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Properties of SuperHyperGraph and Neutrosophic SuperHyperGraph

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Abstract. New setting is introduced to study dominating, resolving, coloring, Eulerian(Hamiltonian) neutrosophic path, n-Eulerian(Hamiltonian) neutrosophic path, zero forcing number, zero forcing neutrosophic-number, independent number, independent neutrosophic-number, clique number, clique neutrosophic-number, matching number, matching neutrosophic-number, girth, neutrosophic girth, 1-zero-forcing number, 1-zero-forcing neutrosophic-number, failed 1-zero-forcing number, failed 1-zero-forcing neutrosophic-number, global-offensive alliance, t-offensive alliance, t-defensive alliance, t-powerful alliance, and global-powerful alliance in SuperHyperGraph and Neutrosophic SuperHyperGraph. Some Classes of SuperHyperGraph and Neutrosophic SuperHyperGraph are cases of study. Some results are applied in family of SuperHyperGraph and Neutrosophic SuperHyperGraph.

Keywords: SuperHyperGraph; Neutrosophic SuperHyperGraph; Classes; Families

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1. Introduction

Fuzzy set in [11], neutrosophic set in [2], related definitions of other sets in [2, 8, 10], hypergraphs and new notions on them in [6], neutrosophic graphs in [3], studies on neutrosophic graphs in [1], relevant definitions of other graphs based on fuzzy graphs in [7], are proposed. Also, some studies and researches about neutrosophic graphs, are proposed as a book in [5].

2. SuperHyperGraph

Definition 2.1. (Smarandache in 2019 and 2020, [9]).
An ordered pair $(G \subseteq P(V), E \subseteq P(V))$ is called by **SuperHyperGraph** and it's denoted by SHG.

Definition 2.2. (Smarandache in 2019 and 2020, [9]).
An ordered pair $(G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is called by **n-SuperHyperGraph** and it's denoted by n-SHG.

Definition 2.3. (Dominating, Resolving and Coloring).
Assume SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$.

(a) : SuperHyper-dominating set and number are defined as follows.

- (i) : A SuperVertex X_n **SuperHyper-dominates** a SuperVertex Y_n if there's at least one SuperHyperEdge which have them.
- (ii) : A set S is called **SuperHyper-dominating set** if for every $Y_n \in G_n \setminus S$, there's at least one SuperVertex X_n which SuperHyper-dominates SuperVertex Y_n.
- (iii) : If $\mathcal{S}$ is set of all sets of SuperHyper-dominating sets, then

$$|X| = \min_{S \in \mathcal{S}} |\{\cup X_n | X_n \in S\}|$$

is called **optimal-SuperHyper-dominating number** and X is called **optimal-SuperHyper-dominating set.**

(b) : SuperHyper-resolving set and number are defined as follows.

- (i) : A SuperVertex x **SuperHyper-resolves** SuperVertices y, w if

$$d(x, y) \neq d(x, w).$$

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- (ii) : A set S is called **SuperHyper-resolving set** if for every $Y_n \in G_n \setminus S$, there's at least one SuperVertex X_n which SuperHyper-resolves SuperVertices Y_n, W_n.
- (iii) : If $\mathcal{S}$ is set of all sets of SuperHyper-resolving sets, then

$$|X| = \min_{S \in \mathcal{S}} |\{\cup X_n | X_n \in S\}|$$

is called **optimal-SuperHyper-resolving number** and X is called **optimal-SuperHyper-resolving set.**

(c) : SuperHyper-coloring set and number are defined as follows.

- (i) : A SuperVertex X_n **SuperHyper-colors** a SuperVertex Y_n differently with itself if there's at least one SuperHyperEdge which is incident to them.
- (ii) : A set S_n is called **SuperHyper-coloring set** if for every $Y_n \in G_n \setminus S_n$, there's at least one SuperVertex X_n which SuperHyper-colors SuperVertex Y_n.
- (iii) : If $\mathcal{S}_n$ is set of all sets of SuperHyper-coloring sets, then

$$|X| = \min_{S_n \in \mathcal{S}_n} |\{\cup X_n | X_n \in S_n\}|$$

is called **optimal-SuperHyper-coloring number** and X is called **optimal-SuperHyper-coloring set.**

Proposition 2.4. *Assume SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$. S *is maximum set of SuperVertices which form a SuperHyperEdge. Then optimal-SuperHyper-coloring set has as cardinality as* S *has.*

Proposition 2.5. *Assume SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$. *If optimal-SuperHyper-coloring number is* $|V|$, *then for every SuperVertex there's at least one SuperHyperEdge which contains has all members of* V.

Proposition 2.6. *Assume SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$. *If there's at least one SuperHyperEdge which has all members of* V, *then optimal-SuperHyper-coloring number is* $|V|$.

Proposition 2.7. *Assume SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$. *If optimal-SuperHyper-dominating number is* $|V|$, *then there's one member of* V, *is contained in, at least one SuperVertex which doesn't have incident to any SuperHyperEdge.*

Proposition 2.8. *Assume SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$. *Then optimal-SuperHyper-dominating number is* $< |V|$.

Proposition 2.9. *Assume SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$. *If optimal-SuperHyper-resolving number is* $|V|$, *then every given SuperVertex doesn't have incident to any SuperHyperEdge.*

Proposition 2.10. *Assume SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$. *Then optimal-SuperHyper-resolving number is* $< |V|$.

Proposition 2.11. *Assume SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$. *If optimal-SuperHyper-coloring number is* $|V|$, *then all SuperVertices which have incident to at least one SuperHyperEdge.*

Proposition 2.12. *Assume SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$. *Then optimal-SuperHyper-coloring number isn't* $< |V|$.

Proposition 2.13. *Assume SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$. *Then optimal-SuperHyper-dominating set has cardinality which is greater than* $n - 1$ *where* n *is the cardinality of the set* V.

Proposition 2.14. *Assume SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$. S *is maximum set of SuperVertices which form a SuperHyperEdge. Then* S *is optimal-SuperHyper-coloring set and* $|\{\cup X_n \mid X_n \in S\}|$ *is optimal-SuperHyper-coloring number.*

Proposition 2.15. *Assume SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$. *If* S *is SuperHyper-dominating set, then* D *contains* S *is SuperHyper-dominating set.*

Proposition 2.16. *Assume SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$. *If* S *is SuperHyper-resolving set, then* D *contains* S *is SuperHyper-resolving set.*

Proposition 2.17. *Assume SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$. *If* S *is SuperHyper-coloring set, then* D *contains* S *is SuperHyper-coloring set.*

Proposition 2.18. *Assume SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$. *Then* G_n *is SuperHyper-dominating set.*

Proposition 2.19. *Assume SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$.*Then* G_n *is SuperHyper-resolving set.*

Proposition 2.20. *Assume SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$. *Then* G_n *is SuperHyper-coloring set.*

Proposition 2.21. *Assume* $\mathcal{G}$ *is a family of SuperHyperGraph. Then* G_n *is SuperHyper-dominating set for all members of* $\mathcal{G}$, *simultaneously.*

Proposition 2.22. *Assume* $\mathcal{G}$ *is a family of SuperHyperGraph. Then* G_n *is SuperHyper-resolving set for all members of* $\mathcal{G}$, *simultaneously.*

Proposition 2.23. *Assume* $\mathcal{G}$ *is a family of SuperHyperGraph. Then* G_n *is SuperHyper-coloring set for all members of* $\mathcal{G}$, *simultaneously.*

Proposition 2.24. *Assume* $\mathcal{G}$ *is a family of SuperHyperGraph. Then* $G_n \backslash \{X_n\}$ *is SuperHyper-dominating set for all members of* $\mathcal{G}$, *simultaneously.*

Proposition 2.25. *Assume* $\mathcal{G}$ *is a family of SuperHyperGraph. Then* $G_n \backslash \{X_n\}$ *is SuperHyper-resolving set for all members of* $\mathcal{G}$, *simultaneously.*

Proposition 2.26. *Assume* $\mathcal{G}$ *is a family of SuperHyperGraph. Then* $G_n \setminus \{X_n\}$ *isn't SuperHyper-coloring set for all members of* $\mathcal{G}$, *simultaneously.*

Proposition 2.27. *Assume* $\mathcal{G}$ *is a family of SuperHyperGraph. Then union of SuperHyper-dominating sets from each member of* $\mathcal{G}$ *is SuperHyper-dominating set for all members of* $\mathcal{G}$, *simultaneously.*

Proposition 2.28. *Assume* $\mathcal{G}$ *is a family of SuperHyperGraph. Then union of SuperHyper-resolving sets from each member of* $\mathcal{G}$ *is SuperHyper-resolving set for all members of* $\mathcal{G}$, *simultaneously.*

Proposition 2.29. *Assume* $\mathcal{G}$ *is a family of SuperHyperGraph. Then union of SuperHyper-coloring sets from each member of* $\mathcal{G}$ *is SuperHyper-coloring set for all members of* $\mathcal{G}$, *simultaneously.*

