A Note On Neutrosophic Chaotic Continuous Functions

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Abstract. Many real time problems are based on uncertainty and chaotic environment. To demonstrate this ambiguous situation more precisely we intend to amalgamate the ideas of chaos theory and neutrosophy. Neutrosophy is a flourishing arena which conceptualizes the notions of true, falsity and indeterminacy attributes of an event. Chaos theory is another branch which brings out the concepts of periodic point, orbit and sensitive of a set. Hence in this paper we focus on the introducing the idea of chaotic periodic points, orbit sets, sensitive functions under neutrosophic settings. We start with defining a neutrosophic chaotic space and enlist its properties, As a further extension we coin neutrosophic chaotic continuous functions and discuss its characterizations and their interrelationships. We have also illustrated the above said concepts with suitable examples.

Keywords: Neutrosophic periodic points, neutrosophic orbit sets, neutrosophic chaotic sets, neutrosophic sensitive functions, neutrosophic orbit extremally disconnected spaces.

1 Introduction

The introduction of the idea of fuzzy set was introduced in the year 1965 by Zadeh[16]. He proposed that each element in a fuzzy set has a degree of membership. Following this concept K.Atanassov[1,2,3] in 1983 introduced the idea of intuitionistic fuzzy set on a universe X as a generalization of fuzzy set. Here besides the degree of membership a degree of non-membership for each element is also defined. Smarandache[11,12] originally gave the definition of a neutrosophic set and neutrosophic logic. The neutrosophic logic is a formal frame trying to measure the truth, indeterminacy and falsehood. The significance of neutrosophy is that it finds indispensable place in decision making. Several authors[7, 8, 9, 10] have done remarkable achievements in this area. One of the prime discoveries of the 20th century which has been widely investigated with significant progress and achievements is the theory of Chaos and fractals.It has become an exciting emerging interdisciplinary area in which a broad spectrum of technologies and methodologies have emerged to deal with large-scale, complex and dynamical systems and problems. In 1989, R.L. Devaney[4] defined chaotic function in general metric space. A breakthrough in the conventional general topology was initiated by T. Thrivikraman and P.B. Vinod Kumar[15] by defining Chaos and fractals in general topological spaces. M. Kousalyaparasakthi, E. Roja, M.K. Uma[6] introduced the above said idea to intuitionistic chaotic continuous functions. Tethering around this concept we introduce neutrosophic periodic points, neutrosophic orbit sets, neutrosophic sensitive functions, neutrosophic clopen chaotic sets and neutrosophic chaos spaces. The concepts of neutrosophic chaotic continuous functions, neutrosophic chaotic’ continuous functions, neutrosophic chaotic” continuous functions, neutrosophic chaotic’’ continuous functions are introduced and studied. Some interrelation are discussed with suitable examples. Also the concept of neutrosophic orbit extremely disconnected spaces, neutrosophic chaotic extremally disconnected spaces, neutrosophic orbit irresolute function are discussed.

2 Preliminaries

2.1 Definition [12]

Let X be a non empty set. A neutrosophic set (NS for short) V is an object having the form V = <x, V¹, V², V³> where V¹, V², V³ represent the degree of membership, the degree of indeterminacy and the degree of non-membership respectively of each element x ∈ X to the set V.

2.2 Definition [12]

Let X be a non empty set, U = <x, U¹, U², U³> and V = <x, V¹, V², V³> be neutrosophic sets on X, and let {Vi; i ∈ J} be an arbitrary family of neutrosophic sets in X, where V¹ = <x, V¹, V², V³>

(i) U ⊆ V ⇔ U¹ ≤ V¹, U² ≥ V² and U³ ≥ V³
(ii) U = V ⇔ U ⊆ V and V ⊆ U.

(iii) V = <x, V^1, V^2, V^3>

(iv) U ∩ V = <x, U^1 ∩ V^1, U^2 ∩ V^2, U^3 ∩ V^3>

(v) U ∪ V = <x, U^1 ∪ V^1, U^2 ∪ V^2, U^3 ∪ V^3>

(vi) U_i ∩ V_i = <x, U^1_i ∩ V^1_i, U^2_i ∩ V^2_i, U^3_i ∩ V^3_i>

(vii) U_i ∪ V_i = <x, U^1_i ∪ V^1_i, U^2_i ∪ V^2_i, U^3_i ∪ V^3_i>

(viii) U − V = U ∩ V.

