } (X^{crp}, \mathcal{I}, \partial) - NSb \text{ compact. We have to prove that } (t^{crp}, \partial) \text{ is } (X^{crp}, \mathcal{I}, \partial) - NS \text{ compact.}
\text{ Let } \{(t, \partial)_1, (t, \partial)_2, (t, \partial)_3, (t, \partial)_4, \ldots\} \text{ be soft } (X^{crp}, \mathcal{I}, \partial) - NS \text{ b-open cover of } (t^{crp}, \partial), \text{ so that } (t^{crp}, \partial) \subset \bigcup_{t=1}^{n} (g, \partial)_i \text{ from which } (t^{crp}, \partial) \cap (t^{crp}, \partial) \subset (t^{crp}, \partial) \cap (g, \partial)_i \text{ for } (t^{crp}, \partial) \subset \bigcup_{t=1}^{n} (g, \partial)_i \text{ meaning there by } (t^{crp}, \partial) \text{ is }
(X^{crp}, \mathcal{I}, \partial) - NSb \text{ open soft cover of } (t^{crp}, \partial) \text{ which is known to be } (t^{crp}, \mathcal{I}, \partial) - NS b \text{ compact hence this sof cover must be reducible to a finite soft sub-cover, say, } \{|(t, \partial)_r; 1 \leq r \leq n\}. \text{ This } \Rightarrow (t^{crp}, \partial) \subset \bigcup_{t=1}^{n} (g, \partial)_r \Rightarrow \\
(t^{crp}, \partial) \subset \bigcup_{t=1}^{n} ((g, \partial)_r \cap (t^{crp}, \partial) \subset \bigcup_{t=1}^{n} (g, \partial)_r, \text{ or }
(t^{crp}, \partial) \subset \bigcup_{t=1}^{n} (g, \partial)_r) \text{ or }
(t^{crp}, \partial) \subset \bigcup_{t=1}^{n} (g, \partial)_r). \text{ This proves that } \{|(g, \partial)_r; 1 \leq r \leq n\} \text{ is a finite soft sub-cover of the soft cover } (g, \partial)_r.
\text{ Starting from an arbitrary } (X^{crp}, \mathcal{I}, \partial) - NS \text{ b-open soft cover of } (t^{crp}, \partial), \text{ we are able to show that this soft }
\text{ neutrosophic b-open cover is freezable to a finite soft sub-cover, showing there by } (t^{crp}, \partial) \text{ is } (X^{crp}, \mathcal{I}, \partial) - Nb \text{ compact.}
Theorem 7.8. Let $(X^{cr,p}, μ)$ be a NSTS and let $(\{e^{r,(a,b,c)}_{(\text{parameter})}\}_{n})$ be a NS sequence in $(X^{cr,p}, μ)$ such that it converges to a point $(x^{r,(a,b,c)}_{(\text{parameter})})_0$ then the soft set $(μ_1, μ_2, \ldots )$ consisting of the points $(x^{r,(a,b,c)}_{(\text{parameter})})_{n_0}$ and $(x^{r,(a,b,c)}_{(\text{parameter})})_{n_0}(n = 1, 2, 3, \ldots)$ is soft NSb compact.

Proof: Given $(X^{cr,p}, μ)$ be a NSTS and let $(\{e^{r,(a,b,c)}_{(\text{parameter})}\}_{n})$ be a NS sequence in $(X^{cr,p}, μ)$ such that it converges to a point $(x^{r,(a,b,c)}_{(\text{parameter})})_{n_0}$ that is $(x^{r,(a,b,c)}_{(\text{parameter})})_{n_0} \rightarrow (x^{r,(a,b,c)}_{(\text{parameter})})_{n_0} \in (X^{cr,p})$. Let

$$(g, δ) = \{(e^{r,(a,b,c)}_{(\text{parameter})})_{n_0}, \{e^{r,(a,b,c)}_{(\text{parameter})}\}_{n_0}, \{e^{r,(a,b,c)}_{(\text{parameter})}\}_{n_0}, \cdots \}.$$ Let $(\{ξ, δ\}_{a} : a \in A)$ be NS b-open covering of $(g, δ)$ so that $(g, δ) \in \bigcup_{a} \{ξ, δ\}_{a}$. According to the definition of soft convergence $(x^{r,(a,b,c)}_{(\text{parameter})})_{n_0} \in (ξ, δ)_{a_0}$ then the soft set $(x^{r,(a,b,c)}_{(\text{parameter})})_{n_0} \in (ξ, δ)_{a_0}$. Evidently, $(ξ, δ)_{a_0}$ contains the points $(x^{r,(a,b,c)}_{(\text{parameter})})_{n_0}$, $(x^{r,(a,b,c)}_{(\text{parameter})})_{n_0+1}$, $(x^{r,(a,b,c)}_{(\text{parameter})})_{n_0+2}$, \ldots Look carefully at the points and train them in a way as,

$$(x^{r,(a,b,c)}_{(\text{parameter})})_{n_0}, (x^{r,(a,b,c)}_{(\text{parameter})})_{n_0+1}, (x^{r,(a,b,c)}_{(\text{parameter})})_{n_0+2}, \ldots,$$

$(x^{r,(a,b,c)}_{(\text{parameter})})_{n_0+1}, \ldots \varepsilon$... generating a finite soft set. Let $1 \leq n_{0-1}$. Then $(x^{r,(a,b,c)}_{(\text{parameter})})_{i} \in (g, δ)$. For this $i$, $(x^{r,(a,b,c)}_{(\text{parameter})})_{i} \in (g, δ)$. Hence $\exists a_i \in A$ s.t. $(x^{r,(a,b,c)}_{(\text{parameter})})_{i} \in (ξ, δ)_{a_i}$. Evidently $(g, δ) \in \bigcup_{a_i} \{ξ, δ\}_{a_i}$. This shows that $(\{ξ, δ\}_{a_i} : 0 \leq n_{0-1})$ is NS b-open cover of $(g, δ)$. Thus any arbitrary soft neutrosophic open cover $(\{ξ, δ\}_{a_i} : a \in A)$ of $(g, δ)$ is reducible to a finite NS sub-cover $(\{ξ, δ\}_{a_i} : i = 0, 1, 2, 3, \ldots n_{0-1})$, it follows that $(g, δ)$ is soft NSb compact.

Theorem 7.9. Let $(X^{cr,p}, μ)$ be a NSTS such that it is soft countably compact. Then every NSb closed subset of $(X^{cr,p}, μ)$ is NSb countably compact.

