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An Introduction to Neutrosophic Vague Refined set

A.Angel Marina[1], A. Francina Shalini[2]

[1]Research Scholar, Nirmala College for Women, Coimbatore, Tamil Nadu; angelmarinamat2020@gmail.com

[2]Assistant Professor, Nirmala College for Women, Coimbatore, Tamil Nadu; francshalu@gmail.com

*Correspondence: angelmarinamat2020@g-mail.com;

Abstract. The aim of this paper is to introduce a new set namely neutrosophic vague refined set to handle inconsistent and indeterminate information. Set theoretic operations depend on neutrosophic vague refined set are discussed along with desired properties. Also, algebraic operations on neutrosophic vague refined set are studied with suitable examples.

Keywords: Neutrosophic set, Vague set, Neutrosophic refined set, Neutrosophic Vague set, Neutrosophic vague refined set.

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1. **Introduction**

Many of the philosophers make an attempt to model unsure information using classical mathematics might not achieve success manner. This is because the uncertainty that is just too convoluted and in addition not understandably outlined. Several scientists place an effort to search out applicable solutions to some mathematical issues that can't be resolved by ancient strategies. These issues have an effect truth that traditional strategies cannot work out the issues of uncertainty in engineering, economy, decision-making, medicine etc. Therefore, many alternative theories was developed to clear up uncertainty and vagueness together with the fuzzy set theory[28], vague set theory[6] and rough set theory [9]. But, these theories cannot deal with uncertainty and consistent info.

In the theory of classical set, the membership of the elements in a non-empty set is of two conditions. As per that conditions, an element either belongs to the set or doesn't belong to the set. In fuzzy set theory, an element is assigned by the grade of membership value within the closed unit interval [0,1]. Therefore, a fuzzy set A in the universal set of discourse X is a function f: A $\rightarrow$ [0,1] and often this function is mentioned as the membership function.

Later, because of the generalisation of fuzzy set Atanassov in 1986 introduced the intuitionistic fuzzy set[1] where an element is assigned by values of truth membership and false membership ranges from [0,1] individually. Vague set theory[6] in 1993 was initially found by

Gau et.al. which is an extension of theory of fuzzy set. Vague sets are taken into account as an efficient tool to influence uncertainty since it provides additional info as compared to fuzzy sets[28]. Many studies have unconcealed that, several researchers have combined vague set with other theories.

Neutrosophic set was developed by Smarandache[21] in 1998 considered as the generalization of probability set, fuzzy set[28] and intuitionistic fuzzy set[1]. The neutrosophic set has 3 independent membership functions. Not like fuzzy set and intuitionistic fuzzy set, the membership functions in neutrosophic sets are truth, indeterminate and falsity.

As a generalisation of set theory, Yager[25] first of all introduced new theory named as theory of bags which in turn gives the concept of multiset (i.e) the element may take place more than once. Subsequently, the idea of multiset was originally initiated by Blizard[2] and Calude et al.[3]. Many authors now and then created variety of generalization of theory of set. As the generalisation of multiset, many researchers [4, 8, 16, 17, 18, 22, 24] discussed additional properties on fuzzy multiset which takes place more than once with possibly same or different membership values. Shinoj et.al.[20] created associate extension of combining the concept of fuzzy multiset by an intuitionstic fuzzy set, which is known as intuitionstic fuzzy multiset. By the study on intuitionstic fuzzy multiset, loads of excellent results are achieved by researchers [5, 10, 11, 12, 13, 14, 15]. The ideas of fuzzy multiset and intuitionistic fuzzy multiset fails to influence uncertainty. Chatterjee et. al.[3] initiated single valued neutrosophic multiset very well.

In 2013, by extending classical neutrosophic logic Smarandache[21] gave n-valued refined neutrosophic logic and its applications in which the neutrosophic components T,I,F are refined to n number of components. Neutrosophic Refined set are defined by the generalisation of fuzzy multi set, intuitionistic fuzzy multi set. Irfan Deli[7] in 2016 extended refined neutrosophic set to refined neutrosophic soft set. Ye and Ye[26] developed single valued neutrosophic set and operations with laws. Ye et. al.[27] originated generalized distance measure and its similarity measure between single valued neutrosophic multi sets. Additionally, they applied the measures to a diagnosing medical side with incomplete, indeterminate and inconsistent info.

This paper is organized as follows: The essential definitions of Fuzzy set, Intuitionistic fuzzy set, Vague set, Neutrosophic set, Neutrosophic vague set and few of its properties that are helpful for the discussion have been presented. We have established a new concept by extending neutrosophic vague set namely neutrosophic vague refined(multi) set and few of its properties and operations on neutrosophic vague refined(multi) set are discussed. Additionally, algebraic operations of neutrosophic vague refined(multi) set along with some examples are given.

2. Preliminaries

In this section, we recall the useful basic definitions and related results for developing the desired set.

Definition 2.1. [28](Fuzzy Set)
Let X be a non-empty set in the universe of discourse. A fuzzy set A_F on X is defined as follows:

$$A_F= \{\langle x, \alpha_A(x)\rangle : x \in X\}$$

where $\alpha_A(x)$: $X \rightarrow [0,1]$ is represented as a grade of membership function of the fuzzy set A_F. It is denoted as A_F.

Definition 2.2. [1](Intuitionistic Fuzzy Set)
Let X be a non-empty set in the universe of discourse. A intuitionistic fuzzy set A_{IF} on X is an object defined as follows:

$$A_{IF}= \{\langle x, \alpha_A(x), \beta_A(x)\rangle : x \in X\}$$

where $\alpha_A(x) : X \rightarrow [0,1]$, $\beta_A(x) : X \rightarrow [0,1]$ represents the degree of membership function and degree of non-membership function respectively of the element x∈X to the set A_{IF}, which is a subset of X. And also for every element x∈X, membership functions $\alpha_A(x)$, $\beta_A(x)$ ranges $0 \leq \alpha_A(x) + \beta_A(x) \leq 1$. It is denoted as A_{IF}.

Definition 2.3. [6](Vague Set)
Let X be a non-empty set in the universe of discourse. A Vague set A_V in X is defined as follows:

$$A_V = \{\langle x, \alpha_A(x), 1 - \beta_A(x)\rangle : x \in X\}$$

where α_A: X→[0,1], β_A: X→[0,1] denotes truth membership function and false membership function respectively and $\alpha_A(x) + \beta_A(x) \leq 1$. Here $\alpha_A(x)$ is a lower bound on the grade of membership of x derived from the evidence for x and $\beta_A(x)$ is a lower bound on the grade of membership of the negation of x derived from the evidence against x.

Definition 2.4. [21](Neutrosophic set)
Let X be a non-empty set in the universe of discourse. A neutrosophic set A_N in X is defined as follows:

$$A_N= \{\langle x, \mu_A(x), \eta_A(x), \nu_A(x)\rangle : x \in X\}$$

where $\mu_A(x), \eta_A(x), \nu_A(x)$ represents truth membership function, indeterminate membership function and a falsity membership function of the element x∈ X to the set A_N respectively and the condition $^{-}0 \leq \mu_A(x) + \eta_A(x) + \nu_A(x) \leq 3^{+}$ holds.

Definition 2.5. [19](Neutrosophic Vague set)

Let X be the non-empty set in universe of discourse. A Neutrosophic vague set (M_{NV}in short) is of the form,

$$M_{NV} = \{\langle x, (\hat{T}_{M_{NV}}(x), \hat{I}_{M_{NV}}(x), \hat{F}_{M_{NV}}(x)\rangle : x \in X\}$$

whose truth membership, indeterminacy membership and false membership functions is defined as

$$\hat{T}_{M_{NV}}(x) = [T^-, T^+], \hat{I}_{M_{NV}}(x) = [I^-, I^+], \hat{F}_{M_{NV}}(x) = [F^-, F^+]$$

where,

$$T^+ = 1 - F^-, F^+ = 1 - T^-$$

and the condition $^-0 \leq T^- + I^- + F^- \leq 2^+$, $^-0 \leq T^+ + I^+ + F^+ \leq 2^+$ holds.

Definition 2.6. [19]

Let M_{NV} and N_{NV} be two neutrosophic vague set of the universe U. For x∈X, M_{NV} is included by N_{NV} if $\hat{T}_{M_{NV}}(x) \leq \hat{T}_{N_{NV}}(x); \hat{I}_{M_{NV}}(x) \geq \hat{I}_{N_{NV}}(x); \hat{F}_{M_{NV}}(x) \geq \hat{F}_{N_{NV}}(x)$. It is denoted as $M_{NV} \subseteq N_{NV}$.