(ix) φ_N = <x, φ, X, X>; X_N = <x, X, φ, φ>.

2.3 Definition [14]

A neutrosophic topology (NT for short) on a nonempty set X is a family τ of neutrosophic set in X satisfying the following axioms:

(i) φ_N, X_N ∈ τ.

(ii) T^1 ∩ T^2 ∈ τ for any T^1, T^2 ∈ τ.

(iii) ∪_{i ∈ I} T^i ∈ τ for any arbitrary family {T^i : i ∈ J} ⊆ τ.

In this case the pair (X, τ) is called a neutrosophic topological space (NTS for short) and any neutrosophic set in τ is called a neutrosophic open set (NOS for short) in X. The complement V of a neutrosophic open set V is called a neutrosophic closed set (NCS for short) in X.

2.4 Definition [14]

Let (X, τ) be a neutrosophic topological space and V = <X, V^1, V^2, V^3> be a set in X. Then the closure and interior of V are defined by

Ncl(V) = ∩{M : M is a neutrosophic closed set in X and V ⊆ M},

Nint(V) = ∪{N : N is a neutrosophic open set in X and N ⊆ V}.

It can be also shown that Ncl(V) is a neutrosophic closed set and Nint(V) is a neutrosophic open set in X, and V is a neutrosophic closed set in X iff Ncl(V) = V; and V is a neutrosophic open set in X iff Nint(V) = V.

Where Ncl - neutrosophic closure and Nint – neutrosophic interior

2.5 Definition [5]

(a) If V = <y, V^1, V^2, V^3> is a neutrosophic set in Y, then the preimage of V under f, denoted by f^−1(V), is the neutrosophic set in X defined by f^−1(V) = <x, f^−1(V)^1, f^−1(V)^2, f^−1(V)^3>.

(b) If U = <x, U^1, U^2, U^3> is a neutrosophic set in X, then the image of U under f, denoted by f(U), is the neutrosophic set in Y defined by f(U) = <y, f(U)^1, f(U)^2, Y−f(X−U)^3> where

f(U^1) = \begin{cases} \sup_{x \in f^−1(y)} U^1 & \text{if } f^−1(y) \neq \phi \\ 0 & \text{otherwise} \end{cases}

f(U^2) = \begin{cases} \sup_{x \in f^−1(y)} U^2 & \text{if } f^−1(y) \neq \phi \\ 0 & \text{otherwise} \end{cases}

Y−f(X−U)^3 = \begin{cases} \inf_{x \in f^−1(y)} U^3 & \text{if } f^−1(y) \neq \phi \\ 1 & \text{otherwise} \end{cases}

2.6 Definition [13]

Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces and let f : X → Y be a function. Then f is said to be continuous if and only if the preimage of each neutrosophic set in σ is a neutrosophic set in τ.

2.7 Definition [13]

Let (X, τ) and (Y, σ) be two neutrosophic topological spaces and let f : (X, τ) → (Y, σ) be a function. Then f is said to be open iff the image of each neutrosophic set in τ is a neutrosophic set in σ.

2.8 Definition [4]

Orbit of a point x in X under the mapping f is O_f(x) = {x, f(x), f^2(x), ..., f^n(x),...}.

2.9 Definition [4]

x in X is called a periodic point of f if f^n(x) = x, for some n ∈ Z^+. Smallest of these n is called period of x.

2.10 Definition [4]
f is sensitive if for each \( \delta > 0 \) \( \exists (a) \varepsilon > 0 (b) y \subseteq X \) and (c) \( n \subseteq Z \), \( d(x,y) < \delta \) and \( d(f^n(x), f^n(y)) > \varepsilon \).

