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# Neutrosophic Weakly Generalized open and Closed Sets

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**Abstract:** Smarandache presented and built up the new idea of Neutrosophic concepts from the Neutrosophic sets. A.A. Salama presented Neutrosophic topological spaces by utilizing the Neutrosophic sets. Point of this paper is we present and concentrate the ideas Neutrosophic Weakly Generalized Closed Set in Neutrosophic topological spaces and its Properties are talked about subtleties

**Keywords:** Neutrosophic Generalized closed sets, Neutrosophic Weakly Generalized Closed Sets, Neutrosophic Weakly Generalized open Sets, Neutrosophic topological spaces

## 1. Introduction

Smarandache's neutrosophic framework have wide scope of constant applications for the fields of Electrical & Electronic, Artificial Intelligence, Mechanics, Computer Science ,Information Systems, Applied Mathematics , basic leadership. Prescription and Management Science and so forth,.In 1965 ,Zadeh proposed Fuzzy set(FS), and Atanassov [1] proposed intuitionistic Fuzzy set (IFS) in 1983 .Topology is an old style subject, as a speculation topological spaces many sort of topological spaces presented over the year. Smarandache [5] characterized the Neutrosophic set on three segment Neutrosophic sets(T Truth, I-Indeterminacy, F-Falsehood). Neutrosophic topological spaces(NS-T-S) presented by Salama [10] et al., R.Dhavaseelan [3], SaiedJafari are introduced Neutrosophic generalized closed sets Point of this paper is we present and concentrate the ideas Neutrosophic Weakly Generalized Closed Set in Neutrosophic topological spaces and its Properties are talked about subtleties

## 2. Preliminaries

In this part, we review required essential definition and results of Neutrosophic sets

**Definition 2.1 [5]** Let $N_X^*$ be a non-empty fixed set. A Neutrosophic set $R_1^*$ is a object having the form

$$R_1^* = \{< r, \mu_{R_1^*}(r), \sigma_{R_1^*}(r), \gamma_{R_1^*}(r) >: r \in N_X^*\},$$

$\mu_{R_1^*}(r)$-represents the degree of membership function

$\sigma_{R_1^*}(r)$-represents degree indeterminacy function and then

$\gamma_{R_1^*}(r)$-represents the degree of non-membership function

**Definition 2.2 [5]** Neutrosophic set $R_1^* = \{< r, \mu_{R_1^*}(r), \sigma_{R_1^*}(r), \gamma_{R_1^*}(r) >: r \in N_X^*\}$, on $N_X^*$ and $\forall r \in N_X^*$ then complement of $R_1^*$ is $R_1^{*C} = \{< r, \gamma_{R_1^*}((r)), 1 - \sigma_{R_1^*}(r), \mu_{R_1^*}(r) >: r \in N_X^*\}$

**Definition 2.3 [5]** Let $R_1^*$ and $R_2^*$ are two Neutrosophic sets, $\forall r \in N_X^*$

$R_1^* = \{< r, \mu_{R_1^*}(r), \sigma_{R_1^*}(r), \gamma_{R_1^*}(r) >: r \in N_X^*\}$,$R_2^* = \{< r, \mu_{R_2^*}(r), \sigma_{R_2^*}(r), \gamma_{R_2^*}(r) >: r \in N_X^*\}$

Then $R_1^* \subseteq R_2^* \Leftrightarrow \mu_{R_1^*}(r) \leq \mu_{R_2^*}(r), \sigma_{R_1^*}(r) \leq \sigma_{R_2^*}(r) \& \gamma_{R_1^*}(r) \geq \gamma_{R_2^*}(r)\}$

**Definition 2.4 [5]** Let $N_X^*$ be a non-empty set, and Let $R_1^*$and $R_2^*$ be two Neutrosophic sets are

$R_1^* = \{< r, \mu_{R_1^*}(r), \sigma_{R_1^*}(r), \gamma_{R_1^*}(r) >: r \in N_X^*\}$, $R_2^* = \{< r, \mu_{R_2^*}(r), \sigma_{R_2^*}(r), \gamma_{R_2^*}(r) >: r \in N_X^*\}$Then

1. $R_1^* \cap R_2^* = \{< r, \mu_{R_1^*}(r) \cap \mu_{R_2^*}(r), \sigma_{R_1^*}(r) \cap \sigma_{R_2^*}(r), \gamma_{R_1^*}(r) \cup \gamma_{R_2^*}(r) >: r \in N_X^*\}$
2. $R_1^* \cup R_2^* = \{< r, \mu_{R_1^*}(r) \cup \mu_{R_2^*}(r), \sigma_{R_1^*}(r) \cup \sigma_{R_2^*}(r), \gamma_{R_1^*}(r) \cap \gamma_{R_2^*}(r) >: r \in N_X^*\}$

