Neutrosophic Vague Binary Sets

Remya.P.B. and Francina Shalini.A.

Abstract: Vague sets and neutrosophic sets play an inevitable role in the developing scenario of mathematical world. In this modern era of artificial intelligence most of the real life situations are found to be immersed with unclear data. Even the newly developed concepts are found to fail with such problems. So new sets like Plithogenic and new combinations like neutrosophic vague arose. Classical set theory dealt with single universe and can be studied by taking its subsets. Situations demand two universes instead of a unique one in certain problems. In this paper two universes are introduced simultaneously and under consideration in a neutrosophic vague environment. It’s basic operations, topology and continuity are also discussed with examples. A real life example is also discussed.

Keywords: binary set, fuzzy binary set, vague binary set, neutrosophic vague binary sets, neutrosophic vague binary topology, neutrosophic vague binary continuity

1. Introduction

Functions are tightly packed but relations are not. They are more general than functions. Decimal system deals with ten digits while binary with two - only with 0 and 1. For detecting electrical signal’s on or off state binary system can be used more effectively. It is the prime reason of selecting binary language in computers. Binary operations in algebra will give another idea! After a binary operation, ‘operands’ produce an element which is also a member of the parent set - means ‘domain and co-domain’ are in the same set. But binary relations are quite different from the ideas mentioned above. They are subsets of the cartesian product of the sets under consideration, taken in a special way. It is clear that binary stands for two. In point-set topology information from elements of topology will give information about subsets of the universal set under consideration. But real life can’t be confined into a single universal set. It may be two or more than two. Being an extension of classical sets [George Cantor, 1874-1897] [27], fuzzy sets (FS’s) [Zadeh, 1965] [29] can deal with partial membership. In intuitionistic fuzzy sets (IFS’s) [Attanassov, 1986] [12] two membership grades are there - truth and false. As an extension of fuzzy sets Gau and Buehrer [9] introduced vague sets in 1993. Neutro-sophy means knowledge of neutral thought. It is a new branch of philosophy introduced by Florentin Smarandache [6] in 1995 - by giving an additional component - indeterminacy. Movement of paradoxism was set up by him in early 1980’s. New concept dealt with the principle of using non-artistic elements to set artistic. Within no time so many hybrid structures developed by using the merits of the newly developed theory. In 2014, Alblowmi. S. A and Mohmed Eisa [1] gave some new concepts of neutrosophic sets. In 1996, Dontchev [5] developed Contra-continuous functions and strongly s-closed spaces. In 2014, Salama A.A, Florentin Smarandache and Valeri Kromov [25] developed neutrosophic closed set and neutrosophic continuous functions.

2. Preliminaries

Definition 2.2. [26] (Neutrosophic vague set)
A neutrosophic vague set $A_{NV}$ ($NVS$ in short) on the universe of discourse $X$ can be written as $A_{NV} = \{x; \hat{T}_{AN}(X), \hat{I}_{AN}(X), \hat{F}_{AN}(X); x \in X\}$ whose truth-membership, indeterminacy-membership and falsity-membership functions are defined as $\hat{T}_{AN}(x)=[T^-, T^+], \hat{I}_{AN}(x)=[I^-, I^+]$ and $\hat{F}_{AN}(x)=[F^-, F^+]$

where (1) $T^+ = 1 - F^-; F^+ = 1 - T^-$ and

(2) $0 \leq T^- + I^- + F^- \leq 2^+$
$0 \leq T^+ + I^+ + F^+ \leq 2^+$

Definition 2.3. [26] (Unit Neutrosophic Vague Set)

Let $\Psi_{NV}$ be a neutrosophic vague set ($NVS$ in short) of the universe $U$ where $\forall u_i \in U$, $\hat{T}_{\Psi_{NV}}(x) = [1, 1], \hat{I}_{\Psi_{NV}}(x) = [0, 0], \hat{F}_{\Psi_{NV}}(x) = [0, 0]$, then $\Psi_{NV}$ is called a unit $NVS$, where $1 \leq i \leq n$

Definition 2.4. [26] (Zero Neutrosophic Vague Set)

Let $\Phi_{NV}$ be a neutrosophic vague set ($NVS$ in short) of the universe $U$ where $\forall u_i \in U$, $\hat{T}_{\Phi_{NV}}(x) = [0, 0], \hat{I}_{\Phi_{NV}}(x) = [1, 1], \hat{F}_{\Phi_{NV}}(x) = [1, 1]$, then $\Phi_{NV}$ is called a zero $NVS$, where $1 \leq i \leq n$

Definition 2.5. [26] (Neutrosophic vague subset)

Let $A_{NV}$ and $B_{NV}$ be two $NVS's$ of the universe $U$.

1. $\hat{T}_{A_{NV}}(u_i) \leq \hat{T}_{B_{NV}}(u_i)$
2. \( I_{ANV} (u_i) \geq I_{BNV} (u_i) \) and 
3. \( F_{ANV} (u_i) \geq F_{BNV} (u_i) \)

then the NVS \( A_{NV} \) are included by \( B_{NV} \) denoted by \( A_{NV} \subseteq B_{NV} \)

Definition 2.6. [26] (Complement of a Neutrosophic vague set)
The complement of a NVS \( A_{NV} \) is denoted by \( A_{NV}^c \) and is defined by 
\[
\hat{T}_{A_{NV}} (x) = [1-T^- , T^- , T^- ] , \hat{I}_{A_{NV}} (x) = [1-I^+, I^+, I^+] \text{ and } \hat{F}_{A_{NV}} (x) = [1-F^+ , 1-F^+ , 1-F^+] 
\]

Definition 2.7. [26] (Union of Neutrosophic vague sets)
Union of two NVS’s \( A_{NV} \) and \( B_{NV} \) is a NVS \( C_{NV} \) written as \( C_{NV} = A_{NV} \cup B_{NV} \) whose truth-membership, indeterminacy-membership and false-membership functions are related to those of \( A_{NV} \) and \( B_{NV} \) given by 
\[
\hat{T}_{C_{NV}} (x) = [\max (T^- A_{NV}(x) , T^- B_{NV}(x)), \max (T^+ A_{NV}(x), T+ B_{NV}(x))] \\
\hat{I}_{C_{NV}} (x) = [\min (I^- A_{NV}(x) , I^- B_{NV}(x)), \min (I^+ A_{NV}(x), I+ B_{NV}(x))] \\
\hat{F}_{C_{NV}} (x) = [\min (F^- A_{NV}(x), F^- B_{NV}(x)), \min (F^+ A_{NV}(x), F^+ B_{NV}(x))]
\]

Definition 2.8. [26] (Intersection of Neutrosophic vague sets)
Intersection of two NVS’s \( A_{NV} \) and \( B_{NV} \) is a NVS \( C_{NV} \) written as \( D_{NV} = A_{NV} \cap B_{NV} \) whose truth-membership, indeterminacy-membership and false-membership functions are related to those of \( A_{NV} \) and \( B_{NV} \) given by 
\[
\hat{T}_{D_{NV}} (x) = [\min (T^- A_{NV}(x) , T^- B_{NV}(x)), \min (T^+ A_{NV}(x), T+ B_{NV}(x))] \\
\hat{I}_{D_{NV}} (x) = [\max (I^- A_{NV}(x) , I^- B_{NV}(x)), \max (I^+ A_{NV}(x), I+ B_{NV}(x))] \\
\hat{F}_{D_{NV}} (x) = [\max (F^- A_{NV}(x), F^- B_{NV}(x)), \max (F^+ A_{NV}(x), F^+ B_{NV}(x))]
\]