2.11 Definition [4]
f is chaotic on \((X,d)\) if (i) Periodic points of \( f \) are dense in \( X \) (ii) Orbit of \( x \) is dense in \( X \) for some \( x \) in \( X \) and (iii) \( f \) is sensitive.

2.12 Definition [15]
Let \((X, \tau)\) be a topological space and \( f : (X, \tau) \to (X, \tau) \) be continuous map. Then \( f \) is sensitive at \( x \) if given any open set \( U \) containing \( x \) \( \exists (i) y \in U \) (ii) \( n \in Z^\ast \) and (iii) an open set \( V \) \( f^n(x) \in V \), \( f^n(y) \notin cl(V) \). We say that \( f \) is sensitive on a \( F \) if \( f|F \) is sensitive at every point of \( F \).

2.13 Definition [15]
Let \((X, \tau)\) be a topological space and \( F \subseteq K(X) \). Let \( f : F \to F \) be a continuous. Then \( f \) is chaotic on \( F \) if
(i) \( cl(O(x)) = F \) for some \( x \in F \).
(ii) periodic points of \( f \) are dense in \( F \).
(iii) \( f \) is sensitive.

2.14 Definition [15]
(i) \( C(F) = \{f : F \to F \mid f \) is chaotic on \( F\} \) and (ii) \( CH(X) = \{F \in NK(X) \mid C(F) \neq \phi \} \).

2.15 Definition [15]
A topological space \((X, \tau)\) is called a chaos space if \( CH(X) \neq \phi \). The members of \( CH(X) \) are called chaotic sets.

3 Characterizations of neutrosophic chaotic continuous functions

3.1 Definition
Let \((X, \tau)\) be a neutrosophic topological space. An orbit of a point \( x \) in \( X \) under the function \( f : (X, \tau) \to (X, \tau) \) is denoted and defined as \( O(x) = \{x, f(x), f^2(x),... f^n(x)\} \) for \( x \in X \) and \( n \in Z^\ast \).

3.2 Definition
Let \((X, \tau)\) be a neutrosophic topological space. An orbit set \( O(x) \) in \( X \) under the function \( f : (X, \tau) \to (X, \tau) \) is denoted and defined as \( NO(x) = \{x, f(x), f^2(x),... f^n(x)\} \) for \( x \in X \) and \( n \in Z^\ast \).

3.3 Example
Let \( X = \{p,q,r\} \). Let \( f : X \to X \) be a function defined by \( f(p) = q, f(q) = r, \) and \( f(r) = p \). If \( n = 1 \), then the orbit points \( O(p) = \{p,q\}, O(q) = \{q,r\} \) and \( O(r) = \{r,p\}. \) If \( n = 2 \), then the orbit points \( O(p) = X, O_1(q) = X \) and \( O_2(r) = X \).

3.4 Definition
Let \((X, \tau)\) be a neutrosophic topological space. A neutrosophic orbit set in \( X \) under the function \( f : (X, \tau) \to (X, \tau) \) is denoted and defined as \( NO_1(x) = \{x, O(x), O_2(x), O_3(x)\} \) for \( x \in X \).

3.5 Example
Let \( X = \{p,q,r,s\} \). Let \( f : X \to X \) be a function defined by \( f(p) = q, f(q) = r, f(r) = s \) and \( f(s) = r,p \). If \( n = 1 \), then the neutrosophic orbit sets \( NO_1(p) = \{x[p,q],[p,s],[p,q]\}, NO_1(q) = \{x[p,q],[q,p],[q,r]\}, NO_1(r) = \{x[p,r],[q,r],[r,s]\} \) and \( NO_1(s) = \{x[p,s],[r,s],[p,s]\}. \) If \( n = 2 \), then the neutrosophic orbit sets \( NO_2(p) = \{x[p,q],[p,s],[p,q],[p,q]\}, NO_2(q) = \{x[p,q],[q,p],[q,r],[q,r]\}, NO_2(r) = \{x[p,r],[p,q],[p,q],[p,q],[p,q]\} \) and \( NO_2(s) = \{x[p,s],[r,s],[p,s],[p,s]\}. \) If \( n = 3 \), then the neutrosophic orbit sets \( NO_3(a) = \{x,X,X,X\}, NO_3(b) = \{x,X,X,X\}, NO_3(c) = \{x,X,X,X\} \) and \( NO_3(d) = \{x,X,X,X\}. \)