Proposition 2.30. *Assume $\mathcal{G}$ is a family of SuperHyperGraph. For every given SuperVertex, there's one SuperHyperGraph such that the SuperVertex has another SuperVertex which are incident to a SuperHyperEdge. If for given SuperVertex, all SuperVertices have a common SuperHyperEdge in this way, then $G_n \setminus \{X_n\}$ is optimal-SuperHyper-dominating set for all members of $\mathcal{G}$, simultaneously.*

Proposition 2.31. *Assume $\mathcal{G}$ is a family of SuperHyperGraph. For every given SuperVertex, there's one SuperHyperGraph such that the SuperVertex has another SuperVertex which are incident to a SuperHyperEdge. If for given SuperVertex, all SuperVertices have a common SuperHyperEdge in this way, then $G_n \setminus \{X_n\}$ is optimal-SuperHyper-resolving set for all members of $\mathcal{G}$, simultaneously.*

Proposition 2.32. *Assume $\mathcal{G}$ is a family of SuperHyperGraph. For every given SuperVertex, there's one SuperHyperGraph such that the SuperVertex has another SuperVertex which are incident to a SuperHyperEdge. If for given SuperVertex, all SuperVertices have a common SuperHyperEdge in this way, then G_n is optimal-SuperHyper-coloring set for all members of $\mathcal{G}$, simultaneously.*

Proposition 2.33. *Let SHG be a SuperHyperGraph. An $(k-1)$-set from an k-set of twin SuperVertices is subset of a SuperHyper-resolving set.*

Corollary 2.34. *Let SHG be a SuperHyperGraph. The number of twin SuperVertices is $n-1$. Then SuperHyper-resolving number is $n-2$.*

Corollary 2.35. *Let SHG be SuperHyperGraph. The number of twin SuperVertices is $n-1$. Then SuperHyper-resolving number is $n-2$. Every $(n-2)$-set including twin SuperVertices is SuperHyper-resolving set.*

Proposition 2.36. *Let SHG be SuperHyperGraph such that it's complete. Then SuperHyper-resolving number is $n-1$. Every $(n-1)$-set is SuperHyper-resolving set.*

Proposition 2.37. *Let $\mathcal{G}$ be a family of SuperHyperGraphs with common super vertex set G_n. Then simultaneously SuperHyper-resolving number of $\mathcal{G}$ is $|V|-1$*

Proposition 2.38. *Let $\mathcal{G}$ be a family of SuperHyperGraphs with common SuperVertex set G_n. Then simultaneously SuperHyper-resolving number of $\mathcal{G}$ is greater than the maximum SuperHyper-resolving number of n-$SHG \in \mathcal{G}$.*

Proposition 2.39. *Let $\mathcal{G}$ be a family of SuperHyperGraphs with common SuperVertex set G_n. Then simultaneously SuperHyper-resolving number of $\mathcal{G}$ is greater than simultaneously SuperHyper-resolving number of $\mathcal{H} \subseteq \mathcal{G}$.*

Theorem 2.40. *Twin SuperVertices aren't SuperHyper-resolved in any given SuperHyperGraph.*

Proposition 2.41. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a SuperHyperGraph. If SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ is complete, then every couple of SuperVertices are twin SuperVertices.*

Theorem 2.42. *Let $\mathcal{G}$ be a family of SuperHyperGraphs $SHG = (G \subseteq P(V), E \subseteq P(V))$ with SuperVertex set G_n and n-$SHG \in \mathcal{G}$ is complete. Then simultaneously SuperHyper-resolving number is $|V| - 1$. Every $(n-1)$-set is simultaneously SuperHyper-resolving set for $\mathcal{G}$.*

Corollary 2.43. *Let $\mathcal{G}$ be a family of SuperHyperGraphs $SHG = (G \subseteq P(V), E \subseteq P(V))$ with SuperVertex set G_n and n-$SHG \in \mathcal{G}$ is complete. Then simultaneously SuperHyper-resolving number is $|V| - 1$. Every $(|V|-1)$-set is simultaneously SuperHyper-resolving set for $\mathcal{G}$.*

Theorem 2.44. *Let $\mathcal{G}$ be a family of SuperHyperGraphs $SHG = (G \subseteq P(V), E \subseteq P(V))$ with SuperVertex set G_n and for every given couple of SuperVertices, there's a n-$SHG \in \mathcal{G}$ such that in that, they're twin SuperVertices. Then simultaneously SuperHyper-resolving number is $|V| - 1$. Every $(|V|-1)$-set is simultaneously SuperHyper-resolving set for $\mathcal{G}$.*

Theorem 2.45. *Let $\mathcal{G}$ be a family of SuperHyperGraphs $SHG = (G \subseteq P(V), E \subseteq P(V))$ with SuperVertex set G_n. If $\mathcal{G}$ contains three SuperHyper-stars with different SuperHyper-centers, then simultaneously SuperHyper-resolving number is $|V| - 2$. Every $(|V|-2)$-set is simultaneously SuperHyper-resolving set for $\mathcal{G}$.*

Corollary 2.46. *Let $\mathcal{G}$ be a family of SuperHyperGraphs $SHG = (G \subseteq P(V), E \subseteq P(V))$ with SuperVertex set G_n. If $\mathcal{G}$ contains three SuperHyper-stars with different SuperHyper-centers, then simultaneously SuperHyper-resolving number is $|V| - 2$. Every $(|V|-2)$-set is simultaneously SuperHyper-resolving set for $\mathcal{G}$.*

Proposition 2.47. *Consider two antipodal SuperVertices X_n and Y_n in any given even SuperHyper-cycle. Let U_n and V_n be given SuperVertices. Then $d(X_n, U_n) \neq d(X_n, V_n)$ if and only if $d(Y_n, U_n) \neq d(Y_n, V_n)$.*

Proposition 2.48. *Consider two antipodal SuperVertices X_n and Y_n in any given even cycle. Let U_n and V_n be given SuperVertices. Then $d(X_n, U_n) = d(X_n, V_n)$ if and only if $d(Y_n, U_n) = d(Y_n, V_n)$.*

Proposition 2.49. *The set contains two antipodal SuperVertices, isn't SuperHyper-resolving set in any given even SuperHyper-cycle.*

Proposition 2.50. *Consider two antipodal SuperVertices X_n and Y_n in any given even SuperHyper-cycle. X_n SuperHyper-resolves a given couple of SuperVertices, Z_n and Z'_n, if and only if Y_n does.*

Proposition 2.51. *There are two antipodal SuperVertices aren't SuperHyper-resolved by other two antipodal SuperVertices in any given even SuperHyper-cycle.*

Proposition 2.52. *For any two antipodal SuperVertices in any given even SuperHyper-cycle, there are only two antipodal SuperVertices don't SuperHyper-resolve them.*

Proposition 2.53. *In any given even SuperHyper-cycle, for any SuperVertex, there's only one SuperVertex such that they're antipodal SuperVertices.*

Proposition 2.54. *Let SuperHyperGraphs $SHG = (G \subseteq P(V), E \subseteq P(V))$ be an even SuperHyper-cycle. Then every couple of SuperVertices are SuperHyper-resolving set if and only if they aren't antipodal SuperVertices.*

Corollary 2.55. *Let SuperHyperGraphs $SHG = (G \subseteq P(V), E \subseteq P(V))$ be an even SuperHyper-cycle. Then SuperHyper-resolving number is two.*

Corollary 2.56. *Let SuperHyperGraphs $SHG = (G \subseteq P(V), E \subseteq P(V))$ be an even SuperHyper-cycle. Then SuperHyper-resolving set contains couple of SuperVertices such that they aren't antipodal SuperVertices.*

Corollary 2.57. *Let $\mathcal{G}$ be a family SuperHyperGraphs $SHG = (G \subseteq P(V), E \subseteq P(V))$ be an odd SuperHyper-cycle with common SuperVertex set G_n. Then simultaneously SuperHyper-resolving set contains couple of SuperVertices such that they aren't antipodal SuperVertices and SuperHyper-resolving number is two.*

Proposition 2.58. *In any given SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ which is odd SuperHyper-cycle, for any SuperVertex, there's no SuperVertex such that they're antipodal SuperVertices.*

Proposition 2.59. *Let SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ be an odd SuperHyper-cycle. Then every couple of SuperVertices are SuperHyper-resolving set.*

Proposition 2.60. *Let SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ be an odd cycle. Then SuperHyper-resolving number is two.*

Corollary 2.61. *Let SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ be an odd cycle. Then SuperHyper-resolving set contains couple of SuperVertices.*

Corollary 2.62. *Let $\mathcal{G}$ be a family of SuperHyperGraphs $SHG = (G \subseteq P(V), E \subseteq P(V))$ which are odd SuperHyper-cycles with common SuperVertex set G_n. Then simultaneously SuperHyper-resolving set contains couple of SuperVertices and SuperHyper-resolving number is two.*

Proposition 2.63. *Let SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a SuperHyper-path. Then every SuperHyper-leaf forms SuperHyper-resolving set.*

Proposition 2.64. *Let SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a SuperHyper-path. Then a set including every couple of SuperVertices is SuperHyper-resolving set.*

Proposition 2.65. *Let SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a SuperHyper-path. Then an 1-set contains leaf is SuperHyper-resolving set and SuperHyper-resolving number is one.*

