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Neutrosophic special dominating set in neutrosophic graphs

Sadegh Banitalebi[1], Rajab Ali Borzooei[2,*]

[1]Department of Knowledge and Cognitive Intelligence, Imam Hossein University, Tehran, Iran; krbanitalebi@ihu.ac.ir

[2]Department of Mathematics, Faculty of Mathematical Sciences, Shahid Beheshti University, Tehran, Iran; borzooei@sbu.ac.ir

*Correspondence: borzooei@sbu.ac.ir; Tel.: (optional; include country code)

Abstract. The neutrosophic graph is a new version of graph theory that has recently been proposed as an extension of fuzzy graph and intuitionistic fuzzy graph that provides more precision compatibility and flexibility than a fuzzy graph and an intuitionistic fuzzy graph in structuring and modelling many real-life problems. The aim of this paper is to offer for the first time the new concepts of neutrosophic highly strong arc, neutrosophic special dominating set, neutrosophic special domination numbers, neutrosophic special cobondage set and neutrosophic special cobondage numbers in the neutrosophic graph, as well as expressing some of relation and results of them and reduce the effect of adding a neutrosophic highly strong arc on neutrosophic special domination number parameter in a neutrosophic graph. Finally, an application related to decision making based on agents affecting the performance of the organization is provided.

Keywords: Neutrosophic graph, neutrosophic special dominating set, neutrosophic special domination number, neutrosophic special cobondage set, neutrosophic special cobondage number.)

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1. Introduction

The concept of neutrosophic sets (NSs) was offered by Smarandache [22] as a of the fuzzy sets [27], intuitionistic fuzzy sets [3], interval valued fuzzy set [26] and interval-valued intuitionistic fuzzy sets [4] theories. The neutrosophic set is a powerful mathematical tool for dealing with incomplete, indeterminate and inconsistent information in real world. The neutrosophic sets are characterized by a truth-membership function T, an indeterminacy-membership function I and a falsitymembership function F independently, which are within the real standard or nonstandard unit interval $]^{-}0, 1^{+}[$. Graph theory has now become a major branch of applied mathematics and it is generally regarded as a branch of combinatorics. Graph is a widely used tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, optimization and computer science. If one has uncertainty regarding either the set of nodes or arcs, or both, the model becomes a fuzzy graph. But, if the relations betwixt nodes (or nodes) in problems are indeterminate, the fuzzy graphs and their extensions fail. For this purpose, Smarandache [23, 24] given two main categories of neutrosophic graphs. In another study, Satham Hussain, Jahir Hussain and Smarandache [21] proposed the notion of domination in neutrosophic soft graphs. By considering the above-mentioned studies, the present paper seek to offer the concepts of neutrosophic special dominating set, neutrosophic special domination numbers, neutrosophic special cobondage set and neutrosophic special cobondage numbers in neutrosophic graphs.

2. Preliminaries

A *fuzzy graph* $G = (\phi, \psi)$ on simple graph $G^* = (\mathbb{V}, \mathbb{E})$ is a pair of functions $\phi : \mathbb{V} \to [0, 1]$ and $\psi : \mathbb{E} \to [0, 1]$ where, for each $zw \in \mathbb{E}$, $\psi(zw) \preceq \min\{\phi(z), \phi(w)\}$.

Definition 2.1. [22] If $\mathbb{V}$ is a space of points (objects) with general elements in $\mathbb{V}$ symbolized by z, then the neutrosophic set H is an object having the form

$$H = \{\langle z : T_H(z), I_H(z), F_H(z)\rangle, z \in \mathbb{V}\},$$

where the functions $T, I, F : \mathbb{V} \to]^-0, 1^+[$ describe respectively, the truth-membership function, the indeterminacy-membership function and the falsity-membership function of the element $z \in \mathbb{V}$ to the set H with the condition

$$^-0 \leq T_H(z) + I_H(z) + F_H(z) \leq 3^+,$$

the functions $T_H(z), I_H(z)$ and $F_H(z)$ are real standard or nonstandard subsets of $]^-0, 1^+[$.

