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Neutrosophic Triplet Group Based on Set Valued Neutrosophic Quadruple Numbers

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Abstract: Smarandache introduced neutrosophic quadruple sets and neutrosophic quadruple numbers [45] in 2015. These sets and numbers are real or complex number valued. In this study, we firstly introduce set valued neutrosophic quadruple sets and numbers. We give some known and special operations for set valued neutrosophic quadruple numbers. Furthermore, Smarandache and Ali obtained neutrosophic triplet groups [30] in 2016. In this study, we firstly give neutrosophic triplet groups based on set valued neutrosophic quadruple number thanks to operations for set valued neutrosophic quadruple numbers. In this way, we define new structures using the together set valued neutrosophic quadruple number and neutrosophic triplet group. Thus, we obtain new results for set valued neutrosophic quadruple numbers and neutrosophic triplet groups based on set valued neutrosophic quadruple number.

Keywords: Neutrosophic triplet set, neutrosophic triplet group, neutrosophic triplet quadruple set, neutrosophic triplet quadruple number, set valued neutrosophic triplet quadruple set, set valued neutrosophic triplet quadruple number

1 Introduction

Smarandache defined neutrosophic logic and neutrosophic set [1] in 1998. In neutrosophic logic and neutrosophic sets, there is T degree of membership, I degree of indeterminacy and F degree of non-membership. These degrees are defined independently of each other. It has a neutrosophic value (T, I, F) form. In other words, a condition is handled according to both its accuracy and its inaccuracy and its uncertainty. Therefore, neutrosophic logic and neutrosophic set help us to explain many uncertainties in our lives. In addition, many researchers have made studies on this theory [2 - 27] and [52-57].

In fact, fuzzy logic and fuzzy set [28] were obtained by Zadeh in 1965. In the concept of fuzzy logic and fuzzy sets, there is only a degree of membership. In addition, intuitionistic fuzzy logic and intuitionistic fuzzy set [29] were obtained by Atanassov in 1986. The concept of intuitionistic fuzzy logic and intuitionistic fuzzy set includes membership degree, degree of indeterminacy and degree of non-membership. But these degrees are defined dependently of each other. Therefore, neutrosophic set is a generalized state of fuzzy and intuitionistic fuzzy set.

Furthermore, Smarandache and Ali obtained neutrosophic triplet set (NTS) and neutrosophic triplet groups (NTG) [30]. For every element "x" in NTS A, there exist a neutral of "x" and an opposite of "x". Also, neutral of "x" must different from the classical neutral element. Therefore, the NTS is different from the classical set. Furthermore, a neutrosophic triplet (NT) "x" is showed by <x, neut(x), anti(x)>. Also, many researchers have introduced NT structures [31-44]

Also, Smarandache introduced neutrosophic quadruple sets (NQS) and neutrosophic quadruple number (NQN) [45]. The NQSs are generalized state of neutrosophic set. A NQS is shown by {(x, yT, zI, tF): x, y, z, t ∈ ℝ or ℂ}. Where, x is called the known part and (yT, zI, tF) is called the unknown part and T, I, F have their usual neutrosophic logic means. Recently, researchers studied NQS and NQN. Akinleye, Smarandache, Agboola studied NQ algebraic structures [46]; Jun, Song, Smarandache obtained NQ BCK/BCI-algebras [47]; Muhiuddin, Al-Kenani, Roh, Jun introduced implicative NQ BCK-algebras and ideals [48]; Li, Ma, Zhang, Zhang studied neutrosophic extended triplet group based on NQNs [49]; Ma, Zhang, and Smarandache studied neutrosophic quadruple rings [50]; Kandasamy, Kandasamy and Smarandache obtained neutrosophic quadruple vector spaces and their properties [51].

In this study, we firstly introduce set valued neutrosophic quadruple set (SVNQS) and set valued neutrosophic quadruple number (SVNQN). In the neutrosophic quadruples, real or complex numbers were taken as variables, while in this study we took sets as variables. So, we will expand the applications of neutrosophic quadruples. Because things or variables in any application will be more useful than real numbers or complex numbers. Also we give NT group (NTG) based on SVNQN. In Section 2, we give definitions and properties for NQS, NQN [45] and NTS, NTG [30]. In Section 3, we define SVNQS and SVNQN. Also, we give operations for these structures. In Section 4, we obtain some NTG based on SVNQN thanks to operations for SVNQN. In this way, we define new structures using the together SVNQN and NTG.

