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Neutrosophic Triplet m – Banach Spaces

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Abstract: Neutrosophic triplet theory has an important place in neutrosophic theory. Since the neutrosophic triplet set (Nts), which have the feature of having multiple unit elements, have different units than the classical unit, they have more features than the classical set. Also, Banach spaces are complete normed vector space defined by real and complex numbers that are studied historically in functional analysis. Thus, normed space and Banach space have an important place in functional analysis. In this article, neutrosophic triplet m - Banach spaces (NtmBs) are firstly obtained. Then, some definitions and examples are given for NtmBs. Based on these definitions, new theorems are given and proved. In addition, it is shown that NtmBs is different from neutrosophic triplet Banach space (NtBs). Furthermore, it is shown that relationship between NtmBs and NtBs. So, we added a new structure to functional analysis and neutrosophic triplet theory.

Keywords: neutrosophic triplet set, neutrosophic triplet normed space, neutrosophic triplet Banach space, neutrosophic triplet m - normed space, neutrosophic triplet m – Banach space

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1 Introduction

Neutrosophic theory [1] has also supported the scientific world with more objective solutions by obtaining new solutions and methods in many fields in both application sciences and theoretical sciences. Neutrosophic theory was obtained by Smarandache in order to obtain more objective results by taking into account the effects of uncertainties encountered in science in 1998 [1]. A neutrosophic number is formulated by (T, I, F). Where, T is truth function; I is indeterminacy function and F is falsity function and these functions's values are independently. Thus, neutrosophic theory is generalized of fuzzy theory [2] and intuitionistic fuzzy theory [3] and neutrosophic theory is more useful than fuzzy theory and intuitionistic fuzzy theory. Thus, many researchers studied neutrosophic theory for these reasons [4-6]. Recently, Olgun et al. studied neutrosophic logic on the decision tree

[7]; Şahin et al. obtained decision making application for single – valued neutrosophic set [8] and Uluçay et al. introduced decision making application for neutrosophic soft expert graphs [9]. Uluçay et al. [10] proposed an outranking approach for MCDM-problems with neutrosophic multi-sets. Uluçay and Şahin [11] defined the concepts of neutrosophic multi group. Bakbak et al. [12] developed some new operations. Uluçay et al. [13] introduced a new hybrid distance-based similarity measure for refined neutrosophic sets.

Neutrosophic triplet structures, which are a sub-branch of the neutrosophic theory, also aimed to carry the new advantages of neutrosophy to the algebraic structure. For this reason, many studies have been carried out on neutrosophic triplet structures. Thus, many structures in classical algebra were reconsidered in neutrosophic theory and new features emerged. Thus, a neutrosophic triplet structure has become available in fixed point theory Also, Smarandache and Ali studied neutrosophic triplet set (Nts) [14]. Since the neutrosophic triplet set (Nts), which have the feature of having multiple unit elements, have different units than the classical unit, they have more features than the classical set. A Nts k is formulated by (k, neut(k), anti(k)). Furthermore, the sets of have been studied by the scholars, on neutrosophic sets [15-20] neutrosophic triplet structures in neutrosophic triplet algebraic structures [21-26], some metric spaces on neutrosophic triplet [27-32]. Recently, Şahin et al. studied Nt m - metric space [33]; Zhang et al. obtained cyclic associative neutrosophic extended triplet groupoids [34]; Sahin et al. obtained Nt normed space [22], Shalla et al. introduced direct and semi-direct product of neutrosophic extended triplet group [35]; Şahin et al. Nt partial bipolar metric space [36]; Şahin et al. Nt partial g-metric space [37], Kandasamy et al. obtained Nts in neutrosophic rings [38], Shalla et al. introduced neutrosophic extended triplet group action [44].

Metric spaces, normed spaces and Banach spaces have an important place in classical mathematics. Metric spaces are widely used, especially in fixed point theory. Thus, Asadi, Karapınar and Salimi introduced m - metric spaces [39] in 2014. m - metric space is a generalized form of classical metric space and classical p - metric space. The m - metric spaces have an important role in fixed point theory. Recently, Souayah et al. obtained fixed point theorems for m – metric space [40]; Patle et al. studied mappings in m – metric space [41] and Pitchaimani et. al introduced $-contraction on m – metric space [42]. Also, Normed spaces and Banach spaces, which are special cases of normed spaces, have an important usage area, especially in the field of analysis.