**Definition 2.5 [11]** Let $N_X^*$ be non-empty set and $NS_\tau$ be the collection of Neutrosophic subsets of $N_X^*$satisfying, the accompanying properties:
1.$0_N, 1_N \in NS_\tau$
2. $N_{T_1} \cap N_{T_2} \in NS_\tau$ for any $N_{T_1}, N_{T_2} \in NS_\tau$
3. $\cup N_{T_i} \in NS_\tau$ for every $\{N_{T_i}: i \in j\} \subseteq NS_\tau$
Then the space $(N_X^*, NS_\tau)$, is called a Neutrosophic topological spaces (NS-T-S).
The component of of $NS_\tau$ are called NS-OS (Neutrosophic open set)
and its complement is NS-CS(Neutrosophic closed set)

**Example 2.6.** Let $N_X^*$ =$\{r\}$ and $\forall r \in N_X^*$

$R_1^* = \langle r, \frac{5}{10}, \frac{5}{10}, \frac{4}{10} \rangle$, $R_2^* = \langle r, \frac{4}{10}, \frac{6}{10}, \frac{8}{10} \rangle$

$R_3^* = \langle r, \frac{5}{10}, \frac{6}{10}, \frac{4}{10} \rangle$ ,$R_4^* = \langle r, \frac{4}{10}, \frac{5}{10}, \frac{8}{10} \rangle$

Then the collection $NS_\tau = \{0_N, R_1^*, R_2^*, R_3^*, R_4^* 1_N\}$ is called a NS-T-S on X.

**Definition 2.7** Let $(N_X^*, NS_\tau)$, be a NS-T-S and $R_1^* = \{< r, \mu_{R_1^*}(r), \sigma_{R_1^*}(r), \gamma_{R_1^*}(r) >: r \in N_X^*\}$ be a Neutrosophic set in $N_X^*$. Then $R_1^*$ is said to be

1. Neutrosophic α-closed set [2] (NS- αCS in short) NS-cl(NS-in(NS-cl($R_1^*$)))⊆ $R_1^*$,
2. Neutrosophic pre-closed set [14] (NS-PCS in short) NS-cl(NS-in($R_1^*$))⊆ $R_1^*$,
3. Neutrosophic regular closed set [5] (NS-RCS in short) NS-cl(NS-in($R_1^*$)) = $R_1^*$,
4. Neutrosophic semi closed set [7] (NS-SCS in short) NS-in(NS-cl($R_1^*$))⊆ $R_1^*$,
5. Neutrosophic generalized closed set [3] (NS-GCS in short) NS-cl($R_1^*$ ⊆H whenever $R_1^*$ ⊆H and H is a NS-OS,
6. Neutrosophic generalized pre closed set [9] (NS-GPCS in short) NS-pcl($R_1^*$) ⊆ H whenever $R_1^*$ ⊆ H and H is a NS-OS,

7. Neutrosophic α generalized closed set [8] (NS- αGCS in short) NS-α-cl($R_1^*$)⊆H whenever $R_1^*$ ⊆H And H is aNS-OS,
8. Neutrosophic generalized semi closed set [13](NS-GSCS in short) NS-Scl($R_1^*$)⊆H whenever $R_1^*$ ⊆H and H is aNS-OS.

**Definition 2.8.** ($N_X^*$, $NS_\tau$), be a NS-T-S and $R_1^* = \{< r, \mu_{R_1^*}(r), \sigma_{R_1^*}(r), \gamma_{R_1^*}(r) >: r \in N_X^*\}$ $N_X^*$.Then

Neutrosophic closure of $R_1^*$ is

NS-Cl($R_1^*$)= ∩ {H:H is a NS-CS in $N_X^*$ and $R_1^*$ ⊆H}

Neutrosophic interior of $R_1^*$ is

NS-Int($R_1^*$)= ∪{M:M is a NS-OS in $N_X^*$ and M⊆ $R_1^*$}.

**Definition 2.9.** Let ($N_X^*$, $NS_\tau$), be a NS-T-S and $R_1^* = \{< r, \mu_{R_1^*}(r), \sigma_{R_1^*}(x), \gamma_{R_1^*}(r) >: r \in N_X^*\}$

NS-Sint($R_1^*$)= ∪{ G/G is a NS-SOS in $N_X^*$ and G⊆ $R_1^*$},
NS-Scl($R_1^*$)= ∩{ K/K is a NS-SCS in $N_X^*$ and $R_1^*$ ⊆K }.
NS-$\alpha$int($R_1^*$)= ∪{ G/G is a NS-$\alpha$OS in $N_X^*$ and G⊆ $R_1^*$},
NS-$\alpha$cl($R_1^*$)= ∩{ K/K is a NS-$\alpha$CS in $N_X^*$ and $R_1^*$ ⊆K }.