Definition 2.2. [8] A neutrosophic graph on simple graph $G^* = (\mathbb{V}, \mathbb{E})$ is symbolized by $G = (K, L)$, where $K = (T_K, I_K, F_K)$ such that $T_K, I_K, F_K : \mathbb{V} \to [0, 1]$ with the condition

$$0 \leq T_K(z) + I_K(z) + F_K(z) \leq 3,$$

for all $z \in \mathbb{V}$ and $L = (T_L, I_L, F_L)$ where $T_L, I_L, F_L : \mathbb{E} \to [0, 1]$ with conditions

$$T_L(zw) \leq T_K(z) \wedge T_K(w),$$

$$I_L(zw) \geq I_K(z) \vee I_K(w),$$

$$F_L(zw) \geq F_K(z) \vee F_K(w),$$

and $0 \leq T_L(zw) + I_L(zw) + F_L(zw) \leq 3$ for all $zw \in \mathbb{E}$.

Definition 2.3. [10] Put $G = (K, L)$ be a neutrosophic graph on simple graph $G^* = (\mathbb{V}, \mathbb{E})$ and $u, v \in \mathbb{V}$. Then,

(i) *T-strength* of connectedness betwixt u and v is

$$T_L^{\infty}(uv) = \sup\{T_L^n(uv)\big|n = 1, 2, \cdots, m\},$$

and

$$T_L^n(uv) = \min\left\{T_L(uz_1), T_L(z_1z_2), \cdots, T_L(z_{n-1}v)\big|u, z_1, \cdots, z_{n-1}, v \in \mathbb{V}, n = 1, 2, \cdots, m\right\}.$$

(ii) *I-strength* of connectedness betwixt u and v is

$$I_L^{\infty}(uv) = \inf\{I_L^n(uv)\big|n = 1, 2, \cdots, m\},$$

and

$$I_L^k(uv) = \max\left\{I_L(uz_1), I_L(z_1z_2), \cdots, I_L(z_{n-1}v)\big|u, z_1, \cdots, z_{n-1}, v \in \mathbb{V}, n = 1, 2, \cdots, m\right\}.$$

(iii) *F-strength* of connectedness betwixt u and v is

$$F_L^{\infty}(uv) = \inf\{F_L^n(uv)\big|n = 1, 2, \cdots, m\},$$

and

$$F_L^n(uv) = \max\{F_L(uz_1), F_L(z_1z_2), \cdots, F_L(z_{n-1}v)\big|u, z_1, \cdots, z_{n-1}, v \in \mathbb{V}, n = 1, 2, \cdots, m\}.$$

Definition 2.4. [10] Put $G = (K, L)$ be a neutrosophic graph on simple graph $G^* = (\mathbb{V}, \mathbb{E})$. An arc $zw \in \mathbb{E}$ said to be a neutrosophic strong arc if

$$T_L(zw) \geq T_L^{\infty}(zw)\ ,\ I_L(zw) \leq I_L^{\infty}(zw) \text{ and } I_L(zw) \leq I_L^{\infty}(zw).$$

Notation 1. *From now on, in this paper we put $G = (K, L)$ be a neutrosophic graph on simple graph $G^* = (\mathbb{V}, \mathbb{E})$ and symbolized by NG.*

3. Study of neutrosophic special dominating set by addition of neutrosophic highly strong arcs

In this part, we describe the notions of neutrosophic highly strong arc, neutrosophic slightly isolated node, neutrosophic special dominating set, neutrosophic special domination numbers, neutrosophic slightly independent set and neutrosophic slightly independent numbers on neutrosophic graphs and we investigate some related results. Also we discuss about neutrosophic special domination of neutrosophic graph by adding a neutrosophic highly strong arc to this neutrosophic graph.

Definition 3.1. Put $G = (K, L)$ be a NG. Then,

(i) the neutrosophic order of G is given by,

$$|\mathbb{V}| = \sum_{v_i \in \mathbb{V}} \left(\frac{3 + T_K(v_i) - (I_K(v_i) + F_K(v_i))}{2} \right),$$

(ii) the neutrosophic size of G is given by,

$$|\mathbb{E}| = \sum_{v_i v_j \in \mathbb{E}} \left(\frac{3 + T_L(v_i v_j) - (I_L(v_i v_j) + F_L(v_i v_j))}{2} \right),$$

(iii) the neutrosophic cardinality of G is given by,

$$|G| = |\mathbb{V}| + |\mathbb{E}|,$$

(iv) for each $U \subset \mathbb{V}$, the neutrosophic node cardinality of U is symbolized by $O(U)$ and given by,

$$O(U) = \sum_{v_i \in U} \left(\frac{3 + T_K(v_i) - (I_K(v_i) + F_K(v_i))}{2} \right),$$