Definition 2.9. [14]
Let \((X, t)\) be a topological space. A subset \(A\) of \(X\) is called:
(i) Semi-closed set if \(\text{int}(\overline{A}) \subseteq A\)
(ii) Pre-closed set if \(\text{cl}(\text{int}(A)) \subseteq A\)
(iii) Semi-pre-closed set if \(\text{int}(\text{cl}(\text{int}(A))) \subseteq A\)
(iv) Regular-closed set if \(A = \text{cl}((\text{int}(A))\)
(v) Generalized semi-closed set if \(\text{scl}(A) \subseteq \text{cl}(\text{U})\) whenever \(A \subseteq \text{cl}(\text{U})\) and \(U\) is open in \(X\)

Definition 2.10. [4] (Image and Pre-image of neutrosophic vague sets)
Let \(X_{NV}\) and \(Y_{NV}\) be two non-empty neutrosophic vague sets and \(f: X_{NV} \rightarrow Y_{NV}\) be a function, then the following statements hold:
(1) If \(B_{NV} = \{ (x, \hat{T}_B(x); \hat{I}_B(x); \hat{F}_B(x)); x \in X_{NV}\} \) is a NVS in \(Y_{NV}\), then the preimage of \(B_{NV}\) under \(f\), denoted by \(f^{-1}(B_{NV})\), is the NVS in \(X_{NV}\) defined by
\[
f^{-1}(B_{NV}) = \{ (x, \hat{T}_B(x)); f^{-1}(T_B(x)); f^{-1}(I_B(x)); f^{-1}(F_B(x)); x \in X_{NV}\}
\]
(2) If \(A_{NV} = \{ (x, \hat{T}_A(x); \hat{I}_A(x); \hat{F}_A(x)); x \in X_{NV}\} \) is a NVS in \(X_{NV}\), then the image of \(A_{NV}\) under \(f\), denoted by \(f(A_{NV})\), is the NVS in \(Y_{NV}\) defined by
\[
f(A_{NV}) = \{ (y, f_{\sup}(\hat{T}_A(y)); f_{\inf}(\hat{I}_A(y)); f_{\inf}(\hat{F}_A(y)); y \in Y_{NV}\}
\]
where
\[
f_{\sup}(\hat{T}_A(y)) = \begin{cases} 
\sup_{x \in f^{-1}(y)}\hat{T}_A(x), & \text{if } f^{-1}(y) \neq \emptyset \\
0, & \text{otherwise}
\end{cases}
\]
\[
f_{\inf}(\hat{I}_A(y)) = \begin{cases} 
\sup_{x \in f^{-1}(y)}\hat{I}_A(x), & \text{if } f^{-1}(y) \neq \emptyset \\
0, & \text{otherwise}
\end{cases}
\]
\[
f_{\inf}(\hat{F}_A(y)) = \begin{cases} 
\sup_{x \in f^{-1}(y)}\hat{F}_A(x), & \text{if } f^{-1}(y) \neq \emptyset \\
0, & \text{otherwise}
\end{cases}
\]
for each \(y \in Y_{NV}\)

Definition 2.11. [5] (Strongly continuous functions)

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A function \( f : (X, \tau) \to (Y, \sigma) \) to be strongly continuous if \( f(\bar{A}) \subset \bar{f(A)} \), \( \forall \) subset \( A \) of \( X \) or equivalently, if the inverse image of every set in \( Y \) is clopen in \( X \).

**Definition 2.12.** [14] (Neutrosophic Vague Continuous Mapping)

Let \( (X, \tau) \) and \( (Y, \sigma) \) be any two neutrosophic vague topological spaces. A map \( f : (X, \tau) \to (Y, \sigma) \) is said to be neutrosophic vague continuous (\( NV \) continuous) if \( f^{-1}(V) \) is neutrosophic vague closed set in \( (X, \tau) \) for every neutrosophic vague closed set \( V \) of \( (Y, \sigma) \).

**Definition 2.13** [14] (Neutrosophic Vague semi-continuous mapping)

Let \( (X, \tau) \) and \( (Y, \sigma) \) be any two neutrosophic vague topological spaces. A map \( f : (X, \tau) \to (Y, \sigma) \) is said to be neutrosophic vague semi-continuous if \( f^{-1}(V) \) is neutrosophic vague semi-closed set in \( (X, \tau) \) for every neutrosophic vague semi-closed set \( V \) of \( (Y, \sigma) \).

**Definition 2.14** [14] (Neutrosophic Vague pre-continuous mapping)

Let \( (X, \tau) \) and \( (Y, \sigma) \) be any two neutrosophic vague topological spaces. A map \( f : (X, \tau) \to (Y, \sigma) \) is said to be neutrosophic vague pre-continuous if \( f^{-1}(V) \) is neutrosophic vague pre-closed set in \( (X, \tau) \) for every neutrosophic vague pre-closed set \( V \) of \( (Y, \sigma) \).

**Definition 2.15** [14] (Neutrosophic Vague regular continuous mapping)

Let \( (X, \tau) \) and \( (Y, \sigma) \) be any two neutrosophic vague topological spaces. A map \( f : (X, \tau) \to (Y, \sigma) \) is said to be neutrosophic vague regular-continuous if \( f^{-1}(V) \) is neutrosophic vague regular-closed set in \( (X, \tau) \) for every neutrosophic vague regular closed set \( V \) of \( (Y, \sigma) \).

**Definition 2.16** [14] (Neutrosophic Vague semi pre-continuous mapping)

Let \( (X, \tau) \) and \( (Y, \sigma) \) be any two neutrosophic vague topological spaces. A map \( f : (X, \tau) \to (Y, \sigma) \) is said to be neutrosophic vague semi pre-continuous if \( f^{-1}(V) \) is neutrosophic vague semi pre-closed set in \( (X, \tau) \) for every neutrosophic vague semi pre-closed set \( V \) of \( (Y, \sigma) \).

### 3. Neutrosophic Vague Binary Sets

In this section neutrosophic vague binary sets are discussed with examples. For this as a preliminary tool fuzzy binary sets and vague binary sets are discussed as a general case by taking all members instead of taking a subset of cartesian product in a confined manner.

**Definition 3.1.** (Binary Set)

Binary set \( A \) over a common universe \( \big\{ U_1 = \{ x_j | 1 \leq j \leq n \}; U_2 = \{ y_k | 1 \leq k \leq p \} \big\} \) is an object of the form \( \bar{A} = \{ (x_j), (y_k) \} \).