**3. Neutrosophic Weakly Generalized Closed Set**

In this section we introduce Neutrosophic weakly generalized closed set and have studied some of its properties.

**Definition 3.1** An (NS)S $R_1^*$ in an (NS)TS ($N_X^*$, $NS_\tau$), is said to be aNeutrosophic weakly generalized closed set ((NS-WG)CS) NS-cl(NS-in($R_1^*$))⊆U whenever $R_1^*$ ⊆U, U is (NS)OS in $N_X^*$.

The family of all (NS-WG)CSs of an (NS)TS ($N_X^*$, $NS_\tau$), is denoted by (NS-WG)CS($N_X^*$).

**Example 3.2:** Let $N_X^*$ = {$r_1^*$, $r_2^*$} and let $NS_\tau$ = {0~, $N_T$, 1~} be a(NS)T on $N_X^*$

Where $N_T = < r, \left(\frac{1}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{6}{10}\right) >$.

Then the (NS)S $R_1^* = < r, \left(\frac{1}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{6}{10}\right) >$ is a(NS-WG)CS in $N_X^*$.

**Theorem 3.3:** Every (NS)CS is a(NS-WG)CS but not conversely.

**Proof:** Let $R_1^*$ be a(NS)CS in ($N_X^*$, $NS_\tau$),.Let U be a Neutrosophic open set such that $R_1^*$ ⊆ U. Since $R_1^*$ is Neutrosophic closed, NS-cl($R_1^*$) = $R_1^*$ and hence NS-cl($R_1^*$) ⊆U. But NS-cl(NS-in($R_1^*$)) ⊆NS-cl(A)⊆U. Therefore NS-cl(NS-in($R_1^*$))⊆U. Hence $R_1^*$ is a(NS-WG)CS in $N_X^*$.

**Example 3.4:** Let $N_X^*$ = {$r_1^*$, $r_2^*$} and let $NS_\tau$ = {0~, $N_T$, 1~} be a(NS) $N_T$ on $N_X^*$,

where $N_T = < r, \left(\frac{3}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{4}{10}\right) >$.

Then the (NS)S $R_1^* = < r, \left(\frac{1}{10}, \frac{5}{10}, \frac{7}{10}\right), \left(\frac{2}{10}, \frac{5}{10}, \frac{6}{10}\right) >$ is a(NS-WG)CS in $N_X^*$

but not an (NS)CS in $N_X^*$ since NS-cl($R_1^*$) =T<sup>C</sup>≠ $R_1^*$

**Theorem 3.5:** Every (NS) CS is a(NS-WG)CS but not conversely.

**Proof:** Let $R_1^*$ be a(NS) CS in $N_X^*$ and let $R_1^*$ ⊆U and U is a(NS)OS in ($N_X^*$, $NS_\tau$),. By hypothesis, NS-cl(NS-in(NS-cl($R_1^*$))) ⊆ $R_1^*$. Therefore NS-cl(NS-in(($R_1^*$))⊆NS-cl(NS-in(NS-cl($R_1^*$)))⊆ $R_1^*$ ⊆U. Therefore NS-cl(NS-in(($R_1^*$))⊆U. Hence $R_1^*$ is a(NS-WG)CS in $N_X^*$.

**Example 3.6:** Let $N_X^*$ = {$r_1^*$, $r_2^*$} and

let $NS_\tau$ = {0~, $N_T$, 1~} be a(NS)Ton $N_X^*$ where

$N_T$ =$< r, \left(\frac{4}{10},\frac{5}{10},\frac{4}{10}\right), \left(\frac{2}{10},\frac{5}{10},\frac{6}{10}\right) >$ .

Then the (NS)S $R_1^* = < r, \left(\frac{3}{10},\frac{5}{10},\frac{5}{10}\right), \left(\frac{1}{10},\frac{5}{10},\frac{7}{10}\right) >$ is a(NS-WG)CS

but not an (NS) CS in $N_X^*$

since $R_1^* \subseteq N_T$ but NS-cl(NS-in(NS-cl($R_1^*$)))=$< r, \left(\frac{4}{10},\frac{5}{10},\frac{4}{10}\right), \left(\frac{6}{10},\frac{5}{10},\frac{2}{10}\right) > \nsubseteq R_1^*$.