(v) for each $F \subset \mathbb{E}$, the neutrosophic arc cardinality of F is symbolized by $S(F)$ and given by,

$$S(F) = \sum_{v_i v_j \in F} \left(\frac{3 + T_L(v_i v_j) - (I_L(v_i v_j) + F_L(v_i v_j))}{2} \right).$$

Definition 3.2. An arc $e = zw$ in G is called a neutrosophic highly strong arc (NHStA), if

$$T_L(zw) > T_L^{\infty}(zw)\ ,\ I_L(zw) < I_L^{\infty}(zw)\ ,\ F_L(zw) < F_L^{\infty}(zw).$$

Definition 3.3. The neutrosophic highly strong neighborhood of $z \in \mathbb{V}$ is symbolized by $N_{hs}(z)$ and given as follows:

$$N_{hs}(z) = \{w \in \mathbb{V} \mid zw \text{ is a highly strong arc in } G\}.$$

Example 3.4. Investigate a NG G as Figure 1. Then, u_1u_3 and u_3u_4 are NHStAs and it is clear that $N_{hs}(u_3) = \{u_1, u_4\}$ and $N_{hs}(u_1) = N_{hs}(u_4) = \{u_3\}$.

Definition 3.5. Put G be a NG on simple graph $G^* = (\mathbb{V}, \mathbb{E})$ and $z, w \in \mathbb{V}$. Then:
(i) we say that z specially dominate w in G, if there is a NHStA betwixt z and w.
(ii) $S \subset \mathbb{V}$ said to be a neutrosophic special dominating set (NSpDS) in G, if for each $w \in \mathbb{V} \setminus S$, there is $z \in S$ where z specially dominates w.
(iii) A NSpDS S in G said to be a minimal neutrosophic special dominating set if no proper subset of S is a neutrosophic special dominating set.
(iv) Minimum neutrosophic node cardinality amongst all minimal NSpDSs of G said to be

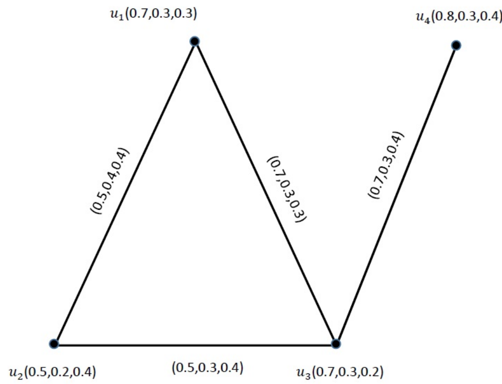

FIGURE 1. NG G.

lower neutrosophic special domination number of G and is symbolized by $(nsdn)_{\mathbb{V}}(G)$.
(v) Maximum neutrosophic node cardinality amongst all minimal NSpDSs of G said to be upper neutrosophic special domination number of G and is symbolized by $(NSDN)_{\mathbb{V}}(G)$.
(vi) The neutrosophic special domination number of G is symbolized by $NS\Delta N(G)$ and given by

$$NS\Delta N(G) = \frac{(nsdn)_{\mathbb{V}}(G) + (NSDN)_{\mathbb{V}}(G)}{2}.$$

Theorem 3.6. *A NSpDS D of a neutrosophic graph G is a minimal NSpDS iff for each node $z \in D$, one of the following conditions hold.*
(i) $N_{hs}(z) \cap D = \emptyset$,
(ii) there is a node $w \in \mathbb{V} \setminus D$ *where* $N_{hs}(w) \cap D = \{z\}$.

Proof. Suppose that D is a minimal NSpDS of G. Then, for each node $z \in D$, $D \setminus \{z\}$ is not a NSpDS. Thus there is $w \in \mathbb{V} \setminus (D \setminus \{z\})$ that is not specially dominated by any node in $D \setminus \{z\}$. If $w = z$, then w is not a neutrosophic strong neighbor of any node in D. If $w \neq z$, then w is not specially dominated by $D \setminus \{z\}$, but is specially dominated by D.
Conversely, consider that D is a NSpDS and for each node $z \in D$, one of the two conditions hold. Suppose D is not a minimal NSpDS. Then there is a node $z \in D$ where $D \setminus \{z\}$ is a NSpDS. Then z is a neutrosophic highly strong neighbor to at least one node in $D \setminus \{z\}$, and so (i) does not true. Also, every node w in $\mathbb{V} \setminus D$ is a neutrosophic highly strong neighbor to at least one node in $D \setminus \{z\}$. Thus (ii) does not true, that is a contradiction. Therefore, D is a minimal NSpDS. □