Proof: Let $(X^{cr,p}, μ)$ be a NSTS such that it is NSb countably compact and suppose $(f, δ)$ be a NSb closed sub-set of $(X^{cr,p}, μ)$. Let $C = \{(ξ_1, δ), (ξ_2, δ), (ξ_3, δ), (ξ_4, δ), (ξ_5, δ), (ξ_6, δ), \ldots \}$. That is $(\{ξ_n, δ\} : n \in N)$ be any countably NS b-open covering of $(f, δ)$. Then, $(f, δ) \in \bigcup (\{ξ_n, δ\})$. This qualifying us to write $(X^{cr,p}, μ) = (f, δ) \cup (f, δ) \subseteq \bigcup (\{ξ_n, δ\}) \cup (f, δ)$. This guarantee that the collection $(\{ξ_n, δ\} : n \in N)$ is a NS countable b-open.
covering of \((X^{cr}, \mathcal{I}, \mathcal{P})\). But \((X^{cr}, \mathcal{I}, \mathcal{P})\) being soft countably NSb compact and \((\mathcal{G}, \mathcal{D})\) being obviously absorbing no piece of \(\langle \mathcal{G}, \mathcal{D} \rangle\). It follows that there exists finite soft number of indices \(n_{1}, n_{2}, n_{3}, ..., n_{k}\) such that

\[ \langle \mathcal{G}, \mathcal{D} \rangle \in \bigcup_{i=1}^{k} (\mathcal{G}_{nu}, \mathcal{D}). \]

This shows that \(\{G_{nu}, D\}; i = 1, 2, 3, ..., k\) is a finite soft neutrosophic sub-covering of \(\mathcal{C}\).

**Theorem 7.10.** If \((X^{cr}, \mathcal{I}, \mathcal{P})\) is NSSTS such that it has the characteristics of soft neutrosophic sequentially compactness. Then, \((X^{cr}, \mathcal{I}, \mathcal{P})\) is safely NSb countably compact.

**Proof:** Let \((X^{cr}, \mathcal{I}, \mathcal{P})\) NSSTS and let \((\mathcal{G}, \mathcal{D})\) be finite soft sub-set of \((X^{cr}, \mathcal{I}, \mathcal{P})\). Let

\[ (x_{e(a,b)}, d_{\text{parameter}})_{1}, (x_{e(a,b)}, d_{\text{parameter}})_{2}, \ldots, (x_{e(a,b)}, d_{\text{parameter}})_{n}. \]

\[ \langle (x_{e(a,b)}, d_{\text{parameter}})_{1}, (x_{e(a,b)}, d_{\text{parameter}})_{2}, \ldots, (x_{e(a,b)}, d_{\text{parameter}})_{n} \rangle \]

be soft sequence of soft points of \((X^{cr}, \mathcal{I}, \mathcal{P})\). Then we have to prove \((\mathcal{G}, \mathcal{D})\) is soft sequentially compact. Hence, \(\langle (x_{e(a,b)}, d_{\text{parameter}})_{1}, (x_{e(a,b)}, d_{\text{parameter}})_{2}, \ldots, (x_{e(a,b)}, d_{\text{parameter}})_{n} \rangle\) is soft sub-sequence of \((X^{cr}, \mathcal{I}, \mathcal{P})\). It follows that there exists finite soft constant sequence and repeatedly constructed by single soft number \((x_{e(a,b)}, d_{\text{parameter}})_{1}\) and we know that a soft constant sequence converges on itself. So it converges to \((x_{e(a,b)}, d_{\text{parameter}})_{0}\) which belongs to \((\mathcal{G}, \mathcal{D})\). Hence, \((\mathcal{G}, \mathcal{D})\) is soft sequentially NSb compact.

**Theorem 7.11.** Let \((X^{cr}, \mathcal{I}, \mathcal{P})\) NSSTS and \((Y^{cr}, \mathcal{J}, \mathcal{P})\) be another NSSTS. Let \((\mathcal{G}, \mathcal{D})\) be a soft continuous mapping of a soft neutrosophic sequentially compact NSb space \((X^{cr}, \mathcal{I}, \mathcal{P})\) into \((Y^{cr}, \mathcal{J}, \mathcal{P})\). Then, \((\mathcal{G}, \mathcal{D})(X^{cr}, \mathcal{I}, \mathcal{P})\) is NSb sequentially compact.

**Proof:** Given \((X^{cr}, \mathcal{I}, \mathcal{P})\) NSSTS and \((Y^{cr}, \mathcal{J}, \mathcal{P})\) be another NSSTS. Let \((\mathcal{G}, \mathcal{D})\) be a soft continuous mapping of a NSb sequentially compact space \((X^{cr}, \mathcal{I}, \mathcal{P})\) into \((Y^{cr}, \mathcal{J}, \mathcal{P})\). Then we have to prove \((\mathcal{G}, \mathcal{D})(X^{cr}, \mathcal{I}, \mathcal{P})\) is NSb sequentially compact. For this we proceed as \(\langle (x_{e(a,b)}, d_{\text{parameter}})_{1}, (x_{e(a,b)}, d_{\text{parameter}})_{2}, \ldots, (x_{e(a,b)}, d_{\text{parameter}})_{n} \rangle\) be a soft sequence of \(NS\) points in \((\mathcal{G}, \mathcal{D})(X^{cr}, \mathcal{I}, \mathcal{P}))\). Then for each \(n \in \mathbb{N}\),

\[ \exists \langle (x_{e(a,b)}, d_{\text{parameter}})_{1}, (x_{e(a,b)}, d_{\text{parameter}})_{2}, \ldots, (x_{e(a,b)}, d_{\text{parameter}})_{n} \rangle \in (X^{cr}, \mathcal{I}, \mathcal{P}) \]
prove in this case because it is then automatically continuity. Thus we obtain a soft sequence

\[
\left( y^{e/\langle a',b',c',d'\rangle}_{1,2}, y^{e/\langle a',b',c',d'\rangle}_{1,2} \right), \ldots
\]

\[
\left( y^{e/\langle a',b',c',d'\rangle}_{n,2}, y^{e/\langle a',b',c',d'\rangle}_{n,2} \right), \ldots
\]

\[
\left( x^{e,\text{parameter}}_{\langle a,b,c,d\rangle_{1,2}}, x^{e,\text{parameter}}_{\langle a,b,c,d\rangle_{1,2}} \right), \ldots
\]

\[
\left( x^{e,\text{parameter}}_{\langle a,b,c,d\rangle_{n,2}}, x^{e,\text{parameter}}_{\langle a,b,c,d\rangle_{n,2}} \right), \ldots
\]

\[
\left( y^{e/\langle a',b',c',d'\rangle}_{1,2}, y^{e/\langle a',b',c',d'\rangle}_{1,2} \right), \ldots
\]

\[
\left( y^{e/\langle a',b',c',d'\rangle}_{n,2}, y^{e/\langle a',b',c',d'\rangle}_{n,2} \right), \ldots
\]

\[
\left( x^{e,\text{parameter}}_{\langle a,b,c,d\rangle_{1,2}}, x^{e,\text{parameter}}_{\langle a,b,c,d\rangle_{1,2}} \right), \ldots
\]