2 Preliminaries

Definition 2.1: [45] A NQN is a number of the form (x, yT, zI, tF), where T, I, F have their usual neutrosophic logic means and x, y, z, t ∈ ℝ or ℂ. The NQS defined by NQ = {(x, yT, zI, tF): x, y, z, t ∈ ℝ or ℂ}.

For a NQN (x, yT, zI, tF), representing any entity which may be a number, an idea, an object, etc., x is called the known part and (yT, zI, tF) is called the unknown part.

Definition 2.2: [45] Let a = (a_1, a_2T, a_3I, a_4F) and b = (b_1, b_2T, b_3I, b_4F) ∈ NQ be NQNs. We define the following:

a + b = (a_1 +b_1, (a_2+b_2)T, (a_3+b_3)I, (a_4+b_4)F)

a - b = (a_1 - b_1, (a_2 - b_2)T, (a_3 - b_3)I, (a_4 - b_4)F)

Definition 2.3: [45] Consider the set {T, I, F}. Suppose in an optimistic way we consider the prevalence order T>I>F. Then we have:

TI = IT = max{T, I} = T,

TF = FT = max{T, F} = T,

FI = IF = max{F, I} = I,

TT = T^2 = T,

II = I^2 = I,

FF = F^2 = F.

Analogously, suppose in a pessimistic way we consider the prevalence order T < I < F. Then we have:

TI = IT = max{T, I} = I,

TF = FT = max{T, F} = F,

FI = IF = max{F, I} = F,

TT = T^2 = T,

II = I^2 = I,

FF = F^2 = F.

Definition 2.4: [45] Let

a = (a_1, a_2T, a_3I, a_4F),

b = (b_1, b_2T, b_3I, b_4F) ∈ NQ;

T < I < F.

Then a*b = (a_1 , a_2 T, a_3 I, a_4 F)* (b_1 , b_2 T, b_3 I, b_4 F) = (a_1b_1 , (a_1b_2 + a_2b_1 + a_2b_2)T, (a_1b_3 + a_2b_3 + a_3b_1 + a_3b_2 + a_3b_3)I, (a_1b_4 + a_2b_4 + a_3b_4 + a_4b_1 + a_4b_2 + a_4b_3 + a_4b_4)F)

Definition 2.5: [45] Let

a = (a_1, a_2T, a_3I, a_4F),

b = (b_1, b_2T, b_3I, b_4F) ∈ NQ,

T > I > F

Then a#b = (a_1, a_2T, a_3I, a_4F) # (b_1, b_2T, b_3I, b_4F) = (a_1b_1, (a_1b_2 + a_2b_1 + a_2b_2 + a_3b_2 + a_4b_2 + a_2b_3 + a_2b_4)T, (a_1b_3 + a_3b_3 + a_3b_4 + a_4b_3)I, (a_1b_4 + a_4b_1 + a_4b_4)F)

Definition 2.6: [30]: Let # be a binary operation. A NTS (X, #) is a set such that for x ∈ X,

i) There exists neutral of "x" such that x#neut(x) = neut(x)#x = x,

ii) There exists anti of "x" such that x#anti(x) = anti(x)#x = neut(x).

Also, a neutrosophic triplet "x" is showed with (x, neut(x), anti(x)).

Definition 2.7: [30] Let (X, #) be a NT set. Then, X is called a NTG such that

a) for all a, b ∈ X, a*b ∈ X.

b) for all a, b, c ∈ X, (a*b)*c = a*(b*c)

3 Set Valued Neutrosophic Quadruple Numbers

Definition 3.1: Let N be a non – empty set and P(N) be power set of N. A SVNQN shown by the form (A_1, A_2T, A_3I, A_4F). Where, T, I and F are degree of membership, degree of undeterminacy, degree of non-membership in neutrosophic theory, respectively. Also, A_1, A_2, A_3, A_4 ∈ P(N). Then, a SVNQS shown by N_q= {(A_1, A_2T, A_3I, A_4F): A_1, A_2, A_3, A_4 ∈ P(N)}.