In this article, we have defined Ntmns with a more specific structure than neutrosophic triplet m-metric spaces. We discussed the properties of this structure and proved the theorems related to this structure. We also discussed the relationship between this structure and the Ntmms. In addition, with the help of some definitions in Ntmms, we obtained important definitions such as convergence and Cauchy sequence in Ntmns. Also, we have defined MtmBs. We compared these structures with previously obtained neutrosophic triplet structures. Thanks to this comparison, we have determined that the structures we have obtained have different and new features than others. Thus, we added a new structure to the neutrosophic triplet theory and prepared the ground for new structures that can be obtained. In Section 2, we give definitions and properties for neutrosophic structures [14], [36] and [37]. In Section 3, we define Ntmns and NtmBs and we give some properties for Ntmns and NtmBs. Furthermore, we obtain neutrosophic triplet m – metric space (NTmms) reduced by Ntmns. Also, we show that Ntns are different from the Ntmns due to triangular inequality. Then, we examine relationship between Ntmns and Ntns. In Section 4, we give conclusions.

2 Preliminaries

Definition 2.1 [14]: Let μ be a binary operation. An Nts (L, μ) is a set such that for l ∈ L,

i) There is neutral of "l" such that l μneut(l) = neut(l)μl = l,

ii) There is anti of "l" such that lμanti(l) = anti(l)μl = neut(l).

Also, an Nt "l" is showed with (l, neut(l), anti(l)).

Furthermore, neut(l) must different from classical unit element.

Definition 2.2: [43] Let (L, μ, π) be an Nts with two binary operations μ and π. Then (L, μ, π) is called Ntf if the following conditions are satisfied.

1. (L, μ) is a commutative Nt group with respect to μ.

2. (L, π) is an Nt group with respect to π.

3. kπ(l μ m) = (k πl)μ(kπm); (l μm)πk = (lπk)μ(mπk) for all k, l, m ∈ L.

Definition 2.3: [22] Let (L, μ_1, π_1) be an Ntf and let (V, μ_2, π_2) be an Nts with binary operations "μ_2" and "π_2". Then (V,μ_2, π_2) is called an Ntvs if the following conditions are satisfied.

i) mμ_2n ∈ V and m $\#_2$s ∈ V; m, n ∈ V and s∈ L;

ii) $(m\mu_2 n)\ \mu_2 l = m\mu_2\ (n\mu_2 l)$; m, n, l ∈ V;

iii) $m\mu_2 n = n\mu_2 m$; m, n ∈ V;

iv) $(m\mu_2 n)\ \pi_2 s = (m\pi_2 s)\ \mu_2(n\pi_2 s)$; s ∈ L and m, n ∈ V;

v) $(s\mu_1 p)\ \pi_2 m = (s\pi_2 m)\ \mu_1(p\pi_2 m)$; s, p ∈ L and m ∈ V;

vi) $(s\pi_1 p)\ \pi_2 m = s\pi_1(p\pi_2 m)$; s, p ∈ L and m ∈ V;

vii) there exists at least an element s ∈ L for each element m such that

$m\pi_2$ neut(s)= neut(s) π_2 m = m; m ∈ V.

Definition 2.4: [22] Let (V, μ_2, π_2) be an Ntvs on (L, μ_1, π_1) Ntf. If the function $\|.\| : V \to \mathbb{R}^+ \cup \{0\}$ is satisfied the following properties, then the function ‖.‖ is an Ntn.

Where,

f: L X V $\to \mathbb{R}^+ \cup \{0\}$ is a function such that

f(k, l) = f(k, anti(l)) and

if l = neut(l), then f(k, l) = 0; k ∈ L, l ∈ V

a) ‖l‖ ≥ 0,

b) If l = neut(l), then ‖l‖ = 0,

c) ‖$k\ \pi_2$ l‖ = f(k, l).‖l‖,

d) ‖anti(l)‖ = ‖l‖,

e) If there exists at least an element m ∈ N for k, l ∈ V pair such that

‖kμ_2 l‖ ≤ ‖kμ_2l$\mu_2$neut(m)‖, then ‖kμ_2l$\mu_2$neut(m)‖ ≤ ‖k‖+‖l‖; k, l, m ∈ V.

Also, ((V, μ_2, π_2), ‖.‖) is called a Ntns.

Definition 2.5: [22] Let ((V, μ_2, π_2), ‖.‖) be an Ntns. Let $\{a_n\}$ be a sequence in ((V, μ_2, π_2), ‖.‖) and let m be an NTm reduced by ((V, μ_2, π_2), ‖.‖). If each $\{a_n\}$ Nt Cauchy sequence in ((V, μ_2, π_2), ‖.‖) is Nt convergent according to Ntmm, then ((V, μ_2, π_2), ‖.‖) is called an Nt Banach space (NtBs).

Theorem 2.6:[22] Let (L, μ) be an NT group without zero divisors, with respect to μ. For l ∈ L;

(i) neut(neut(l)) = neut(l),
(ii) anti(neut(l)) = neut(l)),
(iii) anti(anti(l)) = l,
(iv) neut(anti(l)) = neut(l).