\[
\left( x^{e,\text{parameter}}_{\langle a,b,c,d\rangle_{n,2}}, x^{e,\text{parameter}}_{\langle a,b,c,d\rangle_{n,2}} \right), \ldots
\]

compact, there is a NS sub-sequence \((x^{e,\text{parameter}}_{\langle a,b,c,d\rangle_{n_{i}}})\) of

\[
\left( x^{e,\text{parameter}}_{\langle a,b,c,d\rangle_{n_{i}}} \right) \text{ such that } \left( x^{e,\text{parameter}}_{\langle a,b,c,d\rangle_{n_{i}}} \right) \rightarrow \left( x^{e,\text{parameter}}_{\langle a,b,c,d\rangle_{n}} \right) \in (X^{cr\text{-ip}}, \mathcal{I}, \partial). \text{ So, by NS continuity of } (\mathcal{F}, \partial), (x^{e,\text{parameter}}_{\langle a,b,c,d\rangle_{n_{i}}}) \rightarrow (x^{e,\text{parameter}}_{\langle a,b,c,d\rangle_{n}}) \rightarrow (\mathcal{F}, \partial)((x^{e,\text{parameter}}_{\langle a,b,c,d\rangle_{n_{i}}})) \rightarrow
\]

\[
(\mathcal{F}, \partial)((x^{e,\text{parameter}}_{\langle a,b,c,d\rangle_{n}})) \in (\mathcal{F}, \partial)((X^{cr\text{-ip}}, \mathcal{I}, \partial)). \text{ Thus, } (\mathcal{F}, \partial)((x^{e,\text{parameter}}_{\langle a,b,c,d\rangle_{n_{i}}})) \text{ is a soft sub-sequence of } (y^{e/\langle a',b',c',d'\rangle}_{n_{i}}), y^{e/\langle a',b',c',d'\rangle}_{n_{i}}) \text{ converges to } ((\mathcal{F}, \partial)(\partial)) \text{ in } (\mathcal{F}, \partial)((X^{cr\text{-ip}}, \mathcal{I}, \partial)).
\]

Hence, \((\mathcal{F}, \partial)((X^{cr\text{-ip}}, \mathcal{I}, \partial))\) is NSb sequentially compact.

**Theorem 7.12.** Let \((X^{cr\text{-ip}}, \mathcal{I}, \partial)\) be NSSTS such that is NS sequentially compact then it is guaranteed that it must be NSb countably compact.

**Proof:** Suppose \((X^{cr\text{-ip}}, \mathcal{I}, \partial)\) is NS sequentially compact. If \((X^{cr\text{-ip}}, \mathcal{I}, \partial)\) is finite, then nothing to prove in this case because it is then automatically NSb countably compact. Suppose \((X^{cr\text{-ip}}, \mathcal{I}, \partial)\) be in-finite. We prove the contrapositive of the statement given in the theorem. Let \(\{(G_{i}, \partial) : i \in N\}\) that is

\[
\{(G_{1}, \partial), (G_{2}, \partial), (G_{3}, \partial), (G_{4}, \partial), \ldots : i \in N\}
\]

be a NS b-open cover of \((X^{cr\text{-ip}}, \mathcal{I}, \partial)\) which has no finite soft...
Choose \( (x_{\epsilon(a,b,c),d}^{\text{parameter}})_1, (x_{\epsilon(a,b,c),d}^{\text{parameter}})_2, \)
\( (x_{\epsilon(a,b,c),d}^{\text{parameter}})_3, (x_{\epsilon(a,b,c),d}^{\text{parameter}})_4, \)
\( (x_{\epsilon(a,b,c),d}^{\text{parameter}})_5, (x_{\epsilon(a,b,c),d}^{\text{parameter}})_6, \ldots \)
sub-cover. Now, we generate a soft sequence \( (x_{\epsilon(a,b,c),d}^{\text{parameter}})_1, (x_{\epsilon(a,b,c),d}^{\text{parameter}})_2, \)
\( (x_{\epsilon(a,b,c),d}^{\text{parameter}})_3, (x_{\epsilon(a,b,c),d}^{\text{parameter}})_4, \)
\( (x_{\epsilon(a,b,c),d}^{\text{parameter}})_5, (x_{\epsilon(a,b,c),d}^{\text{parameter}})_6, \ldots \)
which may be soft monotone. That is soft monotonically non-increasing or soft monotonically strictly increasing or soft monotonically non-decreasing or soft monotonically strictly decreasing. Whatever the case may be we proceed as follows. Let \( n_1 \) be the smallest positive integer such that \( (X^{\text{crisp}}, \mathcal{I}, d) \cap (G_{n_1}, d) \neq \emptyset \). Choose \( (x_{\epsilon(a,b,c),d}^{\text{parameter}})_1 \in (X^{\text{crisp}}, \mathcal{I}, d) \cap (G_{n_1}, d) \). Now let \( n_2 \) be the least positive integer greater than \( n_1 \) such that \( (X^{\text{crisp}}, \mathcal{I}, d) \cap (G_{n_2}, d) \neq \emptyset \). Choose \( (x_{\epsilon(a,b,c),d}^{\text{parameter}})_2 \in (X^{\text{crisp}}, \mathcal{I}, d) \cap (G_{n_2}, d) \). It is important to be noted that such a point \( (x_{\epsilon(a,b,c),d}^{\text{parameter}})_3 \) always exists, for otherwise \( G_{n_2} \) will be a soft cover of \((X^{\text{crisp}}, \mathcal{I}, d)\). Choose \( (x_{\epsilon(a,b,c),d}^{\text{parameter}})_4 \in (X^{\text{crisp}}, \mathcal{I}, d) \cap (G_{n_3}, d) \). Choose \( (x_{\epsilon(a,b,c),d}^{\text{parameter}})_5 \in (X^{\text{crisp}}, \mathcal{I}, d) \cap (G_{n_4}, d) \). It is important to be noted that such a point \( (x_{\epsilon(a,b,c),d}^{\text{parameter}})_3 \) always exists, for otherwise \( G_{n_2} \) will be a soft cover of \((X^{\text{crisp}}, \mathcal{I}, d)\). Choose \( (x_{\epsilon(a,b,c),d}^{\text{parameter}})_n \in (X^{\text{crisp}}, \mathcal{I}, d) \cap (G_{n_n}, d) \). We get the soft sequence
\[
\begin{pmatrix}
(x_{\epsilon(a,b,c),d}^{\text{parameter}})_1, & (x_{\epsilon(a,b,c),d}^{\text{parameter}})_2, \\
(x_{\epsilon(a,b,c),d}^{\text{parameter}})_3, & (x_{\epsilon(a,b,c),d}^{\text{parameter}})_4, \\
(x_{\epsilon(a,b,c),d}^{\text{parameter}})_5, & (x_{\epsilon(a,b,c),d}^{\text{parameter}})_6, \\
& \ldots
\end{pmatrix}
\]
having the characteristics that, for each \( i \in \mathbb{N} \), \( (x_{\epsilon(a,b,c),d}^{\text{parameter}})_i \) does not belong to \( \bigcup_{k=1, 2, 3, \ldots, n-1} \{(G_{n_k}, d) \cap (G_{n_{n_k}}, d) \} \), \( m \geq n_i - 1 \). It can be seen that \( (x_{\epsilon(a,b,c),d}^{\text{parameter}})_n \) supposes no soft convergent sub-sequence in \((X^{\text{crisp}}, \mathcal{I}, d)\). For let \( (x_{\epsilon(a,b,c),d}^{\text{parameter}})_n \in (X^{\text{crisp}}, \mathcal{I}, d) \). Then there exists \( (G_{n_i}, d) \in V((G_{n_{n-1}}, d) : n \in \mathbb{N}) \) such
that \((x^e_{(a,b,c)},\widehat{\text{parameter}}) \in (G_{m_0},\partial)\). Since \((X^{cr^p},\mathcal{I},\partial)\) \(\cap (G_{m_0},\partial) \neq 0\) \((\mathcal{I}),\widehat{\text{parameter}}\), there exists \(k_0 \in N\) such that \((G_{n_k},\partial) = (G_{m_0},\partial)\). But by the choice of \(k_0\) of the soft sequence