**Definition 3.2.** (Fuzzy Binary Set)

Fuzzy binary set \( A \) over a common universe \( \big\{ U_1 = \{ x_j | 1 \leq j \leq n \}; U_2 = \{ y_k | 1 \leq k \leq p \} \big\} \) is an object of the form \( \bar{A}_F = \left\{ \left( \mu_A(x_j), \nu_A(x_j) \right) ; \forall x_j \in U_1, \left( \mu_A(y_k), \nu_A(y_k) \right) ; \forall y_k \in U_2 \right\} \), where \( \mu_A(x_j) : U_1 \to [0, 1] \) gives the truth membership value of the elements \( x_j \) in \( U_1 \); \( \nu_A(y_k) : U_2 \to [0, 1] \) gives the truth membership values of the elements \( y_k \) in \( U_2 \).

**Example 3.3.**

\( \bar{A}_F = \left\{ \left( \frac{0.2}{0.5}, \frac{0.4}{0.7}, \frac{0.1}{0.9} \right), \left( \frac{0.6}{0.1}, \frac{0.5}{0.3} \right) \right\} \) represents the fuzzy binary set.

**Definition 3.4.** (Vague Binary Set)

Vague binary set \( A \) over a common universe \( \big\{ U_1 = \{ x_j | 1 \leq j \leq n \}; U_2 = \{ y_k | 1 \leq k \leq p \} \big\} \) is an object of the form \( \bar{A}_V = \left\{ \left( \varphi_A(x_j) ; \forall x_j \in U_1, \right), \left( \varphi_A(y_k) ; \forall y_k \in U_2 \right) \right\} \); \( \varphi_A(x_j) : U_1 \to [0, 1] \); \( \varphi_A(y_k) : U_2 \to [0, 1] \).

**Example 3.5.**

\( \bar{A}_V = \left\{ \left( \frac{0.2}{0.6}, \frac{0.4}{0.7}, \frac{0.1}{0.9} \right), \left( \frac{0.6}{0.9}, \frac{0.1}{0.4} \right) \right\} \) is a vague binary set where \( U_1 = \{ h_1^0, h_2^0, h_3^0 \}, U_2 = \{ h_1^1, h_2^1 \} \).

**Definition 3.6.** (Neutrosophic binary set)
Neutrosophic binary set $A_N$ over a common universe $\{U_1 = \{x_j/1 \leq j \leq n\}; U_2 = \{y_k/1 \leq k \leq p\}\}$ is an object of the form

$$\tilde{A}_N = \left\{ \left( \tau_{A}(x_j), \iota_{A}(x_j), \tilde{F}_A(x_j) \right) \mid \forall x_j \in U_1, \left( \tau_{A}(y_k), \iota_{A}(y_k), \tilde{F}_A(y_k) \right) \mid \forall y_k \in U_2 \right\}$$

$T_A(x_j), I_A(x_j), F_A(x_j): U_1 \rightarrow [0, 1]$ gives the 'truth, indeterminacy and false' membership values of the elements $x_j$ in $U_1$ and $T_A(y_k), I_A(y_k), F_A(y_k): U_2 \rightarrow [0, 1]$ gives the 'truth, indeterminacy and false' membership values of the elements $y_k$ in $U_2$

**Example 3.9. (Neutrosophic vague binary set)**

A neutrosophic vague binary set $M_{NVB}$ (NVBS in short) over a common universe $\{U_1 = \{x_j/1 \leq j \leq n\}; U_2 = \{y_k/1 \leq k \leq p\}\}$ is an object of the form

$$M_{NVB} = \left\{ \left( \tilde{T}_{M_{NVB}}(x_j), \tilde{I}_{M_{NVB}}(x_j), \tilde{F}_{M_{NVB}}(x_j) \right) \mid \forall x_j \in U_1, \left( \tilde{T}_{M_{NVB}}(y_k), \tilde{I}_{M_{NVB}}(y_k), \tilde{F}_{M_{NVB}}(y_k) \right) \mid \forall y_k \in U_2 \right\}$$

is defined as

$$\tilde{T}_{M_{NVB}}(x_j) = [T^-(x_j), T^+(x_j)], \tilde{I}_{M_{NVB}}(x_j) = [I^-(x_j), I^+(x_j)] \text{ and } \tilde{F}_{M_{NVB}}(x_j) = [F^-(x_j), F^+(x_j)]; x_j \in U_1 \text{ and } \tilde{T}_{M_{NVB}}(y_k) = [T^-(y_k), T^+(y_k)], \tilde{I}_{M_{NVB}}(y_k) = [I^-(y_k), I^+(y_k)] \text{ and } \tilde{F}_{M_{NVB}}(y_k) = [F^-(y_k), F^+(y_k)]; y_k \in U_2$$

where (1) $T^+(x_j) = 1 - F^-(x_j); F^+(x_j) = 1 - T^-(x_j); \forall x_j \in U_1$ and $T^+(y_k) = 1 - F^-(y_k); F^+(y_k) = 1 - T^-(y_k); \forall y_k \in U_2$

(2) $0 \leq T^-(x_j) + I^-(x_j) + F^-(x_j) \leq 2^+$; $0 \leq T^-(y_k) + I^-(y_k) + F^-(y_k) \leq 2^+$

or

$0 \leq T^-(x_j) + I^-(x_j) + F^-(x_j) + T^-(y_k) + I^-(y_k) + F^-(y_k) \leq 4^+$

and

$0 \leq T^+(x_j) + I^+(x_j) + F^+(x_j) \leq 2^+$; $0 \leq T^+(y_k) + I^+(y_k) + F^+(y_k) \leq 2^+$

or

$0 \leq T^+(x_j) + I^+(x_j) + F^+(x_j) + T^+(y_k) + I^+(y_k) + F^+(y_k) \leq 4^+$

(3) $T^-(x_j), I^-(x_j), F^-(x_j): V(U_1) \rightarrow [0, 1]$ and $T^-(y_k), I^-(y_k), F^-(y_k): V(U_2) \rightarrow [0, 1]$

$T^+(x_j), I^+(x_j), F^+(x_j): V(U_1) \rightarrow [0, 1]$ and $T^+(y_k), I^+(y_k), F^+(y_k): V(U_2) \rightarrow [0, 1]$

Here $V(U_1), V(U_2)$ denotes power set of vague sets on $U_1, U_2$ respectively.

**Example 3.9.**

Let $U_1 = \{x_1, x_2, x_3\}, U_2 = \{y_1, y_2\}$ be the common universe under consideration.

A NVBS is given below:

$$M_{NVB} = \left\{ \left( \begin{array}{ccc}
X_1 & X_2 & X_3 \\
0.2, 0.3 & 0.6, 0.7, 0.8 & 0.3, 0.7, 0.9 \\
0.3, 0.7 & 0.5, 0.6 & 0.3, 0.7, 0.9 \\
0.1, 0.9 & 0.4, 0.8 & 0.2, 0.7, 0.6, 0.9, 0.3, 0.8 \\
\end{array} \right) \right\}$$

**Definition 3.10. (Zero neutrosophic vague binary set and Unit Neutrosophic vague binary set)**

Let $\{U_1 = \{x_j/1 \leq j \leq n\}; U_2 = \{y_k/1 \leq k \leq p\}\}$ be two universes under consideration.