Definition 2.7:[33] Let L be a nonempty set and $m^d: L \times L \to \mathbb{R}^+ \cup \{0\}$ be a function. Then,

(i) $m^d{}_{ln} = \min\{m(l, l), m(n, n)\} = m(l, l) \vee m(n, n)$; l, n ∈L

(ii) $M^d{}_{ln} = \max\{m(l, l), m(n, n)\} = m(l, l) \wedge m(n, n)$; l, n ∈L

Definition 2.8: [33] Let (L, μ) be an Nts and m:LxL→ $\mathbb{R}^+ \cup \{0\}$ be a function. If (L, μ) and m satisfy the following properties, then m is called an Nt m –metric space.

a) For all l, n ∈ L, l μ n ∈ L,

b) If m(l, l) = m(n, n) = m(l, n) = 0, then l = n,

c) $m^d{}_{ln} \le m(l, n)$,

d) m(l, n) = m(l, n),

e) If there exists at least an element s ∈ L for each pair k, l ∈ L such that

m(k, l) ≤ m(k, l μ neut(s)), then (m(k, l μ neut(s))) – m_{kl}) ≤ (m(k, l) – m_{ks})+(m(l, s) – m_{ls}).

Also, ((L, μ), m) is called an Ntmms.

3 Neutrosophic Triplet m – Normed Space

In this section, we have defined Ntmns with a more specific structure than Ntmms. We discussed the properties of this structure and proved the theorems related to this structure. We also discussed the relationship between this structure and the Ntmms. In addition, with the help of some definitions in Ntmms, we obtained important definitions such as convergence and Cauchy sequence in Ntmns.

Definition 3.1: Let V be an Nts and let $m: V \times V \to \mathbb{R}^+ \cup \{0\}$ be a function. Then,

(i) $m_{ln} = \min\{\|neut(l)\|_m, \|neut(n)\|_m\}$,

(ii) $M_{ln} = \max\{\|neut(l)\|_m, \|neut(n)\|_m\}$.

Definition 3.2: Let (V, μ_2, π_2) be an Ntv on the Ntf (L, μ_1, π_1). Then function $\|.\|_m$:V → $\mathbb{R}^+ \cup \{0\}$ is an Nt m – norm (Ntmn) such that

a) $0 \le \|neut(l)\|_m \le \|l\|_m$,

b) If $\|l\ \mu_2 neut(n)\|_m = \|neut(l)\|_m = \|neut(n)\|_m = 0$, then l = n.

c) $\|\beta\pi_2\ l\|_m = f(\beta, l). \|l\|_m$. Where, f: L x V → $\mathbb{R}^+ \cup \{0\}$, f(β, l) = f(β, anti(l)) is a function.

d) $\|anti(l)\|_m = \|l\|_m$,

(e) If there exists at least l p ∈ V for each l, n ∈ V pair of elements such that

$\|l\ \mu_2\ n\|_m \le \|l\ \mu_2\ n\ \mu_2\ neut(p)\|_m$; then

$\|l\ \mu_2 n\ \mu_2 neut(p)\|_m - m_{ln} \le \|l\|_m + \|n\|_m - m_{lp} - m_{np}$.

In this case, ((V, μ_2, π_2), $\|.\|_m$) is called Nt m – normed space (Ntmns).

Corollary 3.3: From Definition 3.2, we obtain that $m_{ln} \le \|l\|_m$ and $m_{ln} \le \|n\|_m$

since $0 \le \|neut(l)\|_m \le \|l\|_m$ and $0 \le \|neut(n)\|_m \le \|n\|_m$.

Corollary 3.4: From Definition 2.4 and Definition 3.2, an Ntmns is different from an Ntns since the triangle inequalities are different in these definitions.

Example 3.5: Let P(M) = {∅, {l}, {n}, {l, n}} be a set and μ be binary operation such that

$$K\ \mu\ N = \begin{cases} N \backslash K, & \text{if } s(K)<s(N) \\ K \backslash N, & \text{if } s(K)>s(N) \\ M, & \text{if } s(K)=s(N) \wedge K \neq N \\ K, & \text{if } K=N \end{cases}.$$

Where, it is clear that (P(M)\ ∅, μ) be an Nts. Also,

neut({l}) = {l}, anti({l}) = {l}; neut({n}) = {n}, anti({n}) = {n}; neut({l, n}) = {l, n}, anti({l, n}) = {l, n}.

Then, from Definition 2.2, (P(M)\ ∅, μ, ∪) is an Ntf. Furthermore, from Definition 2.3, (P(M)\ ∅, μ, ∪) is an Ntvs.

Now, we show that

$\|K\|_m$: P(M)\∅→ $\mathbb{R}^+ \cup \{0\}$, $\|K\|_m = \begin{cases} 2^{s(K)} & , \text{if } K \neq \{l, n\} \\ 2^{s(K)-1} & , \text{if } K = \{l, n\} \end{cases}$ is an Ntmn such that f: $P(M)\setminus\emptyset$ 1 $P(M)\setminus\emptyset \rightarrow \mathbb{R}^{+} \cup\{0\}$, $f(K,N) = \begin{cases} \frac{2^{s(K\cup N)-1}}{2^{s(N)}} & , \text{if } K\cup N = \{l, n\} \\ \frac{2^{s(K\cup N)}}{2^{s(N)}} & , \text{if } K\cup N \neq \{l, n\} \end{cases}$. Where s(K) is the number of elements of the set K.