\[
\left( x^e_{(a,b,c)},\widehat{\text{parameter}} \right)_1, (x^e_{(a,b,c)},\widehat{\text{parameter}})_2, (x^e_{(a,b,c)},\widehat{\text{parameter}})_3, \ldots
\]

we have \(i > k_0\) this implies that

\(x^e_{(a,b,c)},\widehat{\text{parameter}}\) does not \((G_{m_0},\partial)\). Since \((G_{m_0},\partial)\) is NS \(b\)-open set containing \((x^e_{(a,b,c)},\widehat{\text{parameter}})\), Since \((x^e_{(a,b,c)},\widehat{\text{parameter}})\) was arbitrary, \((X^{cr^p},\mathcal{I},\partial)\) is not NS sequentially compact, which is planely contradiction. Hence \((X^{cr^p},\mathcal{I},\partial)\) is NSb countably compact.

**Theorem 7.13.** Every NS co-finite NSTS \((X^{cr^p},\mathcal{I},\partial)\) is NSb separable.

**Proof:** case1: If \((X^{cr^p},\mathcal{I},\partial)\) is NS countable, then clearly \((X^{cr^p},\mathcal{I},\partial)\) is a NSb countable dense soft sub-set of \((X^{cr^p},\mathcal{I},\partial)\) and therefore, in this case, \((X^{cr^p},\mathcal{I},\partial)\) is NSb separable.

Case2: suppose \((X^{cr^p},\mathcal{I},\partial)\) is NS uncountable. Then, there exists an infinite NSb countable soft sub-set \((\mathcal{F},\partial)\) of \((X^{cr^p},\mathcal{I},\partial)\). Now \((\mathcal{F},\partial)\) is the smallest NSb closed superset of \((\mathcal{F},\partial)\) and in the soft co-finite space \((X^{cr^p},\mathcal{I},\partial)\), the only NSb closed sub-sets of \((X^{cr^p},\mathcal{I},\partial)\) are \((X^{cr^p},\mathcal{I},\partial)\) and finite soft sets. Results in, \((\mathcal{F},\partial) = (X^{cr^p},\mathcal{I},\partial)\). Thus, \((\mathcal{F},\partial)\) is a NSb countable dense soft sub-set of \((X^{cr^p},\mathcal{I},\partial)\). This signifies that \((X^{cr^p},\mathcal{I},\partial)\) is soft separable. Hence, every NSb co-finite NSTS.

**Theorem 7.14.** If \((X^{cr^p},\mathcal{I},\partial)\) is NSSTS such that it is NS second countable then it has the characteristics NS separability.

**Proof:** Suppose \((X^{cr^p},\mathcal{I},\partial)\) be NS second countable space.

Let \(\mathcal{B} = (B_1, B_2, B_3, B_4, \ldots, B_n; n \in N)\) be a NSb countable base for \((X^{cr^p},\mathcal{I},\partial)\). Choose \((x^e_{(a,b,c)},\widehat{\text{parameter}})_n \in B_n\) for each \(n\). Then, the set \((\mathcal{F},\partial) = \{n: (x^e_{(a,b,c)},\widehat{\text{parameter}})_n \in B_n\}\) is NS countable. Only remaining to prove that \((\mathcal{F},\partial)\) is soft dense in \((X^{cr^p},\mathcal{I},\partial)\). Suppose \((x^e_{(a,b,c)},\widehat{\text{parameter}}) \in (X^{cr^p},\mathcal{I},\partial)\) and let \((G,\partial)(x^e_{(a,b,c)},\widehat{\text{parameter}})\) be NS b open set absorbing \((x^e_{(a,b,c)},\widehat{\text{parameter}})\). Then, \(\mathcal{B}\) being a NSbase, there exists a NSb open set \(B_0\) in \(\mathcal{B}\) such that \((x^e_{(a,b,c)},\widehat{\text{parameter}}) \in B_0 \subseteq (G,\partial)(x^e_{(a,b,c)},\widehat{\text{parameter}})\). But, by our choice of \((\mathcal{F},\partial)\), the soft set \(B_0\) contains a point \((x^e_{(a,b,c)},\widehat{\text{parameter}})_0\) of \((\mathcal{F},\partial)\) that is every NSb open set containing \((x^e_{(a,b,c)},\widehat{\text{parameter}})_0\) contain at least one point of \((\mathcal{F},\partial)\). So, \((x^e_{(a,b,c)},\widehat{\text{parameter}})\) is soft adherent point of...
Thus, every point of \((X^{crsp}, \mathcal{I}, \partial)\) is soft adherent point of \((\bar{f}, \partial)\) that is \((\bar{f}, \partial) = (X^{crsp}, \mathcal{I}, \partial)\). It follows, therefore, that \((\bar{f}, \partial)\) is soft countable dense soft sub-set of \((X^{crsp}, \mathcal{I}, \partial)\). Hence, \((X^{crsp}, \mathcal{I}, \partial)\) is NSb separable.

**Theorem 7.15.** Let \((X^{crsp}, \mathcal{I}, \partial)\) be a second soft neutrosophic countable NS space is NS Lindelof space.

