



Neutrosophic Statistical Analysis of Temperature of Different Cities of Pakistan

Ishmal Shahzadi

Undergraduate student at Department of Statistics and Actuarial Science, University of the Punjab, Lahore 54000, Pakistan

Email: ishahzadiaslam@gmail.com

Abstract: In this paper, neutrosophic statistical analysis of temperature data of five different cities of Pakistan is given. The neutrosophic mean and neutrosophic coefficient of variation are computed using the temperature data. From the analysis, it is concluded that on average the temperature of Lahore city is higher than the temperature of other cities. Also, the temperature of Karachi city is more consistent compared to other cities. In addition, the neutrosophic results are compared with results under classical statistics. The neutrosophic statistical analysis is found to be more informative than classical statistics results.

Keywords: temperature; indeterminacy; uncertainty; statistical; analysis

1. Introduction

The weather temperature has a serious effect on the human body. An increase in weather temperature can be harmful to humans and other living things. It also affects the reduction of productivity of agriculture (Janjua et al., 2020). Recently, Pakistan has been experiencing an unexpected extreme change in climate which cause a lot of damage to health and livelihoods in the country (Eckstein, 2018). In recent years many heat strokes have been recorded which has caused many problems in the environment. The animals are dying because of water due to environmental change. The statistical methods have been widely applied for the prediction and estimation of temperature. Several researchers also studied different aspects of temperature. Iqbal et al. (2016) presented a study on the recent changes in maximum and minimum temperatures in Pakistan. This analysis deals with the trends in both variables at a monthly, seasonal, and annual resolution. Dawood (2017) presented a Spatio-statistical analysis of temperature fluctuations, this analysis deals with the variation in temperature such that positive tends with mean maximum temperature and negative tends with mean minimum temperature and slope magnitude. Amin et al. (2018) dealt with the analysis of historical temperature (1996–2015) and projected (2030–2060) climate in Pakistan, presented the possible variations for both minimum and maximum temperature. Khan et al. (2019) presented the analysis of both minimum and maximum temperature trends and the significant increase in a heat wave. This analysis shows that the intense heat wave occurred in southwest Pakistan. Abid et al. (2019) presented the Farmers perception of climate change, observed trends, and adaptation of agriculture in Pakistan. This analysis deals with the perception of increasing mean temperature with locally recorded data. Tariq et al. (2020) presented the analysis of seasonal land surface temperature and land use land cover change using optical multi-temporal satellite data of Faisalabad, Pakistan. Saleem et al. (2021) presented the annual and seasonal trends of extreme temperature and pacific variability during 1980-2019. Rafiq et al. (2022) presented the analysis of the variability of mean monthly, seasonal and annual temperature of Baluchistan province, Pakistan.

These analyses are done by using classical statistics. More information on analysis can be seen in Iqbal and Quamar (2011).

Classical statistics deals with determinate and exact data, crisp arguments charts, diagrams, probability distributions, algorithms, functions, parametric and non-parametric whereas; neutrosophic statistics is an advanced form of classical statistics that deal with indeterminacy, uncertainty, unclear and incomplete form of data, and also a generalization of interval statistics, see Smarandache (2014). According to Smarandache (2022) "Neutrosophic Statistics is an extension of the Interval Statistics, since it may deal with all types of indeterminacies (with respect to the data, inferential procedures, probability distributions, graphical representations, etc.), it allows the reduction of indeterminacy, and it uses the neutrosophic probability that is more general than imprecise and classical probabilities, and has more detailed corresponding probability density functions. While Interval Statistics only deals with indeterminacy that can be represented by intervals. Not all indeterminacies (uncertainties) may be represented by intervals. Also, in some applications, we should better use hesitant sets (that have less indeterminacy) instead of intervals. Neutrosophic statistics is a generalization of interval statistics, because of, among others, while interval statistics is based on interval analysis, neutrosophic statistics is based on set Analysis (meaning all kinds of sets, not only intervals)" To deal with neutrosophic data or data in intervals the various applications can be viewed in Broumi and Smarandache (2014) presented the neutrosophic set of new cosine similarities between two intervals. Aslam and Khan (2021) worked on the normality test of temperature in Jeddah city using Cochran's test under indeterminacy. Afzal et al. (2021) presented the analysis of resistance depending on the temperature variance of conducting material under the neutrosophic statistical analysis. Further, Janjua et al. (2022) worked on the climate variability and wheat crop under a neutrosophic environment. Afzal et al. (2022) presented the work on the fabrication of temperature flexibility on robot skin.