Since anti({l}) = {l}, anti({n}) = {n}, anti({l, n}) = {l, n}; it is clear that f(K, N) = f(K, anti(N)) for K, N ∈ $P(M)\setminus\emptyset$.

a) Since neut({l}) = {l}, neut({n}) = {n}, neut({l, n}) = {l, n}; it is clear that $0 \leq \|neut(l)\|_m \leq \|l\|_m$ for K, N ∈ $P(M)\setminus\emptyset$.

b) There are not K, N ∈ $P(M)\setminus\emptyset$ such that $\|K\,\mu\, neut(N)\|_m = \|neut(K)\|_m = \|neut(N)\|_m = 0$.

c) We assume that $K\cup N = \{l, n\}$. Thus, $\|K \cup N\|_m = 2^{s(K\cup N)-1} = \frac{2^{s(K\cup N)-1}}{2^{s(N)}} \cdot 2^{s(N)}$ = f(K, N). $\|N\|_m$ for K, N ∈ $P(M)\setminus\emptyset$.

We assume that $K\cup N \neq \{l, n\}$. Thus, $\|K \cup N\|_m = 2^{s(K\cup N)} = \frac{2^{s(K\cup N)}}{2^{s(N)}} \cdot 2^{s(N)}$ = f(K, N). $\|N\|_m$ for K, N ∈ $P(M)\setminus\emptyset$.

d) Since anti({l}) = {n}, anti({n}) = {n}, anti({l, n}) = {l, n}; we obtain that $\|anti(l)\|_m = \|l\|_m$ for K, N ∈ $P(M)\setminus\emptyset$.

e) For {l} ∈ $P(M)\setminus\emptyset$, we obtain that $\|\{l\}\,\mu\,\{l\}\|_m \leq \|(\{l\}\,\mu\,\{l\}\,\mu\, neut(\{n\})\|_m$. Thus,

$\|\{l\}\,\mu\,\{l\}\,\mu\, neut(\{n\})\|_m \leq \|\{l\}\|_m + \|\{l\}\|_m + m_{\{l\}\{l\}} - m_{\{l\}\{n\}} - m_{\{l\}\{n\}})$.

For {n} ∈ $P(M)\setminus\emptyset$, we obtain that $\|\{n\}\,\mu\,\{n\}\|_m \leq \|(\{n\}\,\mu\,\{n\}\,\mu\, neut(\{n\})\|_m$. Thus,

$\|\{n\}\,\mu\,\{n\}\,\mu\, neut(\{l\})\|_m \leq \|\{n\}\|_m + \|\{n\}\|_m + m_{\{n\}\{n\}} - m_{\{l\}\{n\}} - m_{\{l\}\{n\}})$.

For {l, n} ∈ $P(M)\setminus\emptyset$, we obtain that $\|\{l,n\}\,\mu\,\{l,n\}\|_m \leq \|(\{l,n\}\,\mu\,\{l,n\}\,\mu\, neut(\{n\})\|_m$. Thus,

$\|\{l,n\}\,\mu\,\{l,n\}\,\mu\, neut(\{n\})\|_m \leq \|\{l,n\}\|_m + \|\{l,n\}\|_m + m_{\{l,n\}\{l,n\}} - m_{\{l,n\}\{n\}} - m_{\{l,n\}\{n\}})$.

For {l}, {n} ∈ $P(M)\setminus\emptyset$, we obtain that $\|\{l\}\,\mu\,\{n\}\|_m \leq \|(\{l\}\,\mu\,\{n\}\,\mu\, neut(\{l,n\})\|_m$. Thus,

$\|\{l\}\,\mu\,\{n\}\,\mu\, neut(\{l,n\})\|_m \leq \|\{l\}\|_m + \|n\}\|_m + m_{\{l\}\{n\}} - m_{\{l,n\}\{n\}} - m_{\{l,n\}\{l\}})$.

For {n}, {l} ∈ $P(M)\setminus\emptyset$, we obtain that $\|\{n\}\,\mu\,\{l\}\|_m \leq \|(\{n\}\,\mu\,\{l\}\,\mu\, neut(\{l,n\})\|_m$. Thus,

$\|\{n\}\,\mu\,\{l\}\,\mu\, neut(\{l,n\})\|_m \leq \|\{n\}\|_m + \|\{l\}\|_m + m_{\{l\}\{n\}} - m_{\{l,n\}\{n\}} - m_{\{l,n\}\{l\}})$.