**Theorem 3.7:** Every (NS)GCS is a(NS-WG)CS but not conversely.

**Proof:** Let $R_1^*$ be a(NS)GCS in $N_X^*$ and let $R_1^*$ ⊆U and U is a(NS)OS in ($N_X^*$, $NS_\tau$). Since NS-cl($R_1^*$) ⊆U, NS-cl(NS-in($R_1^*$)) ⊆ NS-cl($R_1^*$).That is NS-cl(NS-in($R_1^*$))⊆NS-cl($R_1^*$)⊆U. Therefore NS-cl(NS-in($R_1^*$))⊆U.Hence $R_1^*$ is a(NS-WG)CS in $N_X^*$.

**Example 3.8:** Let $N_X^*$ = {$r_1^*$, $r_2^*$} and

let $NS_\tau$ = {0~, $N_T$, 1~} be a(NS) $N_T$ on $N_X^*$

where $N_T$ =$< r, \left(\frac{2}{10},\frac{5}{10},\frac{6}{10}\right), \left(\frac{3}{10},\frac{5}{10},\frac{5}{10}\right) >$ .

Then (NS)S $R_1^* = < r, \left(\frac{1}{10},\frac{5}{10},\frac{7}{10}\right), \left(\frac{3}{10},\frac{5}{10},\frac{5}{10}\right) >$is a(NS-WG)CS

but not an (NS)GCS in $N_X^*$

since $R_1^* \subseteq N_T$ but NS-cl($R_1^*$) = $< r, \left(\frac{6}{10},\frac{5}{10},\frac{2}{10}\right), \left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right) > \nsubseteq N_T$.

**Theorem 3.9:** Every (NS)RCS is a(NS-WG)CS but not conversely.

**Proof:** Let $R_1^*$ be a(NS)RCS in $N_X^*$ and let $R_1^*$ ⊆U and U is a(NS)OS in ($N_X^*$, $NS_\tau$),.Since $R_1^*$ is (NS)RCS, NS-cl(NS-in($R_1^*$)) = $R_1^*$ ⊆ U. This implies NS-cl(NS-in($R_1^*$))⊆U. Hence $R_1^*$ is a(NS-WG)CS in $N_X^*$.

**Example 3.10:**

Let $N_X^*$ = {$r_1^*$, $r_2^*$} and

let $NS_\tau$ = {0~, $N_T$, 1~} be a(NS) Ton $N_X^*$, where

$N_T$ = $< r, \left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right), \left(\frac{7}{10},\frac{5}{10},\frac{1}{10}\right) >$ .

The (NS)S $R_1^* = < r, \left(\frac{1}{10},\frac{5}{10},\frac{6}{10}\right), \left(\frac{1}{10},\frac{5}{10},\frac{7}{10}\right) >$is a(NS-WG)CS

but not an (NS)RCS in $N_X^*$ since NS-cl(NS-in($R_1^*$)) $\neq$ 0~ $R_1^*$.

**Theorem 3.11:** Every (NS)PCS is a(NS-WG)CS but not conversely.

**Proof:** Let $R_1^*$ be a(NS)PCS in $N_X^*$ and let $R_1^*$ ⊆U and U is a(NS)OS in ($N_X^*$, $NS_\tau$). By Definition, NS-cl(NS-in($R_1^*$))⊆ $R_1^*$ and $R_1^*$ ⊆U. Therefore NS-cl(NS-in($R_1^*$)⊆U. Hence $R_1^*$ is a(NS-WG)CS in $N_X^*$.

**Example 3.12:**

Let $N_X^*$ = {$r_1^*$, $r_2^*$} and

let $NS_\tau$ = {0~, $N_T$, 1~} be a(NS)T on $N_X^*$,

$N_T$ = $< r, \left(\frac{4}{10},\frac{5}{10},\frac{4}{10}\right), \left(\frac{2}{10},\frac{5}{10},\frac{6}{10}\right) >$.

Then the (NS)S $R_1^* =< r, \left(\frac{8}{10}, \frac{5}{10}, \frac{0}{10}\right), \left(\frac{2}{10}, \frac{5}{10}, \frac{5}{10}\right) >$ is a(NS-WG)CS

but not an (NS)PCS in $N_X^*$
since NS-cl(NS-in($R_1^*$)) =$T^C \nsubseteq R_1^*$.