3.6 Definition
Let \((X, \tau)\) be a neutrosophic topological space and \( f : (X, \tau) \to (X, \tau) \) be a neutrosophic continuous function. Then \( f \) is said to be neutrosophic sensitive at \( x \in X \) if given any neutrosophic open set \( U = \{x,U^1,U^2,U^3\} \) containing \( x \) \( \exists \) a neutrosophic open set \( V = \{x,V^1,V^2,V^3\} \) \( f^n(x) \in V \), \( f^n(y) \notin \text{Necl}(V) \) and \( y \in U \), \( n \in Z^\ast \). We say that \( f \) is neutrosophic sensitive on a neutrosophic compact set \( F = \{x,F^1,F^2,F^3\} \) if \( f|F \) is neutrosophic sensitive at every point of \( F \).

3.7 Example
Let \( X = \{p,q,r,s\}. \) Then the neutrosophic sets \( P, Q, R \) and \( S \) are defined by \( P = \{x,[p,r],[p,q],[p,r],[p,s]\} \), \( Q = \{x,[r,s],[p,r],[p,s]\} \), \( R = \{x,[r,s],[p,q],[p,s]\} \) and \( S = \{x,[r,s],[p,s],[p,s]\}. \) Then the family \( \tau = \{X_0,\phi_P,\phi_Q,\phi_R\} \) is neutrosophic topology on \( X. \) Clearly, \((X, \tau)\) is an neutrosophic topological space. Let \( f : (X,
τ) → (X, τ) be a function defined by f(p) = <r,q,s>, f(q) = <r,s,t>, f(r) = <q,p,r> and f(s) = <p,r,q>. Let x = p and y = r. If n = 1, 3, 5, then the neutrosophic open set P = <x, {p,r,s}, {p,q,r}, {p,r,s}> containing x there exists an neutrosophic open set R = <x, {r,s}, {p,q,r}, {p,r,s}> such that f^n(x) ∈ R, f^n(y) ∉ Ncl(R) and y ∈ P. Hence the function f is called neutrosophic sensitive.

3.8 Notation
Let (X, τ) be a neutrosophic topological space. Let F = <x, F_1, F_2, F_3> ⊆ X, then S(F) = <x, S(F)_1, S(F)_2, S(F)_3>, where S(F)_1 = {f | f is neutrosophic sensitive on F}, S(F)_2 = {f | f is indeterminacy neutrosophic sensitive on F} and S(F)_3 = {f | f is not neutrosophic sensitive on F}.

3.9 Definition
Let (X, τ) be a two neutrosophic topological space. Let f : (X, τ) → (X, τ) be a function. A neutrosophic periodic set is denoted and defined as NP_f(x) = <x, {x ∈ X | f^n(x) = x}, {x ∈ X | f^n(x) = x}, {x ∈ X | f^n(x) = x}>

3.10 Example
Let X = {p, q, r}. Let f : X → X be a function defined by f(p) = <p, q, r>, f(q) = <r, p, q> and f(r) = <q, r, p>. If n = 1, then the neutrosophic periodic set NP_f(p) = <x, {p}, {q}, {r}, {x}>, NP_f(q) = <x, {r}, {p}, {q}, {r}>, and NP_f(r) = <x, {q}, {r}, {p}, {q}>. If n = 2, then the neutrosophic periodic sets NP_f(p) = <x, {p}, {p}, {p}, {x}>, NP_f(q) = <x, {q}, {q}, {q}, {x}> and NP_f(r) = <x, {r}, {r}, {r}, {x}>

3.11 Definition
Let (X, τ) be a neutrosophic topological space. A neutrosophic set V = <x, V_1, V_2, V_3> of X is said to be a neutrosophic dense in X, if Ncl(V) = X.