For {n}, {l, n} ∈ $P(M)\setminus\emptyset$, we obtain that $\|\{n\}\,\mu\,\{l,n\}\|_m \leq \|(\{n\}\,\mu\,\{l,n\}\,\mu\, neut(\{l\})\|_m$. Thus,

$\|\{n\}\,\mu\,\{l,n\}\,\mu\, neut(\{l\})\|_m \leq \|\{n\}\|_m + \|\{l,n\}\|_m + m_{\{n\}\{l,n\}} - m_{\{l,n\}\{x\}} - m_{\{n\}\{l\}})$.

For {l, n}, {n} ∈ $P(M)\setminus\emptyset$, we obtain that $\|\{l,n\}\,\mu\,\{n\}\|_m \leq \|(\{l,n\}\,\mu\,\{n\}\,\mu\, neut(\{l\})\|_m$. Thus,

$\|\{l, n\}\ \mu\ \{ n\}\ \mu\ \text{neut}(\{l\})\|_m \ \le\ \|\{l, n\}\|_m + \|\{ n\}\|_m + m_{\{l,n\}\{n\}} - m_{\{n\}\{l\}} - m_{\{l,n\}\{l\}})$.

For {l}, {l, n} ∈ P(M)\∅, we obtain that $\|\{l\}\ \mu\ \{l, n\}\|_m \le \|(\{l\}\ \mu\ \{l, n\}\ \mu\ \text{neut}(\{n\})\|_m$. Thus,

$\|\{l\}\ \mu\ \{l, n\}\ \mu\ \text{neut}(\{n\})\|_m \ \le\ \|\{l\}\|_m + \|\{l, n\}\|_m + m_{\{l\}\{l,n\}} - m_{\{l,n\}\{n\}} - m_{\{n\}\{l\}})$.

For {l, n}, {l} ∈ P(M)\∅, we obtain that $\|\{l, n\}\ \mu\ \{l\}\|_m \le \|(\{l, n\}\ \mu\ \{l\}\ \mu\ \text{neut}(\{n\})\|_m$. Thus,

$\|\{l, n\}\ \mu\ \{ n\}\ \mu\ \text{neut}(\{n\})\|_m \ \le\ \|\{l, n\}\|_m + \|\{ l\}\|_m + m_{\{l,n\}\{l\}} - m_{\{l,n\}\{n\}} - m_{\{l\}\{n\}})$.

Hence, if there exists at least a P ∈ P(M)\∅ for each K, N ∈ P(M)\∅ pair of elements such that

$\|K\ \mu\ N\|_m \le \|K\ \mu\ N\ \mu\ \text{neut}(P)\|_m$; then

$\|K\ \mu\ N\ \mu\ \text{neut}(P)\|_m - m_{KN} \ \le\ \|K\|_m + \|N\|_m - m_{KP} - m_{PN}$.

Therefore, $\|K\|_m$ is an Ntmns.

Theorem 3.6: Let $((V, \mu_2, \pi_2)\ , \|.\|_m)$ be an Ntmns. Then, the function

$m: V \times V \to \mathbb{R}^+ \cup \{0\}$ defined by $m(l, n) = \|l\ \mu_2\ \text{anti}(n)\|_m$ is an Ntmms.

Proof: Let l, n, p ∈ V. From Definition 3.2,

a) Since (V, μ_2, π_2) is Ntvs, we obtain that l μ n ∈ V, for all l, n ∈ V.

b) If $m(l, n) = \|l\ \mu_2\ \text{anti}(n)\|_m = \|l\ \mu_2\ \text{anti}(l)\|_m = \|\text{neut}(l)\|_m = \|n\ \mu_2\ \text{anti}(n)\|_m = \|\text{neut}(n)\|_m = 0$, then l = n.

(c) We show that

$m^d_{ln} = \min\{m(l, l), m(n, n)\} = \min\{\|\text{neut}(l)\|_m, \|\text{neut}(n)\|_m\} \le m(l, n) = \|l\ \mu_2\ \text{anti}(n)\|_m$.

We assume that $\|l\ \mu_2\ \text{anti}(n)\|_m \le \|l\ \mu_2\ \text{anti}(n)\ \mu_2\ \text{neut}(p)\|_m$. From Definition 3.2,

$m(l, n) = \|l\ \mu_2\ \text{anti}(n)\|_m \le \|l\|_m + \|\text{anti}(n)\|_m + m_{a\text{anti}(n)} - m_{lp} - m_{\text{anti}(n)p}$. Also, since $\|\text{anti}(n)\|_m = \|n\|_m$ and Theorem 2.5; we obtain that

$$m(l, n) = \|l\ \mu_2\ \text{anti}(n)\|_m \le \|l\|_m + \|n\|_m + m_{ln} - m_{lp} - m_{np}. \quad (1)$$

There are two cases:

1) Let $\|\text{neut}(l)\|_m \le \|\text{neut}(n)\|_m$.

Then we show that $\|\text{neut}(l)\|_m \le m(l, n) \le \|l\|_m + \|n\|_m + m_{ln} - m_{lp} - m_{np}$.