Definition 2.7. [19]

The complement of neutrosophic vague set M_{NV} is defined by $\hat{T}^c_{M_{NV}}(x) = [1 - T^+, 1 - T^-]$, $\hat{I}^c_{M_{NV}}(x) = [1 - I^+, 1 - I^-]$, $\hat{F}^c_{M_{NV}}(x) = [1 - F^+, 1 - F^-]$.It is denoted as M^c_{NV},

Definition 2.8. [19]

The union of two neutrosophic vague sets M_{NV} and N_{NV} is given by P_{NV} where P_{NV}= $M_{NV} \cup N_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are

$$\hat{T}_{P_{NV}}(\mathrm{x}) = \Big[max(T^-_{M_{NV_x}}, T^-_{N_{NV_x}}), max(T^+_{M_{NV_x}}, T^+_{N_{NV_x}})\Big]$$
$$\hat{I}_{P_{NV}}(\mathrm{x}) = \Big[min(I^-_{M_{NV_x}}, I^-_{N_{NV_x}}), min(I^+_{M_{NV_x}}, I^+_{N_{NV_x}})\Big]$$
$$\hat{F}_{P_{NV}}(\mathrm{x}) = \Big[min(F^-_{M_{NV_x}}, F^-_{N_{NV_x}}), min(F^+_{M_{NV_x}}, F^+_{N_{NV_x}})\Big]$$

Definition 2.9. [19]

The intersection of two neutrosophic vague sets M_{NV} and N_{NV} is given by P_{NV} where P_{NV}= $M_{NV} \cap N_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are

$$\hat{T}_{P_{NV}}(\mathrm{x}) = \Big[min(T^-_{M_{NV_x}}, T^-_{N_{NV_x}}), min(T^+_{M_{NV_x}}, T^+_{N_{NV_x}})\Big]$$
$$\hat{I}_{P_{NV}}(\mathrm{x}) = \Big[max(I^-_{M_{NV_x}}, I^-_{N_{NV_x}}), max(I^+_{M_{NV_x}}, I^+_{N_{NV_x}})\Big]$$
$$\hat{F}_{P_{NV}}(\mathrm{x}) = \Big[max(F^-_{M_{NV_x}}, F^-_{N_{NV_x}}), max(F^+_{M_{NV_x}}, F^+_{N_{NV_x}})\Big]$$

Definition 2.10. [22](Neutrosophic Refined set)

Let X be a non empty set of universe of discourse. A neutrosophic refined set (briefly NRS) on X can be defined as follows:

$A_{NR}=\{\langle x,(T^1_{A_{NR}}(x),T^2_{A_{NR}}(x),...T^p_{A_{NR}}(x)),(I^1_{A_{NR}}(x),I^2_{A_{NR}}(x),...I^p_{A_{NR}}(x)),(F^1_{A_{NR}}(x),F^2_{A_{NR}}(x),$
$...F^p_{A_{NR}}(x))\rangle : x\in X\}$

where

$$T^1_{A_{NR}}(x),T^2_{A_{NR}}(x),...T^p_{A_{NR}}(x) : X\to [0,1],$$
$$I^1_{A_{NR}}(x),I^2_{A_{NR}}(x),...I^p_{A_{NR}}(x) : X\to [0,1],$$
$$F^1_{A_{NR}}(x),F^2_{A_{NR}}(x),...F^p_{A_{NR}}(x) : X\to [0,1]$$

such that $0\leq T^j_{A_{NR}}(x)+I^j_{A_{NR}}(x)+F^j_{A_{NR}}(x)\leq 3$ for j=1,2,...,p for any x∈X, $T^1_{A_{NR}}(x),T^2_{A_{NR}}(x),...T^p_{A_{NR}}(x)$, $I^1_{A_{NR}}(x),I^2_{A_{NR}}(x),...I^p_{A_{NR}}(x)$ and $F^1_{A_{NR}}(x),F^2_{A_{NR}}(x),...F^p_{A_{NR}}(x)$ is the truth-membership sequence, indeterminate-membership sequence and falsity-membership sequence of the element x, respectively. Also, p is called the dimension of neutrosophic refined set A_{NR}.

3. Neutrosophic Vague Refined sets

In this section, we introduce the new set namely neutrosophic vague refined set with suitable example.

Definition 3.1. (Neutrosophic vague refined set)

Let X be a non empty set in the universe of discourse. A neutrosophic vague refined set (in short NVRS)D_{NVR} on X can be defined by the form

$D_{NVR}=\{\langle x,(\hat{T}^1_{D_{NVR}}(x),\hat{T}^2_{D_{NVR}}(x),....\hat{T}^u_{D_{NVR}}(x)),(\hat{I}^1_{D_{NVR}}(x),\hat{I}^2_{D_{NVR}}(x),....\hat{I}^u_{D_{NVR}}(x)),$
$(\hat{F}^1_{D_{NVR}}(x),\hat{F}^2_{D_{NVR}}(x),....\hat{F}^u_{D_{NVR}}(x))\rangle : x\in X\}$

The truth-membership, indeterminacy-membership and falsity-membership functions of neutrosophic vague refined set is defined as,

$$\hat{T}^j_{D_{NVR}}=[T^-_j,T^+_j],\hat{I}^j_{D_{NVR}}=[I^-_j,I^+_j],\hat{F}^j_{D_{NVR}}=[F^-_j,F^+_j]$$

and

$$T^+_j=1-F^-_j,F^+_j=1-T^-_j$$

where

$\hat{T}^1_{D_{NVR}}(x),\hat{T}^2_{D_{NVR}}(x),...,\hat{T}^u_{D_{NVR}}(x) : X\longrightarrow P[0,1],\hat{I}^1_{D_{NVR}}(x),\hat{I}^2_{D_{NVR}}(x),...,\hat{I}^u_{D_{NVR}}(x) : X\longrightarrow P[0,1],\hat{F}^1_{D_{NVR}}(x),\hat{F}^2_{D_{NVR}}(x),...,\hat{F}^u_{D_{NVR}}(x) : X\longrightarrow P[0,1]$ such that $^-0\leq \hat{T}^j_{D_{NVR}}(x)+\hat{I}^j_{D_{NVR}}(x)+\hat{F}^j_{D_{NVR}}(x)\leq 2^+$ for j=1,2,...,u for any element x ∈ X and P[0,1] is the power set of [0,1].

Here, $\hat{T}^1_{D_{NVR}}(x),\hat{T}^2_{D_{NVR}}(x),...,\hat{T}^u_{D_{NVR}}(x),\hat{I}^1_{D_{NVR}}(x),\hat{I}^2_{D_{NVR}}(x),...,\hat{I}^u_{D_{NVR}}(x)$, $\hat{F}^1_{D_{NVR}}(x),\hat{F}^2_{D_{NVR}}(x),...,\hat{F}^u_{D_{NVR}}(x)$ is represented as truth membership sequence, indeterminate membership sequence and falsity membership sequence of the element x respectively. And also, u is called the dimension of neutrosophic vague refined set D_{NVR}.

Remark 3.2.

We arrange the membership sequence in decreasing order but the corresponding indeterminate sequence and non-membership sequence may not be in increasing or decreasing order.

Example 3.3.

Let X={a,b} be any non empty set. Then

D_{NVR}=$\{\langle a, ([0.1, 0.6], [0.5, 0.8], [0.1, 0.8]), ([0.3, 0.5], [0.8, 0.9], [0.6, 0.8]), ([0.4, 0.9], [0.2, 0.5],$
$[0.2, 0.9]), b, ([0.5, 0.8], [0.3, 0.9], [0.2, 0.5]), ([0.7, 0.8], [0.3, 0.4], [0.2, 0.3]), ([0.2, 0.5],$
$[0.1, 0.7], [0.5, 0.8])\rangle\}$

is said to be a neutrosophic vague refined subset of X.

Definition 3.4. (Null set)

A neutrosophic vague refined sets on the universe X is said to be a null neutrosophic vague refined set denoted by 0_{NVR} if

$$\hat{T}^j_{P_{NVR}}(x) = [0, 0], \hat{I}^j_{P_{NVR}}(x) = [1, 1], \hat{F}^j_{P_{NVR}}(x) = [1, 1]$$

for j=1,2,3,...u and x∈ X.

Definition 3.5. (Absolute set)

A neutrosophic vague refined sets on the universe X is said to be a absolute neutrosophic vague refined set denoted by 1_{NVR} if

$$\hat{T}^j_{P_{NVR}}(x) = [1, 1], \hat{I}^j_{P_{NVR}}(x) = [0, 0], \hat{F}^j_{P_{NVR}}(x) = [0, 0]$$

for j=1,2,3,...u and x∈ X.

4. Set Theoretic Operations on Neutrosophic Vague Refined Set

In this section, we study some of the set theoretic operations on neutrosophic vague refined set with suitable examples.