In this paper, we will apply neutrosophic statistics to the temperature data collected from different cities in Pakistan. We will present the neutrosophic statistical analysis of the temperature data of Gujranwala, Lahore, Karachi, Islamabad and Sialkot enumerated by Pakistan Meteorological Department. We will compare the result of classical statistics with the result of neutrosophic statistics using the temperature data of different cities in Pakistan.

2. Methodology

Suppose that $X_N = X_L + X_U I_N; I_N \in [I_L, I_U]$ be a neutrosophic random variable which represents the temperature of different cities of Pakistan, where X_L is the lower temperature and $X_U I_N$ is the upper temperature and $I_N \in [I_L, I_U]$ be the interval of indeterminacy. By following, Chen et al. (2017), Chen et al. (2017) and Aslam (2019), the neutrosophic average of temperature data $X_N \in [X_L, X_U]$ can be

calculated as $\bar{X}_N = \bar{X}_L + \bar{X}_U I_N; I_N \in [I_L, I_U]$, where $\bar{X}_L = \frac{1}{n_N} \sum_{i=1}^{n_N} X_{iL}$, $\bar{X}_U = \frac{1}{n_N} \sum_{i=1}^{n_N} X_{iU}$ and $n_N \in [n_L, n_U]$ be a neutrosophic sample. The neutrosophic standard deviation can be computed as follows

$$\sum_{i=1}^{n_N} (X_i - \bar{X}_{iN})^2 = \sum_{i=1}^{n_N} \left[\begin{array}{l} \min \left((a_i + b_i I_L)(\bar{a} + \bar{b} I_L), (a_i + b_i I_L)(\bar{a} + \bar{b} I_U) \right) \\ (a_i + b_i I_U)(\bar{a} + \bar{b} I_L), (a_i + b_i I_U)(\bar{a} + \bar{b} I_L) \\ \max \left((a_i + b_i I_L)(\bar{a} + \bar{b} I_L), (a_i + b_i I_L)(\bar{a} + \bar{b} I_U) \right) \\ (a_i + b_i I_U)(\bar{a} + \bar{b} I_L), (a_i + b_i I_U)(\bar{a} + \bar{b} I_U) \end{array} \right], I \in [I_L, I_U] \quad (1)$$

Note that $a_i = X_L$ and $b_i = X_U$. We will use the symbols a_i and b_i to present the lower and upper values, respectively throughout the paper. The neutrosophic sample variance can be computed by;

$$S_N^2 = \frac{\sum_{i=1}^{n_N} (X_i - \bar{X}_{iN})^2}{n_N}; S_N^2 \in [S^2_L, S^2_U] \quad (2)$$

The neutrosophic form of $S_N^2 \in [S^2_L, S^2_U]$ can be written as

$$a_S + b_S I_{NS}; I_{NS} \in [I_{LS}, I_{US}] \quad (3)$$

The neutrosophic coefficient of variation (CV_N) can be applied to see the consistency of the temperature in the different cities of Pakistan. A city having a smaller value of CV_N means more consistent than the other city in temperature. The CV_N can be computed by;

$$CV_N = \frac{\sqrt{S_N^2}}{\bar{X}_N} \times 100; CV_N \in [CV_L, CV_U] \quad (4)$$

The neutrosophic form of CV_N is

$$a_V + b_V I_{NV}; I_{NV} \in [I_{LV}, I_{UV}] \quad (5)$$

3. Data collection

We used temperature data of different big cities of Pakistan like Gujranwala, Lahore, Karachi, Islamabad and Sialkot. Our aim is to investigate which city on average has the higher temperature and which city temperature is more consistent. We used daily data of temperature for the month of July 2022 from <https://www.gismeteo.com/>. The data is reported in Table 1. Table 1 presents low and high values of the temperature data. The temperature data given in the interval cannot be analysed

using classical statistics. The interval data can be analyzed using neutrosophic statistics. The neutrosophic statistical analysis for the temperature data is shown in Section 4.