We assume that

i) $\|neut(l)\|_m \le \|neut(n)\|_m \le \|neut(p)\|_m$. Thus, from (1),

$$\|neut(l)\|_m \le m(l,n) \le \|l\|_m + \|n\|_m + \|neut(l)\|_m - \|neut(l)\|_m - \|neut(n)\|_m.$$

Hence, $m^d{}_{ln} = \min(m(l, l), m(n, n)) = \min\{\|neut(l)\|_m, \|neut(n)\|_m\} = \|neut(l)\|_m \le m(l, n)$.

ii) $\|neut(l)\|_m \le \|neut(p)\|_m \le \|neut(n)\|_m$. Thus, from (1)

$$\|neut(l)\|_m \le m(l,n) \le \|l\|_m + \|n\|_m + \|neut(l)\|_m - \|neut(l)\|_m - \|neut(p)\|_m.$$ Hence, $m^d{}_{ln} = \min(m(l, l), m(n, n)) = \min\{\|neut(l)\|_m, \|neut(n)\|_m\} = \|neut(l)\|_m \le m(l, n)$.

iii) Let $\|neut(p)\|_m \le \|neut(l)\|_m \le \|neut(n)\|_m$. Thus, from (1)

$$\|neut(l)\|_m \le m(l,n) \le \|l\|_m + \|n\|_m + \|neut(p)\|_m - \|neut(l)\|_m - \|neut(p)\|_m.$$

Therefore, $m^d{}_{ln} = \min(m(l, l), m(n, n)) = \min\{\|neut(l)\|_m, \|neut(n)\|_m\} = \|neut(l)\|_m \le m(l, n)$.

2) Let $\|neut(n)\|_m \le \|neut(l)\|_m$.

Then we show that $\|neut(n)\|_m \le m(l,n) \le \|l\|_m + \|n\|_m + m_{ln} - m_{lp} - m_{np}$.

We assume that

i) $\|neut(n)\|_m \le \|neut(l)\|_m \le \|neut(p)\|_m$. Thus, from (1),

$$\|neut(n)\|_m \le m(l,n) \le \|l\|_m + \|n\|_m + \|neut(n)\|_m - \|neut(n)\|_m - \|neut(l)\|_m.$$

Hence, $m^d{}_{ln} = \min(m(l, l), m(n, n)) = \min\{\|neut(l)\|_m, \|neut(n)\|_m\} = \|neut(n)\|_m \le m(l, n)$.

ii) $\|neut(n)\|_m \le \|neut(p)\|_m \le \|neut(l)\|_m$. Thus, from (1)

$$\|neut(n)\|_m \le m(l,n) \le \|l\|_m + \|n\|_m + \|neut(n)\|_m - \|neut(n)\|_m - \|neut(p)\|_m.$$

Hence, $m^d{}_{ln} = \min(m(l, n), m(n, n)) = \min\{\|neut(l)\|_m, \|neut(n)\|_m\} = \|neut(n)\|_m \le m(l, n)$.

iii) Let $\|neut(p)\|_m \le \|neut(n)\|_m \le \|neut(l)\|_m$. Thus, from (1)

$$\|neut(n)\|_m \le m(l,n) \le \|l\|_m + \|n\|_m + \|neut(n)\|_m - \|neut(p)\|_m - \|neut(p)\|_m.$$

Hence, $m^d{}_{ln} = \min(m(l, l), m(n, n)) = \min\{\|neut(l)\|_m, \|neut(n)\|_m\} = \|neut(n)\|_m \le m(l, n)$.

(d) For any n ∈ V; suppose that $m(l, p) = \|l \mu_2 \text{ anti}(p)\| \le \|l \mu_2 \text{ anti}(p) \mu_2 \text{neut}(n)\|$. Then

$m(l, p) = \|a \mu_2 \text{ anti}(p)\| \le \|l \mu_2 \text{ anti}(p) \mu_2 \text{neut}(n)\| = \|l \mu_2 \text{anti}(p) \mu_2 n \mu_2 \text{anti}(n)\|$.

As V is an Nt commutative group, we obtain that

$\|l\ \mu_2 \text{anti}(p)\ \mu_2\ n\ \mu_2 \text{anti}(n)\| = \|(l\ \mu_2 \text{anti}\ (n))\ \mu_2\ (\text{anti}(p)\ \mu_2\ n\|$

$\leq \|l\ \mu_2\ \text{anti}(n)\| + \|n\ \mu_2 \text{anti}\ (p)\|\| + m_{lp} - m_{ln} - m_{np}$

. $= m(l, n) + m(n, p) + m_{lp} - m_{ln} - m_{np}$

Thus; if m(l, p) ≤ m(l, p μ_2 neut(n)); then

(m(l, p μ neut(n))) – m_{lp}) ≤ (m(l, n) – m_{ln})+(m(n, p) – m_{np}).