Definition 4.1. (Union)

Let P_{NVR} and Q_{NVR} be two neutrosophic vague refined sets in the universal set X, where

$$P_{NVR} = \{\langle(\hat{T}^1_{P_{NVR}}(x), \hat{T}^2_{P_{NVR}}(x), ...\hat{T}^u_{P_{NVR}}(x)), (\hat{I}^1_{P_{NVR}}(x), \hat{I}^2_{P_{NVR}}(x), ...\hat{I}^u_{P_{NVR}}(x)),$$
$$(\hat{F}^1_{P_{NVR}}(x), \hat{F}^2_{P_{NVR}}(x), ...\hat{F}^u_{P_{NVR}}(x))\rangle : x \in X\ \}$$
$$Q_{NVR} = \{\langle(\hat{T}^1_{Q_{NVR}}(x), \hat{T}^2_{Q_{NVR}}(x), ...\hat{T}^v_{Q_{NVR}}(x)), (\hat{I}^1_{Q_{NVR}}(x), \hat{I}^2_{Q_{NVR}}(x), ...\hat{I}^v_{Q_{NVR}}(x)),$$
$$(\hat{F}^1_{Q_{NVR}}(x), \hat{F}^2_{Q_{NVR}}(x), ...\hat{F}^v_{Q_{NVR}}(x))\rangle : x \in X\ \}$$

Then, the neutrosophic vague refined union of two sets P_{NVR} and Q_{NVR} denoted by S_{NVR}

where $S_{NVR} = P_{NVR} \cup Q_{NVR}$, whose truth membership function, indeterminate membership function and false membership function is

$$\hat{T}^j_{S_{NVR}}(x) = max\{\hat{T}^j_{P_{NVR}}(x), \hat{T}^j_{Q_{NVR}}(x)\},$$
$$\hat{I}^j_{S_{NVR}}(x) = min\{\hat{I}^j_{P_{NVR}}(x), \hat{I}_{Q_{NVR}}(x)\},$$
$$\hat{F}^j_{S_{NVR}}(x) = min\{\hat{F}^j_{P_{NVR}}(x), \hat{F}^j_{Q_{NVR}}(x)\}$$

∀ x∈X, j=1,2,...,r.

Example 4.2.

Let X= {a,b} be the non empty set. Let the sets P_{NVR} and Q_{NVR} be defined as:

$$P_{NVR} = \{\langle a, ([0.3, 0.4], [0.4, 0.5], [0.4, 0.6]), ([0.5, 0.6], [0.4, 0.5], [0.1, 0.2]), ([0.6, 0.7], [0.5, 0.6], [0.4, 0.6]), b, ([0.4, 0.6], [0.4, 0.5], [0.4, 0.7]), ([0.5, 0.6], [0.4, 0.5], [0.1, 0.3]), ([0.4, 0.6], [0.5, 0.6], [0.3, 0.6])\rangle\},$$
$$Q_{NVR} = \{\langle a, ([0.1, 0.2], [0.3, 0.5], [0.1, 0.3]), ([0.7, 0.8], [0.6, 0.8], [0.2, 0.3]), ([0.8, 0.9], [0.5, 0.7], [0.7, 0.9]), b, ([0.3, 0.6], [0.7, 0.8], [0.2, 0.4]), ([0.6, 0.8], [0.8, 0.9], [0.2, 0.3]), ([0.4, 0.7], [0.2, 0.3], [0.6, 0.8])\rangle\}$$

Then, the union of two neutrosophic vague refined sets P_{NVR} and Q_{NVR} is

$$S_{NVR} = \{\langle a, ([0.3, 0.4], [0.4, 0.5], [0.4, 0.6]), ([0.5, 0.6], [0.4, 0.5], [0.1, 0.2]), ([0.6, 0.7], [0.5, 0.6], [0.4, 0.6]), b, ([0.4, 0.6], [0.7, 0.8], [0.4, 0.7]), ([0.5, 0.6], [0.4, 0.5], [0.1, 0.3]), ([0.4, 0.6], [0.2, 0.3], [0.3, 0.6])\rangle\}$$

Definition 4.3. (Intersection)

Let P_{NVR} and Q_{NVR} be two neutrosophic vague refined sets in the universal set X, where

$$P_{NVR} = \{\langle(\hat{T}^1_{P_{NVR}}(x), \hat{T}^2_{P_{NVR}}(x), ...\hat{T}^u_{P_{NVR}}(x)), (\hat{I}^1_{P_{NVR}}(x), \hat{I}^2_{P_{NVR}}(x), ...\hat{I}^u_{P_{NVR}}(x)), (\hat{F}^1_{P_{NVR}}(x), \hat{F}^2_{P_{NVR}}(x), ...\hat{F}^u_{P_{NVR}}(x))\rangle : x \in X\ \}.$$
$$Q_{NVR} = \{\langle(\hat{T}^1_{Q_{NVR}}(x), \hat{T}^2_{Q_{NVR}}(x), ...\hat{T}^v_{Q_{NVR}}(x)), (\hat{I}^1_{Q_{NVR}}(x), \hat{I}^2_{Q_{NVR}}(x), ...\hat{I}^v_{Q_{NVR}}(x)), (\hat{F}^1_{Q_{NVR}}(x), \hat{F}^2_{Q_{NVR}}(x), ...\hat{F}^v_{Q_{NVR}}(x))\rangle : x \in X\ \}.$$

Then, the neutrosophic vague refined intersection of two sets P_{NVR} and Q_{NVR} denoted by S_{NVR} where $S_{NVR} = P_{NVR} \cap Q_{NVR}$, whose truth membership function, indeterminate membership function and false membership function is

$$\hat{T}^j_{S_{NVR}}(x) = min\{\hat{T}^j_{P_{NVR}}(x), \hat{T}^j_{Q_{NVR}}(x)\},$$
$$\hat{I}^j_{S_{NVR}}(x) = max\{\hat{I}^j_{P_{NVR}}(x), \hat{I}^j_{Q_{NVR}}(x)\},$$
$$\hat{F}^j_{S_{NVR}}(x) = max\{\hat{F}^j_{P_{NVR}}(x), \hat{F}^j_{Q_{NVR}}(x)\}$$

∀ x∈X, j=1,2,...,r.

Example 4.4.

Let X= {a,b} be the non empty set. Let the sets P_{NVR} and Q_{NVR} be defined as:

$$P_{NVR} = \{\langle a, ([0.1, 0.5], [0.4, 0.7], [0.1, 0.8]), ([0.7, 0.8], [0.4, 0.8], [0.2, 0.7]), ([0.5, 0.9], [0.3, 0.6], [0.2, 0.9]), b, ([0.1, 0.6], [0.3, 0.9], [0.4, 0.7]), ([0.8, 0.9], [0.8, 0.9], [0.5, 0.8]), ([0.4, 0.9], [0.1, 0.7], [0.3, 0.6])\rangle\},$$

$$Q_{NVR} = \{\langle a, ([0.3, 0.8], [0.4, 0.7], [0.2, 0.6]), ([0.5, 0.6], [0.6, 0.8], [0.1, 0.2]), ([0.5, 0.7], [0.4, 0.8], [0.2, 0.7]), b, ([0.2, 0.7], [0.3, 0.6], [0.4, 0.8]), ([0.5, 0.6], [0.4, 0.5], [0.2, 0.3]), ([0.3, 0.5], [0.2, 0.6], [0.3, 0.8])\rangle\}$$

Then intersection of two neutrosophic vague refined sets P_{NVR} and Q_{NVR} is

$$S_{NVR} = \{\langle a, ([0.1, 0.5], [0.4, 0.7], [0.1, 0.6]), ([0.7, 0.8], [0.6, 0.8], [0.2, 0.7]), ([0.5, 0.9], [0.4, 0.8], [0.2, 0.9]), b, ([0.1, 0.6], [0.3, 0.6], [0.4, 0.7]), ([0.8, 0.9], [0.8, 0.9], [0.5, 0.8]), ([0.4, 0.9], [0.2, 0.7], [0.3, 0.8])\rangle\}$$

Definition 4.5. (Inclusion)

A neutrosophic vague refined set P_{NVR} is a subset of another neutrosophic vague refined set Q_{NVR} denoted by $P_{NVR} \subseteq Q_{NVR}$,

$$P_{NVR} = \{\langle(\hat{T}^1_{P_{NVR}}(x), \hat{T}^2_{P_{NVR}}(x), ...\hat{T}^u_{P_{NVR}}(x)), (\hat{I}^1_{P_{NVR}}(x), \hat{I}^2_{P_{NVR}}(x), ...\hat{I}^u_{P_{NVR}}(x)), (\hat{F}^1_{P_{NVR}}(x), \hat{F}^2_{P_{NVR}}(x), ...\hat{F}^u_{P_{NVR}}(x))\rangle : x \in X \}.$$

$$Q_{NVR} = \{\langle(\hat{T}^1_{Q_{NVR}}(x), \hat{T}^2_{Q_{NVR}}(x), ...\hat{T}^v_{Q_{NVR}}(x)), (\hat{I}^1_{Q_{NVR}}(x), \hat{I}^2_{Q_{NVR}}(x), ...\hat{I}^v_{Q_{NVR}}(x)), (\hat{F}^1_{Q_{NVR}}(x), \hat{F}^2_{Q_{NVR}}(x), ...\hat{F}^v_{Q_{NVR}}(x))\rangle : x \in X \}.$$

$P_{NVR} \subseteq Q_{NVR} \Rightarrow \hat{T}^j_{P_{NVR}}(x) \leq \hat{T}^j_{Q_{NVR}}(x), \hat{I}^j_{P_{NVR}}(x) \geq \hat{I}^j_{Q_{NVR}}(x), \hat{F}^j_{P_{NVR}}(x) \geq \hat{F}^j_{Q_{NVR}}(x)\ \forall$ x∈X, j=1,2,...,r.