Corollary 2.66. *Let $\mathcal{G}$ be a family of SuperHyperGraphs $SHG = (G \subseteq P(V), E \subseteq P(V))$ are SuperHyper-paths with common SuperVertex set G_n such that they've a common SuperHyper-leaf. Then simultaneously SuperHyper-resolving number is 1, 1-set contains common leaf, is simultaneously SuperHyper-resolving set for $\mathcal{G}$.*

Proposition 2.67. *Let $\mathcal{G}$ be a family of SuperHyperGraphs $SHG = (G \subseteq P(V), E \subseteq P(V))$ are SuperHyper-paths with common SuperVertex set G_n such that for every SuperHyper-leaf L_n from n-SHG, there's another n-$SHG \in \mathcal{G}$ such that L_n isn't SuperHyper-leaf. Then an 2-set contains every couple of SuperVertices, is SuperHyper-resolving set. An 2-set contains every couple of SuperVertices, is optimal-SuperHyper-resolving set. Optimal-SuperHyper-resolving number is two.*

Corollary 2.68. *Let $\mathcal{G}$ be a family of SuperHyperGraphs $SHG = (G \subseteq P(V), E \subseteq P(V))$ are SuperHyper-paths with common SuperVertex set G_n such that they've no common SuperHyper-leaf. Then an 2-set is simultaneously optimal-SuperHyper-resolving set and simultaneously optimal-SuperHyper-resolving number is 2.*

Proposition 2.69. *Let SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a SuperHyper-t-partite. Then every set excluding couple of SuperVertices in different parts whose cardinalities of them are strictly greater than one, is optimal-SuperHyper-resolving set.*

Corollary 2.70. *Let SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a SuperHyper-t-partite. Let $|V| \geq 3$. Then every $(|V| - 2)$-set excludes two SuperVertices from different parts whose cardinalities of them are strictly greater than one, is optimal-SuperHyper-resolving set and optimal-SuperHyper-resolving number is $|V| - 2$.*

Corollary 2.71. *Let SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a SuperHyper-bipartite. Let* $|V| \geq 3$. *Then every* $(|V|-2)$*-set excludes two SuperVertices from different parts, is optimal-SuperHyper-resolving set and optimal-SuperHyper-resolving number is* $|V| - 2$.

Corollary 2.72. *Let SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a SuperHyper-star. Then every* $(|V|-2)$*-set excludes SuperHyper-center and a given SuperVertex, is optimal-SuperHyper-resolving set and optimal-SuperHyper-resolving number is* $(|V| - 2)$.

Corollary 2.73. *Let SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a SuperHyper-wheel. Let* $|V| \geq 3$. *Then every* $(|V| - 2)$*-set excludes SuperHyper-center and a given SuperVertex, is optimal-SuperHyper-resolving set and optimal-SuperHyper-resolving number is* $|V| - 2$.

Corollary 2.74. *Let* $\mathcal{G}$ *be a family of SuperHyperGraphs* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *which are SuperHyper-t-partite with common SuperVertex set* G_n. *Let* $|V| \geq 3$. *Then simultaneously optimal-SuperHyper-resolving number is* $|V| - 2$ *and every* $(|V| - 2)$*-set excludes two SuperVertices from different parts, is simultaneously optimal-SuperHyper-resolving set for* $\mathcal{G}$.

Corollary 2.75. *Let* $\mathcal{G}$ *be a family of SuperHyperGraphs* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *which are SuperHyper-bipartite with common SuperVertex set* G_n. *Let* $|V| \geq 3$. *Then simultaneously optimal-SuperHyper-resolving number is* $|V| - 2$ *and every* $(|V| - 2)$*-set excludes two SuperVertices from different parts, is simultaneously optimal-SuperHyper-resolving set for* $\mathcal{G}$.

Corollary 2.76. *Let* $\mathcal{G}$ *be a family of SuperHyperGraphs* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *which are SuperHyper-star with common SuperVertex set* G_n. *Let* $|V| \geq 3$. *Then simultaneously optimal-SuperHyper-resolving number is* $|V| - 2$ *and every* $(|V| - 2)$*-set excludes SuperHyper-center and a given SuperVertex, is simultaneously optimal-SuperHyper-resolving set for* $\mathcal{G}$.

Corollary 2.77. *Let* $\mathcal{G}$ *be a family of SuperHyperGraphs* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *which are SuperHyper-wheel with common SuperVertex set* G_n. *Let* $|V| \geq 3$. *Then simultaneously optimal-SuperHyper-resolving number is* $|V| - 2$ *and every* $(|V| - 2)$*-set excludes SuperHyper-center and a given SuperVertex, is simultaneously optimal-SuperHyper-resolving set for* $\mathcal{G}$.

Proposition 2.78. *Let SuperHyperGraphs* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a SuperHyper-complete. Then optimal-SuperHyper-coloring number is* $|V|$.

Proposition 2.79. *Let SuperHyperGraphs* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a SuperHyper-path. Then optimal-SuperHyper-coloring number is two.*

Proposition 2.80. *Let SuperHyperGraphs* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be an even SuperHyper-cycle. Then optimal-SuperHyper-coloring number is two.*

Proposition 2.81. *Let SuperHyperGraphs* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be an odd SuperHyper-cycle. Then optimal-SuperHyper-coloring number is three.*

Proposition 2.82. *Let SuperHyperGraphs* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a SuperHyper-star. Then optimal-SuperHyper-coloring number is two.*

Proposition 2.83. *Let SuperHyperGraphs* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a SuperHyper-wheel such that it has even SuperHyper-cycle. Then optimal-SuperHyper-coloring number is Three.*

Proposition 2.84. *Let SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a SuperHyper-wheel such that it has odd SuperHyper-cycle. Then optimal-SuperHyper-coloring number is four.*

Proposition 2.85. *Let SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a SuperHyper-complete and SuperHyper-bipartite. Then optimal-SuperHyper-coloring number is two.*

Proposition 2.86. *Let SuperHyperGraph* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a SuperHyper-complete and SuperHyper-t-partite. Then optimal-SuperHyper-coloring number is* t.

Proposition 2.87. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be SuperHyperGraph. Then optimal-SuperHyper-coloring number is* 1 *if and only if* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is SuperHyper-empty.*

Proposition 2.88. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be SuperHyperGraph. Then optimal-SuperHyper-coloring number is* 2 *if and only if* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is both SuperHyper-complete and SuperHyper-bipartite.*

Proposition 2.89. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be SuperHyperGraph. Then optimal-SuperHyper-coloring number is* $|V|$ *if and only if* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is SuperHyper-complete.*

Proposition 2.90. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be SuperHyperGraph. Then optimal-SuperHyper-coloring number is obtained from the number of SuperVertices which is* $|G_n|$ *and optimal-SuperHyper-coloring number is at most* $|V|$.

Proposition 2.91. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be SuperHyperGraph. Then optimal-SuperHyper-coloring number is at most* $\Delta + 1$ *and at least* 2.

Proposition 2.92. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be SuperHyperGraph and SuperHyper-r-regular. Then optimal-SuperHyper-coloring number is at most* $r + 1$.

Definition 2.93. (Eulerian(Hamiltonian) Neutrosophic Path).
Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a neutrosophic SuperHyperGraph. Then

(i) **Eulerian(Hamiltonian) neutrosophic path** $\mathcal{M}_e(SHG)(\mathcal{M}_h(SHG))$ for a neutrosophic SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ is a sequence of consecutive edges(vertices) $x_1, x_2, \cdots, x_{\mathcal{S}(SHG)}(x_1, x_2, \cdots, x_{\mathcal{O}(SHG)})$ which is neutrosophic path;

(ii) **n-Eulerian(Hamiltonian) neutrosophic path** $\mathcal{N}_e(SHG)(\mathcal{N}_h(SHG))$ for a neutrosophic SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ is the number of sequences of consecutive edges(vertices) $x_1, x_2, \cdots, x_{\mathcal{S}(SHG)}(x_1, x_2, \cdots, x_{\mathcal{O}(SHG)})$ which is neutrosophic path.

