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Neutrosophic Supra Simply Open Set and Neutrosophic Supra Simply Compact Space

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Abstract: The aim of this article is to procure the notions of neutrosophic supra simply open set, neutrosophic supra simply open cover, neutrosophic supra simply compactness via neutrosophic supra topological spaces. Further, we formulate some results in the form of remarks, theorems, propositions etc.

Keywords: *Neutrosophic Supra Simply Open*; *Neutrosophic Supra Simply b-Open*; *Neutrosophic Supra Compact*; *Neutrosophic Supra Simply compact*.

1. Introduction

Smarandache [30] grounded the notion of neutrosophic set (in short NS) in the year 1998 by extending the concept of intuitionistic fuzzy set [1]. In the year 2010, Wang et al. presented the idea of single valued neutrosophic set. Till now, many researchers around the globe applied the notions of NS and its extensions in the formation of MADM-algorithms [3, 6, 10, 18, 21, 22, 24, 25, etc.]. In the year 2012, Salama and Alblowi [28] established the idea of neutrosophic topological space (in short NTS). Further, Salama and Alblowi [29] presented generalized NS and generalized NTS. The concept of neutrosophic semi-closed set and neutrosophic semi-open set in NTSs was introduced by Iswaraya and Bageerathi [16]. Afterwards, Arokiarani et al. [2] grounded the idea of neutrosophic semi-open functions and established relation between them. The concept of generalized neutrosophic closed sets in NTSs was studied by Dhavaseelan, and Jafari [14]. The neutrosophic generalized closed sets in NTSs was established by Pushpalatha and Nandhini [26]. Thereafter, Ebenanjar et al. [15] presented the neutrosophic *b*-open sets in NTSs. In the year 2018, Maheswari et al. [19] grounded the idea of neutrosophic generalized *b*-closed sets in NTSs. In the year 2020, Das and Pramanik [7] introduced the notion of generalized neutrosophic *b*-open sets in NTSs. Das and Pramanik [8] also established the concept of neutrosophic Φ-open sets and neutrosophic Φ-continuous functions. Mallick and Pramanik [20] introduced the notions of pentapartitioned neutrosophic set and studied their several properties. Later on, Das et al. [5] presented the idea of pentapartitioned neutrosophic *Q*-ideals of *Q*-algebra. In the year 2021, Das et al. [4] grounded the notions of quadripartitioned neutrosophic topological space. Noori and Yousif [23] introduced the idea of soft simply compact space via soft topological spaces in the year 2020. Afterwards, Das and Pramanik [9] presented the concept of neutrosophic simply soft open set in neutrosophic soft topological spaces, and studied the neutrosophic simply soft compactness. Das and Tripathy [12] introduced the neutrosophic simply-b-open set in NTSs. Dhavaseelan et al. [13] grounded the notion of neutrosophic supra topology and studied the neutrosophic α-supra open sets in neutrosophic supra topological spaces (in short NSTS). Later on, Jayaparthasarathy et al. [17] presented an application of neutrosophic supra topology in data mining process.

The main focus of this article is to procure the concept of neutrosophic supra simply open set, neutrosophic supra simply compactness via NSTSs. We formulate some results on neutrosophic supra simply open set, neutrosophic supra simply compactness.

2. Preliminaries and Definitions:

Definition 2.1. [17] A family τ of NSs over a fixed set W is called an neutrosophic supra topology (in short NST) on W if the following holds:

(i) $0_N, 1_N \in \tau$;

(ii) $\cup J_i \in \tau$, for every $\{J_i : I \in \Delta\} \subseteq \tau$.

The pair (W,τ) is called an neutrosophic supra topological space (in short NSTS). If $Y \in \tau$, then Y is called an neutrosophic supra open set (in short NSO-set) and Y^c is called an neutrosophic supra closed set (in short NSC-set).

Remark 2.1. [17] The family of all NSO-sets and NSC-sets in an NSTS (W,τ) may be denoted by NSO(W) and NSC(W) respectively.

Definition 2.2. [17] Suppose that (W,τ) be an NSTS. Then, the neutrosophic supra interior (in short N_{int}^{s}) and neutrosophic supra closure (in short N_{cl}^{s}) of an NS Y over W are defined by

$N_{int}^{s}(Y) = \cup\{R : R$ is an NSO-set in W and $R \subseteq Y\}$;

$N_{cl}^{s}(Y) = \cap\{P : P$ is an NSC-set in W and $Y \subseteq P\}$.