**Proof:** Let \((X^{crsp}, \mathcal{I}, \partial)\) be a second NS countable topological space and let \(\mathcal{B} = \{B_1, B_2, B_3, B_4, \ldots B_n: n \in \mathbb{N}\}\) soft base for \((X^{crsp}, \mathcal{I}, \partial)\). Let \(C = \{(G_1, \partial): i \in N\}\) that is \((G_1, \partial), (G_2, \partial), (G_3, \partial), (G_4, \partial), \ldots i \in N\) be any NSb open cover of \((X^{crsp}, \mathcal{I}, \partial)\). Then for each \(\left(x^{e(a,b,c)}, d_{\text{parameter}}\right) \in (X^{crsp}, \mathcal{I}, \partial)\) there exists a NSb open set \((G_1, \partial)_{a(x^{e(a,b,c)},d_{\text{parameter}})}\) and \(\mathcal{B}\) being a NSb bases, corresponding to each such NSb open set there exists a NSb open set \((G_1, \partial)_{n(x^{e(a,b,c)},d_{\text{parameter}})} \in \mathcal{B}\) such that \(\left(x^{e(a,b,c)}, d_{\text{parameter}}\right) \in B_{n(x^{e(a,b,c)},d_{\text{parameter}})} \in \mathcal{B}\). Therefore, \((X^{crsp}, \mathcal{I}, \partial) = \bigcup \left\{B_{n(x^{e(a,b,c)},d_{\text{parameter}})}: \left(x^{e(a,b,c)}, d_{\text{parameter}}\right) \in X\right\} \in \bigcup \left\{(G_1, \partial)_{n(x^{e(a,b,c)},d_{\text{parameter}})}: \left(x^{e(a,b,c)}, d_{\text{parameter}}\right) \in X\right\}\). Now, \(\left\{(G_1, \partial)_{n(x^{e(a,b,c)},d_{\text{parameter}})}: \left(x^{e(a,b,c)}, d_{\text{parameter}}\right) \in X\right\}\) being s soft sub-family of covering \((X^{crsp}, \mathcal{I}, \partial)\). Thus, every NSb open covering of \((X^{crsp}, \mathcal{I}, \partial)\) is reducible to a soft sub-covering. Hence, \((X^{crsp}, \mathcal{I}, \partial)\) is a NSb Lindelof space.

**Theorem 7.16.** Let \((X^{crsp}, \mathcal{I}, \partial)\) be a NS Lindelof space, then this space need not always be second NScountable.

**Proof:** Let \((X^{crsp}, \mathcal{I}, \partial)\) be a NS co-finite topological space provided \((X^{crsp}, \mathcal{I}, \partial)\) is NS uncountable. Now, Let \(C = \{(G_1, \partial): i \in N\}\) that is \((G_1, \partial), (G_2, \partial), (G_3, \partial), (G_4, \partial), \ldots i \in N\) be any NSb open cover of \((X^{crsp}, \mathcal{I}, \partial)\). \((G_1, \partial)_{a_0}\) be an arbitrary member of \((G_1, \partial), (G_2, \partial), (G_3, \partial), (G_4, \partial), \ldots i \in N\). Then, \((G_1, \partial)_{a_0} \subset \mathcal{C}\) is finite. Let \((G_1, \partial)_{a_0} \subset \mathcal{C}\) be a NSb covered by at the most \((n)\) sets in \((G_1, \partial): i \in N\) that is \((G_1, \partial), (G_2, \partial), (G_3, \partial), (G_4, \partial), \ldots i \in N\) and so \((X^{crsp}, \mathcal{I}, \partial)\) is covered by the most \((n + 1)\) sets in \((G_1, \partial)_{a_0} \subset \mathcal{C}\). Where, \((G_1, \partial)_{a_0} \subset \mathcal{C}\) absorbs \((n)\) points of \((G_1, \partial)_{a_0} \subset \mathcal{C}\).
{(G_i, δ): i ∈ N} that is { (G_1, δ), (G_2, δ), (G_3, δ), (G_4, δ), ... : i ∈ N}. Thus, (X^{cr,p}, ℳ, δ) is NSb compact space and therefore, a NS Lindelof space. Now, if possible, let there be a NS countable base ℳ for (X^{cr,p}, ℳ, δ) . Let \( x^e_{(a,b,c)} \) denote the parameter ∈ X. Then, \( \{ (G_i, δ) | x^e_{(a,b,c)} \} \) ∈ \( \{ (X^{cr,p}, ℳ, δ) \} \) for, if \( (y^{e/}_{(a,b',c')},d) \) ≠ \( (x^e_{(a,b,c)} \) )d, then either \( (y^{e/}_{(a,b',c')},d) \) > \( (x^e_{(a,b,c)} \) )d or \( (y^{e/}_{(a,b',c')},d) \) < \( (x^e_{(a,b,c)} \) )d . Then \( X \setminus \{ (y^{e/}_{(a,b',c')},d) \} \) is clearly NSb open sets containing \( (x^e_{(a,b,c)} \) )d but not \( (y^{e/}_{(a,b',c')},d) \) and therefore any \( (y^{e/}_{(a,b',c')},d) \) different from \( (x^e_{(a,b,c)} \) )d . More-over, \( ℳ = \{ B_1, B_2, B_3, ..., B_n | n ∈ N \} \) being base soft for each (G_i, δ), \( x^e_{(a,b,c)} \) ∃ B_1, B_2, B_3, ..., B_n ∈ ℳ such that \( (x^e_{(a,b,c)} \) )d ∈ B_1, B_2, B_3, ..., B_n ∈ ℳ .

Obviously, \( \{ x^e_{(a,b,c)} \} \) ∈ ℳ .

Let \( (G_i, δ), (x^e_{(a,b,c)} \) )d, \( (y^{e/}_{(a,b',c')},d) \) ∈ ℳ . Then, \( (x^e_{(a,b,c)} \) )d \∈ ℳ or \( (y^{e/}_{(a,b',c')},d) \) \∈ ℳ .

Thus, (X^{cr,p}, ℳ, δ) is a NS Lindelof space which is not second countable.

**Theorem 7.17.** Let (X^{cr,p}, ℳ, δ) be a NS Lindelof space and (Y^{cr,p}, ℳ, δ) be soft sub-space of (X^{cr,p}, ℳ, δ), then guaranteedely, (Y^{cr,p}, ℳ, δ) is NSLindelof space.
Proof: Given \((X^{cr}, \mathfrak{I}, \partial)\) be a NS Lindelof space and \((Y^{cr}, \mathfrak{I}, \partial)\) be soft sub-space of \((X^{cr}, \mathfrak{I}, \partial)\). Let \(\mathcal{C} = \{(\mathcal{H}, \partial)\}\) be any \((Y^{cr}, \mathfrak{I}, \partial)\) NS b open covering \(Y^{cr}\). Then, \(Y^{cr} = \bigcup (\mathcal{H}, \partial)\). Also, \(\mathcal{H}(\mathcal{G}, \delta)\) \(\mathcal{G} \in \mathcal{X}(\mathcal{F}, \mathcal{G})\). Therefore, \(Y = \bigcup (\mathcal{G}, \delta)\). So \((X^{cr}, \mathfrak{I}, \partial)\) = \(Y \bigcup Y^c \in (\mathcal{G}, \delta)\). Thus, \(\mathcal{C}^* = \{(\mathcal{G}, \delta), Y^c\}\) is soft b open covering of the NS Lindelof space of \((X^{cr}, \mathfrak{I}, \partial)\). Since, \(Y^c\) covers any part of \(Y\), so there exists a NS countable number of \((\mathcal{G}, \delta)\) in \(\mathcal{C}^*\) such that \(Y \subseteq \bigcup \{(\mathcal{G}, \delta)\} \in \mathcal{N} \subseteq N\) or \(Y = \bigcup \{(\mathcal{G}, \delta)\} \in \mathcal{N}\). Therefore, \(Y = \bigcup \{(\mathcal{H}, \partial)\} \in N\). This shows that \(\mathcal{C}^*\) is reducible to a NS countable subcovering. Hence, \((X^{cr}, \mathfrak{I}, \partial)\) is also a NS b Lindelof space.