Proposition 2.94. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a complete-neutrosophic SuperHyperGraph with two weakest edges. Then*

$$\mathcal{M}_e(CMT_\sigma) : \textit{Not Existed;}$$

$$\mathcal{M}_h(CMT_\sigma) : v_{\tau(1)}, v_{\tau(2)}, \cdots, v_{\tau(\mathcal{O}(CMT_\sigma)-1)}, v_{\tau(\mathcal{O}(CMT_\sigma))}$$

where τ is a permutation on $\mathcal{O}(CMT_\sigma)$.

$$\mathcal{N}_e(CMT_\sigma) = 0;$$

$$\mathcal{N}_h(CMT_\sigma) = \mathcal{O}(CMT_\sigma)!.$$

Proposition 2.95. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a path-neutrosophic SuperHyperGraph. Then*

$$\mathcal{M}_e(PTH) : v_1, v_2, \cdots, v_{\mathcal{S}(PTH)};$$

$$\mathcal{M}_h(PTH) : v_1, v_2, \cdots, v_{\mathcal{O}(PTH)}.$$

$$\mathcal{N}_e(PTH) = 1;$$

$$\mathcal{N}_h(PTH) = 1.$$

Proposition 2.96. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a cycle-neutrosophic SuperHyperGraph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{M}_e(CYC) : \textit{Not Existed};$$

$$\mathcal{M}_h(CYC) : x_i, x_{i+1}, \cdots, x_{\mathcal{O}(CYC)-1}, x_{\mathcal{O}(CYC)}, \cdots, x_{i-1}.$$

$$\mathcal{N}_e(CYC) = 0;$$

$$\mathcal{N}_h(CYC) = \mathcal{O}(CYC).$$

Proposition 2.97. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a star-neutrosophic SuperHyperGraph with center c. Then*

$$\mathcal{M}_e(STR_{1,\sigma_2}) : v_1, v_2$$

$$\mathcal{M}_h(STR_{1,\sigma_2}) : v_1, c, v_2$$

where $\mathcal{O}(STR_{1,\sigma_2}) \leq 2$;

$$\mathcal{M}_e(STR_{1,\sigma_2}) : \textit{Not Existed}$$

$$\mathcal{M}_h(STR_{1,\sigma_2}) : \textit{Not Existed}$$

where $\mathcal{O}(STR_{1,\sigma_2}) \geq 3$.

$$\mathcal{N}_e(STR_{1,\sigma_2}) = 2$$

$$\mathcal{N}_h(STR_{1,\sigma_2}) = 3$$

where $\mathcal{O}(STR_{1,\sigma_2}) \leq 2$;

$$\mathcal{N}_e(STR_{1,\sigma_2}) = 0$$

$$\mathcal{N}_h(STR_{1,\sigma_2}) = 0$$

where $\mathcal{O}(STR_{1,\sigma_2}) \geq 3$.

Proposition 2.98. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a complete-bipartite-neutrosophic SuperHyperGraph. Then*

$$\mathcal{M}_e(CMC_{\sigma_1,\sigma_2}) : \textit{Not Existed}$$

$$\mathcal{M}_h(CMC_{\sigma_1,\sigma_2}) : v_1, v_2, \cdots, v_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})-1}, v_{\mathcal{O}(CMC_{\sigma_1,\sigma_2})}$$

where $\mathcal{O}(CMC_{\sigma_1,\sigma_2}) \geq 3,\ |V_1| = |V_2|,\ v_{2i+1} \in V_1,\ v_{2i} \in V_2$;

$$\mathcal{M}_e(CMC_{\sigma_1,\sigma_2}) : v_1 v_2$$

$$\mathcal{M}_h(CMC_{\sigma_1,\sigma_2}) : v_1, v_2$$

where $\mathcal{O}(CMC_{\sigma_1,\sigma_2}) = 2$;

$$\mathcal{M}_e(CMC_{\sigma_1,\sigma_2}) : -$$

$$\mathcal{M}_h(CMC_{\sigma_1,\sigma_2}) : v_1$$

where $\mathcal{O}(CMC_{\sigma_1,\sigma_2}) = 1$.

$$\mathcal{N}_e(CMC_{\sigma_1,\sigma_2}) = 0$$

$$\mathcal{N}_h(CMC_{\sigma_1,\sigma_2}) = c$$

where $\mathcal{O}(CMC_{\sigma_1,\sigma_2}) \geq 3,\ |V_1| = |V_2|,\ v_{2i+1} \in V_1,\ v_{2i} \in V_2$;

$$\mathcal{N}_e(CMC_{\sigma_1,\sigma_2}) = 2$$

$$\mathcal{N}_h(CMC_{\sigma_1,\sigma_2}) = 2$$

where $\mathcal{O}(CMC_{\sigma_1,\sigma_2}) = 2$;

$$\mathcal{N}_e(CMC_{\sigma_1,\sigma_2}) = -$$

$$\mathcal{N}_h(CMC_{\sigma_1,\sigma_2}) = 1$$

where $\mathcal{O}(CMC_{\sigma_1,\sigma_2}) = 1$.

Proposition 2.99. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a complete-t-partite-neutrosophic SuperHyperGraph. Then*

$$\mathcal{M}_e(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) : \textit{Not Existed}$$

$$\mathcal{M}_h(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) : v_1, v_2, \cdots, v_{\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t})-1}, v_{\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t})}$$

where $\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) \geq 3,\ |V_i| = |V_j|,\ v_{2i+1} \in V_i,\ v_{2i} \in V_j;$

$$\mathcal{M}_e(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) : v_1v_2$$

$$\mathcal{M}_h(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) : v_1, v_2$$

where $\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2;$

$$\mathcal{M}_e(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) : -$$

$$\mathcal{M}_h(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) : v_1$$

where $\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 1$.

$$\mathcal{N}_e(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 0$$

$$\mathcal{N}_h(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = c$$

where $\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) \geq 3,\ |V_i| = |V_j|,\ v_{2i+1} \in V_i,\ v_{2i} \in V_j;$

$$\mathcal{N}_e(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$$

$$\mathcal{N}_h(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2$$

where $\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 2;$

$$\mathcal{N}_e(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = -$$

$$\mathcal{N}_h(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 1$$

where $\mathcal{O}(CMC_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = 1$.

Proposition 2.100. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a wheel-neutrosophic SuperHyperGraph. Then*

$$\mathcal{M}_h(WHL_{1,\sigma_2}) : x_i, x_{i+1}, \cdots, x_{\mathcal{O}(WHL_{1,\sigma_2})-1}, x_{\mathcal{O}(WHL_{1,\sigma_2})}, x_{i-1}.$$

$$\mathcal{M}_e(WHL_{1,\sigma_2}) : v_1, v_2, v_3$$

where $\mathcal{S}(WHL_{1,\sigma_2}) = 3$.

$$\mathcal{M}_h(WHL_{1,\sigma_2}) : x_i, x_{i+1}, \cdots, x_{\mathcal{O}(WHL_{1,\sigma_2})-1}, x_{\mathcal{O}(WHL_{1,\sigma_2})}, x_{i-1}.$$

$$\mathcal{M}_e(WHL_{1,\sigma_2}) : \textit{Not Existed}$$

where $\mathcal{S}(WHL_{1,\sigma_2}) > 3$.

$$\mathcal{N}_h(WHL_{1,\sigma_2}) = \mathcal{O}(WHL_{1,\sigma_2});$$

$$\mathcal{N}_e(WHL_{1,\sigma_2}) = 3;$$

where $\mathcal{S}(WHL_{1,\sigma_2}) = 3$.

$$\mathcal{N}_h(WHL_{1,\sigma_2}) = \mathcal{O}(WHL_{1,\sigma_2});$$

$$\mathcal{N}_e(WHL_{1,\sigma_2}) = 0;$$

where $\mathcal{S}(WHL_{1,\sigma_2}) > 3$.

3. Neutrosophic SuperHyperGraph

Definition 3.1. (Zero Forcing Number).
Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a neutrosophic SuperHyperGraph. Then

(i) **zero forcing number** $\mathcal{Z}(SHG)$ for a neutrosophic SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ is minimum cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex;

(ii) **zero forcing neutrosophic-number** $\mathcal{Z}_n(SHG)$ for a neutrosophic SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ is minimum neutrosophic cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

Definition 3.2. (Independent Number).
Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a neutrosophic SuperHyperGraph. Then

(i) **independent number** $\mathcal{I}(SHG)$ for a neutrosophic SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ is maximum cardinality of a set S of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously;

(ii) **independent neutrosophic-number** $\mathcal{I}_n(SHG)$ for a neutrosophic SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S aren't endpoints for an edge, simultaneously.

Definition 3.3. (Clique Number).
Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a neutrosophic SuperHyperGraph. Then

(i) **clique number** $\mathcal{C}(SHG)$ for a neutrosophic SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ is maximum cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously;

(ii) **clique neutrosophic-number** $\mathcal{C}_n(SHG)$ for a neutrosophic SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ is maximum neutrosophic cardinality of a set S of vertices such that every two vertices of S are endpoints for an edge, simultaneously.

Definition 3.4. (Matching Number).
Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a neutrosophic SuperHyperGraph. Then

(i) **matching number** $\mathcal{M}(SHG)$ for a neutrosophic SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ is maximum cardinality of a set S of edges such that every two edges of S don't have any vertex in common;

(ii) **matching neutrosophic-number** $\mathcal{M}_n(SHG)$ for a neutrosophic SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ is maximum neutrosophic cardinality of a set S of edges such that every two edges of S don't have any vertex in common.

Definition 3.5. (Girth and Neutrosophic Girth).
Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a neutrosophic SuperHyperGraph. Then

(i) **girth** $\mathcal{G}(SHG)$ for a neutrosophic SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ is minimum crisp cardinality of vertices forming shortest cycle. If there isn't, then girth is ∞;

(ii) **neutrosophic girth** $\mathcal{G}_n(SHG)$ for a neutrosophic SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ is minimum neutrosophic cardinality of vertices forming shortest cycle. If there isn't, then girth is ∞.