Definition 2.3. [13] Assume that (W,τ) be an NSTS. Then Y, an NS over W is called an neutrosophic supra α-open set (in short NS-α-O-set) iff $Y \subseteq N_{int}^{s}(N_{cl}^{s}(N_{int}^{s}(Y)))$.

Definition 2.4. [13] Suppose that (W,τ) be an NSTS. Then Y, an NS over W is called an neutrosophic supra semi-open set if and only if $Y \subseteq N_{cl}^{s}(N_{int}^{s}(Y))$.

Definition 2.5. [13] Let (W,τ) be an NSTS. Let Y be an NS over W. Then, Y is called an neutrosophic supra pre-open set if and only if $Y \subseteq N_{int}^{s}(N_{cl}^{s}(Y))$.

Remark 2.2. Throughout the paper, NS-α-O(W), NSSO(W), NSPO(W) denotes the family of all NS-α-O-sets, NSS-O-sets, NSP-O-sets in NSTS (W,τ).

Definition 2.6. [13] Suppose that ξ be a one to one and onto mapping from an NSTS (W,τ_1) to another NSTS (M,τ_2). Then, ξ is called an

(i) neutrosophic supra continuous (in short NS-Continuous) function if $\xi^{-1}(K)$ is a NSO-set in W, whenever K is a NSO-set in M.

(ii) neutrosophic supra-α-continuous (in short NS-α-Continuous) function if $\xi^{-1}(K)$ is a NS-α-O-set in W, whenever K is a NSO-set in M.

3. Neutrosophic Supra Simply Open Set:

In this section, we procure the notions of neutrosophic supra b-open set, neutrosophic supra b-continuous mapping, neutrosophic supra simply open set, neutrosophic supra simply continuous mapping, neutrosophic supra simply b-continuous mapping, neutrosophic supra simply compactness, and neutrosophic supra simply b-compactness in NSTSs.

Definition 3.1. Suppose that (W,τ) be an NSTS. Then Y, an NS over W is called an neutrosophic supra b-open set (NS-b-O-set) iff $Y \subseteq N_{int}^{s}(N_{cl}^{s}(Y)) \cup N_{cl}^{s}(N_{int}^{s}(Y))$.

Remark 3.1. Throughout the paper, NS-b-O(W) and NS-b-C(W) denotes the family of all NS-b-O-sets and NS-b-C-sets in NSTS (W,τ). Clearly, NSOS(W) $\subseteq$ NS-bO(W) and NSCS(W) $\subseteq$ NS-bC(W).

Definition 3.2. Suppose that ξ be an one to one and onto mapping from an NSTS (W,τ_1) to another NSTS (M,τ_2). Then, ξ is called an neutrosophic supra-b-continuous (in short NS-b-Continuous) function if $\xi^{-1}(K)$ is an NS-b-O-set in W, whenever K is an NSO-set in M.

Definition 3.3. A collection $\{S_\alpha: \alpha\in\Delta\}$ of NSO-sets in (W,τ), where Δ is an index set, is called an neutrosophic supra open cover (in short NSO-cover) of an neutrosophic set S if $S \subseteq \cup_{\alpha\in\Delta} S_\alpha$.

Definition 3.4. A family $\{S_\alpha: \alpha = 1, 2, 3, \dots, n\}$ of NSO-sets in (W,τ) is called an neutrosophic supra open finite sub cover (in short NSO-finite sub cover) of an neutrosophic set S if $S \subseteq \cup_{\alpha=1}^{n} S_\alpha$.

Definition 3.5. An NSTS (W,τ) is called an neutrosophic supra compact space (in short NS-compact-space) if every NSO-cover of W has an NSO-finite sub cover.

Definition 3.6. An neutrosophic subset B of an NSTS (W,τ) is said to be an neutrosophic supra compact set relative to W if every NSO-cover of B has a finite sub-cover.