Theorem 7.18. Let \((X^{cr}, \mathfrak{I}, \partial)\) be a NS b space and \(\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right)\), \(\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right)\) \(\in X^{cr}\) such that \(\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right) > \left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right)\) or \(\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right) < \left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right)\). If \(\mathbb{B}\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right)\) is a NS b local base at \(\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right)\), then there exists at least one member of \(\mathbb{B}\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right)\) which does not supremum \(\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right)\).

Proof: Since \((X^{cr}, \mathfrak{I}, \partial)\) be a NS b space and \(\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right) > \left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right)\) or \(\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right) < \left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right)\), \(\exists\) NS b open sets \((\mathcal{G}, \partial)\) and \((\mathcal{H}, \partial)\) such that \(\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right) \in (\mathcal{G}, \partial)\) but \(\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right) \in (\mathcal{H}, \partial)\) and \(\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right) \in (\mathcal{H}, \partial)\) but \(\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right) \notin (\mathcal{H}, \partial)\). Since, \(\mathbb{B}\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right)\) is NS b local base at \(\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right)\) there exists \(\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right) \in B \in (\mathcal{G}, \partial)\). Since \(\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right) \in (\mathcal{G}, \partial)\) and \(B \in (\mathcal{G}, \partial)\) so \(\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right) \notin B\). Thus, \(B \in \mathbb{B}\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right)\) such that \(\left(\mathfrak{A}_{(a,b,c)}, d_{parameter}\right) \notin B\).

Theorem 7.19. Let \((X^{cr}, \mathfrak{I}, \partial)\) be a NSSTS such that it is NS of \(b_1\) space in which every in-finite soft subset has a soft limit point. Then, \((X^{cr}, \mathfrak{I}, \partial)\) is definitely NS b compact.

Proof: Let \(C\) NS open covering of \((X^{cr}, \mathfrak{I}, \partial)\). Then \((X^{cr}, \mathfrak{I}, \partial)\) being NS Lindelof space, \(C\) is reducible to a NS countable sub-covering, say \(C^* = \{(\mathcal{G}_n, \partial)\} n \in \Lambda \subseteq N\) that is \(\{(\mathcal{G}_1, \partial), (\mathcal{G}_2, \partial), (\mathcal{G}_3, \partial), (\mathcal{G}_4, \partial), \ldots\} n \in \Lambda \subseteq N\). If possible, let \(C^*\) is not reducible to a finite soft subcovering. Then, for any positive integer \(k\), the soft \((\bigcup_{i=1}^{k}(\mathcal{G}_n, \partial))\) is NS b open proper subset of...
\( (\mathcal{X}^{cr,p}, \mathcal{I}, \partial) \) and therefore, its complement, \( F_k = (\bigcup_{i=1}^{n} (G_n, \partial))^c \) is non-empty NS closed subset of \( (\mathcal{X}^{cr,p}, \mathcal{I}, \partial) \). Now, taking \( k = 1, 2, 3, \ldots \) we obtain a nested soft sequence \( (F_k) \) of soft neutrosophic closed subsets of \( (\mathcal{X}^{cr,p}, \mathcal{I}, \partial) \) such that \((F_1, \partial) \supseteq (F_2, \partial) \supseteq (F_3, \partial) \supseteq \ldots \supseteq (F_6, \partial) \supseteq (F_7, \partial) \supseteq \ldots \supseteq (F_n, \partial) \supseteq \ldots \). Let \( A = \{ (x^{e}_{(a,b,c)}, d_{parameter})_k : (x^{e}_{(a,b,c)}, d_{parameter})_k \in F_k \} \), then, the soft set \( A \) is obviously an infinite soft set. So, by the given hypothesis, \( A \) has a soft limit point, suppose \( (x^{e}_{(a,b,c)}, d_{parameter}) \). But \( (\mathcal{X}^{cr,p}, \mathcal{I}, \partial) \) being NSb1 space, so every NSb open set containing \( (x^{e}_{(a,b,c)}, d_{parameter}) \) must therefore contain an infinite number of points of \( A \). Result in \( (x^{e}_{(a,b,c)}, d_{parameter}) \) is a soft limit point of each \( F_k \) that is \((F_1, \partial), (F_2, \partial), (F_3, \partial), (F_4, \partial), (F_5, \partial), \ldots \) But each of \((F_1, \partial), (F_2, \partial), (F_3, \partial), (F_4, \partial), (F_5, \partial), \ldots \) is soft b-closed, \((x^{e}_{(a,b,c)}, d_{parameter}) \in (F_1, \partial), (x^{e}_{(a,b,c)}, d_{parameter}) \in (F_2, \partial), (x^{e}_{(a,b,c)}, d_{parameter}) \in (F_3, \partial), (x^{e}_{(a,b,c)}, d_{parameter}) \in (F_4, \partial), (x^{e}_{(a,b,c)}, d_{parameter}) \in (F_5, \partial), \ldots \) This contradicts the fact that \( C \) as a soft covering of \( (\mathcal{X}^{cr,p}, \mathcal{I}, \partial) \) and hence \( C \) reduces to a finite soft sub covering.

**Theorem 7.20.** Let \( (\mathcal{X}^{cr,p}, \mathcal{I}, \partial) \) be NS regular Lindelop space then it is safely NSb\( n \)ormal.