Definition 4.6. (Equality)

Let P_{NVR} and Q_{NVR} be two neutrosophic vague refined sets. These sets are said to be neutrosophic vague refined equal if P_{NVR} is neutrosophic vague refined subset of Q_{NVR} and Q_{NVR} is neutrosophic vague refined subset of P_{NVR}. It is denoted by $P_{NVR} = Q_{NVR}$.

Definition 4.7. (Complement)

Let P_{NVR} be the neutrosophic vague refined set in the universal set X.

$$P_{NVR} = \{\langle(\hat{T}^1_{P_{NVR}}(x), \hat{T}^2_{P_{NVR}}(x), ...\hat{T}^u_{P_{NVR}}(x)), (\hat{I}^1_{P_{NVR}}(x), \hat{I}^2_{P_{NVR}}(x), ...\hat{I}^u_{P_{NVR}}(x)), (\hat{F}^1_{P_{NVR}}(x), \hat{F}^2_{P_{NVR}}(x), ...\hat{F}^u_{P_{NVR}}(x))\rangle : x \in X \}.$$

Then the complement of the neutrosophic vague refined set denoted by P^c_{NVR} is defined as

$$\hat{T}^{j}_{P_{NVR}}(x)^{c} = 1 - \hat{T}^{j}_{P_{NVR}}(x)$$
$$\hat{I}^{j}_{P_{NVR}}(x)^{c} = 1 - \hat{I}^{j}_{P_{NVR}}(x)$$
$$\hat{F}^{j}_{P_{NVR}}(x)^{c} = 1 - \hat{F}^{j}_{P_{NVR}}(x)$$

∀ x∈X, j=1,2,...,r.

Example 4.8.

Let P_{NVR} be the neutrosophic vague refined set in the universal set X,

$P_{NVR} = \{\langle a, ([0.2, 0.4], [0.6, 0.7], [0.4, 0.9]), ([0.2, 0.5], [0.5, 0.7], [0.8, 0.9]), ([0.6, 0.8], [0.3, 0.4], [0.1, 0.6])\rangle\}$

Then the complement of Neutrosophic vague refined set A is as follows:

$P^{c}_{NVR} = \{\langle a, ([0.8, 0.6], [0.4, 0.3], [0.6, 0.1]), ([0.8, 0.5], [0.5, 0.3], [0.2, 0.1]), ([0.4, 0.2], [0.7, 0.6], [0.9, 0.4])\rangle\}$

Definition 4.9. (Addition)

Let P_{NVR} and Q_{NVR} be two neutrosophic vague refined sets in the universal set X, where

$$P_{NVR} = \{\langle (\hat{T}^{1}_{P_{NVR}}(x), \hat{T}^{2}_{P_{NVR}}(x), ...\hat{T}^{u}_{P_{NVR}}(x)), (\hat{I}^{1}_{P_{NVR}}(x), \hat{I}^{2}_{P_{NVR}}(x), ...\hat{I}^{u}_{P_{NVR}}(x)), (\hat{F}^{1}_{P_{NVR}}(x), \hat{F}^{2}_{P_{NVR}}(x), ...\hat{F}^{u}_{P_{NVR}}(x))\rangle : x \in X\ \}.$$
$$Q_{NVR} = \{\langle (\hat{T}^{1}_{Q_{NVR}}(x), \hat{T}^{2}_{Q_{NVR}}(x), ...\hat{T}^{v}_{Q_{NVR}}(x)), (\hat{I}^{1}_{Q_{NVR}}(x), \hat{I}^{2}_{Q_{NVR}}(x), ...\hat{I}^{v}_{Q_{NVR}}(x)), (\hat{F}^{1}_{Q_{NVR}}(x), \hat{F}^{2}_{Q_{NVR}}(x), ...\hat{F}^{v}_{Q_{NVR}}(x))\rangle : x \in X\ \}.$$

Then the neutrosophic vague refined addition of two sets P_{NVR} and Q_{NVR} denoted by $P_{NVR} \oplus Q_{NVR}$, whose truth membership function, indeterminate membership function and false membership function is

$$\hat{T}^{j}_{P_{NVR} \bigoplus Q_{NVR}}(x) = \hat{T}^{j}_{P_{NVR}}(x) + \hat{T}^{j}_{Q_{NVR}}(x) - \hat{T}^{j}_{P_{NVR}}(x).\hat{T}^{j}_{Q_{NVR}}(x)$$
$$\hat{I}^{j}_{P_{NVR} \bigoplus Q_{NVR}}(x) = \hat{I}^{j}_{P_{NVR}}(x) + \hat{I}^{j}_{Q_{NVR}}(x) - \hat{I}^{j}_{P_{NVR}}(x).\hat{I}^{j}_{Q_{NVR}}(x)$$
$$\hat{F}^{j}_{P_{NVR} \bigoplus Q_{NVR}}(x) = \hat{F}^{j}_{P_{NVR}}(x).\hat{F}^{j}_{Q_{NVR}}(x)$$

∀ x∈X, j=1,2,...,r.

Example 4.10.

Let X= {a,b} be the set in universal set X. Let the sets P_{NVR} and Q_{NVR} be defined as:

$P_{NVR} = \{\langle a, ([0.2, 0.5], [0.8, 0.9], [0.4, 0.7]), ([0.2, 0.3], [0.5, 0.8], [0.4, 0.6]), ([0.5, 0.8], [0.1, 0.2], [0.3, 0.6]), b, ([0.3, 0.8], [0.5, 0.9], [0.2, 0.6]), ([0.3, 0.5], [0.7, 0.9], [0.1, 0.5]), ([0.2, 0.7], [0.1, 0.5], [0.4, 0.8])\rangle\}$,

$Q_{NVR} = \{\langle a, ([0.3, 0.7], [0.2, 0.4], [0.5, 0.7]), ([0.3, 0.6], [0.2, 0.7], [0.5, 0.8]), ([0.3, 0.7], [0.6, 0.8], [0.3, 0.5]), b, ([0.4, 0.8], [0.6, 0.7], [0.1, 0.6]), ([0.5, 0.5], [0.7, 0.9], [0.8, 0.9]), ([0.2, 0.6], [0.3, 0.4], [0.4, 0.9])\rangle\}$

Then the addition of two neutrosophic vague refined sets P_{NVR} and Q_{NVR} is

$P_{NVR} \oplus Q_{NVR}$=

$$\{\langle a, ([0.44, 0.85], [0.84, 0.94], [0.70, 0.91]), ([0.44, 0.72], [0.6, 0.94], [0.70, 0.92]), ([0.15, 0.56], [0.06, 0.16], ([0.09, 0.30]), b, ([0.58, 0.96], [0.80, 0.97], [0.28, 0.84]), ([0.51, 0.75], [0.91, 0.99], [0.82, 0.95]), ([0.04, 0.42], [0.03, 0.20], [0.16, 0.72])\rangle\}$$

Definition 4.11. (Multiplication)

Let P_{NVR} and Q_{NVR} be two neutrosophic vague refined sets in the universal set X, where

$$P_{NVR} = \{\langle(\hat{T}^1_{P_{NVR}}(x), \hat{T}^2_{P_{NVR}}(x), ...\hat{T}^u_{P_{NVR}}(x)), (\hat{I}^1_{P_{NVR}}(x), \hat{I}^2_{P_{NVR}}(x), ...\hat{I}^u_{P_{NVR}}(x)), (\hat{F}^1_{P_{NVR}}(x), \hat{F}^2_{P_{NVR}}(x), ...\hat{F}^u_{P_{NVR}}(x))\rangle : x \in X\ \}.$$

$$Q_{NVR} = \{\langle(\hat{T}^1_{Q_{NVR}}(x), \hat{T}^2_{Q_{NVR}}(x), ...\hat{T}^v_{Q_{NVR}}(x)), (\hat{I}^1_{Q_{NVR}}(x), \hat{I}^2_{Q_{NVR}}(x), ...\hat{I}^v_{Q_{NVR}}(x)), (\hat{F}^1_{Q_{NVR}}(x), \hat{F}^2_{Q_{NVR}}(x), ...\hat{F}^v_{Q_{NVR}}(x))\rangle : x \in X\ \}.$$

Then the neutrosophic vague refined multiplication of two sets P_{NVR} and Q_{NVR} denoted by $P_{NVR} \otimes Q_{NVR}$, whose truth membership function, indeterminate membership function and false membership function is

$$\hat{T}^j_{P_{NVR} \bigotimes Q_{NVR}}(x) = \hat{T}^j_{P_{NVR}}(x).\hat{T}^j_{Q_{NVR}}(x)$$
$$\hat{I}^j_{P_{NVR} \bigotimes Q_{NVR}}(x) = \hat{I}^j_{P_{NVR}}(x).\hat{I}^j_{Q_{NVR}}(x)$$
$$\hat{F}^j_{P_{NVR} \bigotimes Q_{NVR}}(x) = \hat{F}^j_{P_{NVR}}(x) + \hat{F}^j_{Q_{NVR}}(x) - \hat{F}^j_{P_{NVR}}(x).\hat{F}^j_{Q_{NVR}}(x)$$

$\forall$ x$\in$X, j=1,2,...,r.