(i) A zero NVBS denoted as $\Phi_{NVB}$ over this common universe is given by,

$$\Phi_{NVB} = \left\{ \left( \begin{array}{c}
X_j \\
\end{array} \right) \mid \forall x_j \in U_1, \left( \begin{array}{c}
\end{array} \right) \mid \forall y_k \in U_2 \right\}$$

(ii) A unit NVBS denoted as $\Psi_{NVB}$ over this common universe is given by,

$$\Psi_{NVB} = \left\{ \left( \begin{array}{c}
X_j \\
\end{array} \right) \mid \forall x_j \in U_1, \left( \begin{array}{c}
\end{array} \right) \mid \forall y_k \in U_2 \right\}$$

4. Operations on Neutrosophic Vague Binary Sets

In this section some usual set theoretical operations are developed for NVBS's
Definition 4.1. (Subset of Neutrosophic vague binary sets)
Let $M_{NVB}$ and $P_{NVB}$ be two NVBS’s on a common universe $U_1$, $U_2$. Then $M_{NVB}$ is included by $P_{NVB}$ denoted by $M_{NVB} \subseteq P_{NVB}$ if the following conditions found true:

If $\forall x_j \in U_1$ and $1 \leq j \leq n$

(1) $\tilde{T}_{M_{NVB}}(x_j) \leq \tilde{T}_{P_{NVB}}(x_j)$
(2) $\tilde{I}_{M_{NVB}}(x_j) \geq \tilde{I}_{P_{NVB}}(x_j)$
(3) $\tilde{F}_{M_{NVB}}(x_j) \geq \tilde{F}_{P_{NVB}}(x_j)$

and $\forall y_j \in U_2$ and $1 \leq k \leq p$

(1) $\tilde{T}_{M_{NVB}}(y_k) \leq \tilde{T}_{P_{NVB}}(y_k)$
(2) $\tilde{I}_{M_{NVB}}(y_k) \geq \tilde{I}_{P_{NVB}}(y_k)$
(3) $\tilde{F}_{M_{NVB}}(y_k) \geq \tilde{F}_{P_{NVB}}(y_k)$

Example 4.2.
Let $U_1 = \{x_1, x_2\}$, $U_2 = \{y_1\}$ be a common universe. Let

$M_{NVB} = \left\{ \begin{array}{ll}
[0.1, 0.2], & \frac{x_1}{1} \\
[0.6, 0.7], & \frac{x_2}{1} \\
[0.8, 0.9], & \frac{y_1}{1}
\end{array} \right\}$

$P_{NVB} = \left\{ \begin{array}{ll}
[0.2, 0.3], & \frac{x_1}{1} \\
[0.5, 0.6], & \frac{x_2}{1} \\
[0.7, 0.8], & \frac{y_1}{1}
\end{array} \right\}$

Clearly, $M_{NVB} \subseteq P_{NVB}$

Definition 4.3. (Union of two neutrosophic vague binary sets)
Let $M_{NVB}$ and $P_{NVB}$ are two NVBS’s

(i) Union of two NVBS’s, $M_{NVB}$ and $P_{NVB}$ is a NVBS, given as

$M_{NVB} \cup P_{NVB} = S_{NVB}^- \left\{ \tilde{T}_{S_{NVB}}(x_j), \tilde{I}_{S_{NVB}}(x_j), \tilde{F}_{S_{NVB}}(x_j) ; \forall x_j \in U_1 \right\}$

whose truth-membership, indeterminacy-membership and false-membership functions are related to those of $M_{NVB}$ and $P_{NVB}$ is given by

$\tilde{T}_{S_{NVB}}(x_j) = \max \left( T^- M_{NVB}(x_j), T^- P_{NVB}(x_j) \right)$
$\tilde{I}_{S_{NVB}}(x_j) = \min \left( I^- M_{NVB}(x_j), I^- P_{NVB}(x_j) \right)$
$\tilde{F}_{S_{NVB}}(x_j) = \min \left( F^- M_{NVB}(x_j), F^- P_{NVB}(x_j) \right)$

and

$\tilde{T}_{S_{NVB}}(y_k) = \max \left( T^- M_{NVB}(y_k), T^- P_{NVB}(y_k) \right)$
$\tilde{I}_{S_{NVB}}(y_k) = \min \left( I^- M_{NVB}(y_k), I^- P_{NVB}(y_k) \right)$
$\tilde{F}_{S_{NVB}}(y_k) = \min \left( F^- M_{NVB}(y_k), F^- P_{NVB}(y_k) \right)$

Example 4.4.
In example 4.2

$S_{NVB}^- = \left\{ \begin{array}{ll}
[0.2, 0.3], & \frac{x_1}{1} \\
[0.5, 0.6], & \frac{x_2}{1} \\
[0.7, 0.8], & \frac{y_1}{1}
\end{array} \right\}$

Definition 4.5. (Intersection of two neutrosophic vague binary sets)
Let $M_{NVB}$ and $P_{NVB}$ are two NVBS’s

(i) Intersection of two NVBS’s, $M_{NVB}$ and $P_{NVB}$ is a NVBS, given as

$M_{NVB} \cap P_{NVB} = R_{NVB} = \left\{ \tilde{T}_{R_{NVB}}(x_j), \tilde{I}_{R_{NVB}}(x_j), \tilde{F}_{R_{NVB}}(x_j) ; \forall x_j \in U_1 \right\}$

whose truth-membership, indeterminacy-membership and false-membership functions are related to those of $M_{NVB}$ and $P_{NVB}$ is given by

$\tilde{T}_{R_{NVB}}(x_j) = \min \left( T^- M_{NVB}(x_j), T^- P_{NVB}(x_j) \right)$
$\tilde{I}_{R_{NVB}}(x_j) = \max \left( I^- M_{NVB}(x_j), I^- P_{NVB}(x_j) \right)$
$\tilde{F}_{R_{NVB}}(x_j) = \max \left( F^- M_{NVB}(x_j), F^- P_{NVB}(x_j) \right)$

and

$\tilde{T}_{R_{NVB}}(y_k) = \min \left( T^- M_{NVB}(y_k), T^- P_{NVB}(y_k) \right)$
$\tilde{I}_{R_{NVB}}(y_k) = \max \left( I^- M_{NVB}(y_k), I^- P_{NVB}(y_k) \right)$
$\tilde{F}_{R_{NVB}}(y_k) = \max \left( F^- M_{NVB}(y_k), F^- P_{NVB}(y_k) \right)$
Example 4.6.
In example 4.2. $R_{NVB} = \left\{ \left[ 0.1,0.2 \right], [0.6,0.7], [0.7,0.8], [0.2,0.6], [0.5,0.6], [0.4,0.8], [0.1,0.3], [0.6,0.7], [0.7,0.9] \right\}$

Definition 4.7. (Complement of a NVBS)
Let $M_{NVB}$ is defined as in definition 3.1. It’s complement is denoted by $M^{c}_{NVB}$ and is given by

$M^{c}_{NVB} = \left\{ T_{M_{NVB}}(x_j), F_{M_{NVB}}(x_j), S_{M_{NVB}}(x_j); \forall x_j \in U_1 \left( T_{M_{NVB}}(y_k), F_{M_{NVB}}(y_k), S_{M_{NVB}}(y_k); \forall y_k \in U_2 \right) \right\}$

is defined as

$T_{M_{NVB}}(x_j) = 1 - T^+(x_j), 1 - T^-(x_j)$

$F_{M_{NVB}}(x_j) = 1 - F^+(x_j), 1 - F^-(x_j)$

and

$S_{M_{NVB}}(x_j) = 1 - S^+(x_j), 1 - S^-(x_j)$

Example 5.6.