Corollary 3.7: Every Ntmns is an Ntmms.

Definition 3.8: Let ((N, μ_2, π_2), $\|.\|_m$) be an Ntmns. Let m: Vx V→ R be an Ntmm defined by

m(l, n) = $\|l\ \mu_2\ \text{anti}(n)\|_m$.

Then, m is called the Ntmm reduced by ((V, μ_2, π_2), $\|x\|_m$).

Definition 3.9: Let ((V, μ_2, π_2), $\|.\|_m$) be a Ntmns. Let $\{l_n\}$ be a sequence in ((V, μ_2, π_2), $\|.\|_m$) and let m be an Ntmm reduced by ((V, μ_2, π_2,), $\|.\|_m$). For all ε > 0 and l ∈ V if there exists an $n_0 \in \mathbb{N}$ such that for all $n > n_0$

m (l, $\{l_n\}$) $-m_{ll_n} = \|l\ \mu_2 \text{anti}(\{l_n\})\|_m - m_{ll_n} < \varepsilon$, then $\{l_n\}$ m-sequence is said to Nt converge to x. It is denoted by $\lim_{n\to\infty} l_n = l$ or $l_n \to l$.

Definition 3.10: Let ((V, μ_2, π_2), $\|.\|_m$) be an Ntmns. Let $\{l_n\}$ be a sequence in ((V, μ_2, π_2), $\|.\|_m$) and let m be an Ntmm reduced by ((V, μ_2, π_2,), $\|.\|_m$). For all ε > 0 and l ∈ V if there exists an $n_0 \in \mathbb{N}$ such that for all n, m > n_0

m ($\{l_m\}$, $\{l_n\}$) $-m_{l_m l_n} = \|l_m\ \mu_2 \text{anti}(\{l_n\})\|_m - m_{l_m l_n} < \varepsilon$, then the sequence $\{l_n\}$ is called an Nt m - Cauchy sequence.

Definition 3.11: Let ((V, μ_2, π_2), $\|.\|_m$) be an Ntmns. Let $\{l_n\}$ be a sequence in ((V, μ_2, π_2), $\|.\|_m$) and let m be an Ntmm reduced by ((V, μ_2, π_2,), $\|.\|_m$). If each $\{l_n\}$ Nt m - Cauchy sequence in ((V, μ_2, π_2), $\|.\|_m$) is Nt convergent according to Ntmm, then ((V, μ_2, π_2), $\|.\|_m$) is called an Nt m - Banach space.

Corollary 3.12: From Definition 2.5 and Definition 3.11, an NtBs is different from a NtmBs since the triangle inequalities are different in these definitions.

Theorem 3.13: Let $((V, \mu_2, \pi_2), \|.\|)$ be an Ntns and l = neut(l), for all l ∈ V. Then, $\|l\|_m = \|l\| + n$ is an Ntmn. Where, $n \in \mathbb{R}^+$.

Proof:

a) Since, l = neut(l), it is clear that $0 \leq \|neut(l)\| + n \leq \|l\| + n$. Thus, $0 \leq \|neut(l)\|_m \leq \|l\|_m$

b) There are not l, n ∈ V such that

$\|l\ \mu_2 neut(n)\|_m = \|l\ \mu_2 neut(n)\| + n = \|neut(l)\|_m = \|neut(l)\| + n = \|neut(n)\|_m = \|neut(n)\| + n = 0.$

c) Since $((V, \mu_2, \pi_2), \|.\|)$ is an Ntns, we can define $\|k\pi_2\ a\| = f(k, l).\ \|l\|$. Also, we assume that

$\|k\pi_2\ l\|_m = f_m(k, l).\|l\|_m$ and $f_m(k, l) = (f(k, l).\ \|l\| + n)\backslash(\|l\| + n)$. Thus, we obtain that

$\|k\pi_2\ l\| + n = \|k\pi_2\ l\|_m = f(k, a).\ \|l\| + n = f_m(k, l).\ \|l\|_m.$

d) Since $((V, \mu_2, \pi_2), \|.\|)$ is an Ntns, we obtain $\|anti(l)\| = \|l\|$.

Thus, $\|anti(l)\| + n = \|anti(l)\|_m = \|l\| + n = \|l\|_m$.

e) We assume that there exists n ∈ V such that $\|l\ \mu_2 p\| \leq \|l\ \mu_2\ p\ \mu_2\ neut(n)\|$. (2)

Also, we obtain $\|l\ \mu_2\ p\| + n \leq \|l\ \mu_2\ p\ \mu_2\ neut(n)\| + n$.