3.12 Definition
Let (X, τ) be a neutrosophic topological space and F = <x, F_1, F_2, F_3> ∈ NK(X). Let f : F → F be a neutrosophic continuous function. Then f is said to be neutrosophic chaotic on F if
(i) Ncl(NO_f(x)) = F for some x ∈ F.
(ii) neutrosophic periodic points of f are neutrosophic dense in F. That is, Ncl(NP_f(x)) = F.
(iii) f ∈ S(F).

3.13 Notation
Let (X, τ) be a neutrosophic topological space then C(F) = <x, C(F)_1, C(F)_2, C(F)_3> where C(F)_1 = {f : F → F | f is neutrosophic chaotic on F}, C(F)_2 = {f : F → F | f is indeterminacy neutrosophic chaotic on F} and C(F)_3 = {f : F → F | f is not neutrosophic chaotic on F}.

3.14 Notation
Let (X, τ) be a neutrosophic topological space then CH(X) = {F = <x, F_1, F_2, F_3> ∈ NK(X) | C(F) ≠ φ}.

3.15 Definition
A neutrosophic topological space (X, τ) is called a neutrosophic chaos space if CH(X) ≠ φ. The members of CH(X) are called neutrosophic chaotic sets.

3.16 Definition
Let (X, τ) be a neutrosophic topological space. A neutrosophic set V = <x, V_1, V_2, V_3> is neutrosophic clopen if it is both neutrosophic open and neutrosophic closed.

3.17 Definition
Let (X, τ) be a neutrosophic topological space.
(i) A neutrosophic open orbit set is a neutrosophic set which is both neutrosophic open and neutrosophic orbit.
(ii) A neutrosophic closed orbit set is a neutrosophic set which is both neutrosophic closed and neutrosophic orbit.
(iii) A neutrosophic clopen orbit set is a neutrosophic set which is both neutrosophic clopen and neutrosophic orbit.

3.18 Definition
Let (X, τ) be a neutrosophic topological space.
(i) A neutrosophic open chaotic set is a neutrosophic set which is both neutrosophic open and neutrosophic chaotic.
(ii) A neutrosophic closed chaotic set is a neutrosophic set which is both neutrosophic closed and neutrosophic chaotic.
(iii) A neutrosophic clopen chaotic set is a neutrosophic set which is both neutrosophic clopen and neutrosophic chaotic.

3.19 Definition
Let (X, τ) and (X, σ) be any two neutrosophic chaos spaces. A function f : (X, τ) → (X, σ) is said to be neutro-T. Madhumathi, F. Nirmala Irudayam and Florentin Smarandache, A Note on Neutrosophic Chaotic Continuous Function.
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neutrosophic chaotic continuous if for each periodic point \( x \in X \) and each neutrosophic clopen chaotic set \( F = \langle x,F^1,F^2,F^3 \rangle \) of \( f(x) \) \( \exists \) a neutrosophic open orbit set \( NO(x) \) of the periodic point \( x \) \( \ni f(NO(x)) \subseteq F \).

3.20 Example
Let \( X = \{p,q,r,s\} \). Then the neutrosophic sets \( M,N,O,P,Q \) and \( R \) are defined by \( M = \langle \{p,r\},\{q,r\},\{s\} \rangle \), \( N = \langle \{p,q\},\{p,s\} \rangle \), \( O = \langle \{p,q,r\},\{q\} \rangle \), \( P = \langle \{p,q,r\},\{p,s\} \rangle \), \( Q = \langle \{p,q,r\},\{r\} \rangle \), \( R = \langle \{p\},\{p,q\} \rangle \). Let \( \tau = \{X_0,\phi_0,M,N,O,P\} \) and \( \sigma = \{X_0,\phi_0,Q,R\} \) be a neutrosophic topologies on \( X \). Clearly \( (X,\tau) \) and \( (X,\sigma) \) be any two neutrosophic chaos spaces. The function \( f : (X,\tau) \rightarrow (X,\sigma) \) is defined by \( f(p) = \langle q,p,s \rangle \), \( f(q) = \langle s,r,p \rangle \), \( f(r) = \langle q,r,p \rangle \) and \( f(s) = \langle s,p,q \rangle \). Now the function \( f \) is called neutrosophic chaotic continuous.