A neutrosophic vague binary topology on a common universe $U$ is defined as

Definition 5.1. (Neutrosophic vague binary topology)
It's various concepts are also discussed.

Example 5.2.

Example 5.4.

Example 5.8.

5. Neutrosophic vague binary topology
In this section neutrosophic vague binary topology (NVBT in short) is developed for NVBS's. It's various concepts are also discussed.

Definition 5.1. (Neutrosophic vague binary topology)
A neutrosophic vague binary topology on a common universe $U_1$, $U_2$ is a family $\tau^N_{NVB}$ of neutrosophic vague binary sets in $U_1$, $U_2$ satisfying the following axioms:

(1) $\Phi_{NVB}, \Psi_{NVB} \in \tau^N_{NVB}$

(2) For any $M_{NVB}, P_{NVB} \in \tau^N_{NVB}, M_{NVB} \cap P_{NVB} \in \tau^N_{NVB}$

i.e., finite intersection of NVBS's of $\tau^N_{NVB}$ is again a member of $\tau^N_{NVB}$

(3) Let $\{ M_{NVB}; i \in I \} \subseteq \tau^N_{NVB}$ then $\bigcup_{i \in I} \tau^N_{NVB} \subseteq \tau^N_{NVB}$

i.e., arbitrary union of neutrosophic vague binary sets in $\tau^N_{NVB}$ is again a member of $\tau^N_{NVB}$

Example 5.2.

Let $U_1 = \{ x_1, x_2 \}; U_2 = \{ y_1 \}$. Following is a neutrosophic vague binary topology ;

$$\tau^N_{NVB} = \left\{ \begin{array}{c}
\Phi_{NVB} = \{ \left[ 0.2,0.4 \right], \left[ 0.3,0.5 \right], \left[ 0.6,0.8 \right], \left[ 0.1,0.3 \right], \left[ 1,0.1 \right] \},

\Psi_{NVB} = \{ \left[ 0.2,0.6 \right], \left[ 0.3,0.7 \right], \left[ 0.7,0.9 \right], \left[ 0.2,0.4 \right], \left[ 0.1,0.3 \right] \},

M_{NVB} = \{ \left[ 0.2,0.4 \right], \left[ 0.3,0.5 \right], \left[ 0.6,0.8 \right], \left[ 0.1,0.3 \right], \left[ 1,0.1 \right] \},

P_{NVB} = \{ \left[ 0.2,0.6 \right], \left[ 0.3,0.7 \right], \left[ 0.7,0.9 \right], \left[ 0.2,0.4 \right], \left[ 0.1,0.3 \right] \},

K_{NVB} = M_{NVB} \cap P_{NVB} = \{ \left[ 0.2,0.4 \right], \left[ 0.3,0.5 \right], \left[ 0.6,0.8 \right], \left[ 0.1,0.3 \right], \left[ 1,0.1 \right] \},

H_{NVB} = M_{NVB} \cup P_{NVB} = \{ \left[ 0.2,0.4 \right], \left[ 0.3,0.5 \right], \left[ 0.6,0.8 \right], \left[ 0.1,0.3 \right], \left[ 1,0.1 \right] \}

\end{array} \right\}$$

Definition 5.3. (Neutrosophic vague binary open set)
Every elements of a NVBT is known as a neutrosophic vague binary open set (NVBOS in short)

Example 5.4.

In example 5.2. $\Phi_{NVB}, M_{NVB}, P_{NVB}, K_{NVB}, H_{NVB}, \Psi_{NVB}$ are all NVBOS’s

Definition 5.5. (Neutrosophic vague binary closed set)
Complement of a NVBOS is known as a neutrosophic vague binary closed set (NVBCS in short)

Example 5.6.
In example 5.2, \( \Phi_{NVB}, M_{NVB}^c, P_{NVB}^c, K_{NVB}^c, H_{NVB}^c, \Psi_{NVB} \) are all NVBC's, where
\[
\Phi_{NVB} = \left\{ \left( \frac{[0.0,1][1,1]}{x_1}, \frac{[0.0,1][1,1]}{x_1} \right), \left( \frac{[0.0,1][1,1]}{x_2}, \frac{[0.0,1][1,1]}{x_2} \right) \right\} = \Psi_{NVB}
\]
\[
M_{NVB}^c = \left\{ \left( \frac{[0.2,0.4][0.2,0.4]}{x_1}, \frac{[0.2,0.4][0.2,0.4]}{x_1} \right), \left( \frac{[0.2,0.4][0.2,0.4]}{x_2}, \frac{[0.2,0.4][0.2,0.4]}{x_2} \right) \right\}
\]
\[
P_{NVB}^c = \left\{ \left( \frac{[0.3,0.4][0.3,0.4]}{x_1}, \frac{[0.3,0.4][0.3,0.4]}{x_1} \right), \left( \frac{[0.3,0.4][0.3,0.4]}{x_2}, \frac{[0.3,0.4][0.3,0.4]}{x_2} \right) \right\}
\]
\[
K_{NVB}^c = \left\{ \left( \frac{[0.6,0.8][0.6,0.8]}{x_1}, \frac{[0.6,0.8][0.6,0.8]}{x_1} \right), \left( \frac{[0.6,0.8][0.6,0.8]}{x_2}, \frac{[0.6,0.8][0.6,0.8]}{x_2} \right) \right\}
\]
\[
H_{NVB}^c = \left\{ \left( \frac{[0.2,0.4][0.2,0.4]}{x_1}, \frac{[0.2,0.4][0.2,0.4]}{x_1} \right), \left( \frac{[0.2,0.4][0.2,0.4]}{x_2}, \frac{[0.2,0.4][0.2,0.4]}{x_2} \right) \right\}
\]
\[
\Psi_{NVB} = \left\{ \left( \frac{[0.1,1][1,1]}{x_1}, \frac{[0.1,1][1,1]}{x_1} \right), \left( \frac{[0.1,1][1,1]}{x_2}, \frac{[0.1,1][1,1]}{x_2} \right) \right\} = \Phi_{NVB}
\]

**Remark 5.7.**

\( \Phi_{NVB} \) and \( \Psi_{NVB} \) will both acts as NVBOS and NVBCS

**Definition 5.8. (Neutrosophic vague binary topological space)**

The triplet \( (U_1, U_2, \tau^\Delta_{NVB}) \) is known as a neutrosophic vague binary topological space (NVBTS in short), where \( \tau^\Delta_{NVB} \) is a neutrosophic vague binary topology defined as in definition 5.1.

**Example 5.9.**

If \( U_1 = \{ x_1, x_2 \} \); \( U_2 = \{ y_1 \} \); \( \tau^\Delta_{NVB} = \{ \Phi_{NVB}, M_{NVB}, P_{NVB}, K_{NVB}, H_{NVB}, \Psi_{NVB} \} \) defined as in example 5.2.

then the triplet \( (U_1, U_2, \tau^\Delta_{NVB}) \) is clearly a NVBTS.