Example 4.12.

Let X= {a,b} be the set in universal set X. Let the sets P_{NVR} and Q_{NVR} be defined as:

$$P_{NVR} = \{\langle a, ([0.2, 0.4], [0.8, 0.9], [0.4, 0.5]), ([0.2, 0.4], [0.5, 0.8], [0.3, 0.6]), ([0.6, 0.8], [0.1, 0.2], [0.5, 0.6]), b, ([0.3, 0.5], [0.5, 0.7], [0.1, 0.6]), ([0.3, 0.5], [0.4, 0.9], [0.1, 0.7]), ([0.5, 0.7], [0.3, 0.5], [0.4, 0.9])\rangle\},$$

$$Q_{NVR} = \{\langle a, ([0.3, 0.9], [0.2, 0.7], [0.5, 0.8]), ([0.1, 0.6], [0.5, 0.7], [0.3, 0.8]), ([0.1, 0.7], [0.3, 0.8], [0.2, 0.5]), b, ([0.2, 0.8], [0.6, 0.8], [0.3, 0.6]), ([0.4, 0.5], [0.5, 0.9], [0.8, 0.8]), ([0.2, 0.8], [0.2, 0.4], [0.4, 0.7])\rangle\}$$

Then the multiplication of two neutrosophic vague refined sets P_{NVR} and Q_{NVR} is

$P_{NVR} \otimes Q_{NVR}$=

$$\{\langle a, ([0.06, 0.36], [0.16, 0.63], [0.20, 0.40]), ([0.02, 0.24], [0.25, 0.56], [0.09, 0.40]), ([0.64, 0.94],$$

$[0.37, 0.84], ([0.60, 0.80]), b, ([0.06, 0.40], [0.30, 0.56], [0.03, 0.36]), ([0.12, 0.25], [0.20, 0.81], [0.08, 0.56]), ([0.60, 0.94], [0.44, 0.70], [0.64, 0.97])\rangle\}$

Definition 4.13. (AND operation)

Let P_{NVR} and Q_{NVR} be two neutrosophic vague refined sets in the universal set X, where

$$P_{NVR} = \{\langle(\hat{T}^1_{P_{NVR}}(x), \hat{T}^2_{P_{NVR}}(x), ...\hat{T}^u_{P_{NVR}}(x)), (\hat{I}^1_{P_{NVR}}(x), \hat{I}^2_{P_{NVR}}(x), ...\hat{I}^u_{P_{NVR}}(x)), (\hat{F}^1_{P_{NVR}}(x), \hat{F}^2_{P_{NVR}}(x), ...\hat{F}^u_{P_{NVR}}(x))\rangle : x \in X\}.$$
$$Q_{NVR} = \{\langle(\hat{T}^1_{Q_{NVR}}(x), \hat{T}^2_{Q_{NVR}}(x), ...\hat{T}^v_{Q_{NVR}}(x)), (\hat{I}^1_{Q_{NVR}}(x), \hat{I}^2_{Q_{NVR}}(x), ...\hat{I}^v_{Q_{NVR}}(x)), (\hat{F}^1_{Q_{NVR}}(x), \hat{F}^2_{Q_{NVR}}(x), ...\hat{F}^v_{Q_{NVR}}(x))\rangle : x \in X\}.$$

AND operation between these two neutrosophic vague refined sets is denoted by $P_{NVR} \wedge Q_{NVR}$ is the intersection of two neutrosophic vague refined sets P_{NVR} and Q_{NVR}.
$\forall$ x∈X, j=1,2,...,r.

Definition 4.14. (OR operation)

Let P_{NVR} and Q_{NVR} be two neutrosophic vague refined sets in the universal set X, where

$$P_{NVR} = \{\langle(\hat{T}^1_{P_{NVR}}(x), \hat{T}^2_{P_{NVR}}(x), ...\hat{T}^u_{P_{NVR}}(x)), (\hat{I}^1_{P_{NVR}}(x), \hat{I}^2_{P_{NVR}}(x), ...\hat{I}^u_{P_{NVR}}(x)), (\hat{F}^1_{P_{NVR}}(x), \hat{F}^2_{P_{NVR}}(x), ...\hat{F}^u_{P_{NVR}}(x))\rangle : x \in X\}.$$
$$Q_{NVR} = \{\langle(\hat{T}^1_{Q_{NVR}}(x), \hat{T}^2_{Q_{NVR}}(x), ...\hat{T}^v_{Q_{NVR}}(x)), (\hat{I}^1_{Q_{NVR}}(x), \hat{I}^2_{Q_{NVR}}(x), ...\hat{I}^v_{Q_{NVR}}(x)), (\hat{F}^1_{Q_{NVR}}(x), \hat{F}^2_{Q_{NVR}}(x), ...\hat{F}^v_{Q_{NVR}}(x))\rangle : x \in X\}.$$

OR operation between these two neutrosophic vague refined sets is denoted by $P_{NVR} \vee Q_{NVR}$ is the intersection of two neutrosophic vague refined sets P_{NVR} and Q_{NVR}.
$\forall$ x∈X, j=1,2,...,r.

Definition 4.15. (Cartesian Product)

Let P_{NVR} and Q_{NVR} be the two neutrosophic vague refined sets.

$$P_{NVR} = \{\langle(\hat{T}^1_{P_{NVR}}(x), \hat{T}^2_{P_{NVR}}(x), ...\hat{T}^u_{P_{NVR}}(x)), (\hat{I}^1_{P_{NVR}}(x), \hat{I}^2_{P_{NVR}}(x), ...\hat{I}^u_{P_{NVR}}(x)), (\hat{F}^1_{P_{NVR}}(x), \hat{F}^2_{P_{NVR}}(x), ...\hat{F}^u_{P_{NVR}}(x))\rangle : x \in X\}.$$
$$Q_{NVR} = \{\langle(\hat{T}^1_{Q_{NVR}}(y), \hat{T}^2_{Q_{NVR}}(y), ...\hat{T}^v_{Q_{NVR}}(y)), (\hat{I}^1_{Q_{NVR}}(y), \hat{I}^2_{Q_{NVR}}(y), ...\hat{I}^v_{Q_{NVR}}(y)), (\hat{F}^1_{Q_{NVR}}(x), \hat{F}^2_{Q_{NVR}}(y), ...\hat{F}^v_{Q_{NVR}}(y))\rangle : y \in X\}.$$

Then the cartesian product of two sets $P_{NVR} \times Q_{NVR}$ is defined as

$$P_{NVR} \times Q_{NVR} = \{\langle(\hat{T}^1_{P_{NVR} \times Q_{NVR}}(x, y), \hat{T}^2_{P_{NVR} \times Q_{NVR}}(x, y), ...\hat{T}^P_{P_{NVR} \times Q_{NVR}}(x, y)),$$

$$(\hat{I}^1_{P_{NVR}\times Q_{NVR}}(x,y), \hat{I}^2_{P_{NVR}\times Q_{NVR}}(x,y), ...\hat{I}^P_{P_{NVR}\times Q_{NVR}}(x,y))$$
$$(\hat{F}^1_{P_{NVR}\times Q_{NVR}}(x,y), \hat{F}^2_{P_{NVR}\times Q_{NVR}}(x,y), ...\hat{F}^P_{P_{NVR}\times Q_{NVR}}(x,y))\}$$

where

$$\hat{T}^j_{P_{NVR}\times Q_{NVR}}, \hat{I}^j_{P_{NVR}\times Q_{NVR}}, \hat{F}^j_{P_{NVR}\times Q_{NVR}} : X \longrightarrow [0,1]$$

and also,

$$\hat{T}^j_{P_{NVR}\times Q_{NVR}}(x,y) = min\{\hat{T}^j_{P_{NVR}\times Q_{NVR}}(x), \hat{T}^j_{P_{NVR}\times Q_{NVR}}(y)\}$$
$$\hat{I}^j_{P_{NVR}\times Q_{NVR}}(x,y) = max\{\hat{I}^j_{P_{NVR}\times Q_{NVR}}(x), \hat{I}^j_{P_{NVR}\times Q_{NVR}}(y)\}$$
$$\hat{F}^j_{P_{NVR}\times Q_{NVR}}(x,y) = max\{\hat{F}^j_{P_{NVR}\times Q_{NVR}}(x), \hat{F}^j_{P_{NVR}\times Q_{NVR}}(y)\}$$

for all x,y $\in$ X and j= $\{1, 2, ..., r\}$.

Example 4.16.