3.21 Theorem
Let \( (X,\tau) \) and \( (X,\sigma) \) be any two neutrosophic chaos spaces. The function \( f : (X,\tau) \rightarrow (X,\sigma) \) is said to be neutrosophic chaotic continuous if for each periodic point \( x \in X \) of the periodic point \( x \ni f(NO(x))) \subseteq F \).

3.22 Definition
Let \( (X,\tau) \) and \( (X,\sigma) \) be any two neutrosophic chaos spaces. A function \( f : (X,\tau) \rightarrow (X,\sigma) \) is said to be neutrosophic chaotic continuous if for each periodic point \( x \in X \) and each neutrosophic closed chaotic set \( F \) containing \( f(x) \), \( \exists \) neutrosophic open orbit set \( NO(x) \) containing \( x \ni f(NO(x))) \subseteq F \).

3.23 Theorem
A neutrosophic chaotic continuous function is a neutrosophic chaotic continuous function.

Proof Since \( f \) is a neutrosophic chaotic continuous function, \( F \) is a neutrosophic closed chaotic set containing \( f(x) \), \( \exists \) neutrosophic open orbit set \( NO(x) \) containing \( x \ni f(NO(x))) \subseteq F \). Then \( F \ni f(NO(x))) \subseteq F \). Therefore, \( f(NO(x))) \subseteq F \). Hence, \( f(NO(x))) \subseteq F \). This implies that, \( f(NO(x))) \subseteq F \). Hence, \( f \) is a neutrosophic chaotic continuous function.

3.24 Remark
The converse of Theorem 3.23. need not be true as shown in Example 3.25.

3.25 Example
Let \( X = \{p,q,r,s\} \). Then the neutrosophic sets \( M,N,O,P,Q,R,S,T \) and \( \tau \) are defined by \( M = \langle \{p,r\},\{q,r\},\{s\} \rangle \), \( N = \langle \{p,q\},\{p,s\} \rangle \), \( O = \langle \{p,q,r\},\{q\} \rangle \), \( P = \langle \{p,q,r\},\{p,s\} \rangle \), \( Q = \langle \{p,q,r\},\{r\} \rangle \), \( R = \langle \{p\},\{p,q\} \rangle \), \( S = \langle \{q,s\},\{p,s\},\{p,q\} \rangle \), \( T = \langle \{p,q,s\},\{q,r\} \rangle \) and \( \tau = \{X_0,\phi_0,M,N,O,P,Q,R,S,T\} \) be a neutrosophic topologies on \( X \). Clearly \( (X,\tau) \) and \( (X,\sigma) \) be any two neutrosophic chaos spaces. The function \( f : (X,\tau) \rightarrow (X,\sigma) \) is defined by \( f(p) = \langle q,p,s \rangle \), \( f(q) = \langle s,r,p \rangle \), \( f(r) = \langle q,r,p \rangle \) and \( f(s) = \langle s,p,q \rangle \). Now the function \( f \) is neutrosophic chaotic continuous but not neutrosophic chaotic continuous. Hence, neutrosophic chaotic continuous function need not be neutrosophic chaotic continuous function.

3.26 Definition
Let \( (X,\tau) \) and \( (X,\sigma) \) be any two neutrosophic chaos spaces. A function \( f : (X,\tau) \rightarrow (X,\sigma) \) is said to be neutrosophic chaotic continuous if for each periodic point \( x \in X \) and each neutrosophic closed chaotic set \( F \) of \( f(x), \ni \) a neutrosophic open orbit set \( NO(x) \) of the periodic point \( x \ni f(NO(x)))) \subseteq F \).
3.27 Theorem
A neutrosophic chaotic continuous function is a neutrosophic chaotic** continuous function.
Proof Since f is a neutrosophic chaotic continuous function, F is a neutrosophic clopen chaotic set containing f(x), \exists a neutrosophic open orbit set NO_f(x) containing x \rightarrow f(NO_f(x)) \subseteq F. Since F is a neutrosophic open orbit set in (X, \sigma), F = Nint(F). This implies that, f(NO_f(x)) \subseteq Nint(F). Hence, f is a neutrosophic chaotic** continuous function.