Proposition 3.6. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a complete-neutrosophic SuperHyperGraph. Then*

(1)
$$\mathcal{Z}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1.$$

(2)
$$\mathcal{I}(SHG) = 1.$$

(3)
$$\mathcal{C}(SHG) = \mathcal{O}(SHG).$$

(4)
$$\mathcal{M}(SHG) = \lfloor \frac{n}{2} \rfloor.$$

(5)
$$\mathcal{G}(SHG) = 3.$$

Proposition 3.7. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a path-neutrosophic SuperHyperGraph. Then*

(1)
$$\mathcal{Z}(PTH_n) = 1.$$
(2)
$$\mathcal{I}(SHG) = \lceil \frac{\mathcal{O}(SHG)}{2} \rceil.$$
(3)
$$\mathcal{C}(SHG) = 2.$$
(4)
$$\mathcal{M}(SHG) = \lfloor \frac{n}{2} \rfloor.$$
(5)
$$\mathcal{G}(SHG) = \infty.$$

Proposition 3.8. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a cycle-neutrosophic SuperHyperGraph where $\mathcal{O}(CYC) \geq 3$. Then*

(1)
$$\mathcal{Z}(CYC_n) = 2.$$
(2)
$$\mathcal{I}(SHG) = \lfloor \frac{\mathcal{O}(SHG)}{2} \rfloor.$$
(3)
$$\mathcal{C}(SHG) = 2.$$
(4)
$$\mathcal{M}(SHG) = \lfloor \frac{n}{2} \rfloor.$$
(5)
$$\mathcal{G}(SHG) = \mathcal{O}(SHG).$$

Proposition 3.9. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a star-neutrosophic SuperHyperGraph with center c. Then*

(1)
$$\mathcal{Z}(STR_{1,\sigma_2}) = \mathcal{O}(STR_{1,\sigma_2}) - 2.$$
(2)
$$\mathcal{I}(SHG) = \mathcal{O}(SHG) - 1.$$
(3)
$$\mathcal{C}(SHG) = 2.$$
(4)
$$\mathcal{M}(SHG) = 1.$$

(5)

$$\mathcal{G}(SHG) = \infty.$$

Proposition 3.10. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a complete-bipartite-neutrosophic SuperHyperGraph. Then*

(1)

$$\mathcal{Z}(CMT_{\sigma_1,\sigma_2}) = \mathcal{O}(CMT_{\sigma_1,\sigma_2}) - 2.$$

(2)

$$\mathcal{I}(SHG) = \max\{|V_1|, |V_2|\}.$$

(3)

$$\mathcal{C}(SHG) = 2.$$

(4)

$$\mathcal{M}(SHG) = \min\{|V_1|, |V_2|\}.$$

(5)

$$\mathcal{G}(SHG) = 4$$

where $\mathcal{O}(SHG) \geq 4$. *And*

$$\mathcal{G}(SHG) = \infty$$

where $\mathcal{O}(SHG) \leq 3$.

Proposition 3.11. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a complete-t-partite-neutrosophic SuperHyperGraph. Then*

(1)

$$\mathcal{Z}(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - 1.$$

(2)

$$\mathcal{I}(SHG) = \max\{|V_1|, |V_2|, \cdots, |V_t|\}.$$

(3)

$$\mathcal{C}(SHG) = t.$$

(4)

$$\mathcal{M}(SHG) = \min |V_i|_{i=1}^{t}.$$

(5)

$$\mathcal{G}(SHG) = 3$$

where $t \geq 3$.

$$\mathcal{G}(SHG) = 4$$

where $t \leq 2$. *And*

$$\mathcal{G}(SHG) = \infty$$

where $\mathcal{O}(SHG) \leq 2$.

Proposition 3.12. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a complete-neutrosophic SuperHyperGraph. Then*

(1)
$$\mathcal{Z}_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) - \max\{\Sigma_{i=1}^{3}\sigma_i(x)\}_{x\in V}.$$

(2)
$$\mathcal{I}_n(SHG) = \max\{\sum_{i=1}^{3}\sigma_i(x)\}_{x\in V}.$$

(3)
$$\mathcal{C}_n(SHG) = \mathcal{O}_n(SHG).$$

(4)
$$\mathcal{M}_n(SHG) = \max\{\sum_{i=1}^{3}\mu_i(x_0x_1) + \sum_{i=1}^{3}\mu_i(x_1x_2) + \cdots + \sum_{i=1}^{3}\mu_i(x_{j-1}x_j)\}_{j=\lfloor \frac{n}{2}\rfloor}.$$

(5)
$$\mathcal{G}_n(SHG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x) + \sigma_i(y) + \sigma_i(z))\}.$$

Proposition 3.13. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a path-neutrosophic SuperHyperGraph. Then*

(1)
$$\mathcal{Z}_n(PTH_n) = \min\{\Sigma_{i=1}^{3}\sigma_i(x)\}_{x \text{ is a leaf}}.$$

(2)
$$\mathcal{I}_n(SHG) = \max\{\sum_{i=1}^{3}(\sigma_i(x_1) + \sigma_i(x_3) + \cdots + \sigma_i(x_t)),$$
$$\sum_{i=1}^{3}\sigma_i(x_2) + \sigma_i(x_4) + \cdots + \sigma_i(x'_t))\}_{x_ix_{i+1}\in E}.$$

(3)
$$\mathcal{C}_n(SHG) = \max\{\sum_{i=1}^{3}(\sigma_i(x_j) + \sigma_i(x_{j+1}))\}_{x_jx_{j+1}\in E}.$$

(4)
$$\mathcal{M}_n(SHG) = \max\{\sum_{i=1}^{3}\mu_i(x_0x_1) + \sum_{i=1}^{3}\mu_i(x_2x_3) + \cdots + \sum_{i=1}^{3}\mu_i(x_{j-1}x_j)\}_{|S|=\lfloor \frac{n}{2}\rfloor}.$$

(5)
$$\mathcal{G}_n(SHG) = \infty.$$

Proposition 3.14. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a cycle-neutrosophic SuperHyperGraph where* $\mathcal{O}(CYC) \geq 3$. *Then*

(1)

$$\mathcal{Z}_n(CYC_n) = \min\{\Sigma_{i=1}^3 \sigma_i(x) + \Sigma_{i=1}^3 \sigma_i(y)\}_{xy \in E.}.$$

(2)

$$\mathcal{I}_n(SHG) = \max\{\sum_{i=1}^3 (\sigma_i(x_1) + \sigma_i(x_3) + \cdots + \sigma_i(x_t)),$$
$$\sum_{i=1}^3 \sigma_i(x_2) + \sigma_i(x_4) + \cdots + \sigma_i(x'_t))\}_{x_i x_{i+1} \in E}.$$

(3)

$$\mathcal{C}_n(SHG) = \max\{\sum_{i=1}^3 (\sigma_i(x_j) + \sigma_i(x_{j+1}))\}_{x_j x_{j+1} \in E}.$$

(4)

$$\mathcal{M}_n(SHG) = \max\{\sum_{i=1}^3 \mu_i(x_0 x_1) + \sum_{i=1}^3 \mu_i(x_2 x_3) + \cdots + \sum_{i=1}^3 \mu_i(x_{j-1} x_j)\}_{|S| = \lfloor \frac{n}{2} \rfloor}.$$

(5)

$$\mathcal{G}_n(SHG) = \mathcal{O}_n(SHG).$$

Proposition 3.15. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a star-neutrosophic SuperHyperGraph with center c. Then*

(1)

$$\mathcal{Z}_n(STR_{1,\sigma_2}) = \mathcal{O}_n(STR_{1,\sigma_2}) - \max\{\Sigma_{i=1}^3 \sigma_i(c) + \Sigma_{i=1}^3 \sigma_i(x)\}_{x \in V}.$$

(2)

$$\mathcal{I}_n(SHG) = \mathcal{O}_n(SHG) - \sigma(c) = \sum_{i=1}^3 \sum_{x_j \neq c} \sigma_i(x_j).$$

(3)

$$\mathcal{C}_n(SHG) = \sum_{i=1}^3 \sigma_i(c) + \max\{\sum_{i=1}^3 \sigma_i(x_j)\}.$$

(4)

$$\mathcal{M}_n(SHG) = \max\{\sum_{i=1}^3 \mu_i(x_{j-1} x_j)\}_{x_{j-1} x_j \in E}.$$

(5)

$$\mathcal{G}_n(SHG) = \infty.$$

Proposition 3.16. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a complete-bipartite-neutrosophic SuperHyperGraph. Then*

(1)

$$\mathcal{Z}_n(CMT_{\sigma_1,\sigma_2}) = \mathcal{O}_n(CMT_{\sigma_1,\sigma_2}) - \max\{\Sigma_{i=1}^3 \sigma_i(x) + \Sigma_{i=1}^3 \sigma_i(x')\}_{x,x' \in V}.$$

(2)

$$\mathcal{I}_n(SHG) = \max\{(\sum_{i=1}^{3}\sum_{x_j\in V_1}\sigma_i(x_j)),(\sum_{i=1}^{3}\sum_{x_j\in V_2}\sigma_i(x_j))\}.$$

(3)

$$\mathcal{C}_n(SHG) = \max\{\sum_{i=1}^{3}(\sigma_i(x_j)+\sigma_i(x_{j'}))\}_{x_j\in V_1,\ x_{j'}\in V_2}.$$

(4)

$$\mathcal{M}_n(SHG) = \max\{\sum_{i=1}^{3}\mu_i(x_0x_1)+\sum_{i=1}^{3}\mu_i(x_2x_3)+\cdots+\sum_{i=1}^{3}\mu_i(x_{j-1}x_j)\}_{|S|=\min\{|V_1|,|V_2|\}}.$$

(5)

$$\mathcal{G}_n(SHG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x)+\sigma_i(y)+\sigma_i(z)+\sigma_i(w))\}_{x,y\in V_1,\ z,w\in V_2}.$$

where $\mathcal{O}(SHG)\geq 4$ *and* $\min\{|V_1|,|V_2|\}\geq 2$. *Also,*

$$\mathcal{G}_n(SHG)=\infty$$

where $\mathcal{O}(SHG)\leq 3$.