**Proof:** \( (\mathcal{X}^{cr,p}, \mathcal{I}, \partial) \) NSb regular Lindelop space and let \( (\Psi_1, \partial)(\Psi_2, \partial) \) be two NSb closed sub-sets of \( (\mathcal{X}^{cr,p}, \mathcal{I}, \partial) \) such that these are mutually excusive. Then, every NSb closed sub-space of a NS Lindelop space is soft Lindelop space. It is then guaranteed that \( (\Psi_1, \partial)(\Psi_2, \partial) \) are NS Lindelop spaces. Now, by the NSb regularity of \( (\mathcal{X}^{cr,p}, \mathcal{I}, \partial) \), corresponding to NSb closed set \( (\Psi_1, \partial) \) and every \( (x^{e}_{(a,b,c)}, d_{parameter}) \in (\Psi_2, \partial) \exists \) soft NSb open set \( (G, \partial)(x^{e}_{(a,b,c)}, d_{parameter}) \) such that \( (x^{e}_{(a,b,c)}, d_{parameter}) \in (G, \partial) \in (\Psi_1, \partial)(x^{e}_{(a,b,c)}, d_{parameter}) \in (\Psi_2, \partial)^c \). Moreover, the soft family \( \{(G, \partial)(x^{e}_{(a,b,c)}, d_{parameter}) : (x^{e}_{(a,b,c)}, d_{parameter}) \in (\Psi_2, \partial)^c \} \) is clearly NSb open covering of the Lindelop space NS set \( (\Psi_2, \partial) \). So, it must supposes NS countable sub-covering \( \{(G, \partial)(x^{e}_{(a,b,c)}, d_{parameter}) : i \in \mathbb{N} \} \).

Again, by the NSb regularity of \( (\mathcal{X}^{cr,p}, \mathcal{I}, \partial) \), corresponding to the NSb closed set \( (\Psi_2, \partial) \) and every \( (x^{e}_{(a,b,c)}, d_{parameter}) \in (\Psi_1, \partial) \exists \) NSb open set \( (\Psi_3, \partial)(g^{e}_{(a,b,c)}, d_{parameter}) \) such that \( (g^{e}_{(a,b,c)}, d_{parameter}) \in (\Psi_3, \partial)(g^{e}_{(a,b,c)}, d_{parameter}) \in (\Psi_1, \partial)(g^{e}_{(a,b,c)}, d_{parameter}) \in (\Psi_2, \partial)^c \).

Clearly, \( \{(\Psi_3, \partial)(g^{e}_{(a,b,c)}, d_{parameter}) : (g^{e}_{(a,b,c)}, d_{parameter}) \in (\Psi_1, \partial) \} \) is NS open covering off the
Lindelöp space \((\Psi, \partial)\) and therefore, it is frezzable to a NSb countable sub-covering \(\langle \Psi, \partial \rangle : (x^{e_{(a,b,c)}}_{(a,b,c)})_{\text{parameter}} \cap \mathcal{N}S \mathcal{T} = \mathcal{N}S \mathcal{B} \) of parameter \(i \in N\). Let \(\langle \mathcal{M}, \partial \rangle \cap = (\Psi, \partial) : (x^{e_{(a,b,c)}}_{(a,b,c)})_{\text{parameter}} \cap \left\{ (X^{e_{(a,b,c)}}_{(a,b,c)})_{\text{parameter}} : i \leq n \right\} \) and \(\langle \omega, \partial \rangle \cap = (\Psi, \partial) : (x^{e_{(a,b,c)}}_{(a,b,c)})_{\text{parameter}} \cap \left\{ \mathcal{N}S \mathcal{T} = \mathcal{N}S \mathcal{B} \right\} = (\Psi, \partial) : (x^{e_{(a,b,c)}}_{(a,b,c)})_{\text{parameter}} \cap \left\{ (X^{e_{(a,b,c)}}_{(a,b,c)})_{\text{parameter}} : i \leq n \right\} \) is NSB open sets and therefore, so are the sets \(\langle \mathcal{M}, \partial \rangle = \mathcal{N}S \mathcal{T} = \mathcal{N}S \mathcal{B} \) of parameter \(n \in N\). Now, \(\langle \Psi, \partial \rangle \mathcal{U} \cap = (\Psi, \partial) : (x^{e_{(a,b,c)}}_{(a,b,c)})_{\text{parameter}} \cap \left\{ \mathcal{N}S \mathcal{T} = \mathcal{N}S \mathcal{B} \right\} = (\Psi, \partial) : (x^{e_{(a,b,c)}}_{(a,b,c)})_{\text{parameter}} \cap \left\{ (X^{e_{(a,b,c)}}_{(a,b,c)})_{\text{parameter}} : i \leq n \right\} \) is NSB closed of \(\langle \mathcal{M}, \partial \rangle \cap \mathcal{N} \mathcal{S} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S} \). 

**Theorem 7.21.** Let \((X^{e_{(a,b,c)}}_{(a,b,c)})_{\mathcal{N} \mathcal{S} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S}}\) and \(\langle \mathcal{N}S \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S} \rangle\) be two NS continuous function on a NS topological space \((X^{e_{(a,b,c)}}_{(a,b,c)})_{\mathcal{N} \mathcal{S} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S}}\) into a NS set \(\langle \mathcal{N}S \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S}, \partial \rangle\) which is NSB Hausdorff. Then, soft set \(\langle (x^{e_{(a,b,c)}}_{(a,b,c)})_{\mathcal{N} \mathcal{S} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S}} \rangle \cap \mathcal{N} \mathcal{S} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S} \) is NSB closed of \((X^{e_{(a,b,c)}}_{(a,b,c)})_{\mathcal{N} \mathcal{S} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S}}\). 

**Proof:** Let \(\langle (x^{e_{(a,b,c)}}_{(a,b,c)})_{\mathcal{N} \mathcal{S} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S}} \rangle \cap \mathcal{N} \mathcal{S} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S} \) be a NS set of function. If \(\langle (x^{e_{(a,b,c)}}_{(a,b,c)})_{\mathcal{N} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S}} \rangle \cap \mathcal{N} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S} \) is NSB open and therefore, \(\langle (x^{e_{(a,b,c)}}_{(a,b,c)})_{\mathcal{N} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S}} \rangle \cap \mathcal{N} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S} \) is NSB closed, that is nothing is proved in this case. Let us consider the case when \(\langle (x^{e_{(a,b,c)}}_{(a,b,c)})_{\mathcal{N} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S}} \rangle \cap \mathcal{N} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S} \) is NSB closed. Then \(\rho \in \langle (x^{e_{(a,b,c)}}_{(a,b,c)})_{\mathcal{N} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S}} \rangle \cap \mathcal{N} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S} \) and \(\phi \in \langle (x^{e_{(a,b,c)}}_{(a,b,c)})_{\mathcal{N} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S}} \rangle \cap \mathcal{N} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S} \). Result in \(\phi(\rho) \neq \phi(\rho)\). Now, \(\langle \mathcal{N} \mathcal{T} \mathcal{S} \mathcal{T} \mathcal{S}, \partial \rangle\) being...
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NSb Hausdorff space so there exists NSb open sets \((g, \partial)\) and \((\overline{g}, \partial)\) of \((f)(\rho)\) and \((g)(\rho)\) respectively such that \((g, \partial)\) and \((\overline{g}, \partial)\) such that these NS sets such that the possibility of one rules out the possibility of other. By soft continuity of \((f, \partial), (g, \partial), (f, \partial)^{-1}\) as well as \((g, \partial)^{-1}\) is NSb open nhd of \(\rho\) and therefore so is \((f, \partial)^{-1} \cap (g, \partial)^{-1}\) is contained in \(\{(x^{e}_{(a, b, c)}, \text{parameter}) \in X^{cr^p} : (f)((x^{e}_{(a, b, c)}, \text{parameter})) = (g)(x^{e}_{(a, b, c)}, \text{parameter}))\}, \text{ for,} \(x^{e}_{(a, b, c)}, \text{parameter}) \in ((f, \partial)^{-1} \cap (g, \partial)^{-1}) \Rightarrow (f)((x^{e}_{(a, b, c)}, \text{parameter})) \in (g, \partial)\) and \((g)((f)((x^{e}_{(a, b, c)}, \text{parameter})) \neq (g)((x^{e}_{(a, b, c)}, \text{parameter}))\) because \((g, \partial)\) and \((\overline{g}, \partial)\) are mutually exclusive. This implies that \(x\) do not belong to \(\{(x^{e}_{(a, b, c)}, \text{parameter}) \in X^{cr^p} : (f)((x^{e}_{(a, b, c)}, \text{parameter})) = (g)(x^{e}_{(a, b, c)}, \text{parameter})\}\). Therefore \(\{(x^{e}_{(a, b, c)}, \text{parameter}) \in X^{cr^p} : (f)((x^{e}_{(a, b, c)}, \text{parameter})) = (g)(x^{e}_{(a, b, c)}, \text{parameter})\}\) is nhd of each of its points. So, \(\{(x^{e}_{(a, b, c)}, \text{parameter}) \in X^{cr^p} : (f)((x^{e}_{(a, b, c)}, \text{parameter})) = (g)(x^{e}_{(a, b, c)}, \text{parameter})\}\) NSb open and hence \(\{(x^{e}_{(a, b, c)}, \text{parameter}) \in X^{cr^p} : (f)((x^{e}_{(a, b, c)}, \text{parameter})) = (g)(x^{e}_{(a, b, c)}, \text{parameter})\}\) is NSb closed.