3.28 Remark
The converse of Theorem 3.27 need not be true as shown in the Example 3.29.

3.29 Example
Let X={p,q,r,s}. Then the neutrosophic sets M,N,O,P,Q,R,S and T are defined by M=<x,q,r,\{r\},\{p,r\}>, N=<x,p,q,r,\{p,q\},\{p,r\}>, O=<x,q,p,q,r,\{p,q\}> and T=<x,q,r,\{r\},\{p,r\}>. Let f(x) = <p,q,s> \rightarrow <q,r,p> and f(s) = <p,q,q>. Now the function f is neutrosophic chaotic** continuous but not neutrosophic chaotic continuous. Hence, neutrosophic chaotic** continuous function need not be neutrosophic chaotic continuous function.

3.30 Definition
Let (X, \tau) and (X, \sigma) be any two neutrosophic chaos spaces. A function f : (X, \tau) \rightarrow (X, \sigma) is said to be a neutrosophic chaotic*** continuous if for each periodic point x \in X and each neutrosophic closed chaotic set F of f(x) \exists a neutrosophic clopen orbit set NO_f(x) of the periodic point x \rightarrow f(NO_f(x)) \subseteq F.

3.31 Theorem
A neutrosophic chaotic continuous function is a neutrosophic chaotic*** continuous function.
Proof Since f is a neutrosophic chaotic continuous function, F is a neutrosophic clopen chaotic set containing f(x), \exists a neutrosophic open orbit set NO_f(x) containing x \rightarrow f(NO_f(x)) \subseteq F. This implies that, NO_f(x) \subseteq f^{-1}(F). Then, f^{-1}(F) is a neutrosophic clopen orbit set in (X, \sigma). By (iii) of Theorem 3.32, f^{-1}(F) is a neutrosophic chaotic*** continuous function. Hence, f is a neutrosophic chaotic*** continuous function.

3.32 Remark
The converse of Theorem 3.31 need not be true as shown in the Example 3.33.

3.33 Example
Let X={p,q,r,s}. Then the neutrosophic sets M,N,O,P,Q,R,S and T are defined by M=<x,q,r,\{r\},\{p,r\}>, N=<x,p,q,r,\{p,q\},\{p,r\}>, O=<x,q,p,q,r,\{p,q\}> and T=<x,q,r,\{r\},\{p,r\}>. Let f(x) = <p,q,s> \rightarrow <q,r,p> and f(s) = <p,q,q>. Now the function f is neutrosophic chaotic*** continuous but not neutrosophic chaotic continuous. Hence, neutrosophic chaotic*** continuous function need not be neutrosophic chaotic continuous function.

3.34 Remark
The interrelation among the functions introduced are given clearly in the following diagram.

Figure 1:

4 Properties of neutrosophic chaotic continuous functions

4.1 Definition
A neutrosophic chaos space (X, \tau) is said to be a neutrosophic orbit extremely disconnected space if the function f is a neutrosophic chaotic*** continuous function.
neutrosophic closure of every neutrosophic open orbit set is neutrosophic open orbit.

4.2 Theorem
Let (X, τ) and (X, σ) be any two neutrosophic chaos spaces. If f : (X, τ) → (X, σ) is a neutrosophic chaotic continuous function and (X, τ) is a neutrosophic orbit extremely disconnected space then f is a neutrosophic chaotic* continuous function.

Proof Let x be a periodic point and x ∈ X. Since f is neutrosophic chaotic continuous, F = <x,F₁,F²,F³> is a neutrosophic clopen chaotic set of (X, σ), ∃ a neutrosophic open orbit set NO(x) of (X, τ) containing x ∋ f(NO(x)) ⊆ F. Therefore, NO(x) is a neutrosophic open orbit set NO(x) of (X, τ). Since (X, τ) is neutrosophic orbit extremely disconnected, Ncl(NO(x)) is a neutrosophic open orbit set. Therefore, F is a neutrosophic closed chaotic set containing f(x) ∋ a neutrosophic open orbit set Ncl(NO(x)) ∋ f(Ncl(NO(x))) ⊆ F. Hence, f is neutrosophic chaotic continuous.