Where, similar to NQS, A_1 is called the known part and (A_1, A_2T, A_3I, A_4F) is called the unknown part.

Definition 3.2: Let A = (A_1, A_2T, A_3I, A_4F) and B = (B_1, B_2T, B_3I, B_4F) be SVNQNs. We define the following operations, well known operators in set theory, such that

A ∪ B = ($A_1 \cup B_1$, ($A_2 \cup B_2$)T, ($A_3 \cup B_3$)I, ($A_4 \cup B_4$)F)

A ∩ B = ($A_1 \cap B_1$, ($A_2 \cap B_2$)T, ($A_3 \cap B_3$)I, ($A_4 \cap B_4$)F)

A \ B = (A_1 \ B_1, (A_2 \ B_2)T, (A_3 \ B_3)I, (A_4 \ B_4)F)

A' = (A'_1, A'_2T, A'_3I, A'_4F)

Now, we define specific operations for SVNQN.

Definition 3.3: Let A = (A_1, A_2T, A_3I, A_4F), B = (B_1, B_2T, B_3I, B_4F) be SVNQNs and T < I < F. We define the following operations

A*₁B = (A_1, A_2T, A_3I, A_4F) *₁ (B_1, B_2T, B_3I, B_4F) = ($A_1 \cap B_1$, (($A_1 \cap B_2$) ∪ ($A_2 \cap B_1$) ∪ ($A_2 \cap B_2$))T, (($A_1 \cap B_3$) ∪ ($A_2 \cap B_3$) ∪ ($A_3 \cap B_1$) ∪ ($A_3 \cap B_2$) ∪ ($A_3 \cap B_3$))I, (($A_1 \cap B_4$) ∪ ($A_2 \cap B_4$) ∪ ($A_3 \cap B_4$) ∪ ($A_4 \cap B_1$) ∪ ($A_4 \cap B_2$) ∪ ($A_4 \cap B_3$) ∪ ($A_4 \cap B_4$))F) and

A*₂B = (A_1, A_2T, A_3I, A_4F) *₂ (B_1, B_2T, B_3I, B_4F) = ($A_1 \cup B_1$, (($A_1 \cup B_2$) ∩ ($A_2 \cup B_1$) ∩ ($A_2 \cup B_2$))T, (($A_1 \cup B_3$) ∩ ($A_2 \cup B_3$) ∩ ($A_3 \cup B_1$) ∩ ($A_3 \cup B_2$) ∩ ($A_3 \cup B_3$))I, (($A_1 \cup B_4$) ∩ ($A_2 \cup B_4$) ∩ ($A_3 \cup B_4$) ∩ ($A_4 \cup B_1$) ∩ ($A_4 \cup B_2$) ∩ ($A_4 \cup B_3$) ∩ ($A_4 \cup B_4$))F).

Definition 3.4: Let A = (A_1, A_2T, A_3I, A_4F), B = (B_1, B_2T, B_3I, B_4F) be SVNQNs and T > I > F. We define the following operations

A #₁B = (A_1, A_2T, A_3I, A_4F) #₁ (B_1, B_2T, B_3I, B_4F) = ($A_1 \cap B_1$, (($A_1 \cap B_2$) ∪ ($A_2 \cap B_1$) ∪ ($A_2 \cap B_2$) ∪ ($A_3 \cap B_2$) ∪ ($A_4 \cap B_2$) ∪ ($A_2 \cap B_3$) ∪ ($A_2 \cap B_4$))T, (($A_1 \cap B_3$) ∪ ($A_3 \cap B_3$) ∪ ($A_3 \cap B_4$) ∪ ($A_4 \cap B_3$))I, (($A_1 \cap B_4$) ∪ ($A_4 \cap B_2$) ∪ ($A_4 \cap B_4$))F) and

A#₂B = (A_1, A_2T, A_3I, A_4F) #₂ (B_1, B_2T, B_3I, B_4F) = ($A_1 \cup B_1$, (($A_1 \cup B_2$) ∩ ($A_2 \cup B_1$) ∩ ($A_2 \cup B_2$) ∩ ($A_3 \cup B_2$) ∩ ($A_4 \cup B_2$) ∩ ($A_2 \cup B_3$) ∩ ($A_2 \cup B_4$))T, (($A_1 \cup B_3$) ∩ ($A_3 \cup B_3$) ∩ ($A_3 \cup B_4$) ∩ ($A_4 \cup B_3$))I, (($A_1 \cup B_4$) ∩ ($A_4 \cup B_2$) ∩ ($A_4 \cup B_4$))F).