**Theorem 3.13:** Every (NS) GCS is a(NS-WG)CS .

**Proof:** Let $R_1^*$ be a(NS) GCS in $N_X^*$ and let $R_1^* \subseteq$ U and U is a (NS)OS in ($N_X^*$, $NS_\tau$), By Definition, $R_1^* \subseteq$ NS-cl(NS-in(NS-cl($R_1^*$)))⊆U. This implies NS-cl(NS-in(NS-cl($R_1^*$)))⊆U and NS-cl(NS-in($R_1^*$)) ⊆NS-cl(NS-in(NS-cl($R_1^*$)))⊆U. Therefore NS-cl(NS-in($R_1^*$))⊆U. Hence $R_1^*$ is a(NS-WG)CS in $N_X^*$.

**Example 3.14:**
Let $N_X^* = \{r_1^*, r_2^*\}$ and
let $NS_\tau$ = {0~, $N_T$, 1~} be a(NS)T on $N_X^*$,
where $N_T = < r, \left(\frac{4}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{3}{10}\right) >$ .

Then the (NS)S $R_1^* = < r, \left(\frac{2}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{4}{10}\right) >$ is a(NS-WG)CS

but not an (NS) GCS in $N_X^*$ since NS-$\alpha$cl($R_1^*$) = 1~ $\nsubseteq N_T$.

**Proposition 3.15:** (NS)SCS and (NS-WG)CS are independent to each other which can be seen from the following example.

**Example 3.16:** Let $N_X^* = \{r_1^*, r_2^*\}$ and
let $NS_\tau$ = {0~, $N_T$, 1~} be a(NS)T on $N_X^*$

$N_T = < r, \left(\frac{3}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{2}{10}, \frac{5}{10}, \frac{6}{10}\right) >$.

Then (NS)S $R_1^* = N_T$ is a(NS)SCS
but not an (NS-WG)CS in $N_X^*$ since $R_1^* \subseteq N_T$

but NS-cl(NS-in($R_1^*$)) = $< r, \left(\frac{5}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{2}{10}\right) > \nsubseteq N_T$.

**Example 3.17:**
Let $N_X^* = \{r_1^*, r_2^*\}$ and
let $NS_\tau$ = {0~, $N_T$, 1~} be a(NS)T on $N_X^*$,

$N_T =< r, \left(\frac{8}{10}, \frac{5}{10}, \frac{0}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{1}{10}\right) >$.

Then the (NS)S $R_1^* =< r, \left(\frac{6}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{3}{10}\right) >$is a(NS-WG)CS

but not an (NS)SCS in $N_X^*$ since NS-in(NS-cl($R_1^*$))= 1~ $\nsubseteq R_1^*$.

**Proposition 3.18:** (NS)GSCS and (NS-WG)CS are independent to each other.

**Example 3.19:** Let $N_X^* = \{r_1^*, r_2^*\}$ and
let $NS_\tau$ = {0~, $N_T$, 1~} be a(NS)T on $N_X^*$,

where $N_T = < r, \left(\frac{2}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{4}{10}\right) >$ .

Then the (NS)S $R_1^* = N_T$ is a(NS-WG)CS
but not an (NS)GPCS in $N_X^*$ since $R_1^* \subseteq N_T$

but NS-cl(NS-in($R_1^*$)= $< r, \left(\frac{6}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{4}{10}\right) > \nsubseteq R_1^*$.

**Example 3.20:** Let $N_X^*$ = {$r_1^*$, $r_2^*$} and

let $NS_\tau$ = {0~, $N_T$, 1~} be a(NS)T on $N_X^*$, where

$N_T$ =< $r$, $\left(\frac{7}{10},\frac{5}{10},\frac{1}{10}\right)$, $\left(\frac{8}{10},\frac{5}{10},\frac{0}{10}\right)$ > .

Then the (NS)S $R_1^*$ = < $r$, $\left(\frac{6}{10},\frac{5}{10},\frac{2}{10}\right)$, $\left(\frac{7}{10},\frac{5}{10},\frac{1}{10}\right)$ > is a(NS-WG)CS

but not an (NS)GSCS in $N_X^*$

since NS-scl($R_1^*$)) = 1~ $\nsubseteq$ $N_T$.

**Remark 3.21:**

The union of any two (NS-WG)CSs need not be a(NS-WG)CS in general as seen from the following example.

**Example 3.22:**

Let $N_X^*$ = {$r_1^*$, $r_2^*$} be a(NS)TS and

let $N_T$ = < $r$, $\left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right)$, $\left(\frac{7}{10},\frac{5}{10},\frac{1}{10}\right)$ >.