**Definition 5.10. (Neutrosophic vague binary discrete topology and Neutrosophic vague binary discrete topological Space)**

A topology consisting of only empty and unit NVBS's is known as a neutrosophic vague binary discrete topology (NVBDTS in short) and the corresponding neutrosophic vague binary topological space is known as a neutrosophic vague binary discrete topological space (NVBDSTS in short).

i.e., \( \tau^\Delta_{NVB} = \{ \Phi_{NVB}, \Psi_{NVB} \} \)

**Example 5.11.**

In example 5.2.

\( \tau^\Delta_{NVB} = \{ \Phi_{NVB}, \Psi_{NVB} \} \)

is clearly a NVBDT and the corresponding neutrosophic vague topology is the NVBDTS.

**Definition 5.12. (Neutrosophic vague binary discrete topology and Neutrosophic vague binary discrete topological Space)**

A NVBTS defined by it's power set is known as neutrosophic vague binary indirect topology (NVBIDTS in short) and the corresponding neutrosophic vague binary topological space is known as a neutrosophic vague binary discrete topological space (NVBIDSTS in short).

**Definition 5.13. (Neutrosophic vague binary interior and Neutrosophic vague binary closure)**

Let \( (U_1, U_2, \tau^\Delta_{NVB}) \) be a NVBTS and also

let \( M_{NVB} = \{ \frac{\Phi_{NVB}(x), I_{NVB}(x), E_{NVB}(x)}{x} \}; \forall x \in U_1 \} \)

\( \frac{\Phi_{NVB}(y), I_{NVB}(y), E_{NVB}(y)}{y} \}; \forall y \in U_2 \}

is a NVBS over a common universe \( U_1, U_2 \) defined as in definition 3.1. Then it's neutrosophic vague binary interior (denoted as \( M_{NVB}^0 \)) and neutrosophic vague binary closure (denoted as \( M_{NVB} \))

are defined as follows:

\( M_{NVB} = \cup \{ M_{NVB}^i ; i \in I \} \)

\( M_{NVB} = \cap \{ M_{NVB}^i ; i \in I \} \)

**Example 5.14.**

In example 5.2.

\( H_{NVB} = \{ \frac{[0.0,1][1,1]}{x_1}, \frac{[0.0,1][1,1]}{x_1} \}; \frac{[0.0,1][1,1]}{x_2}, \frac{[0.0,1][1,1]}{x_2} \}

= H_{NVB}

From example 5.6.

\( M_{NVB} = \{ \frac{[0.2,0.4][0.2,0.4]}{x_1}, \frac{[0.2,0.4][0.2,0.4]}{x_1} \}; \frac{[0.2,0.4][0.2,0.4]}{x_2}, \frac{[0.2,0.4][0.2,0.4]}{x_2} \}

= M_{NVB}^c

**Proposition 5.15.**
(i) \( M_{NVB} \) is a NVBOS \( \iff \) \( M_{NVB}^0 = M_{NVB} \)
(ii) \( M_{NVB} \) is a NVBCS \( \iff \) \( M_{NVB} = M_{NVB} \)

**Proof**

Proof is clear

**Proposition 5.16.**

(i) \( M_{NVB} \subseteq M_{NVB}^2 \) and \( P_{NVB}^1 \subseteq P_{NVB}^2 \Rightarrow (M_{NVB}^1 \cup P_{NVB}^1) \subseteq (M_{NVB}^2 \cup P_{NVB}^2) \) and 
\( (M_{NVB}^1 \cap P_{NVB}^1) \subseteq (M_{NVB}^2 \cap P_{NVB}^2) \)

(ii) \( M_{NVB} \subseteq M_{NVB}^1 \) and \( M_{NVB} \subseteq M_{NVB}^2 \Rightarrow M_{NVB} \subseteq (M_{NVB}^1 \cap M_{NVB}^2) \) and 
\( M_{NVB} \subseteq M_{NVB} \Rightarrow (M_{NVB}^1 \cup M_{NVB}^2) \subseteq M_{NVB} \)

(iii) \( M_{NVB} = M_{NVB} \)

(iv) \( M_{NVB} \subseteq P_{NVB} \Rightarrow \overline{P_{NVB}} \subseteq \overline{M_{NVB}} \)

(v) \( \Phi_{NVB} = \Psi_{NVB} \)

(vi) \( \Psi_{NVB} = \Phi_{NVB} \)

**Proof**

Proof is clear

6. **Continuous mapping for NVBS’s**

Continuity plays vital role in any topology. In this section image, pre-image and continuity are developed for NVBS’s.

**Definition 6.1.** (Image and Pre-image of neutrosophic vague binary sets)

Let \( M_{NVB} \) and \( P_{NVB} \) be two non-empty NVBS’s defined on two common universes \( U_1, U_2 \) and \( V_1, V_2 \) respectively. Define a function \( f: M_{NVB} \rightarrow P_{NVB} \), then the following statements hold:

(1) If \( D_{NVB} = \left\{ \left( \hat{\Phi}_{NVB}(s_i); \hat{\Psi}_{NVB}(s_i); \hat{\Pi}_{NVB}(s_i) \right); s_i \in V_1 \right\} \) is a NVBS in \( P_{NVB} \), then the preimage of \( D_{NVB} \) under \( f \), denoted by \( f^{-1}(D_{NVB}) \), is a NVBS in \( M_{NVB} \) defined by 
\( f^{-1}(D_{NVB}) = \left\{ \left( \hat{\Phi}_{NVB}(s_i); \hat{\Psi}_{NVB}(s_i); \hat{\Pi}_{NVB}(s_i) \right); s_i \in V_1 \right\} \)

(2) If \( A_{NVB} = \left\{ \left( \hat{\Phi}_{NVB}(s_i); \hat{\Psi}_{NVB}(s_i); \hat{\Pi}_{NVB}(s_i) \right); s_i \in V_1 \right\} \) is a NVBS in \( M_{NVB} \), then the image of \( A_{NVB} \) under \( f \), denoted by \( f(A_{NVB}) \), is a NVBS in \( P_{NVB} \) defined by 
\( f(A_{NVB}) = \left\{ \left( \hat{\Phi}_{NVB}(s_i); \hat{\Psi}_{NVB}(s_i); \hat{\Pi}_{NVB}(s_i) \right); s_i \in V_1 \right\} \)