Definition 3.7. A family $\{S_\alpha: \alpha\in\Delta\}$ of NS-b-O-sets in (W,τ), where Δ is an index set, is called an neutrosophic supra b-open cover (in short NS-b-O-cover) of an neutrosophic set S if $S \subseteq \cup_{\alpha\in\Delta} S_\alpha$.

Definition 3.8. Suppose that (W,τ) be an NSTS. Then, (W,τ) is called an neutrosophic supra b-compact space (in short NS-b-compact-space) if every NS-b-O-cover of W has a finite sub-cover.

Definition 3.9. An neutrosophic subset B of an NSTS (W,τ) is said to be an neutrosophic supra b-compact relative to W if every NS-b-O-cover of B has a finite sub-cover.

Theorem 3.1. Every NS-b-compact-space is an NS-compact-space.

Proof. Suppose that (W,τ) be an NS-b-compact-space. Therefore, every NS-b-O-cover of (W,τ) has a finite sub-cover. Let (W,τ) may not be an NS-compact-space. Then, there exists an NSO-cover $\mathcal{H}$ (say) of W, which has no finite sub-cover. Since, every NSO-set is an NS-b-O-set, so there exists an NS-b-O-cover $\mathcal{H}$ of W, which has no finite sub-cover. This contradicts the fact that (W,τ) is an NS-b-compact-space. Therefore, (W,τ) must be an NS-compact-space.

Definition 3.10. Let (W,τ) be an neutrosophic supra topological space. Then, an NS Z over W is called an neutrosophic supra simply open set (in short NSSO-set) in (W,τ) if and only if it is an NSO-set in (W,τ) with the condition $N_{int}N_{cl}(Z)\subseteq N_{cl}N_{int}(Z)$.

Clearly, every NSSO-set is an NSO-set in (W,τ).

Remark 3.2. In an NSTS (W,τ), both 0_N and 1_N are NSSO-set.

Definition 3.11. Suppose that Z be an neutrosophic set over a fixed set W. Then, Z is called an neutrosophic supra simply b-open set (in short NSS-b-O-set) in the NSTS (W,τ) if and only if it is an NS-b-O-set in (W,τ) with the condition $N_{int}N_{cl}(Z)\subseteq N_{cl}N_{int}(Z)$.

If Y is an NSS-b-O-set, then Y^c is called an neutrosophic supra simply b-closed set (in short NSS-b-C-set). The family of all NSS-b-O-sets and NSS-b-C-sets may be denoted as NSS-b-O(W) and NSS-b-C(W) respectively.

Clearly, every NSS-b-O-set in an NSTS (W,τ), is also an NS-b-O-set.

Remark 3.3. In an NSTS (W,τ), both 0_N and 1_N are NSSO-set.

Theorem 3.2. In an NSTS (W,τ), every NSO-set is an NSS-O-set in an NSTS (W,τ).

Proof: Suppose that J be an NSO-set in an NSTS (W,τ). Therefore, $N_{int}(J) = J$. It is known that, every NSO-set is an NS-b-O-set. Therefore, J is an NS-b-O-set in (W,τ). Further, it is known that $J\subseteq N_{cl}(J)$.

Now, $J\subseteq N_{cl}(J)$

$\Rightarrow J\subseteq N_{cl}N_{int}(J)$

$\Rightarrow N_{cl}(J)\subseteq N_{cl}N_{cl}N_{int}(J)$

$=N_{cl}N_{int}(J)$ [Since $N_{cl}N_{int}(J)$ is a NSC-set in (W,τ)] (1)

Further, we have, $N_{int}N_{cl}(J)\subseteq N_{cl}(J)$ (2)

From eq. (1) and eq. (2), we have, $N_{int}N_{cl}(J)\subseteq N_{cl}N_{int}(J)$.

Therefore, J is an NS-b-O set in (W,τ) and $N_{int}N_{cl}(J)\subseteq N_{cl}N_{int}(J)$. Hence, J is an NS-b-O set in (W,τ).

Proposition 3.1. Every NSO-set is an NSS-b-O-set in an NSTS (W,τ).

Theorem 3.3. Every neutrosophic supra semi-open set is an NSS-b-O-set in an NSTS (W,τ).

Proof. Suppose that Q be an neutrosophic supra semi-open set in an NTS (W,τ). Therefore, $Q\subseteq N_{cl}N_{int}(Q)$. It is known that, every neutrosophic supra semi-open set is an NS-b-O set. This implies, Q is an NS-b-O set in (W,τ).