**Theorem 7.22.** Let \((X^{cr^p}, \mathcal{I}, \partial)\) NSSTS such that it is NSb Hausdorff space and let \((f)\) be soft continuous function of \((X^{cr^p}, \mathcal{I}, \partial)\) into itself. Then, the NS set of fixed points under \((f)\) is a NSb closed set.

**Proof:** Let \(\delta = \{(f)((x^{e}_{(a, b, c)}, \text{parameter})) = (x^{e}_{(a, b, c)}, \text{parameter})\}\). If \(\delta^c = \overline{\delta}\), Then is NSb open and therefore \(\{(f)((x^{e}_{(a, b, c)}, \text{parameter})) = (x^{e}_{(a, b, c)}, \text{parameter})\}\) NSb closed. So, let \(\{(f)((x^{e}_{(a, b, c)}, \text{parameter})) = (x^{e}_{(a, b, c)}, \text{parameter})\}\) and let \(\left(y^{e^j}_{(a', b', c')}, \text{parameter}\right) \in \{(f)((x^{e}_{(a, b, c)}, \text{parameter})) = (x^{e}_{(a, b, c)}, \text{parameter})\}\) . Then, \(\left(y^{e^j}_{(a', b', c')}, \text{parameter}\right)\) does not belong to \(\{(f)((x^{e}_{(a, b, c)}, \text{parameter})) = (x^{e}_{(a, b, c)}, \text{parameter})\}\) and therefore \(\left((y^{e^j}_{(a', b', c')}, \text{parameter})\right) \neq \left(y^{e^j}_{(a', b', c')}, \text{parameter}\right)\). Now, \(\left(y^{e^j}_{(a', b', c')}, \text{parameter}\right)\) and
(f) \left( \left( y^{e/\subscript{a,b,c,d}} \right)^{d \text{parameter}} \right) \) being two distinct points of the NSb Hasdorff space \( (X^{e/\subscript{a,b,c,d}}, X, \partial) \), so there exists NSb open sets \((g, \partial)\) and \((\tilde{S}, \partial)\) such that \( \left( y^{e/\subscript{a,b,c,d}} \right)^{d \text{parameter}} \in (g, \partial) \) and \((\tilde{S}, \partial)\) are disjoint. Also, by the NS continuity of \((f)\), \((f)^{-1}((H, \partial))\) is NS b open set containing \(y\). We pretend that \((g, \partial) \cap (\tilde{S}, \partial)\) are disjoint. Then, \( \mu \in (g, \partial) \cap (\tilde{S}, \partial) \) and \( \mu \notin (f)^{-1}((\tilde{S}, \partial)) \). As \( (g, \partial) \cap (\tilde{S}, \partial) = \emptyset \), \( \mu \notin (f)^{-1}((\tilde{S}, \partial)) \).

\( \mu \) does not belong to \((f)\) \( \left( \left( x^{e/\subscript{a,b,c,d}} \right)^{d \text{parameter}} \right) = (x^{e/\subscript{a,b,c,d}})^{d \text{parameter}} \). Therefore, \( \left( y^{e/\subscript{a,b,c,d}} \right)^{d \text{parameter}} \in (g, \partial) \cap (\tilde{S}, \partial) \). Thus, \( \left( x^{e/\subscript{a,b,c,d}} \right)^{d \text{parameter}} \) is the NS nhd of each of its points.

So, \( \left( (x^{e/\subscript{a,b,c,d}})^{d \text{parameter}} \right)^{c} \) is NS b open and hence \( \left( x^{e/\subscript{a,b,c,d}} \right)^{d \text{parameter}} \) is NS b closed.

8. Conclusion

In this paper, neutrosophic soft points with one point greater than the other and their properties, generalized neutrosophic soft open set known as b-open set, neutrosophic soft separation axioms theoretically and with support of suitable examples with respect to soft points, neutrosophic soft b₀-space engagement with generalized neutrosophic soft closed set, neutrosophic soft b₂-space engagement with generalized neutrosophic soft open set are addressed. In continuation, neutrosophic soft b₀-space behavior as neutrosophic soft b₂-space with the plantation of some extra condition on soft b₀-space, neutrosophic soft b₁-space and related theorems, neutrosophic soft b₃-space, monotonous behavior of neutrosophic soft function with connection of different neutrosophic soft separation axioms, monotonous behavior of neutrosophic soft function with connection of different neutrosophic soft close sets are reflected. Secondly, long touched has been given to neutrosophic soft countability connection with bases and sub-bases, neutrosophic soft product spaces and its engagement through different generalized neutrosophic soft open set and close sets, neutrosophic soft coordinate spaces and its engagement through different generalized neutrosophic soft open set and close sets. Finally, neutrosophic soft countability and its relationship with Bolzano Weirstrass Property through engagement of compactness, neutrosophic soft strongly spaces and its
related theorems, neutrosophic soft sequences and its relation with neutrosophic soft compactness, neutrosophic soft Lindelof space and related theorems are supposed to address.

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Conflicts of Interest
The authors declare that they have no conflict of interest.

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