Definition 3.5: Let A = (A_1, A_2T, A_3I, A_4F), B = (B_1, B_2T, B_3I, B_4F) be SVNQNs. If $A_1 \subset B_1$, $A_2 \subset B_2$, $A_3 \subset B_3$, $A_4 \subset B_4$, then it is called that A is subset of B. It is shown by A⊂ B.

Definition 3.6: Let A = (A_1, A_2T, A_3I, A_4F), B = (B_1, B_2T, B_3I, B_4F) be SVNQNs If A⊂ B and $B \subset A$., then it is called that A is equal to B. It is shown by A = B.

Example 3.7: Let X = {x, y, z} be a set. Thus, we have P(X) ={∅ , {x}, {y}, {z}, {y, z}, {x, z}, {x, y} ,{x, y, z}}. Also, X_q= {(A_1, A_2T, A_3I, A_4F): A_1, A_2, A_3, A_4 ∈ P(X)} is a SVNQS. For example,

A_1 = ({y, z}, {x, y, z}T, {x, y}I, {z}F) and A_2 = ({ z}, {x, z}T, {x, y}I, ∅F) are two SVNQNs in X_q. Furthermore,

$A_1 \cup A_2$ = ({y, z}, {x, y, z}T, {x, y}I, {z}F) = A_1.

$A_1 \cap A_2$ = ({ z}, {x, z}T, {x, y}I, ∅F) = A_2.

Thus, we have $A_2 \subset A_1$. Also,

${A_1}'$ = ({x}, ∅T, {z}I, {x, y}F)

$A_1 \setminus A_2$ = ({y}, { y}T, ∅I, {z}F)

4 Neutrosophic Triplet Group Based on Set Valued Neutrosophic Quadruple Numbers

Theorem 4.1: Let N be a non – empty set and N_q= {(A_1, A_2T, A_3I, A_4F): A_1, A_2, A_3, A_4 ∈ P(N)}be a SVNQS. Then,
a) (N_q, ∪) is a NTS.
b) (N_q, ∩) is a NTS.
Proof:
a) Let A = (A_1, A_2T, A_3I, A_4F) be a SVNQN in N_q. From Definition 3.2, it is clear that
A ∪ A = (A_1, A_2T, A_3I, A_4F) ∪ (A_1, A_2T, A_3I, A_4F) = ($A_1 \cup A_1$, ($A_2 \cup A_2$)T, ($A_3 \cup A$)I, ($A_4 \cup A_4$)F) = (A_1, A_2T, A_3I, A_4F) = A.
Hence, we can take neut(A) = A. Also, if neut(A) = A, then we have anti(A) = A. Thus, (N_q, ∪) is a neutrosophic triplet set with neut(A) = A and anti(A) = A.
b) a) Let A = (A_1, A_2T, A_3I, A_4F) be a SVNQN in N_q. From Definition 3.2, it is clear that
A ∩ A = (A_1, A_2T, A_3I, A_4F) ∩ (A_1, A_2T, A_3I, A_4F) = ($A_1 \cap A_1$, ($A_2 \cap A_2$)T, ($A_3 \cap A$)I, ($A_4 \cap A_4$)F) = (A_1, A_2T, A_3I, A_4F) = A.
Hence, we can take neut(A) = A. Also, if neut(A) = A, then we have anti(A) = A. Thus, (N_q, ∩) is a neutrosophic triplet set with neut(A) = A and anti(A) = A.