Now, $Q\subseteq N_{cl}N_{int}(Q)$

$\Rightarrow N_{cl}(Q)\subseteq N_{cl}N_{cl}N_{int}(Q)$

$=N_{cl}N_{int}(Q)$ [Since $N_{cl}N_{int}(Q)$ is an NSC-set in (W,τ)]

$\Rightarrow N_{cl}(Q)\subseteq N_{cl}N_{int}(Q)$ (3)

It is known that, $N_{int}N_{cl}(Q)\subseteq N_{cl}(Q)$ (4)

From eq. (3) and eq. (4), we have, $N_{int}N_{cl}(Q) \subseteq N_{cl}N_{int}(Q)$. Therefore, Q is an NS-b-O set in (W,τ) and $N_{int}N_{cl}(Q)\subseteq N_{cl}N_{int}(Q)$. Hence Q is an NSS-b-O set in (W,τ).

Theorem 3.4. If an neutrosophic set A is both neutrosophic supra pre open set and NSS-b-O-set in an NSTS (W,τ), then it is also an neutrosophic supra semi open set in (W,τ).

Proof. Let Q_1 be both neutrosophic supra pre-open set and NSS-b-O-set in an NTS (W,τ). Since, Q_1 is an neutrosophic supra pre open set, so $Q_1\subseteq N_{int}N_{cl}(Q_1)$. Further, since Q_1 is an NSS-b-O-set, so Q_1 is an NS-b-O-set and $N_{int}N_{cl}(Q_1)\subseteq N_{cl}N_{int}(Q_1)$. This implies, $Q_1\subseteq N_{cl}N_{int}(Q_1)$. Therefore, Q_1 is an neutrosophic supra semi open set.

Remark 3.4. Suppose that Z_1 and Z_2 be two NSS-b-O-sets. Then, $Z_1\cap Z_2$ may not be an NSS-b-O-set.

Proof. Let Z_1 and Z_2 be two NSS-b-O-sets. Therefore, Z_1, Z_2 are NS-b-O-sets in (W,τ) such that $N_{int}N_{cl}(Z_1)\subseteq N_{cl}N_{int}(Z_1)$, $N_{int}N_{cl}(Z_2)\subseteq N_{cl}N_{int}(Z_2)$. But it is known that the intersection of two NS-b-O-sets may not be an NS-b-O-set in a NSTS (W,τ). Hence, $Z_1\cap Z_2$ may not be an NS-b-O-set in (W,τ). Therefore, $Z_1\cap Z_2$ may not be an NS-b-O-set in (W,τ).

Definition 3.12. An one to one and onto mapping ξ: $(W,\tau_1)\rightarrow(M,\tau_2)$ is said to be an neutrosophic supra simply continuous (in short NSS-Continuous) mapping if $\xi^{-1}(Z)$ is an NSSO-set in W whenever Z is an NSO-set in M.

Remark 3.5. Every NSS-Continuous mapping is an NS-Continuous mapping.

Definition 3.13. An one to one and onto mapping ξ: $(W,\tau_1)\rightarrow(M,\tau_2)$ is said to be an neutrosophic supra simply b-continuous (in short NSS-b-Continuous) mapping if $\xi^{-1}(Z)$ is an NSS-b-O-set in W whenever Z is an NSO-set in M.

Remark 3.6. Every NSS-b-Continuous mapping is an NS-b-Continuous mapping.

Definition 3.14. An one to one and onto mapping ξ: $(W,\tau_1)\rightarrow(M,\tau_2)$ is called an neutrosophic supra simply open mapping (in short NSS-Open-mapping) if $\xi(K)$ is an NSSO-set in M, whenever K is an NSO-set in W.

Definition 3.15. An one to one and onto mapping ξ: $(W,\tau_1)\to(M,\tau_2)$ is called an neutrosophic supra simply *b*-open mapping (in short NSS-*b*-Open-mapping) if $\xi(K)$ is an NSS-*b*-O-set in *M*, whenever *K* is an NSO-set in *W*.