Example 3.7. Investigate a NG G as Figure 2. Then, $D_1 = \{u_1, u_3, u_4\}$, $D_2 = \{u_2, u_4, u_5\}$ and $D_3 = \{u_3, u_4, u_5\}$ are minimal NSpDSs and clearly $(nsdn)_{\mathbb{V}}(G) = 1.55$ and $(NSDN)_{\mathbb{V}}(G) =$

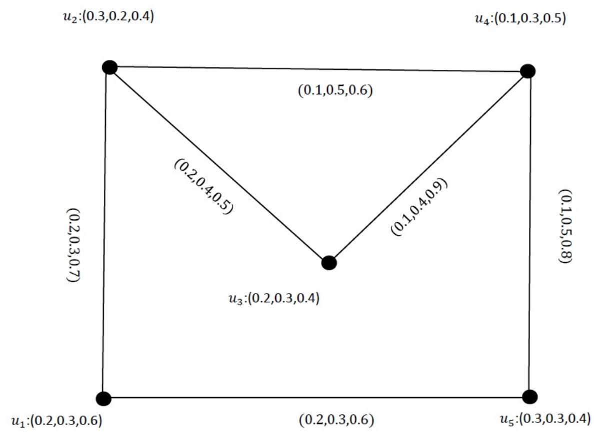

FIGURE 2. NG G.

2.8 and so,

$$NS\Delta N(G) = \frac{1.55 + 2.8}{2} = 2.175.$$

Definition 3.8. A node $z \in \mathbb{V}$ of a NG G is called a neutrosophic slightly isolated node (NSlIN) if does not specially dominate any other node of G and $N_{hs}(z) = \emptyset$.

Example 3.9. Investigate the NG G as Figure 1. Then, u_2 is a NSlIN in G.

If in graph $G^* = (\mathbb{V}, \mathbb{E})$ we add an arc e to $\mathbb{E}$, then we denote it by $\mathbb{E}_e = \mathbb{E} \cup \{e\}$ and $G^*_e = (\mathbb{V}, \mathbb{E}_e)$. Moreover, if neutrosophic graph $G = (K, L)$ on G^* extened on G^*_e, then we symbolized it by $G_e = (K_e, L_e)$.

Notation 2. *If arc e in NG G_e is a NHStA, then we denote $G^{hs}_e = (K^{hs}_e, L^{hs}_e)$ insteade of $G_e = (K_e, L_e)$.*

Theorem 3.10. *Put $e = zw$ be an additional NHStA in G^*_e. Then*
(i) $NS\Delta N(G^{hs}_e) \leq NS\Delta N(G)$.
(ii) $0 \leq NS\Delta N(G) - NS\Delta N(G^{hs}_e) \leq \max\{O(\{z\}), O(\{w\})\}$.

Proof. (i) Suppose that D is a minimal NSpDS of G and $e = zw$ be an additional NHStA in G^*. If z or w is a NSlIN, then $D \setminus \{z\}$ or $D \setminus \{w\}$ is a minimal NSpDS in G^s_e. Otherwise, D is a minimal NSpDS in G^s_e. Hence, $(nsdn)_{\mathbb{V}}(G^{hs}_e) \leq (nsdn)_{\mathbb{V}}(G)$ and $(NSDN)_{\mathbb{V}}(G^{hs}_e) \leq (NSDN)_{\mathbb{V}}(G)$. Therefore, $NS\Delta N(G^{hs}_e) \leq NS\Delta N(G)$.
(ii) By the proof of (i), we have:

$$0 \leq (nsdn)_{\mathbb{V}}(G) - (nsdn)_{\mathbb{V}}(G^{hs}_e) \leq \max\{O(\{z\}), O(\{w\})\},$$

and

$$0 \leq (NSDN)_{\mathbb{V}}(G) - (NSDN)_{\mathbb{V}}(G^{hs}_e) \leq \max\{O(\{z\}), O(\{w\})\}.$$

Then,

$$0 \leq NS\Delta N(G) - NS\Delta N(G_e^{hs}) \leq \max\{O(\{z\}), O(\{w\})\}.$$

□

Theorem 3.11. *Put G be a NG and e be an additional arc in G_e^*. Then e is a NHStA in G_e iff there exist nodes z and w where $z - w$ neutrosophic path of G_e that includes e is the unique strongest neutrosophic path betwixt two nodes z and w.*