Let X= {a,b} be the set in universal set X. Let the sets P_{NVR} and Q_{NVR} be defined as:

$$P_{NVR} = \{\langle a, ([0.3, 0.7], [0.4, 0.5], [0.4, 0.7]), ([0.2, 0.5], [0.4, 0.9], [0.3, 0.8]), ([0.3, 0.7], [0.5, 0.9], [0.3, 0.6]), b, ([0.4, 0.6], [0.3, 0.8], [0.1, 0.5]), ([0.3, 0.7], [0.5, 0.9], [0.2, 0.6]), ([0.4, 0.6], [0.2, 0.7], [0.5, 0.9])\rangle\},$$
$$Q_{NVR} = \{\langle a, ([0.1, 0.5], [0.5, 0.7], [0.5, 0.8]), ([0.2, 0.7], [0.5, 0.8], [0.4, 0.8]), ([0.5, 0.9], [0.3, 0.5], [0.2, 0.5]), b, ([0.4, 0.9], [0.1, 0.5], [0.5, 0.6]), ([0.3, 0.6], [0.4, 0.9], [0.5, 0.8]), ([0.1, 0.6], [0.5, 0.9], [0.4, 0.5])\rangle\}$$

Then the cartesian product of two neutrosophic vague refined sets P_{NVR} and Q_{NVR} is as follows

$P_{NVR} \times Q_{NVR} =$
$\{\langle(a, a)([0.1, 0.5], [0.1, 0.5], [0.4, 0.7]), ([0.2, 0.7], [0.5, 0.9], [0.4, 0.8]), ([0.5, 0.9], [0.5, 0.9], [0.3, 0.6])\rangle,$
$\langle(a, b)([0.3, 0.7], [0.1, 0.5], [0.4, 0.6]), ([0.3, 0.6], [0.4, 0.9], [0.5, 0.8]), ([0.1, 0.6], [0.5, 0.9], [0.4, 0.6])\rangle,$
$\langle(b, a)([0.1, 0.5], [0.3, 0.7], [0.1, 0.5]), ([0.3, 0.7], [0.5, 0.9], [0.4, 0.8]), ([0.5, 0.9], [0.3, 0.7], [0.5, 0.9])\rangle,$
$\langle(b, b)([0.4, 0.6], [0.1, 0.5], [0.1, 0.5]), ([0.3, 0.7], [0.5, 0.9], [0.5, 0.8]), ([0.4, 0.6], [0.5, 0.9], [0.5, 0.9])\rangle\}$

5. Algebraic properties of neutrosophic vague refined set operations

In this section, we study algebraic properties of above operations in neutrosophic vague refined set with examples.

Proposition 5.1. (Identity Law)

For any neutrosophic vague refined set P_{NVR} defined on the absolute neutrosophic vague refined set X.

(1)$P_{NVR} \cup \emptyset_{NVR}$= P_{NVR}

(2)$P_{NVR} \cap X_{NVR}$= P_{NVR}

Proof.

(1)Let P_{NVR} be the two neutrosophic vague refined set.

$$P_{NVR} = \{\langle(\hat{T}^1_{P_{NVR}}(x), \hat{T}^2_{P_{NVR}}(x), ...\hat{T}^u_{P_{NVR}}(x)), (\hat{I}^1_{P_{NVR}}(x), \hat{I}^2_{P_{NVR}}(x), ...\hat{I}^u_{P_{NVR}}(x)), (\hat{F}^1_{P_{NVR}}(x), \hat{F}^2_{P_{NVR}}(x), ...\hat{F}^u_{P_{NVR}}(x))\rangle : x \in X \}.$$

$\emptyset_{NVR}$ is defined as follows

$$\emptyset_{NVR}=\{\langle a, ([0,0],[0,0], ...[0,0]), ([1,1],[1,1], ...[1,1]), ([1,1],[1,1], ..., [1,1]) \; b, ([0,0],[0,0], ...[0,0]), ([1,1],[1,1], ...[1,1]), ([1,1],[1,1], ..., [1,1])\rangle\}.$$

So, $P_{NVR} \cup \emptyset_{NVR}$=

$$\{< (max\{\hat{T}^1_{P_{NVR}}(x), [0,0]\}, min\{\hat{T}^2_{P_{NVR}}(x), [0,0]\}, ..., min\{\hat{T}^u_{P_{NVR}}(x), [0,0]\}), (max\{\hat{I}^1_{P_{NVR}}(x), [1,1]\}, min\{\hat{I}^2_{P_{NVR}}(x), [1,1]\}, ..., min\{\hat{T}^u_{P_{NVR}}(x), [1,1]\}), (max\{\hat{F}^1_{P_{NVR}}(x), [1,1]\}, min\{\hat{F}^2_{P_{NVR}}(x), [1,1]\}, ..., min\{\hat{u}^2_{P_{NVR}}(x), [1,1]\}) >\}$$

Therefore,

$$P_{NVR} \cup \emptyset_{NVR} = \{\langle(\hat{T}^1_{P_{NVR}}(x), \hat{T}^2_{P_{NVR}}(x), ...\hat{T}^u_{P_{NVR}}(x)), (\hat{I}^1_{P_{NVR}}(x), \hat{I}^2_{P_{NVR}}(x), ...\hat{I}^u_{P_{NVR}}(x)), (\hat{F}^1_{P_{NVR}}(x), \hat{F}^2_{P_{NVR}}(x), ...\hat{F}^u_{P_{NVR}}(x))\rangle : x \in X\}.$$

(2)Proof is simitar to (1). □

Example 5.2.

(1)Let P_{NVR} be neutrosophic vague refined set in the universal set X.

$$P_{NVR} = \{\langle a, ([0.2, 0.4], [0.8, 0.9], [0.4, 0.5]), ([0.2, 0.4], [0.5, 0.8], [0.3, 0.6]), ([0.6, 0.8], [0.1, 0.2], [0.5, 0.6]), b, ([0.3, 0.5], [0.5, 0.7], [0.1, 0.6]), ([0.3, 0.5], [0.4, 0.9], [0.1, 0.7]), ([0.5, 0.7], [0.3, 0.5], [0.4, 0.9])\rangle\},$$

$$\emptyset_{NVR} = \{\langle a, ([0,0],[1,1],[1,1]), ([0,0],[1,1],[1,1]), ([0,0],[1,1],[1,1])\rangle\}.$$

Then,

$$P_{NVR} \cup \emptyset_{NVR} =\{\langle a, ([0.2, 0.4], [0.8, 0.9], [0.4, 0.5]), ([0.2, 0.4], [0.5, 0.8], [0.3, 0.6]), ([0.6, 0.8], [0.1, 0.2], [0.5, 0.6]), b, ([0.3, 0.5], [0.5, 0.7], [0.1, 0.6]), ([0.3, 0.5], [0.4, 0.9], [0.1, 0.7]), ([0.5, 0.7], [0.3, 0.5], [0.4, 0.9])\rangle\},$$

(2) Obvious.

Proposition 5.3. (Domination Law)

For any neutrosophic vague refined set P_{NVR} defined on absolute neutrosophic vague refined set X

(1) $P_{NVR} \cap \emptyset_{NVR} = \emptyset_{NVR}$

(2) $P_{NVR} \cup X_{NVR} = X_{NVR}$

Proof.

(1)Let P_{NVR} be the two neutrosophic vague refined set.

$$P_{NVR} = \{\langle(\hat{T}^1_{P_{NVR}}(x), \hat{T}^2_{P_{NVR}}(x), ...\hat{T}^u_{P_{NVR}}(x)), (\hat{I}^1_{P_{NVR}}(x), \hat{I}^2_{P_{NVR}}(x), ...\hat{I}^u_{P_{NVR}}(x)),$$

$$(\hat{F}^1_{P_{NVR}}(x), \hat{F}^2_{P_{NVR}}(x), ...\hat{F}^u_{P_{NVR}}(x))\rangle : x \in X \}.$$

$\emptyset_{NVR}$ is defined as follows

$\emptyset_{NVR}=\{\langle a, ([0,0],[0,0],....[0,0]),([1,1],[1,1],....[1,1]),([1,1],[1,1],...,[1,1])$
$b, ([0,0],[0,0],....[0,0]),([1,1],[1,1],....[1,1]),([1,1],[1,1],...,[1,1])\rangle\}.$

So, $P_{NVR} \cap \emptyset_{NVR}=$

$$\{< (min\{\hat{T}^1_{P_{NVR}}(x),[0,0]\}, max\{\hat{T}^2_{P_{NVR}}(x),[0,0]\}, ..., max\{\hat{T}^u_{P_{NVR}}(x),[0,0]\}),$$
$$(min\{\hat{I}^1_{P_{NVR}}(x),[1,1]\}, max\{\hat{I}^2_{P_{NVR}}(x),[1,1]\}, ..., max\{\hat{T}^u_{P_{NVR}}(x),[1,1]\}),$$
$$(min\{\hat{F}^1_{P_{NVR}}(x),[1,1]\}, max\{\hat{F}^2_{P_{NVR}}(x),[1,1]\}, ..., max\{\hat{F}^u_{P_{NVR}}(x),[1,1]\}) >\}$$

Therefore,

$P_{NVR} \cap \emptyset_{NVR} = \{\langle a, ([0,0],[0,0],....[0,0]),([1,1],[1,1],....[1,1]),([1,1],[1,1],...,[1,1])\rangle\}.$

(2)Proof is similar to (1). □

Example 5.4.

(1) Let P_{NVR} be neutrosophic vague refined set in the universal set X.