Then $NS_\tau$ = { 0~,$N_T$, 1~ }is a(NS)T on $N_X^*$ and the (NS)Ss

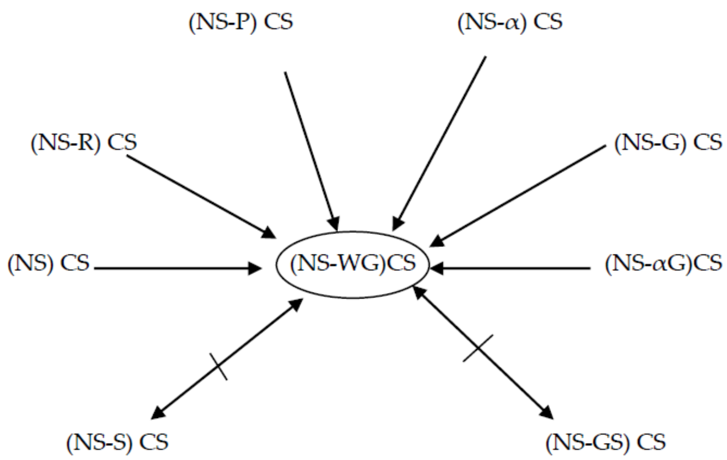
$R_1^*$ =< $r$, $\left(\frac{0}{10},\frac{5}{10},\frac{8}{10}\right)$ , $\left(\frac{7}{10},\frac{5}{10},\frac{1}{10}\right)$ >,

$R_2^*$ =< $r$, $\left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right)$, $\left(\frac{6}{10},\frac{5}{10},\frac{2}{10}\right)$ >

are (NS-WG)CSs but $R_1^* \cup R_2^*$ is not an (NS-WG)CS in $N_X^*$.

The following implications are true:

Fig.1




## 4. NEUTROSOPHIC WEAKLY GENERALIZED OPEN SET

In this section we introduce Neutrosophic weakly generalized open set and have studied some of its properties.

**Definition 4.1:** An (NS)S $R_1^*$ is said to be a Neutrosophic weakly generalized open set ((NS-WG)OS in short) in ($N_X^*$, $NS_\tau$), (NS) the complement $(R_1^*)^C$ is a(NS-WG)CS in $N_X^*$.

The family of all (NS-WG)OS of an (NS)TS ($N_X^*$, $NS_\tau$), is denoted by (NS-WG)O($N_X^*$).

**Example 4.2:** Let $N_X^*$ = {$r_1^*$, $r_2^*$} and

let $NS_\tau$ = {0~, $N_T$, 1~} be a(NS)T on $N_X^*$, where $N_T = < r, \left(\frac{6}{10},\frac{5}{10},\frac{1}{10}\right), \left(\frac{5}{10},\frac{5}{10},\frac{4}{10}\right) >$.

Then the (NS)S $R_1^* =< r, \left(\frac{7}{10},\frac{5}{10},\frac{1}{10}\right), \left(\frac{6}{10},\frac{5}{10},\frac{1}{10}\right) >$ is a(NS-WG)OS in $N_X^*$.

**Theorem 4.3:** For any (NS)TS ($N_X^*$, $NS_\tau$), we have the following:

(i)Every (NS)OS is a(NS-WG)OS.

(ii)Every (NS)SOS is a(NS-WG)OS.

(iii)Every (NS)$\alpha$OS is a(NS-WG)OS.

(iv)Every (NS)GOS is a(NS-WG)OS. But the converses are not true in general.

**Proof:** Straight forward.

The converse of the above statement need not be true in general which can be seen from the following examples.

**Example 4.4:** Let $N_X^*= \{r_1^*, r_2^*\}$ and

$N_T = < r, \left(\frac{6}{10},\frac{5}{10},\frac{2}{10}\right), \left(\frac{4}{10},\frac{5}{10},\frac{3}{10}\right) >$ .

Then $NS_\tau = \{0\sim, N_T, 1\sim\}$ is a(NS)T on $N_X^*$. The (NS)S

$R_1^* =< r, \left(\frac{7}{10},\frac{5}{10},\frac{0}{10}\right), \left(\frac{6}{10},\frac{5}{10},\frac{1}{10}\right) >$ is a(NS-WG)OS in ($N_X^*$, $NS_\tau$),

but not an (NS)OS in $N_X^*$.

$N_T= < r, \left(\frac{0}{10},\frac{5}{10},\frac{8}{10}\right), \left(\frac{1}{10},\frac{5}{10},\frac{6}{10}\right) >$.

Then the (NS)S$R_1^* = < r, \left(\frac{2}{10},\frac{5}{10},\frac{6}{10}\right), \left(\frac{3}{10},\frac{5}{10},\frac{5}{10}\right) >$is a(NS-WG)OS

but not an (NS)SOS in $N_X^*$.