Proposition 3.17. *Let* $SHG=(G\subseteq P(V),E\subseteq P(V))$ *be a complete-t-partite-neutrosophic SuperHyperGraph. Then*

(1)

$$\mathcal{Z}_n(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}_n(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - \max\{\Sigma_{i=1}^{3}\sigma_i(x)\}_{x\in V}.$$

(2)

$$\mathcal{I}_n(SHG) = \max\{(\sum_{i=1}^{3}\sum_{x_j\in V_1}\sigma_i(x_j)),(\sum_{i=1}^{3}\sum_{x_j\in V_2}\sigma_i(x_j)),\cdots,$$
$$(\sum_{i=1}^{3}\sum_{x_j\in V_t}\sigma_i(x_j))\}.$$

(3)

$$\mathcal{C}_n(SHG) = \max\{\sum_{i=1}^{3}(\sigma_i(x_{j_1})+\sigma_i(x_{j_2})+\cdots+\sigma_i(x_{j_t}))\}_{x_{j_1}\in V_1,x_{j_2}\in V_2,\cdots,\ x_{j_t}\in V_t}.$$

(4)

$$\mathcal{M}_n(SHG) = \max\{\sum_{i=1}^{3}\mu_i(x_0x_1)+\sum_{i=1}^{3}\mu_i(x_2x_3)+\cdots+\sum_{i=1}^{3}\mu_i(x_{j-1}x_j)\}_{|S|=\min|V_i|_{i=1}^{t}}.$$

(5)

$$\mathcal{G}_n(SHG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x)+\sigma_i(y)+\sigma_i(z))\}_{x\in V_1,\ y\in V_2,\ z\in V_3}.$$

where $t\geq 3$.

$$\mathcal{G}_n(SHG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x)+\sigma_i(y)+\sigma_i(z)+\sigma_i(w))\}_{x,y\in V_1,\ z,w\in V_2}.$$

where $t \leq 2$. *And*

$$\mathcal{G}_n(SHG) = \infty$$

where $\mathcal{O}(SHG) \leq 2$.

3.1. *Setting of Neutrosophic 1-Zero-Forcing Number*

Definition 3.18. (1-Zero-Forcing Number).
Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a neutrosophic SuperHyperGraph. Then

(i) **1-zero-forcing number** $\mathcal{Z}(SHG)$ for a neutrosophic SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ is minimum cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex. The last condition is as follows. For one time, black can change any vertex from white to black.

(ii) **1-zero-forcing neutrosophic-number** $\mathcal{Z}_n(SHG)$ for a neutrosophic SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ is minimum neutrosophic cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex. The last condition is as follows. For one time, black can change any vertex from white to black.

Definition 3.19. (Failed 1-Zero-Forcing Number).
Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a neutrosophic SuperHyperGraph. Then

(i) **failed 1-zero-forcing number** $\mathcal{Z}'(SHG)$ for a neutrosophic SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ is maximum cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ isn't turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex. The last condition is as follows. For one time, Black can change any vertex from white to black. The last condition is as follows. For one time, black can change any vertex from white to black;

(ii) **failed 1-zero-forcing neutrosophic-number** $\mathcal{Z}'_n(SHG)$ for a neutrosophic SuperHyperGraph $SHG = (G \subseteq P(V), E \subseteq P(V))$ is maximum neutrosophic cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ isn't turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex. The last condition is as follows. For one time, Black can change any vertex from white to black. The last condition is as follows. For one time, black can change any vertex from white to black.

Proposition 3.20. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a complete-neutrosophic SuperHyperGraph. Then*

$$\mathcal{Z}(CMT_{\sigma}) = \mathcal{O}(CMT_{\sigma}) - 2.$$

Proposition 3.21. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a path-neutrosophic SuperHyperGraph. Then*

$$\mathcal{Z}(PTH_n) = 1.$$

Proposition 3.22. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a cycle-neutrosophic SuperHyperGraph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{Z}(CYC_n) = 1.$$

Proposition 3.23. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a star-neutrosophic SuperHyperGraph with center c. Then*

$$\mathcal{Z}(STR_{1,\sigma_2}) = \mathcal{O}(STR_{1,\sigma_2}) - 3.$$

Proposition 3.24. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a complete-bipartite-neutrosophic SuperHyperGraph. Then*

$$\mathcal{Z}(CMT_{\sigma_1,\sigma_2}) = \mathcal{O}(CMT_{\sigma_1,\sigma_2}) - 3.$$

Proposition 3.25. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a complete-t-partite-neutrosophic SuperHyperGraph. Then*

$$\mathcal{Z}(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - 2.$$

3.2. *Setting of 1-Zero-Forcing Neutrosophic-Number*

Proposition 3.26. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a complete-neutrosophic SuperHyperGraph. Then*

$$\mathcal{Z}_n(CMT_{\sigma}) = \mathcal{O}_n(CMT_{\sigma}) - \max\{\Sigma_{i=1}^{3}\sigma_i(x) + \Sigma_{i=1}^{3}\sigma_i(y)\}_{x,y\in V}.$$

Proposition 3.27. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a path-neutrosophic SuperHyperGraph. Then*

$$\mathcal{Z}_n(PTH_n) = \min\{\Sigma_{i=1}^{3}\sigma_i(x)\}_{x \text{ is a vertex}}.$$

Proposition 3.28. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a cycle-neutrosophic SuperHyperGraph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{Z}_n(CYC_n) = \min\{\Sigma_{i=1}^{3}\sigma_i(x)\}_{x \text{ is a vertex}}.$$

Proposition 3.29. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a star-neutrosophic SuperHyperGraph with center c. Then*

$$\mathcal{Z}_n(STR_{1,\sigma_2}) = \mathcal{O}_n(STR_{1,\sigma_2}) - \max\{\Sigma_{i=1}^3 \sigma_i(c) + \Sigma_{i=1}^3 \sigma_i(x) + \Sigma_{i=1}^3 \sigma_i(y)\}_{x,y \in V}.$$

Proposition 3.30. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a complete-bipartite-neutrosophic SuperHyperGraph. Then*

$$\mathcal{Z}_n(CMT_{\sigma_1,\sigma_2}) = \mathcal{O}_n(CMT_{\sigma_1,\sigma_2}) - \max\{\Sigma_{i=1}^3 \sigma_i(x) + \Sigma_{i=1}^3 \sigma_i(x') + \Sigma_{i=1}^3 \sigma_i(x'')\}_{x,x',x'' \in V}.$$

Proposition 3.31. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a complete-t-partite-neutrosophic SuperHyperGraph. Then*

$$\mathcal{Z}_n(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}_n(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - \max\{\Sigma_{i=1}^3 \sigma_i(x) + \Sigma_{i=1}^3 \sigma_i(x')\}_{x,x' \in V}.$$

3.3. *Setting of Neutrosophic Failed 1-Zero-Forcing Number*

Proposition 3.32. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a complete-neutrosophic SuperHyperGraph. Then*

$$\mathcal{Z}'(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 3.$$

Proposition 3.33. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a path-neutrosophic SuperHyperGraph. Then*

$$\mathcal{Z}'(PTH_n) = 0.$$

Proposition 3.34. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a cycle-neutrosophic SuperHyperGraph where $\mathcal{O}(CYC) \geq 3$.*

$$\mathcal{Z}'(CYC_n) = 0.$$

Proposition 3.35. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a star-neutrosophic SuperHyperGraph with center c. Then*

$$\mathcal{Z}'(STR_{1,\sigma_2}) = \mathcal{O}(STR_{1,\sigma_2}) - 4.$$

Proposition 3.36. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a complete-bipartite-neutrosophic SuperHyperGraph. Then*

$$\mathcal{Z}'(CMT_{\sigma_1,\sigma_2}) = \mathcal{O}(CMT_{\sigma_1,\sigma_2}) - 4.$$

Proposition 3.37. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a complete-t-partite-neutrosophic SuperHyperGraph. Then*

$$\mathcal{Z}'(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - 3.$$

3.4. *Setting of Failed 1-Zero-Forcing Neutrosophic-Number*

Proposition 3.38. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a complete-neutrosophic SuperHyperGraph. Then*

$$\mathcal{Z}'_n(CMT_\sigma) = \mathcal{O}_n(CMT_\sigma) -$$
$$\min\{\Sigma_{i=1}^3 \sigma_i(x) + \Sigma_{i=1}^3 \sigma_i(y) + \Sigma_{i=1}^3 \sigma_i(z)\}_{x,y,z\in V}.$$