Theorem 4.2: Let N be a non – empty set and N_q= {(A_1, A_2T, A_3I, A_4F): A_1, A_2, A_3, A_4 ∈ P(N)}be a SVNQS. Then,
a) (N_q, ∪) is a NTG.
b) (N_q, ∩) is a NTG.
Proof:
a) From Theorem 4.1, (N_q, ∪) is a NTS with neut(A) = A and anti(A) = A. Let A = (A_1, A_2T, A_3I, A_4F), B = (B_1, B_2T, B_3I, B_4F) and C = (C_1, C_2T, C_3I, C_4F) ∈ N_q.
i) We have that A ∪ B ∈ N_q since P(N) is power set of N and A, B ∈ P(N). Because, if A, B ∈ P(X), then A ∪ B ∈ P(N).
ii) (A ∪ B) ∪ C = [($A_1 \cup B_1$, ($A_2 \cup B_2$)T, ($A_3 \cup B_3$)I, ($A_4 \cup B_4$)F)] ∪ (C_1, C_2T, C_3I, C_4F) =
[($A_1 \cup B_1$) ∪ C_1, (($A_2 \cup B_2$) ∪ C_2)T, (($A_3 \cup B_3$) ∪ C_3)I, (($A_4 \cup B_4$) ∪ C_4))F)] =
[$A_1 \cup (B_1 \cup C_1)$, ($A_2 \cup (B_2 \cup C_2)$)T, ($A_3 \cup (B_3 \cup C_3)$)I, ($A_4 \cup (B_4 \cup C_4)$)F)] = A ∪ (B ∪ C).
Thus, (N_q, ∪) is a NTG.
b) From Theorem 4.1, (N_q, ∩) is a NTS with neut(A) = A and anti(A) = A. Let A = (A_1, A_2T, A_3I, A_4F), B = (B_1, B_2T, B_3I, B_4F) and C = (C_1, C_2T, C_3I, C_4F) ∈ N_q.
i) We have that A ∩ B ∈ N_q since P(N) is power set of N and A, B ∈ P(N). Because, if A, B ∈ P(N), then A ∩ B ∈ P(N).
iii) (A ∩ B) ∩ C = [($A_1 \cap B_1$, ($A_2 \cap B_2$)T, ($A_3 \cap B_3$)I, ($A_4 \cap B_4$)F)] ∩ (C_1, C_2T, C_3I, C_4F) =[($A_1 \cap B_1$) ∩ C_1, (($A_2 \cap B_2$) ∩ C_2)T, (($A_3 \cap B_3$) ∩ C_3)I, (($A_4 \cap B_4$) ∩ C_4))F)] = [$A_1 \cap (B_1 \cap C_1)$, ($A_2 \cap (B_2 \cap C_2)$)T, ($A_3 \cap (B_3 \cap C_3)$)I, ($A_4 \cap (B_4 \cap C_4)$)F)] = A ∩ (B ∩ C).
Thus, (N_q, ∩) is a NTG.

Theorem 4.3: Let N be a non – empty set and N_q= {(A_1, A_2T, A_3I, A_4F): A_1, A_2, A_3, A_4 ∈ P(N)}be a SVNQS. Then,
a) (N_q, $*_1$) is a NTS with binary operation $*_1$ in Definition 3.3.
b) (N_q, $*_2$) is a NTS with binary operation $*_2$ in Definition 3.3.
Proof:
a) Let A = (A_1, A_2T, A_3I, A_4F) be a SVNQN in N_q. From Definition 3.3, we obtain
A $*_1$ A = (A_1, A_2T, A_3I, A_4F) $*_1$ (A_1, A_2T, A_3I, A_4F) =