where

\[
\begin{align*}
& f_{\sup} \left( \hat{\Phi}_{NVB}(s_i) \right) = \begin{cases} 
\sup_{x \notin f^{-1}(s_i)} \hat{\Phi}_{NVB}(x), & \text{if } f^{-1}(s_i) \neq \emptyset \\
0, & \text{otherwise}
\end{cases} \\
& f_{\sup} \left( \hat{\Psi}_{NVB}(s_i) \right) = \begin{cases} 
\sup_{y \notin f^{-1}(t_r)} \hat{\Psi}_{NVB}(y), & \text{if } f^{-1}(t_r) \neq \emptyset \\
0, & \text{otherwise}
\end{cases} \\
& f_{\inf} \left( \hat{\Phi}_{NVB}(s_i) \right) = \begin{cases} 
\inf_{x \notin f^{-1}(s_i)} \hat{\Phi}_{NVB}(x), & \text{if } f^{-1}(s_i) \neq \emptyset \\
0, & \text{otherwise}
\end{cases} \\
& f_{\inf} \left( \hat{\Psi}_{NVB}(s_i) \right) = \begin{cases} 
\inf_{y \notin f^{-1}(t_r)} \hat{\Psi}_{NVB}(y), & \text{if } f^{-1}(t_r) \neq \emptyset \\
0, & \text{otherwise}
\end{cases} \\
& f_{\inf} \left( \hat{\Pi}_{NVB}(s_i) \right) = \begin{cases} 
\inf_{x \notin f^{-1}(s_i)} \hat{\Pi}_{NVB}(x), & \text{if } f^{-1}(s_i) \neq \emptyset \\
0, & \text{otherwise}
\end{cases} \\
& f_{\inf} \left( \hat{\Pi}_{NVB}(s_i) \right) = \begin{cases} 
\inf_{y \notin f^{-1}(t_r)} \hat{\Pi}_{NVB}(y), & \text{if } f^{-1}(t_r) \neq \emptyset \\
0, & \text{otherwise}
\end{cases}
\end{align*}
\]

for each \( s_i \in V_1 \) and for each \( t_r \in V_2 \)

**Definition 6.2.** (Neutrosophic Vague strongly continuous mapping)
Let \((X, \tau)\) and \((Y, \sigma)\) be any two neutrosophic vague topological spaces. A map \(f : (X, \tau) \to (Y, \sigma)\) is said to be neutrosophic vague strongly continuous if inverse image of every neutrosophic vague set in \((Y, \sigma)\) is neutrosophic vague open set \(\{\text{a set which acts simultaneously as neutrosophic vague open set and neutrosophic vague closed set}\}\) in \((X, \tau)\)

**Definition 6.3.**

(i) Neutrosophic Vague Binary Continuity:
Let \((U_1, U_2, \tau^1_{NVB})\) and \((V_1, V_2, \sigma^1_{NVB})\) be any two NVBS’s.
A map \(f : (U_1, U_2, \tau^1_{NVB}) \to (V_1, V_2, \sigma^1_{NVB})\) is said to be neutrosophic vague binary continuous (NVB continuous) if for every NVBOS (or NVBCS) \(M^1_{NVB}\) of \((V_1, V_2, \sigma^1_{NVB})\), \(f^{-1}(M^1_{NVB})\) is a NVBOS (or NVBCS) in \((U_1, U_2, \tau^1_{NVB})\).

(ii) Various kinds of Continuities for NVBS’s
Let \((U_1, U_2, \tau^2_{NVB})\) and \((V_1, V_2, \sigma^2_{NVB})\) be any two NVBTS’s. A map \(f : (U_1, U_2, \tau^2_{NVB}) \to (V_1, V_2, \sigma^2_{NVB})\) is said to be

1. Neutrosophic vague binary semi-continuous (NVBSC): if for every neutrosophic vague binary open set \((NVBOS\ in\ short)\) in \((NVBCS\ in\ short)\) \(M^2_{NVB}\) of \((V_1, V_2, \sigma^2_{NVB})\), \(f^{-1}(M^2_{NVB})\) is a neutrosophic vague binary semi-open set \((NVBOS\ in\ short)\) in \((NVBCS\ in\ short)\) in \((U_1, U_2, \tau^2_{NVB})\).
2. Neutrosophic vague binary pre-continuous (NVBPC) continuous): if for every NVBOS (or NVBCS) \(M^2_{NVB}\) of \((V_1, V_2, \sigma^2_{NVB})\) \(f^{-1}(M^2_{NVB})\) is a neutrosophic vague binary pre-open set \((NVBPOS\ in\ short)\) in \((NVBPCS\ in\ short)\) in \((U_1, U_2, \tau^2_{NVB})\).
3. Neutrosophic vague binary strongly-continuous (NVBSC continuous): if inverse image of every neutrosophic vague binary set in \((V_1, V_2, \sigma^2_{NVB})\) is neutrosophic vague binary clopen set \(\{\text{a set which acts simultaneously as neutrosophic vague binary open set and neutrosophic vague binary closed set}\}\) in \((U_1, U_2, \tau^2_{NVB})\).
4. Neutrosophic vague binary regular-continuous (NVBRC continuous): if for every NVBOS (or NVBCS) \(M^2_{NVB}\) of \((V_1, V_2, \sigma^2_{NVB})\) \(f^{-1}(M^2_{NVB})\) is a neutrosophic vague binary regular-open set \((NVBROS\ in\ short)\) in \((NVBPCS\ in\ short)\) in \((U_1, U_2, \tau^2_{NVB})\).
5. Neutrosophic vague binary semi-pre-continuous (NVBRC continuous): if for every NVBOS (or NVBCS) \(M^2_{NVB}\) of \((V_1, V_2, \sigma^2_{NVB})\) \(f^{-1}(M^2_{NVB})\) is a neutrosophic vague binary generalized semi-open set \((NVBGSOS\ in\ short)\) in \((NVBGSCS\ in\ short)\) in \((U_1, U_2, \tau^2_{NVB})\).

**Example 6.4.**

Let \(f = (g, h) : M^2_{NVB} \to P^2_{NVB}\) be a function defined as \(f(\Phi^1_{NVB}) = \Phi^2_{NVB}, f(M^2_{NVB}) = P^2_{NVB}, f(M^2_{NVB}) = P^2_{NVB}\), \(f(\Psi^1_{NVB}) = \Psi^2_{NVB}\) where \(g : U_1 \to V_1\) and \(h : U_2 \to V_2\) be two functions with \(g(x_1) = s_2, g(x_2) = s_1\) and \(h(y_1) = t_1\), \(U_1 = \{x_1, x_2\}, U_2 = \{y_1\}\) and \(V_1 = \{s_1, s_2\}, U_2 = \{t_1\}\).

Let \(\tau^2_{NVB} = \{\Phi^1_{NVB}, M^1_{NVB}, M^2_{NVB}, M^3_{NVB}, M^4_{NVB}, M^5_{NVB}, M^6_{NVB}, M^7_{NVB}, M^8_{NVB}, M^9_{NVB}, M^{10}_{NVB}, M^{11}_{NVB}, \Psi^2_{NVB}\} \) and \(\sigma^2_{NVB} = \{\Phi^2_{NVB}, P^2_{NVB}, \Psi^2_{NVB}\}\) be their respective NVBT’s.