Table 1: The temperature data (in C°) of different cities in Pakistan

Day	Date	Gujranwala		Lahore		Karachi		Islamabad		Sialkot	
		Low	High	Low	High	Low	High	Low	High	Low	High
Monday	4	29	40	29	40	28	36	28	35	28	37
Tuesday	5	34	43	36	43	29	34	31	39	31	41
Wednesday	6	29	39	31	38	28	31	26	36	30	35
Thursday	7	28	36	29	39	28	33	27	35	28	36
Friday	8	31	41	31	41	29	32	29	37	31	41
Saturday	9	30	35	29	34	29	33	28	34	29	35
Sunday	10	28	37	29	39	30	35	26	33	29	37
Monday	11	28	37	29	37	29	33	27	35	28	37
Tuesday	12	30	36	31	37	29	31	28	34	27	34
Wednesday	13	27	34	28	36	29	32	28	34	27	34
Thursday	14	28	37	27	38	28	31	27	33	26	36
Friday	15	27	36	26	35	29	32	27	35	27	36
Saturday	16	29	38	29	37	29	33	27	35	29	37
Sunday	17	29	38	30	40	29	36	27	32	28	38
Monday	18	31	40	32	40	28	32	26	37	31	40
Tuesday	19	33	41	33	41	28	31	30	37	31	40
Wednesday	20	33	40	34	41	28	30	29	37	29	36
Thursday	21	30	38	32	40	28	31	25	35	26	33
Friday	22	29	33	31	35	28	32	27	34	26	29
Saturday	23	28	34	30	36	28	32	26	34	25	30
Sunday	24	28	36	30	38	28	32	26	35	25	32
Monday	25	29	39	31	42	29	32	27	37	26	34
Tuesday	26	32	41	35	43	29	31	28	39	28	36
Wednesday	27	32	40	35	43	29	31	29	39	28	35
Thursday	28	32	39	35	42	29	30	29	38	28	35
Friday	29	29	36	31	39	28	30	28	34	26	31
Saturday	30	29	37	32	40	28	30	26	35	25	32
Sunday	31	29	37	32	40	28	30	26	34	25	32

4. Result and interpretation

We performed the neutrosophic statistical analysis using the temperature data. The neutrosophic mean of temperature is shown in Table 2. The neutrosophic standard deviation is shown in Table 3. The neutrosophic coefficient variation is shown in Table 4. From Table 2, the neutrosophic form of the average temperature of Gujranwala city is

$29.6 + 37.79I_N; I_N \in [0, 0.20]$. It means that the average temperature of Gujranwala city is between 29.68 to 37.79 and the measure of indeterminacy is 0.20, a neutrosophic form of average temperature of Lahore city is $30.97 + 39.08I_N; I_N \in [0, 0.20]$. It means that the average temperature of Lahore city is between 30.97 to 39.08 and the measure of indeterminacy is 0.20, a neutrosophic form of the average temperature of Karachi city is $28.52 + 32.00I_N; I_N \in [0, 0.10]$.

It means that the average temperature of Karachi city is between 28.52 to 32.00 and the measure of indeterminacy is 0.10, a neutrosophic form of the average temperature of Islamabad city is $27.41 + 35.41I_N; I_N \in [0, 0.26]$. It means that the average temperature of Islamabad city is between 27.41 to 35.41 and the measure of indeterminacy is 0.26 and, neutrosophic form of average temperature of Sialkot city is $27.75 + 35.31I_N; I_N \in [0, 0.20]$. It means that the average temperature of between 27.75 to 35.31 and the measure of indeterminacy is 0.20. On average, the temperature of Lahore city is higher than other cities as it has the maximum monthly average temperature as compared to Gujranwala city.

Table 2. The neutrosophic mean of the temperature of different cities of Pakistan.

Cities	\bar{X}_N	$a_{\bar{X}} + b_{\bar{X}}I_N; I_N \in [I_{L\bar{X}}, I_{U\bar{X}}]$
Gujranwala	[29.68, 37.79]	$29.6 + 37.79I_N; I_N \in [0, 0.20]$
Lahore	[30.97, 39.08]	$30.97 + 39.08I_N; I_N \in [0, 0.20]$
Karachi	[28.52, 32.00]	$28.52 + 32.00I_N; I_N \in [0, 0.10]$
Islamabad	[27.41, 35.41]	$27.41 + 35.41I_N; I_N \in [0, 0.26]$
Sialkot	[27.75, 35.31]	$27.75 + 35.31I_N; I_N \in [0, 0.20]$

Table 3. The neutrosophic standard deviation of the temperature of different cities of Pakistan.