Definition 3.16. An one to one and onto family $\{Z_\alpha: \alpha\in\Delta\}$, where Δ is an index set and for each $\alpha\in\Delta$, Z_α is an NS-*b*-O-set in a NTS (W,τ) is said to be an neutrosophic supra *b*-open cover of an neutrosophic set *Z* if $Z\subseteq\cup_{\alpha\in\Delta} Z_\alpha$.

Definition 3.17. An NSTS (W,τ) is called an neutrosophic supra simply compact space if every neutrosophic simply open cover of *W* has a finite sub-cover.

Definition 3.18. An neutrosophic subset *K* of an NSTS (W,τ) is called an neutrosophic supra simply compact set relative to *W* if every neutrosophic supra simply open cover of *K* has a finite sub-cover.

Theorem 3.5. Every neutrosophic supra simply closed subset of an neutrosophic supra simply compact space (W,τ) is an neutrosophic supra simply compact set relative to *W*.

Proof: Suppose that (W,τ) be an neutrosophic supra simply compact space and *K* be an neutrosophic supra simply closed set in (W,τ). Therefore, K^c is an NSS-O-set in (W,τ). Suppose that $U = \{U_i: i\in\Delta$ and $U_i\in$NSS-O$(W)\}$ be an neutrosophic supra simply open cover of *K*. Therefore, $\mathcal{H}=\{K^c\}\cup U$ is an neutrosophic supra simply open cover of *X*. Since, *X* is an neutrosophic supra simply compact space, so it has a finite sub-cover say $\{H_1, H_2, H_3,\ldots,H_n, K^c\}$. This implies, $\{H_1, H_2, H_3,\ldots, H_n\}$ is a finite neutrosophic supra simply open cover of *K*. Hence, *K* is an neutrosophic supra simply compact set relative to *W*.

Definition 3.19. An NSTS (W,τ) is called an neutrosophic supra simply *b*-compact space if each neutrosophic simply *b*-open cover of *W* has a finite sub-cover.

Definition 3.20. An neutrosophic subset *K* of (W,τ) is called an neutrosophic supra simply *b*-compact set relative to *W* if every neutrosophic supra simply *b*-open cover of *K* has a finite sub-cover.

Theorem 3.6. Every neutrosophic supra simply *b*-closed subset of an neutrosophic supra simply *b*-compact space (W,τ) is an neutrosophic supra simply *b*-compact set relative to *W*.

Proof: Suppose that (W,τ) be an neutrosophic supra simply *b*-compact space and *K* be an neutrosophic supra simply *b*-closed set in (W,τ). Therefore, K^c is an NSS-*b*-O-set in (W,τ). Suppose that $U = \{U_i: i\in\Delta$ and $U_i\in$NSS-*b*-O$(W)\}$ be an neutrosophic supra simply *b*-open cover of *K*. Therefore, $\mathcal{H}=\{K^c\}\cup U$ is an neutrosophic supra simply *b*-open cover of *X*. Since, *X* is an neutrosophic supra simply *b*-compact space, so it has a finite sub-cover say $\{H_1, H_2, H_3,\ldots,H_n, K^c\}$. This implies, $\{H_1, H_2, H_3,\ldots, H_n\}$ is a finite neutrosophic supra simply *b*-open cover of *K*. Hence, *K* is an neutrosophic supra simply *b*-compact set relative to *W*.

Theorem 3.7.

(i) Every neutrosophic supra *b*-compact space is an neutrosophic supra simply *b*-compact space.

(ii) Every neutrosophic supra simply *b*-compact space is an neutrosophic supra compact space.

Proof. (i) Let (W,τ) be an neutrosophic supra *b*-compact space. Suppose that (W,τ) is not an neutrosophic supra simply *b*-compact space. Then there exists an neutrosophic supra simply *b*-open cover $\mathcal{H}$ (say) of *W*, which has no finite sub-cover. Since, every neutrosophic supra simply *b*-open set is an neutrosophic supra *b*-open set, so we have an neutrosophic supra *b*-open cover $\mathcal{H}$ of *W*, which has no finite sub-cover. This contradicts our assumption. Hence, (W,τ) is an neutrosophic supra simply *b*-compact space.