Proof. Put $e = xy$ be a NHStA in G_e. Then,

$$T_L(zw) > T_L^\infty(zw),\ I_L(zw) < I_L^\infty(zw),\ F_L(zw) < F_L^\infty(zw).$$

If we let $z = x$ and $w = y$, then the proof is clear.
Conversely, if there exist nodes z, w where $z - w$ neutrosophic path P_e of G_e that includes $e = xy$ is the unique strongest neutrosophic path betwixt two nodes z and w, then for each $x - y$ neutrosophic path P without arc $e = xy$ in G, we have:

$$T_L(zw) > T_P(zw),\ I_L(zw) < I_P(zw),\ F_L(zw) < F_P(zw).$$

Hence,

$$T_L(zw) > T_L^\infty(zw)\ ,\ I_L(zw) < I_L^\infty(zw)\ ,\ F_L(zw) < F_L^\infty(zw).$$

Therefore, $e = xy$ is a NHStA in G_e. □

Definition 3.12. An arc e in a NG G said to be a/an
(i) T-bridge if deleting e reduces the T-strength of connectedness betwixt some pair of nodes.
(ii) I-bridge if deleting e increases the I-strength of connectedness betwixt some pair of nodes.
(iii) F-bridge if deleting e increases the F-strength of connectedness betwixt some pair of nodes.
(iv) neutrosophic bridge if it is a T-bridge, I-bridge and F-bridge.

Theorem 3.13. *An arc $e = zw$ in G^* is a NHStA iff $e = zw$ is a neutrosophic bridge in G.*

Proof. Put $e = zw$ be a NHStA in G_e. Then,

$$T_L(zw) > T_L^\infty(zw),\ I_L(zw) < I_L^\infty(zw),\ F_L(zw) < F_L^\infty(zw).$$

It is clear that $e = zw$ is the unique strongest neutrosophic path betwixt z and w. Thus, deleting $e = zw$ reduces the T-strength and also increases I-strength and F-strength of connectedness betwixt z and w. Therefore $e = zw$ is a neutrosophic bridge in G.
Conversely, if we let $e = zw$ as a neutrosophic bridge in G, then the proof is clear. □

Example 3.14. Investigate the NG G as Figure 2. Then, $e_1 = u_1u_2$, $e_2 = u_1u_5$ and $e_3 = u_2u_3$ are NHStAs and so neutrosophic bridges in G.

Definition 3.15. Put G be a NG. Then:
(i) Two nodes $z,\ w \in \mathbb{V}$ are called neutrosophic slightly independent if there is not any NHStA betwixt them.
(ii) $S \subset \mathbb{V}$ is called a neutrosophic slightly independent set (NSlIS) in G if for each $z, w \in S$, $T_L(uv) \leq T_L^{\infty}(zw)$, $I_L(zw) \geq I_L^{\infty}(zw)$ and $F_L(zw) \geq F_L^{\infty}(zw)$.
(iii) A NSlIS S in G is called a maximal NSlIS if for each node $w \in \mathbb{V} \setminus S$, the set $S \cup \{w\}$ is not NSlIS.
(iv) Minimum neutrosophic node cardinality amongst all maximal NSlISs said to be lower neutrosophic slightly independent number of G and is symbolized by $(ni)_{\mathbb{V}}(G)$.
(v) Maximum neutrosophic node cardinality amongst all maximal NSlISs said to be upper neutrosophic slightly independent number of G and is symbolized by $(NI)_{\mathbb{V}}(G)$.
(vi) The neutrosophic slightly independent number of G is symbolized by $NI(G)$ and given as follows,

$$NI(G) = \frac{(ni)_{\mathbb{V}}(G) + (NI)_{\mathbb{V}}(G)}{2}.$$

Theorem 3.16. *Every maximal NSlIS in G is a minimal NSpDS.*

Proof. Assume that M is a maximal NSlIS in G. Then any node $v \in \mathbb{V} \setminus M$ is a NHSN to at least one node in M. Hence, M is a NSpDS in G. On the other hand, for each node $d \in M$, $N_{hs}(d) \cap D = \emptyset$. Therefore, by Theorem 3.6, M is a minimal neutrosophic special dominating set. □

Example 3.17. In Figure 2, $D_3 = \{u_3, u_4, u_5\}$ is a maximal NSlIS and so minimal NSpDS in G.