Cities	S_N	$a_S + b_S I_N; I_N \in [I_{LS}, I_{US}]$
Gujranwala	[1.887, 2.439]	$1.887 + 2.439I_N; I_N \in [0, 0.23]$
Lahore	[2.487, 2.538]	$2.487 + 2.538I_N; I_N \in [0, 0.02]$
Karachi	[0.575, 1.678]	$0.575 + 1.678I_N; I_N \in [0, 0.66]$

Islamabad	[1.398,1.893]	$1.398 + 1.893I_N; I_N \in [0,0.26]$
Sialkot	[1.935,3.104]	$27.75 + 35.31I_N; I_N \in [0,0.38]$

Table 4. Neutrosophic coefficient of variation of temperature of different cities in Pakistan.

Cities	CV_N	$a_v + b_v I_{Nv}; I_{Nv} \in [I_{Lv}, I_{Uv}]$
Gujranwala	[6.36,6.45]	$6.36+6.45I_{Nv}; I_{Nv} \in [0,0.0149]$
Lahore	[8.03,6.49]	$8.03-6.49I_{Nv}; I_{Nv} \in [0,0.2365]$
Karachi	[2.02,5.24]	$2.02+5.24I_{Nv}; I_{Nv} \in [0,0.6155]$
Islamabad	[5.10,5.35]	$5.10+5.35I_{Nv}; I_{Nv} \in [0,0.0459]$
Sialkot	[6.97,8.79]	$6.97+8.79I_{Nv}; I_{Nv} \in [0,0.2068]$

From Table 4, the coefficient of variation of temperature of Gujranwala city is between 6.36 to 6.45, coefficient of variation of temperature of Lahore city is between 8.03 to 6.49, coefficient of variation of temperature of Karachi city is between 2.02 to 5.24, coefficient of variation of temperature of Islamabad city is between 5.10 to 5.35, coefficient of variation of temperature of Sialkot city is between 6.97 to 8.79. The measures of indeterminacy associated with the coefficient of variation are also shown in Table 4. Based on the analysis, it can be concluded that the values of the coefficient of variation of temperature in Karachi is minimum. Therefore, the temperature of Karachi city is more consistent than the other cities in Pakistan.

5. Comparative study

The neutrosophic statistical analysis is the generalization of the classical statistical analysis. The neutrosophic statistical analysis reduces to classical statistical analysis when no indeterminacy is found in the data or data is not recorded in the intervals. Note here that temperature data is always recorded in intervals and therefore adequately analysed by the neutrosophic statistics. We now compare the results obtained using neutrosophic statistics with the results of classical statistics. The neutrosophic form of the temperature of Karachi city is $CV_N = 2.02+5.24I_{Nv}; I_{Nv} \in [0,0.6155]$. The first value (determinate) 2.02 of this neutrosophic shows the analysis from the classical statistics while the second part $5.24I_{Nv}$ of the neutrosophic form shows the indeterminate part. From the analysis, it can be seen that the values CV_N ranges from 2.02% to 5.24% with the measure of indeterminacy or uncertainty at 0.6155. Note that when $I_{Nv}=0$, the neutrosophic statistical results reduce to the results under classical statistics. Based on the comparative study, it can be concluded that neutrosophic statistical results are more adequate, flexible and more informative than the classical statistics

5. Concluding remarks

In this paper, we applied neutrosophic statistical analysis to temperature data of different cities of Pakistan. We observed that neutrosophic analysis of temperature data provided the estimated results of temperature in intervals rather the result of temperature in exact values. Therefore, the neutrosophic result is more flexible than classical statistics result. The government of Pakistan should take serious steps to reduce global warming by planting more trees, especially in Lahore city. The neutrosophic statistical analysis can be applied to analyse the interval data more adequately than classical statistics.

6. Acknowledgements

The author is deeply thankful to the editor and reviewers for their valuable suggestions to improve the quality and presentation of the paper. We also thank Dr. Azhar Ali Janjua for his valuable suggestions to improve the quality and presentation of the paper.