**Example 4.6:** Let $N_X^* = \{r_1^*, r_2^*\}$ and let $NS_\tau = \{0\sim, N_T, 1\sim\}$ be a(NS)T on $N_X^*$, where $N_T = < r,$ (0.5, 0.7), (0.5, 0.3) $>$ .

Then the (NS)S $R_1^* = < r, \left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right), \left(\frac{7}{10},\frac{5}{10},\frac{1}{10}\right) >$ is a(NS-WG)OS

but not an (NS)$\alpha$OS in $N_X^*$.

**Example 4.7:** Let $N_X^* = \{r_1^*, r_2^*\}$ and

$N_T = < r, \left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right), \left(\frac{4}{10},\frac{5}{10},\frac{4}{10}\right) >$.

Then $NS_\tau = \{0\sim, N_T, 1\sim\}$ is a(NS)T on $N_X^*$.

The (NS)S $R_1^* =< r, \left(\frac{6}{10},\frac{5}{10},\frac{2}{10}\right), \left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right) >$ is a(NS-WG)OS

but not an (NS)POS in $N_X^*$.

**Remark 4.8:**

The intersection of any two (NS-WG)OSs need not be a(NS-WG)OS in general.

**Example 4.9:** Let $N_X^* = \{r_1^*, r_2^*\}$ be a(NS)TS and

let $N_T =< r, \left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right), \left(\frac{7}{10},\frac{5}{10},\frac{1}{10}\right) >$.

Then $NS_\tau = \{0\sim, N_T, 1\sim\}$ is a(NS)T on $N_X^*$.

The (NS)Ss $R_1^*= < r,\ \left(\frac{8}{10},\frac{5}{10},\frac{0}{10}\right),\left(\frac{1}{10},\frac{5}{10},\frac{7}{10}\right) >$ and

$R_2^* = < r,\ \left(\frac{3}{10},\frac{5}{10},\frac{5}{10}\right),\left(\frac{2}{10},\frac{5}{10},\frac{6}{10}\right)$ >are (NS-WG)OS's but $R_1^* \cap R_2^*$ is not an (NS-WG)OS in $N_X^*$.

**Theorem 4.10:**

An (NS)S $R_1^*$ of an (NS)TS ($N_X^*$, $NS_\tau$), is a(NS-WG)OS (NS) and only (NS) F⊆NS-in(NS-cl($R_1^*$)) whenever F is a(NS)CS and F⊆ $R_1^*$.

**Proof:**

Necessity:

Suppose $R_1^*$ is a(NS-WG)OS in $N_X^*$. Let F be a(NS)CS and F ⊆ $R_1^*$. Then $F^C$ is a(NS)OS in $N_X^*$ such that $(R_1^*)^C$ $F^C$. Since $(R_1^*)^C$ is a(NS-WG)CS,NS-cl(NS-in($(R_1^*)^C$)) ⊆ $F^C$ .Hence (NS-in(NS-cl($R_1^*$)))$^C$ ⊆ $F^C$.This implies F ⊆ NS-in(NS-cl($R_1^*$)).

Sufficiency:

Let $R_1^*$ be a(NS)S of $N_X^*$ and let F ⊆ NS-in(NS-cl($R_1^*$)) whenever F is a(NS)CS and F⊆ $R_1^*$. Then $(R_1^*)^C$ ⊆ $F^C$ and $F^C$ is a(NS)OS. By hypothesis, (NS-in(NS-cl($R_1^*$)))$^C$ ⊆ $F^C$. Hence NS-cl(NS-in($(R_1^*)^C$)) ⊆ $F^C$ .

Hence $R_1^*$ is a(NS-WG)OS of $N_X^*$.

## 5. APPLICATIONS

In this section, we introduce Neutrosophic$wT^{\frac{1}{2}}$ space and wgqT space, which utilize Neutrosophic weakly generalized closed set and its characterizations are proved.

**Definition 5.1**:

An (NS)TS ($N_X^*$, $NS_\tau$), is called an Neutrosophic $wT^{\frac{1}{2}}$ ($(NS)w\,T^{\frac{1}{2}}$in short) space (NS) every (NS-WG)CS in $N_X^*$ is a(NS)CS in $N_X^*$.

**Definition 5.2:**

An (NS)TS ($N_X^*$, $NS_\tau$), is called an Neutrosophic wgqT ((NS-WG)qT in short) space (NS) every (NS-WG)CS in $N_X^*$ is a(NS)PCS in $N_X^*$.