Theorem 3.18. *Put e be an additional NHStA in G_e^*. Then $NI(G_e^{hs}) \leq NI(G)$.*

Proof. Straightforward □

4. Neutrosophic special cobondage numbers of a NG.

In this part, we offer the concepts of neutrosophic special cobondage set and neutrosophic special cobondage numbers on NGs and investigated some related results.

Definition 4.1. (i) The neutrosophic special cobondage set (NSpCS) of a NG G is the set C of additional NHStAs to G, that reduces the neutrosophic special domination number, i.e,

$$NS\Delta N(G_C) < NS\Delta N(G).$$

(ii) A NSpCS C of G said to be a minimal NSpCS if no proper subset of C is a NSpCS.
(iii) Minimum neutrosophic arc cardinality amongst all minimal NSpCSs of G said to be lower neutrosophic special cobondage number of G and symbolized by $(nsbn)_{\mathbb{E}}(G)$.
(iv) Maximum neutrosophic arc cardinality amongst all minimal NSpCSs of G said to be upper neutrosophic special cobondage number of G and symbolized by $(NSBN)_{\mathbb{E}}(G)$.

Example 4.2. Investigate a NG G as Figure 3. Obviously $D_1^* = \{a, d\}$ and $D_2^* = \{b, c\}$ are

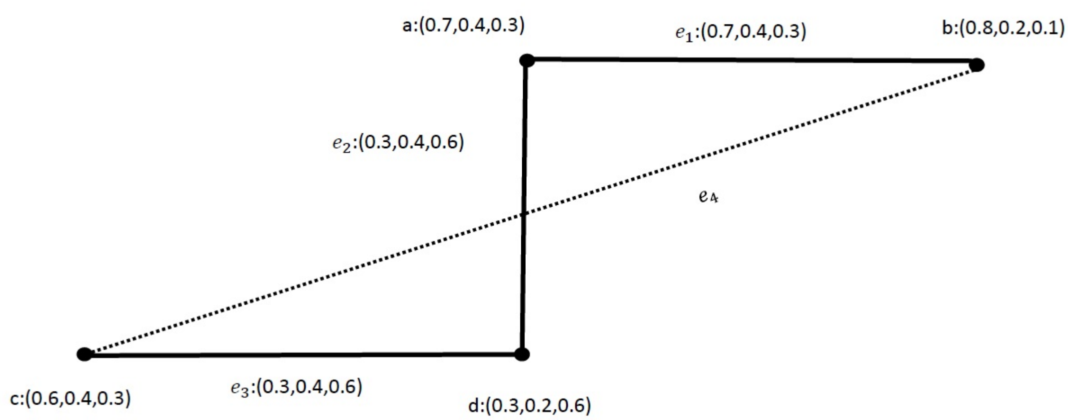

FIGURE 3. NG G.

the minimal NSpDSs of G ($(ndn)_{\mathbb{V}}(G) = 1.83$, $(NDN)_{\mathbb{V}}(G) = 2.67$ and $N\Delta N(G) = 2.25$). In this case, by adding $e_4 = (0.5, 0.4, 0.5)$, the set $D_1 = \{c, d\}$ is a minimal NSpDS with the neutrosophic node cardinality of 1.8. Then, by adding e_5 as $bd = (0.3, 0.2, 0.6)$, the set $D_2 = \{d\}$ is a minimal NSpDS with the neutrosophic node cardinality of 0.83, so $x_2 = \{e_5\}$ is a minimal NSpCS, and by adding e_6 as $ac = (0.6, 0.4, 0.3)$, the set $D_3 = \{a\}$ is a minimal NSpDS with the neutrosophic node cardinality of 1. Thus, $x_3 = \{e_6\}$ is a minimal NSpCS and so $(nbn)_{\mathbb{E}}(G)$ and $(NBN)_{\mathbb{E}}(G)$ are 0.83 and 0.97, respectively.