(ii) Let (W,τ) be an neutrosophic supra simply b-compact space. Suppose that (W,τ) is not an neutrosophic supra compact space. Therefore, there exists an neutrosophic supra open cover $\mathfrak{R}$(say) of W, which has no finite sub-cover. Since, every neutrosophic supra open set is an neutrosophic supra simply b-open set, so we have an neutrosophic supra simply b-open cover $\mathfrak{R}$ of W, which has no finite sub-cover. This contradicts our assumption. Hence, (W,τ) is an neutrosophic supra compact space.

Theorem 3.8. If $\xi:(W,\tau_1)\to(M,\tau_2)$ is an neutrosophic supra open function and (M,τ_2) is an neutrosophic supra compact space, then (W,τ_1) is an neutrosophic supra compact space.

Proof. Assume that $\xi:(W,\tau_1)\to(M,\tau_2)$ be an neutrosophic supra open function and (M,τ_2) be an neutrosophic supra compact space. Suppose $\mathcal{H}$ = $\{H_i$: $i\in\Delta$ and $H_i\in NSO(W)\}$ be an neutrosophic supra open cover of W. Therefore, $\xi(\mathcal{H})=\{\xi(H_i)$: $i\in\Delta$ and $\xi(H_i)\in NSO(M)\}$ is an neutrosophic supra open cover of M. Since, (M,τ_2) is an neutrosophic supra compact space, so there exists a finite sub-cover say $\{\xi(H_1), \xi(H_2), \ldots\ldots., \xi(H_n)\}$ such that $M \subseteq\cup\{\xi(H_i)$: i=1, 2, …, $n\}$. This implies, $\{H_1, H_2, \ldots, H_n\}$ is a finite sub-cover for W. Hence, (W,τ_1) is an neutrosophic supra compact space.

Theorem 3.9. Suppose that (W,τ_1) and (M,τ_2) be two NSTSs.

(i) If $\xi:(W,\tau_1)\to(M,\tau_2)$ is an neutrosophic supra b-open function and (M,τ_2) is an neutrosophic supra b-compact space, then (W,τ_1) is an neutrosophic supra compact space.

(ii) If $\xi:(W,\tau_1)\to(M,\tau_2)$ is an neutrosophic supra simply b-open function and (M,τ_2) is an neutrosophic supra simply b-compact space, then (W,τ_1) is also an neutrosophic supra b-compact space.

Proof. (i) Let $\xi:(W,\tau_1)\to(M,\tau_2)$ be an neutrosophic supra b-open function and (M,τ_2) be an neutrosophic supra b-compact space. Let $\mathcal{H}=\{H_i$: $i\in\Delta$ and $H_i\in$N-bO$(W)\}$ be an neutrosophic supra open cover of W. Therefore, $\xi(\mathcal{H})=\{\xi(H_i)$: $i\in\Delta$ and $\xi(H_i)\in$N-bO$(M)\}$ is an neutrosophic supra b-open cover of M. Since, (M,τ_2) is an neutrosophic supra b-compact space, so there exists a finite sub-cover say $\{\xi(H_1), \xi(H_2), \ldots\ldots., \xi(H_n)\}$ such that $M \subseteq\cup\{\xi(H_i)$: i=1, 2, …, $n\}$. This implies, $\{H_1, H_2, \ldots, H_n\}$ is a finite sub-cover for W. Hence, (W,τ_1) is an neutrosophic supra compact space.

(ii) Suppose that $\xi:(W,\tau_1)\to(M,\tau_2)$ be an neutrosophic supra simply b-open function and (M,τ_2) be an neutrosophic supra simply b-compact space. Let $\mathcal{H}=\{K_i$: $i\in\Delta$ and $K_i\in$N-bO$(W)\}$ be an neutrosophic supra b-open cover of W. Therefore, $\xi(\mathcal{H})=\{\xi(K_i)$: $i\in\Delta$ and $\xi(K_i)\in$NSS-b-O$(M)\}$ is an neutrosophic supra simply b-open cover of M. Since, (M,τ_2) is an neutrosophic supra simply b-compact space, so there exists a finite sub-cover say $\{\xi(K_1), \xi(K_2), \ldots\ldots., \xi(K_n)\}$ such that $M\subseteq\cup\{\xi(K_i)$: i=1, 2, …, $n\}$. Therefore, $\{K_1, K_2, \ldots, K_n\}$ is a finite sub-cover for W. Hence, $(W,\tau_1)$is an neutrosophic supra b-compact space.