$P_{NVR} = \{\langle a, ([0.1, 0.2], [0.3, 0.5], [0.1, 0.3]), ([0.7, 0.8], [0.6, 0.8], [0.2, 0.3]), ([0.8, 0.9], [0.5, 0.7],$
$[0.7, 0.9]), b, ([0.3, 0.6], [0.7, 0.8], [0.2, 0.4]), ([0.6, 0.8], [0.8, 0.9], [0.2, 0.3]), ([0.4, 0.7],$
$[0.2, 0.3], [0.6, 0.8])\rangle\}$

$\emptyset_{NVR}=\{\langle a, ([0,0],[0,0],....[0,0]),([1,1],[1,1],....[1,1]),([1,1],[1,1],...,[1,1])$
$b, ([0,0],[0,0],....[0,0]),([1,1],[1,1],....[1,1]),([1,1],[1,1],...,[1,1])\rangle\}.$

Then,

$P_{NVR} \cap \emptyset_{NVR}=$

$\{\langle a, ([0,0],[0,0],....[0,0]),([1,1],[1,1],....[1,1]),([1,1],[1,1],...,[1,1])$
$b, ([0,0],[0,0],....[0,0]),([1,1],[1,1],....[1,1]),([1,1],[1,1],...,[1,1])\rangle\}.$

(2) Obvious.

Proposition 5.5. (Idempotent Law)

For any neutrosophic vague refined set P_{NVR} defined on absolute neutrosophic vague refined set X.

(1) $P_{NVR} \cap P_{NVR}= P_{NVR}$

(2) $P_{NVR} \cup P_{NVR}= P_{NVR}$

Proof.

The proof is obvious. □

Example 5.6.

(1)Let P_{NVR} be neutrosophic vague refined set in the universal set X.

$P_{NVR} = \{\langle a, ([0.1, 0.5], [0.5, 0.7], [0.5, 0.8]), ([0.2, 0.7], [0.5, 0.8], [0.4, 0.8]), ([0.5, 0.9], [0.3, 0.5], [0.2, 0.5]), b, ([0.4, 0.9], [0.1, 0.5], [0.5, 0.6]), ([0.3, 0.6], [0.4, 0.9], [0.5, 0.8]), ([0.1, 0.6], [0.5, 0.9], [0.4, 0.5])\rangle\}$

Then,

$P_{NVR} \cap P_{NVR} = \{\langle a, ([0.1, 0.5], [0.5, 0.7], [0.5, 0.8]), ([0.2, 0.7], [0.5, 0.8], [0.4, 0.8]), ([0.5, 0.9], [0.3, 0.5], [0.2, 0.5]), b, ([0.4, 0.9], [0.1, 0.5], [0.5, 0.6]), ([0.3, 0.6], [0.4, 0.9], [0.5, 0.8]), ([0.1, 0.6], [0.5, 0.9], [0.4, 0.5])\rangle\}$

Hence, $P_{NVR} \cap P_{NVR} = P_{NVR}$.

(2) Obvious.

Proposition 5.7. (Commutative Law)

For any neutrosophic vague refined set P_{NVR} defined on absolute neutrosophic vague refined set X

(1) $P_{NVR} \cup Q_{NVR} = Q_{NVR} \cup P_{NVR}$

(2) $P_{NVR} \cap Q_{NVR} = Q_{NVR} \cap P_{NVR}$

Proof. Proof is obvious. □

Proposition 5.8. (Associative Law)

For any neutrosophic vague refined sets P_{NVR}, Q_{NVR} and R_{NVR} defined on absolute neutrosophic vague refined set X.

(1) $(P_{NVR} \cup Q_{NVR}) \cup R_{NVR} = P_{NVR} \cup (Q_{NVR} \cup R_{NVR})$

(2) $(P_{NVR} \cap Q_{NVR}) \cap R_{NVR} = P_{NVR} \cap (Q_{NVR} \cap R_{NVR})$

Proof.

(1) Let P_{NVR}, Q_{NVR} and R_{NVR} be three neutrosophic vague refined sets defined as follows:

$$P_{NVR} = \{\langle(\hat{T}^1_{P_{NVR}}(x), \hat{T}^2_{P_{NVR}}(x), ...\hat{T}^u_{P_{NVR}}(x)), (\hat{I}^1_{P_{NVR}}(x), \hat{I}^2_{P_{NVR}}(x), ...\hat{I}^u_{P_{NVR}}(x)), (\hat{F}^1_{P_{NVR}}(x), \hat{F}^2_{P_{NVR}}(x), ...\hat{F}^u_{P_{NVR}}(x))\rangle : x \in X\ \}.$$

$$Q_{NVR} = \{\langle(\hat{T}^1_{Q_{NVR}}(x), \hat{T}^2_{Q_{NVR}}(x), ...\hat{T}^v_{Q_{NVR}}(x)), (\hat{I}^1_{Q_{NVR}}(x), \hat{I}^2_{Q_{NVR}}(x), ...\hat{I}^v_{Q_{NVR}}(x)), (\hat{F}^1_{Q_{NVR}}(x), \hat{F}^2_{Q_{NVR}}(x), ...\hat{F}^v_{Q_{NVR}}(x))\rangle : x \in X\ \}.$$

$$R_{NVR} = \{\langle(\hat{T}^1_{R_{NVR}}(x), \hat{T}^2_{R_{NVR}}(x), ...\hat{T}^w_{R_{NVR}}(x)), (\hat{I}^1_{R_{NVR}}(x), \hat{I}^2_{R_{NVR}}(x), ...\hat{I}^w_{R_{NVR}}(x)), (\hat{F}^1_{R_{NVR}}(x), \hat{F}^2_{R_{NVR}}(x), ...\hat{F}^w_{R_{NVR}}(x))\rangle : x \in X\}.$$

Then, $(P_{NVR} \cup Q_{NVR}) \cup R_{NVR}$

$$=\{\langle(\hat{T}^1_{P_{NVR}}(x), \vee\hat{T}^1_{Q_{NVR}}(x)), ..., (\hat{T}^u_{P_{NVR}}(x) \vee \hat{T}^v_{Q_{NVR}}(x)), (\hat{I}^1_{P_{NVR}}(x) \vee \hat{I}^1_{Q_{NVR}}(x)), ..., (\hat{I}^u_{P_{NVR}}(x) \vee \hat{I}^v_{Q_{NVR}}(x)), (\hat{F}^1_{P_{NVR}}(x) \vee \hat{F}^1_{Q_{NVR}}(x)), ..., (\hat{F}^u_{P_{NVR}}(x) \vee \hat{F}^v_{Q_{NVR}}(x))\rangle : x \in X\} \cap$$

$\{\langle(\hat{T}^1_{R_{NVR}}(x),...,\hat{T}^w_{R_{NVR}}(x)),(\hat{I}^1_{R_{NVR}}(x),...,\hat{I}^w_{R_{NVR}}(x)),(\hat{F}^1_{R_{NVR}}(x),..,\hat{F}^w_{R_{NVR}}(x))\rangle \ : \ x \ \in X\}$

$=\{\langle(((\hat{T}^1_{P_{NVR}}(x)\vee\hat{T}^1_{Q_{NVR}}(x))\vee\hat{T}^1_{R_{NVR}}(x)).....((\hat{T}^P_{P_{NVR}}(x)\vee\hat{T}^P_{Q_{NVR}}(x))\vee\hat{T}^P_{R_{NVR}}(x)),((\hat{I}^1_{P_{NVR}}(x) \vee (\hat{I}^1_{Q_{NVR}}(x))\vee\hat{I}^1_{R_{NVR}}(x)).....((\hat{I}^P_{P_{NVR}}(x)\vee\hat{I}^P_{Q_{NVR}}(x))\vee\hat{I}^P_{R_{NVR}}(x)),((\hat{F}^1_{P_{NVR}}(x)\vee (\hat{F}^1_{Q_{NVR}}(x))\vee\hat{F}^1_{R_{NVR}}(x)).....((\hat{F}^u_{P_{NVR}}(x)\vee\hat{F}^v_{Q_{NVR}}(x))\vee\hat{F}^w_{R_{NVR}}(x))\rangle : x\in X\}$

$=\{\langle(\hat{T}^1_{P_{NVR}}(x)\vee(\hat{T}^1_{Q_{NVR}}(x)\vee\hat{T}^1_{R_{NVR}}(x))).....(\hat{T}^u_{P_{NVR}}(x)\vee(\hat{T}^v_{Q_{NVR}}(x)\vee\hat{T}^w_{R_{NVR}}(x))), (\hat{I}^1_{P_{NVR}}(x)\vee(\hat{I}^1_{Q_{NVR}}(x)\vee\hat{I}^1_{R_{NVR}}(x))).....(\hat{I}^u_{P_{NVR}}(x)\vee(\hat{I}^v_{Q_{NVR}}(x)\vee\hat{I}^w_{R_{NVR}}(x))),(\hat{F}^1_{P_{NVR}}(x)\vee (\hat{F}^1_{Q_{NVR}}(x)\vee\hat{F}^1_{R_{NVR}}(x))).....(\hat{F}^u_{P_{NVR}}(x)\vee(\hat{F}^v_{Q_{NVR}}(x)\vee\hat{F}^w_{R_{NVR}}(x)))\rangle : x\in X\}$

$=P_{NVR}\cup(Q_{NVR}\cup R_{NVR})$

(2) The proof is similar to (1) □

Proposition 5.9. (Distributive Law)

For any neutrosophic vague refined sets P_{NVR}, Q_{NVR} and R_{NVR} defined on absolute neutrosophic vague refined set X.