Here
\[
\Phi^1_{NVB} = \left\{\frac{[0.0, 0.1, 1.1]}{x_1}, \frac{[0.0, 1.1, 1.1]}{x_2}, \frac{[0.0, 1.1, 1.1]}{y_1}\right\}
\]
\[
M^1_{NVB} = \left\{\frac{[0.3, 0.4]}{x_1}, \frac{[0.7, 0.8]}{x_2}, \frac{[0.6, 0.7]}{y_1}\right\}
\]
\[
M^2_{NVB} = \left\{\frac{[0.1, 0.6]}{x_1}, \frac{[0.6, 0.9]}{x_2}, \frac{[0.4, 0.9]}{y_1}\right\}
\]
\[
M^3_{NVB} = \left\{\frac{[0.6, 0.8]}{x_1}, \frac{[0.1, 0.5]}{x_2}, \frac{[0.2, 0.4]}{y_1}\right\}
\]
\[
M^4_{NVB} = \left\{\frac{[0.3, 0.6]}{x_1}, \frac{[0.6, 0.8]}{x_2}, \frac{[0.4, 0.7]}{y_1}\right\}
\]
\[
M^5_{NVB} = \left\{\frac{[0.2, 0.7]}{x_1}, \frac{[0.2, 0.9]}{x_2}, \frac{[0.3, 0.8]}{y_1}\right\}
\]
\[
M^6_{NVB} = \left\{\frac{[0.2, 0.7]}{x_1}, \frac{[0.2, 0.9]}{x_2}, \frac{[0.3, 0.8]}{y_1}\right\}
\]

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Let $U_1 = \{x_1, x_2, \ldots, x_n\}; \; U_2 = \{y_1, y_2, \ldots, y_p\}$ be the common universe. Also let $M_{NVB}$ and $P_{NVB}$ be two $NVBS's$.

(i) Hamming distance between them is defined as

$$d^H_{NVB}(M_{NVB}, P_{NVB}) =$$

$$= \frac{1}{n} \sum_{i=1}^{n} (|f_{M_{NVB}}(x_i) - f_{P_{NVB}}(x_i)| + |f_{M_{NVB}}(x_i) - f_{P_{NVB}}(x_i)| + |f_{M_{NVB}}(x_i) - f_{P_{NVB}}(x_i)| + |f_{M_{NVB}}(x_i) - f_{P_{NVB}}(x_i)| + |f_{M_{NVB}}(x_i) - f_{P_{NVB}}(x_i)| + |f_{M_{NVB}}(x_i) - f_{P_{NVB}}(x_i)|)$$

It is got that, $f^{-1}(\Phi^2_{NVB}) = \Phi^1_{NVB}, \; f^{-1}(P^1_{NVB}) = M^2_{NVB}, \; f^{-1}(\Psi^2_{NVB}) = \Psi^1_{NVB}$. Then clearly $f$ is a neutrosophic vague binary continuous mapping.

7. Distance Measures for $NVBS's$
(ii) Normalized Hamming distance between them is defined as

\[
d_{H,NVBS}(M_{NVBS}, P_{NVBS}) = \sum_{i=1}^{n} \left| \frac{T_{NVBS}(x_i) - T_{NVBS}(y_i)}{\min(T_{NVBS}(x_i), T_{NVBS}(y_i))} \right|
\]

(iii) Euclidean distance between them is defined as

\[
d_{E,NVBS}(M_{NVBS}, P_{NVBS}) = \sqrt{\sum_{i=1}^{n} \left( T_{NVBS}(x_i) - T_{NVBS}(y_i) \right)^2}
\]

(iv) Normalized Euclidean distance between them is defined as

\[
d_{NE,NVBS}(M_{NVBS}, P_{NVBS}) = \sqrt{\sum_{i=1}^{n} \left( \frac{T_{NVBS}(x_i) - T_{NVBS}(y_i)}{\min(T_{NVBS}(x_i), T_{NVBS}(y_i))} \right)^2}
\]

8. NVBS’s in Medical Diagnosis

This section deals with an application of NVBS’s in medical diagnosis. Following table describes datas collected from three patients after conducting liver function test. First set of sample is collected before treatment which describes the first universe. Second set of sample is collected after treatment which describes the second universe. \( P_1^{AT}, P_2^{AT}, P_3^{AT} \) are three NVBS’s formed, based on the datas of the three patients under consideration

<table>
<thead>
<tr>
<th>Before Treatment (BT)</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albumin</td>
<td>[0.042, 0.052]</td>
<td>[0.025, 0.052]</td>
<td>[0.052, 0.064]</td>
</tr>
<tr>
<td>Globulin Serum</td>
<td>[0.035, 0.045]</td>
<td>[0.033, 0.035]</td>
<td>[0.011, 0.035]</td>
</tr>
<tr>
<td>Bilirubin Total</td>
<td>[0.045, 0.100]</td>
<td>[0.070, 0.100]</td>
<td>[0.093, 0.100]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>After Treatment (AT)</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albumin</td>
<td>[0.031, 0.052]</td>
<td>[0.036, 0.052]</td>
<td>[0.052, 0.064]</td>
</tr>
<tr>
<td>Globulin Serum</td>
<td>[0.021, 0.035]</td>
<td>[0.035, 0.042]</td>
<td>[0.019, 0.035]</td>
</tr>
<tr>
<td>Bilirubin Total</td>
<td>[0.025, 0.100]</td>
<td>[0.017, 0.100]</td>
<td>[0.099,0.100]</td>
</tr>
</tbody>
</table>

Data collected from 3 persons are converted to NVBS’s as given below:

\[
P_{NVBS} = \begin{pmatrix}
P_{NVBS}^{\text{Albumin}} & P_{NVBS}^{\text{Globulin Serum}} & P_{NVBS}^{\text{Bilirubin Total}} \\
P_{NVBS}^{\text{Albumin}} & P_{NVBS}^{\text{Globulin Serum}} & P_{NVBS}^{\text{Bilirubin Total}} \\
P_{NVBS}^{\text{Albumin}} & P_{NVBS}^{\text{Globulin Serum}} & P_{NVBS}^{\text{Bilirubin Total}} \\
\end{pmatrix}
\]

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under a liver function test for albumin, Globulin serum and Bilirubin Total is given as follows:

Continuity has an important role in topology. It is also developed for this new concept. Practical concepts. Real life situations demand binary and higher dimensional universes than a unique one.

9. Conclusions

Neutrosophic vague binary euclidean distance measure can be used to diagnose which patient is more suffering with liver problems even after treatment. Following table gives the neutrosophic vague binary euclidean difference between each of the patients from $D_{LFT}^{LFT}$

$$d_{ED}^{NVBS}(P_{avg}^{1}, D_{LFT}^{NVBS})$$

Below datas are converted to NVBS as below.

Neutrosophic vague binary euclidean distance measure can be used to diagnose which patient is more suffering with liver problems even after treatment. Following table gives the neutrosophic vague binary euclidean difference between each of the patients from $D_{LFT}^{LFT}$

$$d_{ED}^{NVBS}(P_{avg}^{1}, D_{LFT}^{NVBS})$$

Lowest neutrosophic vague binary euclidean difference is for patient I. So patient I suffers more with liver problems even after treatment.

### 9. Conclusions

Neutrosophic vague binary sets are developed in this paper with some examples and basic concepts. Real life situations demand binary and higher dimensional universes than a unique one. Being the vital concept to homeomorphism - 'which is the underlying principle to any topology' - continuity has an important role in topology. It is also developed for this new concept. Practical
applications are tremendous for binary concept in day today life. One real life example in medical diagnosis is discussed above. Several situations demand combined result than ‘a unique separate one’- to compare and deal situations in a more fast manner. Neutrosophic vague binary sets is a good tool for comparison in such cases. It could be made use in surveys, case studies and in some other sort of similar situations. Topology are special type of subsets to a universal set- based on which study of all other subsets of the universal set is possible. New study will produce a combined result or net effect than taking a single result. This work could be extended by taking subsets of the common universe.

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