Theorem 3.10. Let (W,τ_1) and (M,τ_2) be two NSTSs.

(i) If $\xi:(W,\tau_1)\to(M,\tau_2)$ is an neutrosophic supra b-continuous function, then $\xi(Q)$ is an neutrosophic supra simply b-compact set in M whenever Q is an neutrosophic supra b-compact set relative to W.

(ii) If $\xi:(W,\tau_1)\to(M,\tau_2)$ is an neutrosophic supra b-continuous function, then $\xi(Z)$ is an neutrosophic supra compact set in M whenever Z is an neutrosophic supra b-compact set relative to W.

Proof. (i) Suppose that $\xi:(W,\tau_1)\to(M,\tau_2)$ be an neutrosophic supra b-continuous function and Q be an neutrosophic supra b-compact set relative to W. Let $\mathcal{H}=\{H_i$: $i\in\Delta$ and $H_i\in N^s$-bO$(M)\}$ be an neutrosophic supra simply b-open cover of $\xi(Q)$. Since, every NSS-b-O-set is an NS-b-O-set, so $\mathcal{H}=\{H_i$: $i\in\Delta$ and $H_i\in$NSS-b-O$(M)\}$ is an neutrosophic supra b-open cover of $\xi(Q)$. By hypothesis $\xi^{-1}(\mathcal{H})$= $\{\xi^{-1}(H_i)$: $i\in\Delta$ and $\xi^{-1}(H_i)\in$NSS-b-O$(M)\}$ is an neutrosophic supra b-open cover of $\xi^{-1}(\xi(Q))=Q$.

Since, Q is an neutrosophic supra b-compact set relative to W, so there exists a finite sub-cover of Q say $\{H_1, H_2, H_3, \ldots, H_n\}$ such that $Q \subseteq \cup_i \{H_i : i=1,2,\ldots, n\}$.

Now, $Q \subseteq \cup_i \{H_i : i=1,2,\ldots, n\}$

$\Rightarrow \xi(Q) \subseteq \cup_i \{\xi(H_i) : i=1,2,\ldots, n\}$

Therefore, there exist a finite sub-cover $\{\xi(H_1), \xi(H_2), \xi(H_3), \ldots, \xi(H_n)\}$ of $\xi(Q)$ such that $\xi(Q) \subseteq \cup_i \{\xi(H_i) : i=1,2,\ldots, n\}$. Hence, $\xi(Q)$ is an neutrosophic supra simply b-compact set relative to M.

(ii) Let $\xi:(W,\tau_1) \rightarrow (M,\tau_2)$ be an neutrosophic supra b-continuous function and Z be an neutrosophic supra b-compact set relative to W. Let $\mathcal{H}=\{H_i : i \in \Delta$ and $H_i \in N\text{-}bO(M)\}$ be an neutrosophic supra open cover of $\xi(Z)$. By hypothesis $\xi^{-1}(\mathcal{H})=\{\xi^{-1}(H_i) : i \in \Delta$ and $\xi^{-1}(H_i) \in N\text{-}bO(M)\}$ is an neutrosophic supra b-open cover of $\xi^{-1}(\xi(Z))=Z$. Since, Z is an neutrosophic supra b-compact set relative to W, so there exists a finite sub-cover of Z say $\{H_1, H_2, H_3, \ldots, H_n\}$ such that $Z \subseteq \cup_i \{H_i : i=1,2,\ldots, n\}$.

Now, $Z \subseteq \cup_i \{H_i : i=1,2,\ldots, n\}$

$\Rightarrow \xi(Z) \subseteq \cup_i \{\xi(H_i) : i=1,2,\ldots, n\}$

Therefore, there exist a finite sub-cover $\{\xi(H_1), \xi(H_2), \xi(H_3), \ldots, \xi(H_n)\}$ of $\xi(Z)$ such that $\xi(Z) \subseteq \cup_i \{\xi(H_i) : i=1,2,\ldots, n\}$. Hence, $\xi(Z)$ is an neutrosophic supra compact set relative to M.

Theorem 3.11. Every neutrosophic supra simply continuous function from an NSTS (W,τ_1) to another NSTS (M,τ_2) is an neutrosophic supra continuous function.