$(A_1 \cap A_1$, $((A_1 \cap A_2) \cup (A_2 \cap A_1) \cup (A_2 \cap A_2))$T, $((A_1 \cap A_3) \cup (A_2 \cap A_3) \cup (A_3 \cap A_1) \cup (A_3 \cap A_2) \cup (A_3 \cap A_3))$I, $((A_1 \cap A_4) \cup (A_2 \cap A_4) \cup (A_3 \cap A_4) \cup (A_4 \cap A_1) \cup (A_4 \cap A_2) \cup (A_4 \cap A_3) \cup (A_4 \cap A_4))$F) = $(A_1$, A_2T, A_3I, A_4F) = A
since
$A_2 \cap A_2 = A_2$ and $(A_1 \cap A_2)$, $(A_2 \cap A_2) \subset A_2$;
$A_3 \cap A_3 = A_3$ and $(A_1 \cap A_3)$, $(A_2 \cap A_3)$, $(A_3 \cap A_3) \subset A_3$;
$A_4 \cap A_4 = A_4$ and $(A_1 \cap A_4)$, $(A_2 \cap A_4)$, $(A_3 \cap A_4)$, $(A_4 \cap A_4) \subset A_4$.
Hence, we can take neut(A) = A. Also, if neut(A) = A, then we have anti(A) = A. Thus, (N_q, $*_1$) is a NTS with neut(A) = A and anti(A) = A.
b) Let A = (A_1, A_2T, A_3I, A_4F) be a SVNQN in N_q. From Definition 3.3, we obtain
A $*_2$ A = (A_1, A_2T, A_3I, A_4F) $*_2$ (A_1, A_2T, A_3I, A_4F) = $(A_1 \cup A_1$, $((A_1 \cup A_2) \cap (A_2 \cup A_1) \cap (A_2 \cup A_2))$T, $((A_1 \cup A_3) \cap (A_2 \cup A_3) \cap (A_3 \cup A_1) \cap (A_3 \cup A_2) \cap (A_3 \cup A_3))$I, $((A_1 \cup A_4) \cap (A_2 \cup A_4) \cap (A_3 \cup A_4) \cap (A_4 \cup A_1) \cap (A_4 \cup A_2) \cap (A_4 \cup A_3) \cap (A_4 \cup A_4))$F) = ($A_1$, A_2T, A_3I, A_4F) = A
since
$A_2 \cup A_2 = A_2$ and $(A_1 \cup A_2)$, $(A_2 \cup A_2) \supset A_2$;
$A_3 \cup A_3 = A_3$ and $(A_1 \cup A_3)$, $(A_2 \cup A_3)$, $(A_3 \cup A_3) \supset A_3$;
$A_4 \cup A_4 = A_4$ and $(A_1 \cup A_4)$, $(A_2 \cup A_4)$, $(A_3 \cup A_4)$, $(A_4 \cup A_4) \supset A_4$.
Hence, we can take neut(A) = A. Also, if neut(A) = A, then we have anti(A) = A. Thus, (N_q, $*_2$) is a NTS with neut(A) = A and anti(A) = A.

Theorem 4.4: Let N be a non – empty set and N_q= {(A_1, A_2T, A_3I, A_4F): A_1, A_2, A_3, A_4 ∈ P(N)}be a SVNQS. Then,
a) (N_q, $*_1$) is a NTG with binary operation $*_1$ in Definition 3.3.
b) (N_q, $*_2$) is a NTG with binary operation $*_2$ in Definition 3.3.