Since $((V, \mu_2, \pi_2), \|.\|)$ is an Ntns, from (2), $\|l\ \mu_2\ p\ \mu_2\ neut(n)\| \leq \|l\| + \|p\|$. (3)

From (3), we obtain that $\|l\ \mu_2\ p\ \mu_2\ neut(n)\| + n \leq \|l\| + n + \|p\| + n$. Thus, we obtain that

$$\|l\ \mu_2 p\ \mu_2 neut(n)\|_m \leq \|l\|_m + \|p\|_m + n - n. \quad (4)$$

Furthermore, $m_{lp} = m_{ln} = m_{np} = n$, since, l = neut(l) and $\|neut(l)\| = 0$. Thus, from (4),

$\|l\ \mu_2\ p\ \mu_2 neut(n)\|_m - m_{lp} \leq \|l\|_m + \|p\|_m - m_{ln} - m_{np}$.

Corollary 3.14: If l = neut(l), an Ntmns can be obtained from an Ntns.

Theorem 3.15: Let $((V, \mu_2, \pi_2), \|.\|_m)$ be an Ntmns and l = neut(l) for all l ∈ V. If the following condition is satisfied, then $\|.\|_m$ an Ntn.

i) If l = neut(l), then $\|l\|_m = 0$.

ii) If l = neut(l), then f(k, l) = 0.

Proof:

Since $((V, \mu_2, \pi_2), \|.\|_m)$ is an Ntmns, f(k, l) = f(k, anti(l)). Also, from condition ii, f(k, l) = f(k, anti(l)).

a) Since $((V, \mu_2, \pi_2), \|.\|_m)$ is an Ntmns, it is clear that $\|l\|_m \geq 0$.

b) It is clear that from condition i.

c) Since $((V, \mu_2, \pi_2), \|.\|_m)$ is an Ntmns, we obtain that $\|k\pi_2\, l\|_m = f(k, l). \|l\|_m$.

d) Since $((V, \mu_2, \pi_2), \|.\|_m)$ is an Ntmns, we obtain that $\|anti(l)\|_m = \|l\|_m$.

e) From condition i, If l = neut(l), then $\|l\|_m = 0$. Since, l = neut(l), we obtain that $\|neut(l)\|_m = 0$, for all l ∈ V. Thus, we obtain that $m_{ln} = 0$, for all l, n ∈ V. (5)

Also, since $((V, \mu_2, \pi_2), \|.\|_m)$ is an Ntmns, If there exists at least n ∈ V for each l, p ∈ V pair of elements such that

$\|l\ \mu_2\ p\|_m \leq \|l\ \mu_2\ p\ \ \mu_2\ neut(n)\|_m$; then

$\|l\ \mu_2\ p\ \mu_2 neut(n)\|_m - m_{lp} \leq \|l\|_m + \|p\|_m - m_{lp} - m_{np}$.

Thus, from (5),

$\|l\ \mu_2\ p\ \mu_2 neut(n)\|_m \leq \|l\|_m + \|p\|_m$.

Conclusion

Metric spaces, normed spaces and Banach spaces have an important place in classical mathematics. Metric spaces are widely used, especially in fixed point theory. For this purpose, m - metric space [39] has been defined and many studies have been carried out on fixed point theories with this definition. Normed spaces and Banach spaces, which are special cases of normed spaces, have an important usage area, especially in the field of analysis. Neutrosophic theory [1] has also supported the scientific world with more objective solutions by obtaining new solutions and methods in many fields in both application sciences and theoretical sciences. Nt structures, which are a sub-branch of the neutrosophic theory, also aimed to carry the new advantages of neutrosophy to the algebraic structure. For this reason, many studies have been carried out on Nt structures. Thus, many structures in classical algebra were reconsidered in neutrosophic theory and new features emerged. In addition, m - metric space was considered in the Nt theory in 2020 and defined Ntmms [33]. Thus, a Nt structure has become available in fixed point theory. In this study, we have defined Ntmns with a more specific structure than Ntmms. We discussed the properties of this structure and proved the theorems related to this structure. We also discussed the relationship between this structure and the Ntmms. In addition, with the help of some definitions in Ntmms, we obtained important definitions such as convergence and Cauchy sequence in Ntmns. Also, we have defined MtmBs. We compared these structures with previously obtained Nt structures. Thanks to this comparison, we have determined that the structures we have obtained have different and new features than others. Thus, we added a new structure to the Nt theory and prepared the ground for new structures that can be obtained. In addition, by using Ntmns and NtmBs, researchers can obtain Nt m - inner product spaces and Nt m - Hilbert spaces. These structures can be the start of many new buildings.

Abbreviations

Nt: Neutrosophic triplet

Nts: Neutrosophic triplet set

Ntn: Neutrosophic triplet norm

Ntns: Neutrosophic triplet normed space

NtBs: Neutrosophic triplet Banach space

Ntmm: Neutrosophic triplet m - metric

NTmms: Neutrosophic triplet m - metric space

Ntf: Neutrosophic triplet field

Ntvs: Neutrosophic triplet vector space

Ntmn: Neutrosophic triplet m – norm

Ntmns: Neutrosophic triplet m – normed space

NtmBs: Neutrosophic triplet m –Banach space

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