Proposition 3.39. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a path-neutrosophic SuperHyperGraph. Then*

$$\mathcal{Z}'_n(PTH_n) = 0.$$

Proposition 3.40. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a cycle-neutrosophic SuperHyperGraph where $\mathcal{O}(CYC) \geq 3$. Then*

$$\mathcal{Z}'_n(CYC_n) = 0$$

Proposition 3.41. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a star-neutrosophic SuperHyperGraph with center c. Then*

$$\mathcal{Z}'_n(STR_{1,\sigma_2}) = \mathcal{O}_n(STR_{1,\sigma_2}) -$$
$$\min\{\Sigma_{i=1}^3 \sigma_i(c) + \Sigma_{i=1}^3 \sigma_i(x) + \Sigma_{i=1}^3 \sigma_i(y) + \Sigma_{i=1}^3 \sigma_i(z)\}_{x,y,z\in V}.$$

Proposition 3.42. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a complete-bipartite-neutrosophic SuperHyperGraph. Then*

$$\mathcal{Z}'_n(CMT_{\sigma_1,\sigma_2}) = \mathcal{O}_n(CMT_{\sigma_1,\sigma_2}) -$$
$$\min\{\Sigma_{i=1}^3 \sigma_i(x) + \Sigma_{i=1}^3 \sigma_i(x') + \Sigma_{i=1}^3 \sigma_i(x'') + \Sigma_{i=1}^3 \sigma_i(x''')\}_{x,x',x'',x'''\in V}.$$

Proposition 3.43. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a complete-t-partite-neutrosophic SuperHyperGraph. Then*

$$\mathcal{Z}'_n(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) = \mathcal{O}_n(CMT_{\sigma_1,\sigma_2,\cdots,\sigma_t}) - \min\{\Sigma_{i=1}^3 \sigma_i(x) + \Sigma_{i=1}^3 \sigma_i(x') + \Sigma_{i=1}^3 \sigma_i(x'')\}_{x,x'\in V}.$$

3.5. *Global Offensive Alliance*

Definition 3.44. Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a neutrosophic SuperHyperGraph. Then

(i) a set S is called **global-offensive alliance** if

$$\forall a \in V \setminus S,\ |N_s(a) \cap S| > |N_s(a) \cap (V \setminus S)|;$$

(ii) $\forall S' \subseteq S$, S is global offensive alliance but S' isn't global offensive alliance. Then S is called **minimal-global-offensive alliance**;

(iii) **minimal-global-offensive-alliance number** of SHG is

$$\bigwedge_{S \text{ is a minimal-global-offensive alliance.}} |S|$$

and it's denoted by Γ;

(iv) **minimal-global-offensive-alliance-neutrosophic number** of SHG is

$$\bigwedge_{S \text{ is a minimal-global-offensive alliance.}} \Sigma_{s\in S}\Sigma_{i=1}^{3}\sigma_i(s)$$

and it's denoted by Γ_s.

Proposition 3.45. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a strong neutrosophic SuperHyperGraph. If S is global-offensive alliance, then $\forall v \in V \setminus S,\ \exists x \in S$ such that*

(i) $v \in N_s(x)$;

(ii) $vx \in E$.

Definition 3.46. Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a strong neutrosophic SuperHyperGraph. Suppose S is a set of vertices. Then

(i) S is called **dominating set** if $\forall v \in V \setminus S,\ \exists s \in S$ such that either $v \in N_s(s)$ or $vs \in E$;

(ii) $|S|$ is called **chromatic number** if $\forall v \in V,\ \exists s \in S$ such that either $v \in N_s(s)$ or $vs \in E$ implies s and v have different colors.

Proposition 3.47. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a strong neutrosophic SuperHyperGraph. If S is global-offensive alliance, then*

(i) *S is dominating set;*

(ii) *there's $S \subseteq S'$ such that $|S'|$ is chromatic number.*

Proposition 3.48. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a strong neutrosophic SuperHyperGraph. Then*

(i) $\Gamma \leq \mathcal{O}$;

(ii) $\Gamma_s \leq \mathcal{O}_n$.

Proposition 3.49. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a strong neutrosophic SuperHyperGraph which is connected. Then*

(i) $\Gamma \leq \mathcal{O} - 1$;

(ii) $\Gamma_s \leq \mathcal{O}_n - \Sigma_{i=1}^{3}\sigma_i(x)$.

Proposition 3.50. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be an odd path. Then*

(i) *the set $S = \{v_2, v_4, \cdots, v_{n-1}\}$ is minimal-global-offensive alliance;*

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor + 1$ *and corresponded set is $S = \{v_2, v_4, \cdots, v_{n-1}\}$;*

(iii) $\Gamma_s = \min\{\Sigma_{s\in S=\{v_2,v_4,\cdots,v_{n-1}\}}\Sigma_{i=1}^{3}\sigma_i(s), \Sigma_{s\in S=\{v_1,v_3,\cdots,v_{n-1}\}}\Sigma_{i=1}^{3}\sigma_i(s)\}$;

(iv) *the sets* $S_1 = \{v_2, v_4, \cdots, v_{n-1}\}$ *and* $S_2 = \{v_1, v_3, \cdots, v_{n-1}\}$ *are only minimal-global-offensive alliances.*

Proposition 3.51. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be an even path. Then*

(i) *the set* $S = \{v_2, v_4, \cdots .v_n\}$ *is minimal-global-offensive alliance;*

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor$ *and corresponded sets are* $\{v_2, v_4, \cdots .v_n\}$ *and* $\{v_1, v_3, \cdots .v_{n-1}\}$;

(iii) $\Gamma_s = \min\{\Sigma_{s \in S = \{v_2, v_4, \cdots, v_n\}} \Sigma_{i=1}^3 \sigma_i(s), \Sigma_{s \in S = \{v_1, v_3, \cdots .v_{n-1}\}} \Sigma_{i=1}^3 \sigma_i(s)\}$;

(iv) *the sets* $S_1 = \{v_2, v_4, \cdots .v_n\}$ *and* $S_2 = \{v_1, v_3, \cdots .v_{n-1}\}$ *are only minimal-global-offensive alliances.*

Proposition 3.52. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be an even cycle. Then*

(i) *the set* $S = \{v_2, v_4, \cdots, v_n\}$ *is minimal-global-offensive alliance;*

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor$ *and corresponded sets are* $\{v_2, v_4, \cdots, v_n\}$ *and* $\{v_1, v_3, \cdots, v_{n-1}\}$;

(iii) $\Gamma_s = \min\{\Sigma_{s \in S = \{v_2, v_4, \cdots, v_n\}} \sigma(s), \Sigma_{s \in S = \{v_1, v_3, \cdots, v_{n-1}\}} \sigma(s)\}$;

(iv) *the sets* $S_1 = \{v_2, v_4, \cdots, v_n\}$ *and* $S_2 = \{v_1, v_3, \cdots, v_{n-1}\}$ *are only minimal-global-offensive alliances.*

Proposition 3.53. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be an odd cycle. Then*

(i) *the set* $S = \{v_2, v_4, \cdots, v_{n-1}\}$ *is minimal-global-offensive alliance;*

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor + 1$ *and corresponded set is* $S = \{v_2, v_4, \cdots, v_{n-1}\}$;

(iii) $\Gamma_s = \min\{\Sigma_{s \in S = \{v_2, v_4, \cdots .v_{n-1}\}} \Sigma_{i=1}^3 \sigma_i(s), \Sigma_{s \in S = \{v_1, v_3, \cdots .v_{n-1}\}} \Sigma_{i=1}^3 \sigma_i(s)\}$;

(iv) *the sets* $S_1 = \{v_2, v_4, \cdots .v_{n-1}\}$ *and* $S_2 = \{v_1, v_3, \cdots .v_{n-1}\}$ *are only minimal-global-offensive alliances.*

Proposition 3.54. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be star. Then*

(i) *the set* $S = \{c\}$ *is minimal-global-offensive alliance;*

(ii) $\Gamma = 1$;

(iii) $\Gamma_s = \Sigma_{i=1}^3 \sigma_i(c)$;

(iv) *the sets* $S = \{c\}$ *and* $S \subset S'$ *are only global-offensive alliances.*

Proposition 3.55. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be wheel. Then*

(i) *the set* $S = \{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1) \leq n}$ *is minimal-global-offensive alliance;*

(ii) $\Gamma = |\{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1) \leq n}|$;

(iii) $\Gamma_s = \Sigma_{\{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1) \leq n}} \Sigma_{i=1}^3 \sigma_i(s)$;

(iv) *the set* $\{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1) \leq n}$ *is only minimal-global-offensive alliance.*

Proposition 3.56. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be an odd complete. Then*

(i) *the set* $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor+1}$ *is minimal-global-offensive alliance;*

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor + 1$;

(iii) $\Gamma_s = \min\{\Sigma_{s\in S}\Sigma_{i=1}^{3}\sigma_i(s)\}_{S=\{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor+1}}$;

(iv) *the set* $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor+1}$ *is only minimal-global-offensive alliances.*

Proposition 3.57. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be an even complete. Then*

(i) *the set* $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ *is minimal-global-offensive alliance;*

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor$;

(iii) $\Gamma_s = \min\{\Sigma_{s\in S}\Sigma_{i=1}^{3}\sigma_i(s)\}_{S=\{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}}$;

(iv) *the set* $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ *is only minimal-global-offensive alliances.*

Proposition 3.58. *Let* $\mathcal{G}$ *be a* m*-family of neutrosophic stars with common neutrosophic vertex set. Then*

(i) *the set* $S = \{c_1, c_2, \cdots, c_m\}$ *is minimal-global-offensive alliance for* $\mathcal{G}$;

(ii) $\Gamma = m$ *for* $\mathcal{G}$;

(iii) $\Gamma_s = \Sigma_{i=1}^{m}\Sigma_{j=1}^{3}\sigma_j(c_i)$ *for* $\mathcal{G}$;

(iv) *the sets* $S = \{c_1, c_2, \cdots, c_m\}$ *and* $S \subset S'$ *are only minimal-global-offensive alliances for* $\mathcal{G}$.