(1) $P_{NVR}\cup(Q_{NVR}\cap R_{NVR}) = (P_{NVR}\cup Q_{NVR})\cap(P_{NVR}\cup R_{NVR})$

(2) $P_{NVR}\cap(Q_{NVR}\cup R_{NVR}) = (P_{NVR}\cap Q_{NVR})\cup(P_{NVR}\cap R_{NVR})$

Proof.

(1) Let P_{NVR}, Q_{NVR} and R_{NVR} be three neutrosophic vague refined sets defined as follows:

$$P_{NVR} = \{\langle(\hat{T}^1_{P_{NVR}}(x),\hat{T}^2_{P_{NVR}}(x),....\hat{T}^u_{P_{NVR}}(x)),(\hat{I}^1_{P_{NVR}}(x),\hat{I}^2_{P_{NVR}}(x),....\hat{I}^u_{P_{NVR}}(x)), (\hat{F}^1_{P_{NVR}}(x),\hat{F}^2_{P_{NVR}}(x),....\hat{F}^u_{P_{NVR}}(x))\rangle : x\in X\ \}.$$
$$Q_{NVR} = \{\langle(\hat{T}^1_{Q_{NVR}}(x),\hat{T}^2_{Q_{NVR}}(x),....\hat{T}^v_{Q_{NVR}}(x)),(\hat{I}^1_{Q_{NVR}}(x),\hat{I}^2_{Q_{NVR}}(x),....\hat{I}^v_{Q_{NVR}}(x)), (\hat{F}^1_{Q_{NVR}}(x),\hat{F}^2_{Q_{NVR}}(x),....\hat{F}^v_{Q_{NVR}}(x))\rangle : x\in X\ \}.$$
$$R_{NVR} = \{\langle(\hat{T}^1_{R_{NVR}}(x),\hat{T}^2_{R_{NVR}}(x),....\hat{T}^w_{R_{NVR}}(x)),(\hat{I}^1_{R_{NVR}}(x),\hat{I}^2_{R_{NVR}}(x),....\hat{I}^w_{R_{NVR}}(x)), (\hat{F}^1_{R_{NVR}}(x),\hat{F}^2_{R_{NVR}}(x),....\hat{F}^w_{R_{NVR}}(x))\rangle : x\in X\}.$$

Then, $P_{NVR}\cup(Q_{NVR}\cap R_{NVR})$

$= \{\langle(\hat{T}^1_{P_{NVR}}(x),....\hat{T}^u_{P_{NVR}}(x)),(\hat{I}^1_{P_{NVR}}(x),....\hat{I}^u_{P_{NVR}}(x)),\ (\hat{F}^1_{P_{NVR}}(x),....\hat{F}^u_{P_{NVR}}(x))\cup ((\hat{T}^1_{Q_{NVR}}(x)\wedge\hat{T}^1_{R_{NVR}}(x)),....,(\hat{T}^v_{Q_{NVR}}(x)\wedge\hat{T}^w_{R_{NVR}}(x)))\rangle : x\in X\}$

$=\{\langle(\hat{T}^1_{P_{NVR}}(x)\vee(\hat{T}^1_{Q_{NVR}}(x)\wedge\hat{T}^1_{R_{NVR}}(x)),....,(\hat{T}^u_{P_{NVR}}(x)\vee(\hat{T}^v_{Q_{NVR}}(x)\wedge(\hat{T}^w_{R_{NVR}}(x))), (\hat{I}^1_{P_{NVR}}(x)\wedge(\hat{I}^1_{Q_{NVR}}(x)\vee\hat{I}^1_{R_{NVR}}(x)),...,(\hat{I}^u_{P_{NVR}}(x)\wedge(\hat{I}^v_{Q_{NVR}}(x)\vee(\hat{I}^w_{R_{NVR}}(x))), (\hat{F}^1_{P_{NVR}}(x)\wedge(\hat{F}^1_{Q_{NVR}}(x)\vee\hat{F}^1_{R_{NVR}}(x)),...,(\hat{F}^u_{P_{NVR}}(x)\wedge(\hat{F}^v_{Q_{NVR}}(x)\vee(\hat{F}^w_{R_{NVR}}(x)))\rangle : x\in X\}$

$=\{\langle(((\hat{T}^1_{P_{NVR}}(x)\vee(\hat{T}^1_{Q_{NVR}}(x))\wedge(\hat{T}^1_{P_{NVR}}(x)\vee\hat{T}^1_{R_{NVR}}(x)),...,((\hat{T}^u_{P_{NVR}}(x)\vee(\hat{T}^v_{Q_{NVR}}(x))\wedge$
$(\hat{T}^u_{P_{NVR}}(x)\vee\hat{T}^w_{R_{NVR}}(x)))),(((\hat{I}^1_{P_{NVR}}(x)\wedge\hat{I}^1_{Q_{NVR}}(x))\vee(\hat{I}^1_{P_{NVR}}(x)\wedge\hat{I}^1_{R_{NVR}}(x)),...,((\hat{I}^u_{P_{NVR}}(x)\wedge$
$\hat{I}^v_{Q_{NVR}}(x))\vee(\hat{I}^u_{P_{NVR}}(x)\wedge\hat{I}^w_{R_{NVR}}(x)))),(((\hat{F}^1_{P_{NVR}}(x)\wedge\hat{F}^1_{Q_{NVR}}(x))\vee(\hat{F}^1_{P_{NVR}}(x)\wedge$
$\hat{F}^1_{R_{NVR}}(x)),...,((\hat{F}^u_{P_{NVR}}(x)\wedge\hat{F}^v_{Q_{NVR}}(x))\vee(\hat{F}^u_{P_{NVR}}(x)\wedge\hat{F}^w_{R_{NVR}}(x))))\rangle : x\in X\}$

$= (P_{NVR}\cup Q_{NVR})\cap(P_{NVR}\cup R_{NVR})$

(2) The proof is similar to (1) □

Proposition 5.10. (Double Complement Law)

For any neutrosophic vague refined set P_{NVR} defined on absolute neutrosophic vague refined set X

$(P^c_{NVR})^c = P_{NVR}$

Proof.

Let P_{NVR} be neutrosophic vague refined set defined as follows:

$$P_{NVR} = \{\langle(\hat{T}^1_{P_{NVR}}(x),\hat{T}^2_{P_{NVR}}(x),....\hat{T}^u_{P_{NVR}}(x)),(\hat{I}^1_{P_{NVR}}(x),\hat{I}^2_{P_{NVR}}(x),....\hat{I}^u_{P_{NVR}}(x)),$$
$$(\hat{F}^1_{P_{NVR}}(x),\hat{F}^2_{P_{NVR}}(x),....\hat{F}^u_{P_{NVR}}(x))\rangle : x\in X\ \}.$$

Then,

$$P^c_{NVR}=\{\langle(1-\hat{T}^1{}_{P_{NVR}}(x),1-\hat{T}^2{}_{P_{NVR}}(x),....1-\hat{T}^u{}_{P_{NVR}}(x)),(1-\hat{I}^1{}_{P_{NVR}}(x),1-\hat{I}^2{}_{P_{NVR}}(x),....,$$
$$1-\hat{I}^u{}_{P_{NVR}}(x)),(1-\hat{F}^1{}_{P_{NVR}}(x),1-\hat{F}^2{}_{P_{NVR}}(x),....1-\hat{F}^u{}_{P_{NVR}}(x))\rangle : x\in X\}.$$

So,

$$(P^c_{NVR})^c = \{\langle(\hat{T}^1_{P_{NVR}}(x),\hat{T}^2_{P_{NVR}}(x),....\hat{T}^u_{P_{NVR}}(x)),(\hat{I}^1_{P_{NVR}}(x),\hat{I}^2_{P_{NVR}}(x),....\hat{I}^u_{P_{NVR}}(x)),$$
$$(\hat{F}^1_{P_{NVR}}(x),\hat{F}^2_{P_{NVR}}(x),....,\hat{F}^u_{P_{NVR}}(x))\rangle : x\in X\}.$$

Therefore, $(P^c_{NVR})^c = P_{NVR}$ □

Proposition 5.11. (Absorption Law)

For any neutrosophic vague refined set P_{NVR} and Q_{NVR} defined on absolute neutrosophic vague refined set X

(1) $P_{NVR}\cup(P_{NVR}\cap Q_{NVR}) = P_{NVR}$

(2) $P_{NVR}\cap(P_{NVR}\cup Q_{NVR}) = P_{NVR}$

Proof. Proof is obvious. □

6. Conclusions

This paper ensures the work of introducing the new set namely neutrosophic vague refined set by the combination of neutrosophic vague set and neutrosophic refined(multi) set. Several operations and laws have been discussed along with some examples. In future, neutrosophic vague refined topological spaces can be introduced. And also, decision making problems on neutrosophic vague refined sets can be introduced.

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