Proof:
a) From Theorem 4.3, (N_q, $*_1$) is a neutrosophic triplet set. Let
A = (A_1, A_2T, A_3I, A_4F), B = (B_1, B_2T, B_3I, B_4F) and C = (C_1, C_2T, C_3I, C_4F) ∈ N_q,
i) We obtain A $*_1$ B ∈ $N_q$since P(N) is power set of N and A, B ∈ P(N).
ii)
(A $*_1$ B) $*_1$ C =
$(A_1 \cap B_1$, $((A_1 \cap B_2) \cup (A_2 \cap B_1) \cup (A_2 \cap B_2))$T, $((A_1 \cap B_3) \cup (A_2 \cap B_3) \cup (A_3 \cap B_1) \cup (A_3 \cap B_2) \cup (A_3 \cap B_3))$I, $((A_1 \cap B_4) \cup (A_2 \cap B_4) \cup (A_3 \cap B_4) \cup (A_4 \cap B_1) \cup (A_4 \cap B_2) \cup (A_4 \cap B_3) \cup (A_4 \cap B_4))$F) $*_1$ (C_1, C_2T, C_3I, C_4F) =
$([A_1 \cap B_1] \cap C_1$,
$(([A_1 \cap B_1] \cap C_2) \cup ([(A_1 \cap B_2) \cup (A_2 \cap B_1) \cup (A_2 \cap B_2)] \cap C_1) \cup ([(A_1 \cap B_2) \cup (A_2 \cap B_1) \cup (A_2 \cap B_2)] \cap C_2))$T,
$([A_1 \cap B_1] \cap C_3) \cup ([(A_1 \cap B_2) \cup (A_2 \cap B_1) \cup (A_2 \cap B_2)] \cap C_3) \cup ([A_1 \cap B_3) \cup (A_2 \cap B_3) \cup (A_3 \cap B_1) \cup (A_3 \cap B_2) \cup (A_3 \cap B_3)] \cap C_1) \cup ([A_1 \cap B_3) \cup (A_2 \cap B_3) \cup (A_3 \cap B_1) \cup (A_3 \cap B_2) \cup (A_3 \cap B_3)] \cap C_2) \cup ([A_1 \cap B_3) \cup (A_2 \cap B_3) \cup (A_3 \cap B_1) \cup (A_3 \cap B_2) \cup (A_3 \cap B_3)] \cap C_3))$I,
$(([A_1 \cap B_1] \cap C_4) \cup ([(A_1 \cap B_2) \cup (A_2 \cap B_1) \cup (A_2 \cap B_2)] \cap C_4) \cup ([A_1 \cap B_3) \cup (A_2 \cap B_3) \cup (A_3 \cap B_1) \cup (A_3 \cap B_2) \cup (A_3 \cap B_3)] \cap C_4) \cup ([(A_1 \cap B_4) \cup (A_2 \cap B_4) \cup (A_3 \cap B_4) \cup (A_4 \cap B_1) \cup (A_4 \cap B_2) \cup (A_4 \cap B_3) \cup (A_4 \cap B_4)] \cap C_1) \cup ([(A_1 \cap B_4) \cup (A_2 \cap B_4) \cup (A_3 \cap B_4) \cup (A_4 \cap B_1) \cup (A_4 \cap B_2) \cup (A_4 \cap B_3) \cup (A_4 \cap B_4)] \cap C_2) \cup ([(A_1 \cap B_4) \cup (A_2 \cap B_4) \cup (A_3 \cap B_4) \cup (A_4 \cap B_1) \cup (A_4 \cap B_2) \cup (A_4 \cap B_3) \cup (A_4 \cap B_4)] \cap C_3) \cup ([(A_1 \cap B_4) \cup (A_2 \cap B_4) \cup (A_3 \cap B_4) \cup (A_4 \cap B_1) \cup (A_4 \cap B_2) \cup (A_4 \cap B_3) \cup (A_4 \cap B_4)] \cap C_4))$F) =
$(A_1 \cap [B_1 \cap C_1]$,
$((A_1 \cap [(B_1 \cap C_2) \cup (B_2 \cap C_1) \cup (B_2 \cap C_2)]) \cup (A_2 \cap [B_1 \cap C_1]) \cup (A_2 \cap [(B_1 \cap C_2) \cup (B_2 \cap C_1) \cup (B_2 \cap C_2)]))$T,
$((A_1 \cap [(B_1 \cap C_3) \cup (B_2 \cap C_3) \cup (B_3 \cap C_1) \cup (B_3 \cap C_2) \cup (B_3 \cap C_3)]) \cup (A_2 \cap [(B_1 \cap C_3) \cup (B_2 \cap C_3) \cup (B_3 \cap C_1) \cup (B_3 \cap C_2) \cup (B_3 \cap C_3)]) \cup (A_3 \cap [B_1 \cap C_1]) \cup (A_3 \cap [(B_1 \cap C_2) \cup (B_2 \cap C_1) \cup (B_2 \cap C_2)]) \cup (A_3 \cap [(B_1 \cap C_3) \cup (B_2 \cap C_3) \cup (B_3 \cap C_1) \cup (B_3 \cap C_2) \cup (B_3 \cap C_3)]))$ I,
$((A_1 \cap [(B_1 \cap C_4) \cup (B_2 \cap C_4) \cup (B_3 \cap C_4) \cup (B_4 \cap C_1) \cup (B_4 \cap C_2) \cup (B_4 \cap C_3) \cup (B_4 \cap C_4)] \cup (A_2 \cap [(B_1 \cap C_4) \cup (B_2 \cap C_4) \cup (B_3 \cap C_4) \cup (B_4 \cap C_1) \cup (B_4 \cap C_2) \cup (B_4 \cap C_3) \cup (B_4 \cap C_4)]) \cup (A_3 \cap$