Proposition 3.59. *Let* $\mathcal{G}$ *be a* m*-family of odd complete graphs with common neutrosophic vertex set. Then*

(i) *the set* $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor+1}$ *is minimal-global-offensive alliance for* $\mathcal{G}$;

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor + 1$ *for* $\mathcal{G}$;

(iii) $\Gamma_s = \min\{\Sigma_{s\in S}\Sigma_{i=1}^{3}\sigma_i(s)\}_{S=\{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor+1}}$ *for* $\mathcal{G}$;

(iv) *the sets* $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor+1}$ *are only minimal-global-offensive alliances for* $\mathcal{G}$.

Proposition 3.60. *Let* $\mathcal{G}$ *be a* m*-family of even complete graphs with common neutrosophic vertex set. Then*

(i) *the set* $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ *is minimal-global-offensive alliance for* $\mathcal{G}$;

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor$ *for* $\mathcal{G}$;

(iii) $\Gamma_s = \min\{\Sigma_{s\in S}\Sigma_{i=1}^{3}\sigma_i(s)\}_{S=\{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}}$ *for* $\mathcal{G}$;

(iv) *the sets* $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ *are only minimal-global-offensive alliances for* $\mathcal{G}$.

3.6. *Global Powerful Alliance*

Definition 3.61. Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a neutrosophic SuperHyperGraph. Then

(i) a set S of vertices is called **t-offensive alliance** if

$$\forall a \in V \setminus S,\ |N_s(a) \cap S| - |N_s(a) \cap (V \setminus S)| > t;$$

(ii) a t-offensive alliance is called **global-offensive alliance** if $t = 0$;

(iii) a set S of vertices is called **t-defensive alliance** if

$$\forall a \in S,\ |N_s(a) \cap S| - |N_s(a) \cap (V \setminus S)| < t;$$

(iv) a t-defensive alliance is called **global-defensive alliance** if $t = 0$;

(v) a set S of vertices is called **t-powerful alliance** if it's both t-offensive alliance and (t-2)-defensive alliance;

(vi) a t-powerful alliance is called **global-powerful alliance** if $t = 0$;

(vii) $\forall S' \subseteq S$, S is global-powerful alliance but S' isn't global-powerful alliance. Then S is called **minimal-global-powerful alliance**;

$(viii)$ **minimal-global-powerful-alliance number** of SHG is

$$\bigwedge_{S \text{ is a minimal-global-powerful alliance.}} |S|$$

and it's denoted by Γ;

(ix) **minimal-global-powerful-alliance-neutrosophic number** of SHG is

$$\bigwedge_{S \text{ is a minimal-global-offensive alliance.}} \Sigma_{s \in S} \Sigma_{i=1}^{3} \sigma_i(s)$$

and it's denoted by Γ_s.

Proposition 3.62. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a strong neutrosophic SuperHyperGraph. Then following statements hold;*

(i) *if $s \geq t$ and a set S of vertices is t-defensive alliance, then S is s-defensive alliance;*

(ii) *if $s \leq t$ and a set S of vertices is t-offensive alliance, then S is s-offensive alliance.*

Proposition 3.63. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a strong neutrosophic SuperHyperGraph. Then following statements hold;*

(i) *if $s \geq t+2$ and a set S of vertices is t-defensive alliance, then S is s-powerful alliance;*

(ii) *if $s \leq t$ and a set S of vertices is t-offensive alliance, then S is t-powerful alliance.*

Proposition 3.64. *Let $SHG = (G \subseteq P(V), E \subseteq P(V))$ be a r-regular-strong-neutrosophic SuperHyperGraph. Then following statements hold;*

(i) *if $\forall a \in S,\ |N_s(a) \cap S| < \lfloor \frac{r}{2} \rfloor + 1$, then $SHG = (G \subseteq P(V), E \subseteq P(V))$ is 2-defensive alliance;*

(ii) *if $\forall a \in V \setminus S,\ |N_s(a) \cap S| > \lfloor \frac{r}{2} \rfloor + 1$, then $SHG = (G \subseteq P(V), E \subseteq P(V))$ is 2-offensive alliance;*

(*iii*) *if* $\forall a \in S,\ |N_s(a) \cap V \setminus S| = 0$, *then* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is r-defensive alliance;*

(*iv*) *if* $\forall a \in V \setminus S,\ |N_s(a) \cap V \setminus S| = 0$, *then* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is r-offensive alliance.*

Proposition 3.65. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a r-regular-strong-neutrosophic SuperHyperGraph. Then following statements hold;*

(*i*) $\forall a \in S,\ |N_s(a) \cap S| < \lfloor \frac{r}{2} \rfloor + 1$ *if* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is 2-defensive alliance;*

(*ii*) $\forall a \in V \setminus S,\ |N_s(a) \cap S| > \lfloor \frac{r}{2} \rfloor + 1$ *if* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is 2-offensive alliance;*

(*iii*) $\forall a \in S,\ |N_s(a) \cap V \setminus S| = 0$ *if* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is r-defensive alliance;*

(*iv*) $\forall a \in V \setminus S,\ |N_s(a) \cap V \setminus S| = 0$ *if* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is r-offensive alliance.*

Proposition 3.66. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a r-regular-strong-neutrosophic SuperHyperGraph which is complete. Then following statements hold;*

(*i*) $\forall a \in S,\ |N_s(a) \cap S| < \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1$ *if* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is 2-defensive alliance;*

(*ii*) $\forall a \in V \setminus S,\ |N_s(a) \cap S| > \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1$ *if* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is 2-offensive alliance;*

(*iii*) $\forall a \in S,\ |N_s(a) \cap V \setminus S| = 0$ *if* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is* $(\mathcal{O}-1)$*-defensive alliance;*

(*iv*) $\forall a \in V \setminus S,\ |N_s(a) \cap V \setminus S| = 0$ *if* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is* $(\mathcal{O}-1)$*-offensive alliance.*

Proposition 3.67. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a r-regular-strong-neutrosophic SuperHyperGraph which is complete. Then following statements hold;*

(*i*) *if* $\forall a \in S,\ |N_s(a) \cap S| < \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1$, *then* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is 2-defensive alliance;*

(*ii*) *if* $\forall a \in V \setminus S,\ |N_s(a) \cap S| > \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1$, *then* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is 2-offensive alliance;*

(*iii*) *if* $\forall a \in S,\ |N_s(a) \cap V \setminus S| = 0$, *then* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is* $(\mathcal{O}-1)$*-defensive alliance;*

(*iv*) *if* $\forall a \in V \setminus S,\ |N_s(a) \cap V \setminus S| = 0$, *then* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is* $(\mathcal{O}-1)$*-offensive alliance.*

Proposition 3.68. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a r-regular-strong-neutrosophic SuperHyperGraph which is cycle. Then following statements hold;*

(i) $\forall a \in S,\ |N_s(a) \cap S| < 2$ *if* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is 2-defensive alliance;*

(ii) $\forall a \in V \setminus S,\ |N_s(a) \cap S| > 2$ *if* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is 2-offensive alliance;*

(iii) $\forall a \in S,\ |N_s(a) \cap V \setminus S| = 0$ *if* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is 2-defensive alliance;*

(iv) $\forall a \in V \setminus S,\ |N_s(a) \cap V \setminus S| = 0$ *if* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is 2-offensive alliance.*

Proposition 3.69. *Let* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *be a r-regular-strong-neutrosophic SuperHyperGraph which is cycle. Then following statements hold;*

(i) *if* $\forall a \in S,\ |N_s(a) \cap S| < 2$*, then* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is 2-defensive alliance;*

(ii) *if* $\forall a \in V \setminus S,\ |N_s(a) \cap S| > 2$*, then* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is 2-offensive alliance;*

(iii) *if* $\forall a \in S,\ |N_s(a) \cap V \setminus S| = 0$*, then* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is 2-defensive alliance;*

(iv) *if* $\forall a \in V \setminus S,\ |N_s(a) \cap V \setminus S| = 0$*, then* $SHG = (G \subseteq P(V), E \subseteq P(V))$ *is 2-offensive alliance.*

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