Proof. Suppose that $\xi:(W,\tau_1) \rightarrow (M,\tau_2)$ be an neutrosophic supra simply continuous function. Let Q be an NSO-set in (M,τ_2). By hypothesis $\xi^{-1}(Q)$ is an NSSO-set in (W,τ_1). It is known that, every NSSO-set is an NSO-set, so $\xi^{-1}(Q)$ is an NSSO-set in (W,τ_2). Therefore, $\xi^{-1}(Q)$ is an NSSO-set in (W,τ_2), whenever Q is an NSO-set in (M,τ_2). Hence, $\xi:(W,\tau_1) \rightarrow (M,\tau_2)$ is an neutrosophic supra simply continuous function.

Theorem 3.12. Every neutrosophic supra continuous function from an NSTS (W,τ_1) to another NSTS (M,τ_2) is an neutrosophic supra simply b-continuous function.

Proof. Let $\xi:(W,\tau_1) \rightarrow (M,\tau_2)$ be an neutrosophic supra continuous function. Let Q be an NSO-set in (M,τ_2). By hypothesis $\xi^{-1}(Q)$ is an NSO-set in (W,τ_1). Since, every NSO-set is an NSS-b-O set, so $\xi^{-1}(Q)$ is an NSS-b-O set in (W,τ_2). Therefore, $\xi^{-1}(Q)$ is an NSS-b-O set in (W,τ_2), whenever Q is an NSO-set in (M,τ_2). Hence, $\xi:(W,\tau_1) \rightarrow (M,\tau_2)$ is an neutrosophic supra simply b-continuous function.

Theorem 3.13. Every neutrosophic supra simply b-continuous function from an NSTS (W,τ_1) to another NSTS (M,τ_2) is an neutrosophic supra b-continuous function.

Proof. Suppose that $\xi:(W,\tau_1) \rightarrow (M,\tau_2)$ be an neutrosophic supra simply b-continuous function. Let Q be an NSO-set in (M,τ_2). By hypothesis $\xi^{-1}(Q)$ is an NSS-b-O-set in (W,τ_1). Since, every NSS-b-O-set is an NS-b-O-set, so $\xi^{-1}(Q)$ is an NS-b-O-set in (W,τ_2). Therefore, $\xi^{-1}(Q)$ is an NS-b-O-set in (W,τ_2) whenever Q is an NSO-set in (M,τ_2). Hence, $\xi:(W,\tau_1) \rightarrow (M,\tau_2)$ is an neutrosophic supra b-continuous function.

Theorem 3.14. If $\xi:(W,\tau_1) \rightarrow (M,\tau_2)$ be an NSS-b-Continuous mapping and $\gamma:(M,\tau_2) \rightarrow (L,\tau_3)$ be an NS-Continuous mapping, then the composition mapping $\gamma \circ \xi: (W,\tau_1) \rightarrow (L,\tau_3)$ is an NSS-b-Continuous mapping.

Proof. Suppose that Q be an NSO-set in (L,τ_3). Since, $\gamma:(M,\tau_2) \rightarrow (L,\tau_3)$ is an NS-Continuous mapping, so $\gamma^{-1}(Q)$ is an NSO-set in (M,τ_2). Further, since $\xi:(W,\tau_1) \rightarrow (M,\tau_2)$ is an NSS-b-Continuous mapping, so $\xi^{-1}(\gamma^{-1}(Q))= (\gamma \circ \xi)^{-1}(Q)$ is an NSS-b-O-set in (W,τ_1). Hence, $(\gamma \circ \xi)^{-1}(Q)$ is an NSS-b-O-set in (W,τ_1), whenever Q is an NSO-set in (L,τ_3). Therefore, $\gamma \circ \xi: (W,\tau_1) \rightarrow (L,\tau_3)$ is an NSS-b-Continuous mapping.

4. Conclusion: In this article, we have established the notions of neutrosophic supra compactness, neutrosophic supra simply compactness via neutrosophic supra topological spaces. Further, we have proved some theorems on neutrosophic supra compactness, neutrosophic supra simply compactness. We hope that, in future based on these notions of neutrosophic supra simply open set and neutrosophic supra simply compactness many new investigations can be done.

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