$[(B_1 \cap C_4) \cup (B_2 \cap C_4) \cup (B_3 \cap C_4) \cup (B_4 \cap C_1) \cup (B_4 \cap C_2) \cup (B_4 \cap C_3) \cup (B_4 \cap C_4)]) \cup (A_4 \cap [B_1 \cap C_1]) \cup (A_4 \cap[(B_1 \cap C_2) \cup (B_2 \cap C_1) \cup (B_2 \cap C_2)]) \cup (A_4 \cap[(B_1 \cap C_3) \cup (B_2 \cap C_3) \cup (B_3 \cap C_1) \cup (B_3 \cap C_2) \cup (B_3 \cap C_3)]) \cup (A_4 \cap[(B_1 \cap C_4) \cup (B_2 \cap C_4) \cup (B_3 \cap C_4) \cup (B_4 \cap C_1) \cup (B_4 \cap C_2) \cup (B_4 \cap C_3) \cup (B_4 \cap C_4)]$))F)
= A $*_1$ (B $*_1$ C).
Thus, (N_q, $*_1$) is a NTG with binary operation $*_1$ in Definition 3.3.
b) This proof can be made similar to a.

Theorem 4.5: Let N be a non – empty set and N_q= {(A_1, A_2T, A_3I, A_4F): A_1, A_2, A_3, A_4 ∈ P(N)}be a SVNQS. Then,
a) (N_q, $*_1$) is a NTS with binary operation $\#_1$ in Definition 3.4.
b) (N_q, $*_2$) is a NTS with binary operation $\#_2$ in Definition 3.4.
Proof: These proofs can be made similar to Theorem 4.3.

Theorem 4.6: Let N be a non – empty set and N_q= {(A_1, A_2T, A_3I, A_4F): A_1, A_2, A_3, A_4 ∈ P(N)}be a SVNQS. Then,
a) (N_q, $*_1$) is a NTG with binary operation $\#_1$ in Definition 3.4.
b) (N_q, $*_2$) is a NTG with binary operation $\#_2$ in Definition 3.4.
Proof: These proofs can be made similar to Theorem 4.4.

Conclusion

In this study, we firstly obtain set valued neutrosophic quadruple sets and numbers. Also, we introduce some known and special operations for set valued neutrosophic quadruple numbers. In the neutrosophic quadruples, real or complex numbers were taken as variables, while in this study we took sets as variables. So, we will expand the applications of neutrosophic quadruples. Because things or variables in any application will be more useful than real numbers or complex numbers. Furthermore, we give some neutrosophic triplet groups based on set valued neutrosophic quadruple number thanks to operations for set valued neutrosophic quadruple numbers. Thus, we have added a new structure to neutrosophic triplet structures and neutrosophic quadruple structures. Thanks to set valued neutrosophic quadruple sets and numbers other neutrosophic triplet structures can be defined similar to this study. For example, neutrosophic triplet metric space based on set valued neutrosophic quadruple numbers; neutrosophic triplet vector space based on set valued neutrosophic quadruple numbers; neutrosophic triplet normed space based on set valued neutrosophic quadruple numbers. Also, set valued neutrosophic quadruple sets can be used decision making applications due to the its set valued structure. For example, in a medical application in which more than one drug is used, this structure may be used.

Abbreviations

NT: Neutrosophic triplet
NTS: Neutrosophic triplet set
NTG: Neutrosophic triplet group
NQ: Neutrosophic quadruple
NQS: Neutrosophic quadruple set
NQN: Neutrosophic quadruple number
SVNQS: Set valued neutrosophic quadruple set
SVNQN: Set valued neutrosophic quadruple number

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Conflicts of Interest

The authors declare no conflict of interest.

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