Theorem 4.3. *If a NG G has a NSlIN w, then*

$$(nsbn)_{\mathbb{E}}(G) \leq O(\{v\}).$$

Proof. Put w be a NSlIN of G. Then w belongs to every minimal NSpDS D of G. If $z \in D \setminus \{w\}$ and e is an NHStA betwixt w and z, then, $D \setminus \{w\}$ is a minimal NSpDS of G_e^{hs} and $(nsdn)_{\mathbb{V}}(G_e^{hs}) < (nsdn)_{\mathbb{V}}(G)$. Thus, $NS\Delta N(G_e^{hs}) < NS\Delta N(G)$. Also, we have $T_L(e) \leq T_K(w)$, $I_L(e) \geq I_K(w)$ and $F_L(e) \geq F_K(w)$. Hence,

$$S(e) \leq \left(\frac{3 + T_K(w) - (I_K(w) + F_K(w))}{2} \right),$$

and so

$$(nsbn)_{\mathbb{E}}(G) = S(e) \leq \left(\frac{3 + T_K(w) - (I_K(w) + F_K(w))}{2}\right) = O(\{w\}).$$

□

Theorem 4.4. *If G has not a NSlIN and $e = zw$ is a NHStA and $N_{hs}(z) = w$, $N_{hs}(w) = z$, then*

$$(nsbn)_{\mathbb{E}}(G) \leq O(\{z\}) + O(\{w\}).$$

Proof. If $e = zw$ is a NHStA in G where $N_{hs}(z) = w$, $N_{hs}(w) = z$, then one of z or w belongs to every minimal NSpDS D of G. Put $z \in D$ and $t \in D \setminus \{z\}$. By adding the NHStAs $e_1 = (zt)$ and $e_2 = (wt)$, the set $D \setminus \{z\}$ is a minimal NSpDS of G_C, where $C = \{e_1, e_2\}$. Thus $NS\Delta N(G_C) < NS\Delta N(G)$. Therefore,

$$\begin{aligned}(nsbn)_{\mathbb{E}}(G) &= S(C) \leq \left(\frac{3 + T_K(z) - (I_K(z) + F_K(z))}{2}\right) + \left(\frac{3 + T_K(w) - (I_K(w) + F_K(w))}{2}\right) \\ &= O(\{z\}) + O(\{w\}).\end{aligned}$$

□

5. Application

NG models have recently been used to model many real-life problems in several different fields of engineering and science. In this study, we present the idea of NSpDS in NG theory. The NSpDS in the neutrophic network can be used to solve many real problems.

5.1. *Decision making in gray conditions betwixt certainty and uncertainty*

NG models are one of the efficient models in various fields of modeling because they show more flexibility than various fuzzy graph models in dealing with real-life problems. Controlling and ensuring the compliance of decisions in various dimensions of the organization with the desired performance and predetermined performance standards despite the gray conditions betwixt certainty and uncertainty, is one of the main tasks of the leaders of an organization and plays an significant role in increasing the productivity and effectiveness of the organization. Therefore, proper management and modeling and optimization of an organization's success plan based on the agents affecting the performance of the organization in the gray conditions betwixt certainty and uncertainty is one of the significant issues considered by the leaders of an organization. The set of agents affecting the performance of an organization in gray conditions betwixt certainty and uncertainty can be considered as a NG. We describe the T-strength, I-strength and F-strength values in each node and arc (path) as follows. For each

$z, w \in V$ and $zw \in E$, we have:

$T_K(z)$: The weight of the direct effectiveness of agent z on the performance of the organization in gray conditions.

$I_K(z)$: The weight of the ineffectiveness of agent z on the performance of the organization in gray conditions.

$F_K(z)$: The weight of the indirect effectiveness of agent z on the performance of the organization in gray conditions.

$T_L(zw)$: The weight of direct impact zw on the performance of the organization in gray conditions.

$I_L(zw)$: The weight of the ineffectiveness zw on the performance of the organization in gray conditions.

$F_L(zw)$: The weight of indirect impact zw on the performance of the organization in gray conditions.

In this case, the following relations seem logical:

$$T_L(zw) \leq T_K(z) \wedge T_K(w),\ I_L(zw) \geq I_K(z) \vee I_K(w),\ F_L(zw) \geq F_K(z) \vee F_K(w).$$

The relationship betwixt z and w is effective when the xy is a NHStA. Thus, the NSpDS of this graph includes agents that other agents are specialiy dominated by at least one of the elements (agents) of this set. In fact, the NSpDS provides an opportunity for managers and leaders of the organization to focus on the agents of the NSpDS and align decisions with these agents instead of observing and controlling a large number of decision agents in gray conditions. This helps organizational leaders and managers make the best decisions with the utmost confidence in a short period of time. For example, Figure 4, displays the graph of agents affecting the performance of an organization, in which the set of $\{u_2, u_4, u_7\}$ is a minimal NSpDS (with minimum neutrosophic node cardinality 4.35). In other words, instead of controlling the 7 agents, only agents u_2, u_4, u_7 can be controlled and observed and be relatively sure about desirable performance in the decision-making process. It is worth noting that some factors such as common computational indices betwixt two agents, dependent calculation formula, and relationship betwixt the variables of calculating the indices of the agents play significant role in creating an effective relation betwixt the agents. For instance, in Figure 4, illustrates the optimal effective weight of the agent graph($S(F)$ where F is the set of all NHStAs of G) is 6.8 on desirable performance achievement.