**Theorem 5.3:** Every $(NS)$ $wT^{\frac{1}{2}}$ space is a(NS-WG)qT space. But reversal isn't true in general.

**Proof:** Let $N_X^*$ be a$(NS)wT^{\frac{1}{2}}$space and let $R_1^*$ be a(NS-WG)CS in $N_X^*$.By hypothesis $R_1^*$ is a(NS)CS in $N_X^*$. Since every (NS)CS is a(NS)PCS, $R_1^*$ is a(NS)PCS in $N_X^*$. Hence $N_X^*$ is a(NS)wgqT space.But reversal isn't true in general.

**Example 5.4:** Let $N_X^*$= {$r_1^*$, $r_2^*$} and

let $NS_\tau$ = {0~, $N_T$, 1~}

$N_T = < r,\ \left(\frac{9}{10},\frac{5}{10},\frac{1}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{1}{10}\right) >$ . Then ($N_X^*$, $NS_\tau$), is a(NS-WG)qT space.

But it is not an $(NS)wT^{\frac{1}{2}}$ space

since the (NS)S $R_1^* =< r,\left(\frac{2}{10},\frac{5}{10},\frac{8}{10}\right),\left(\frac{3}{10},\frac{5}{10},\frac{7}{10}\right) >$ is (NS-WG)CS but not (NS)CS in $N_X^*$.

**Theorem 5.5:** Let $(N_X^*,\ NS_\tau)$, be a(NS)TS and $N_X^*$ is a$(NS)wT^{\frac{1}{2}}$space then

(i) Any union of (NS-WG)CS is a(NS-WG)CS.

(ii) Any intersection of (NS-WG)OS is a(NS-WG)OS.

**Proof:**

(i): Let {Ai}i∈J be a collection of (NS-WG)CS in an $(NS)wT^{\frac{1}{2}}$space $(N_X^*,\ NS_\tau)$. Therefore every (NS-WG)CS is a(NS)CS. But the union of (NS)CS is a(NS)CS. Hence the Union of (NS-WG)CS is a(NS-WG)CS in $N_X^*$.

(ii): It tends to be demonstrated by taking complement at(i).

**Theorem 5.6:**

An (NS)TS $N_X^*$ is a(NS-WG)qT space (NS) and only (NS) (NS-WG)OS($N_X^*$) = (NS)POS($N_X^*$).

**Proof:**

Necessity:

Let $R_1^*$ be a(NS-WG)OS in $N_X^*$. Then $(R_1^*)^C$ is a(NS-WG)CS in $N_X^*$. By hypothesis $(R_1^*)^C$ is a(NS)PCS in $N_X^*$. Therefore $R_1^*$ is a(NS)POS in $N_X^*$. Hence (NS-WG)OS($N_X^*$) = (NS)POS($N_X^*$).

Sufficiency:

Let $R_1^*$ be a(NS-WG)CS in $N_X^*$. Then $(R_1^*)^C$ is a(NS-WG)OS in $N_X^*$. By hypothesis $(R_1^*)^C$ is a(NS)POS in $N_X^*$. Therefore $R_1^*$ is a(NS)PCS in $N_X^*$. Hence $N_X^*$ is a(NS-WG)qT space.

**Theorem 5.7:** An (NS)TS $N_X^*$ is a$(NS)wT^{\frac{1}{2}}$space (NS) and only (NS) (NS-WG)OS ($N_X^*$) = (NS)OS($N_X^*$).

**Proof:** Necessity:

Let $R_1^*$ be a(NS-WG)OS in $N_X^*$. Then $(R_1^*)^C$ is a(NS-WG)CS in $N_X^*$. By hypothesis $(R_1^*)^C$ is a(NS)CS in $N_X^*$. Therefore $R_1^*$ is a(NS)OS in $N_X^*$. Hence (NS-WG)OS($N_X^*$) = (NS)OS($N_X^*$).

Sufficiency:

Let $R_1^*$ be a(NS-WG)CS in $N_X^*$. Then $(R_1^*)^C$ is a(NS-WG)OS in $N_X^*$. By hypothesis $(R_1^*)^C$ is a(NS)OS in $N_X^*$. Therefore $R_1^*$ is a(NS)CS in $N_X^*$. Hence $N_X^*$ is a $(NS)wT^{\frac{1}{2}}$ space.

## 6. CONCLUSION

In this paper we have presented another class of Neutrosophic closed set to be specific (NS)WG closed set and have examined the connection between Neutrosophic weakly generalized closed set and other existing Neutrosophic closed sets. Likewise we have explored a portion of the properties of Neutrosophic weakly generalized closed set. As an utilization of Neutrosophic weakly generalized closed set we have presented two new spaces specifically $(NS)wT^{\frac{1}{2}}$ space and Neutrosophicwg qT ((NS-WG)qT in short) space and concentrated a portion of their properties.

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