References

1. Abid, M., Scheffran, J., Schneider, U. A., & Elahi, E. (2019). Farmer perceptions of climate change, observed trends and adaptation of agriculture in Pakistan. *Environmental management*, 63(1), 110-123.
2. Afzal, U., Afzal, F., Maryam, K., & Aslam, M. (2022). Fabrication of flexible temperature sensors to explore indeterministic data analysis for robots as an application of Internet of Things. *RSC Advances*, 12(27), 17138-17145.
3. Afzal, U., Afzal, F., Maryam, K., & Aslam, M. (2022). Fabrication of flexible temperature sensors to explore indeterministic data analysis for robots as an application of Internet of Things. *RSC Advances*, 12(27), 17138-17145.
4. Amin, A., Nasim, W., Fahad, S., Ali, S., Ahmad, S., Rasool, A., ... & Paz, J. O. (2018). Evaluation and analysis of temperature for historical (1996–2015) and projected (2030–2060) climates in Pakistan using SimCLIM climate model: Ensemble application. *Atmospheric Research*, 213, 422-436.
5. Aslam, M. (2019). A new method to analyze rock joint roughness coefficient based on neutrosophic statistics. *Measurement*, 146, 65-71.
6. Aslam, M., & Khan, N. (2021). Normality test of temperature in Jeddah city using Cochran's test under indeterminacy. *Mapan*, 36(3), 589-598.
7. Broumi, S., & Smarandache, F. (2014). Cosine similarity measure of interval valued neutrosophic sets. *Infinite Study*.
8. Florentin Smarandache, Neutrosophic Statistics is an extension of Interval Statistics, while Plithogenic Statistics is the most general form of statistics (second version), *International Journal of Neutrosophic Science*, Vol. 19, No. 1, (2022) : 148-165
9. Chen, J., Ye, J., Du, S., & Yong, R. (2017). Expressions of rock joint roughness coefficient using neutrosophic interval statistical numbers. *Symmetry*, 9(7), 123.
10. Eckstein, D., Hutfils, M. L., & Wings, M. (2018). Global climate risk index 2019. Who suffers most from extreme weather events, 36.
11. Iqbal, M. A., Penas, A., Cano-Ortiz, A., Kersebaum, K. C., Herrero, L., & del Río, S. (2016). Analysis of recent changes in maximum and minimum temperatures in Pakistan. *Atmospheric Research*, 168, 234-249.
12. Iqbal, M. J., & Quamar, J. (2011). Measuring temperature variability of five major cities of Pakistan. *Arabian Journal of Geosciences*, 4(3), 595-606.
13. Janjua, A. A., Aslam, M., & Ahmed, Z. (2022). Comparative Analysis of Climate Variability and Wheat Crop under Neutrosophic Environment. *MAPAN*, 37(1), 25-32.
14. Janjua, A. A., Aslam, M., & Sultana, N. (2020). Evaluating the relationship between climate variability and agricultural crops under indeterminacy. *Theoretical and Applied Climatology*, 142(3), 1641-1648.

15. Khan, N., Shahid, S., Ismail, T., Ahmed, K., & Nawaz, N. (2019). Trends in heat wave related indices in Pakistan. *Stochastic environmental research and risk assessment*, 33(1), 287-302.
16. Rafiq, M., Li, Y. C., Cheng, Y., Rahman, G., Ali, A., Iqbal, M., ... & Ullah, R. (2022). Spatio-Statistical Analysis of Temperature and Trend Detection in Baluchistan, Pakistan. *Ecological Questions*, 33(3), 1-21.
17. Saleem, F., Zeng, X., Hina, S., & Omer, A. (2021). Regional changes in extreme temperature records over Pakistan and their relation to Pacific variability. *Atmospheric Research*, 250, 105407.
18. Smarandache, F. (2022). Neutrosophic Statistics is an extension of Interval Statistics, while Plithogenic Statistics is the most general form of statistics, *International Journal of Neutrosophic Science*, (in press)
19. Tariq, A., & Shu, H. (2020). CA-Markov chain analysis of seasonal land surface temperature and land use land cover change using optical multi-temporal satellite data of Faisalabad, Pakistan. *Remote Sensing*, 12(20), 3402.

Received: Sep 17, 2022. Accepted: Dec 20, 2022