Now, if possible, the optimal normal weight of the agent graph on desirable performance achievement can be increased by reinforcing the relation betwixt the agents, which leads to increased accuracy and confidence in the decision-making process and decreasing the neutrosophic node cardinality of the NSpDS. For instance, as shown in Figure 5, the NSpDS of agents decreases to the set $\{u_2, u_4\}$ when establishing an effective relationship is possible betwixt u_2

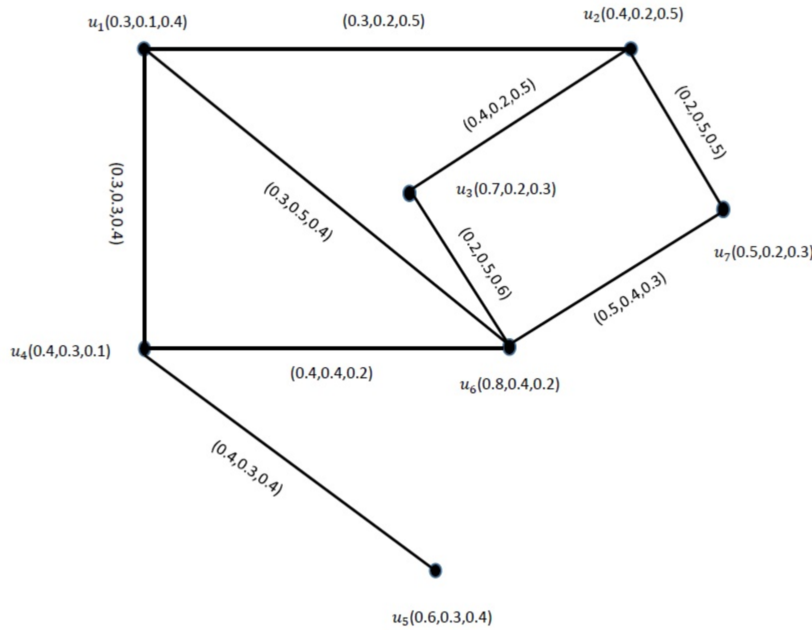

FIGURE 4. Neutrosophic graph G.

and u_7 agents with coordinates $(0.4, 0.2, 0.5)$, while the optimal effective weight of graph upgrades to 8.15.

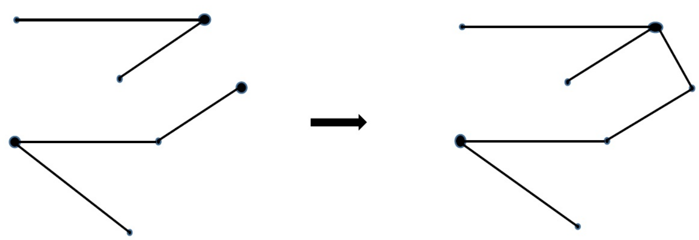

FIGURE 5. D and D'.

Neutrosophic special dominating set	$O(D)$	Optimal effective weight
$D = \{u_2, u_4, u_7\}$	4.35	6.8
$D^{'} = \{u_2, u_4\}$	2.85	8.15

6. Conclusion

Many practical problems of interest can be illustrated with graphs. In general, graph theory has a wide range of applications in various fields. The notion of domination in graph is very important in both theoretical developments and applications. In this paper, for the first time the notions of neutrosophic special dominating set, neutrosophic special domination numbers, neutrosophic special cobondage set and neutrosophic special cobondage numbers in a NG are presented. Finally, by using the concept of neutrosophic special dominating set and the reduction effect of an additional neutrosophic highly strong arc on the neutrosophic special domination number parameter, a model for optimizing the neutrosophic special domination parameter was presented. In future works, we have a decision to study the concepts of neutrosophic special n-dominating set and inverse neutrosophic special dominating set in a NG.

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