



# Neutrosophic Statistical Process Monitoring

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**Abstract:** Woodall et al. [1] raised some issues regarding neutrosophic methodology. In this paper, we will address their comments and questions related to the use of neutrosophic statistics in process monitoring. We provided the responses to some important questions/comments related to neutrosophic statistical process monitoring.

**Keywords:** classical statistics; neutrosophic statistics; control chart; sampling plans; average run length

## 1. Introduction

Neutrosophic statistics as an extension of classical statistics was introduced by Smarandache [2]. Neutrosophic statistics is applied when the data is in the interval and has imprecise observations. Smarandache [37] provided a detailed discussion on the application of neutrosophic statistics. Smarandache [37] proved that neutrosophic statistics is more efficient than classical statistics and interval statistics. Smarandache [37] provided responses to questions of Woodall et al. [1] related to neutrosophic methodology. More applications of neutrosophic statistics can be seen in [3-5].

In this paper, we will address some important questions raised by Woodall et al. [1]. We provided the responses to some important questions/comments related to neutrosophic statistical process monitoring (NSPM)

## 2 Some Comments/Questions

In this section, we selected some important questions/comments from Woodall et al. [1] and provided the responses.

### 2.1 Neutrosophic Sample Size

The control charts using the fuzzy-based approach have been used for decades. These control charts are designed when uncertainty is found in sample size, data, and control chart parameters. Jean [6] presented a detailed discussion on the determination of the sample size for a control chart. Gülbay and Kahraman [7] proposed the control chart for imprecise data. Engin et al. [8] argued that the determination of sample size is a problem in attribute control charts. Moheb Alizadeh et al. [9] proposed control when observations in each sample are a canonical fuzzy number. Turanoğlu et al. [10] presented the sampling plan when the sample size is fuzzy. Yimnak and Intaramo [11] designed a standard deviation control chart when uncertainty (fuzziness) is found in sample size. Haridy et al. [12] contradicted the common belief that the sample size for X-bar and R chart or X-bar and standard deviation should be between [4, 6]. Hesamian et al. [13] proposed the exponentially weighted moving average (EWMA) control chart when the random variable is fuzzy. Zhou et al. [14] proposed control chart by considering the fuzzy sample number. Yalçın and Kaya [15] presented the analysis using the process capability index using a neutrosophic sample [90, 100]. More information on such

as the control chart can be seen in [16] and [17]. Control charts using neutrosophic statistics are the extension of control charts using a fuzzy-based approach. Under the neutrosophic framework, the neutrosophic sample size is fixed before the sample is collected. Like the fuzzy theory, neutrosophic theory is workable under uncertainty where the decision-makers are uncertain about the sample size before the sample is collected. In addition, neutrosophic statistics reduce to classical statistics when the decision-makers are certain about the sample before the sample is collected. Chen et al. [3] and Chen et al. [4] showed the efficiency of neutrosophic statistics over classical statistics.

### **2.2 Extreme Amount of Imprecision in Attribute Control Charts**

In the attribute control chart, during the simulation, it was observed that there is a high jump in average run length while changing the other parameters slightly. Therefore, in attribute control charts, it is expected a high indeterminacy to meet the given conditions.

### **2.3 Neutrosophic Sample Size in Acceptance sampling Plan**

Aslam [18] and Aslam [19] determined the neutrosophic parameters of sampling plans. The real data used in Aslam [18] and Aslam [19] is assumed to have neutrosophic numbers. Therefore, fixed sample size is selected from the indeterminate interval of the sample size. The neutrosophic sample size in the interval can be applied using the same lines when two data sets having neutrosophic numbers are available. The neutrosophic theory is flexible and can be modified according to the situation and underlying studies.

### **2.4 Neutrosophic Control Limits Multiplier**

The development of control limits using the fuzzy-based approach can be seen in [20-22]. The smoothing constant ( $\lambda$ ) is expressed in intervals having the range between 0 and 1. Hunter [23] suggested that the smoothing parameter should be selected from 0.20 to 0.3. On the other hand, Montgomery [24] recommended selecting the values of the smoothing parameter from 0.05 to 0.25. The determination of the smoothing parameter is an important issue in designing the control charts; see [25-26]. Therefore, the decision-makers are not always sure about the value of the smoothing constant to be selected in designing the control chart. In addition, as neutrosophic statistics was applied for the interval data, therefore, it would be justifiable to express the parameters in intervals rather than the exact value. The control charts using neutrosophic statistics are designed when uncertainty is found in the smoothing parameter or moving average span. Note that all neutrosophic parameters of the control chart are fixed in advance. Therefore, it is important to study the behavior of control charts when uncertainty is found in observations or any parameters of the control charts.

### **2.5 Approximation in EWMA Control Charts**

The use of approximations in the evaluation of the average run length of EWMA charts was provided, for example, by [27-29]. More information can be seen in Ziegel [30]. Using the approximation of Aslam et al. [31], it is found that the values of average run length (ARL) are close to Lucas and Saccucci [32] for the larger values of smoothing constant ( $\lambda$ ). It is not recommended to apply the approximation using the  $ARL=1/p$  for EWMA control charts. This type of approximation should be used for Shewhart control charts only.

### **2.6 Efficiency of Control Charts using Repetitive Sampling**

Aslam et al. [33] found that the average sample number for the control chart using repetitive sampling is almost similar to the sample size. It means that the number of repetitions is quite small

(almost close to one). But, from the comparative study of the control chart using repetitive sampling and the control chart using single sampling, the average run length for the control chart using repetitive sampling is smaller than the average run length obtained from the control chart using the single sampling when the average sample number is equal to the competitor chart using single. By increasing the sample size to get smaller values of average run length cannot be encouraged.

### 2.6 Equivalence of Multiple Dependent State Sampling and Runs-Rule

Recently, Woodall et al. [34] showed that multiple dependent state sampling (MDSS) is equivalent to a run-rule chart. Aslam et al. [35] proposed control chart using MDSS when the process is in-control and by following the assumption of MDSS. Designing the MDSS by following [34] has less significant chance that  $(i+1)$  samples are in intermediate region and will not be efficient as the approach adopted by Woodall et al. [34]. As suggested by Riaz et al. [36] "In real applications, having a wider indecisive zone may not be very practical, and hence we have chosen the indecisive regions of practical worth".

### 3 Concluding Remarks

In this paper, we addressed some questions raised by Woodall et al. [1] on the application of control charts using neutrosophic statistics. From the study, it is concluded that like the fuzzy-based approach, the control charts using neutrosophic statistics can be designed and applied in an uncertain environment.

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