4.3 Definition
A neutrosophic chaos space (X, τ) is said to be neutrosophic chaotic 0-dimensional if it has a neutrosophic base consisting of neutrosophic clopen chaotic sets.

4.4 Theorem
Let (X, τ) and (X, σ) be any two neutrosophic chaos spaces. Let f : (X, τ) → (X, σ) be a neutrosophic chaotic*** continuous function. If (X, σ) is neutrosophic chaotic 0-dimensional then f is a neutrosophic chaotic continuous function.

Proof Let the periodic point x ∈ X. Since (X, σ) is neutrosophic chaotic 0-dimensional, ∃ a neutrosophic clopen chaotic set F = <x,F₁,F²,F³> in (X, σ). Since f is a neutrosophic chaotic*** continuous function, ∃ a neutrosophic open orbit set NO(x) of (X, τ) containing x ∋ f(NO(x)) ⊆ F. Since NO(x) is a neutrosophic open orbit set, Nint(NO(x)) = NO(x). This implies that, f(NO(x)) ⊆ F. Therefore, f is neutrosophic chaotic continuous.

4.5 Definition
A neutrosophic chaos space (X, τ) is said to be a neutrosophic orbit connected space if X

4.6 Definition
Let (X, τ) be a neutrosophic chaos space. If a family {NO₁,NO₂,...} of neutrosophic open orbit sets in (X, τ) is neutrosophic chaotic connected, then (X, τ) is called a neutrosophic chaotic connected space.

4.7 Theorem
Let (X, τ) be neutrosophic chaotic disconnected. Let F₁ = <x,F₁¹,F₁²,F₁³> and F₂ = <x,F₂¹,F₂²,F₂³> be a neutrosophic chaotic disconnected sets of (X, τ). Then F₁ ∩ F₂ = φ and F₁ ∪ F₂ = X. Now, X = F₁ ∩ Y₃ = F₁ ∪ F₂ = X. Since f is neutrosophic chaotic continuous, f⁻¹(F₁) and f⁻¹(F₂) are neutrosophic open orbit sets in (X, τ). Also f⁻¹(F₁) ∩ f⁻¹(F₂) = φ. Therefore, (X, τ) is not neutrosophic orbit connected. Which is a contradiction. Hence, (X, τ) is neutrosophic chaotic connected.

4.8 Theorem
Let (X, τ) and (X, σ) be any two neutrosophic chaos spaces. If f : (X, τ) → (X, σ) is a neutrosophic chaotic continuous function and NO(x) is neutrosophic open orbit set then the restriction f(NO(x)) : NO(x) → (X, σ) is neutrosophic chaotic continuous.

Proof Let F = <x,F¹,F²,F³> be a neutrosophic clopen chaotic set in (X, σ). Then, f(NO(x)) = f⁻¹(F) ∩ NO(x). Since f is neutrosophic chaotic continuous, f⁻¹(F) is neutrosophic open orbit in (X, τ) and NO(x) is a neutrosophic open orbit set. This implies that, f⁻¹(F) ∩ NO(x) is a neutrosophic open orbit set. Therefore, (f(NO(x)))⁻¹(F) is neutrosophic open orbit in (X, τ). Hence, f(NO(x)) is neutrosophic chaotic continuous.

4.9 Definition
Let (X, τ) be a neutrosophic chaotic space. If a family {NO(x) : i ∈ J} of neutrosophic open orbit set in (X, τ) satisfies the condition U NO(x) = X, then it is called a neutrosophic open orbit cover of (X, τ).

4.10 Theorem
Let [NO(x) : y ∈ Γ] be any neutrosophic open orbit cover of a neutrosophic chaos space (X, τ). A function f : (X, τ) → (X, σ) is a neutrosophic chaotic continuous function if and only if the restriction f(NO(x)) : NO(x) → (X, σ) is neutrosophic chaotic continuous for each y ∈ Γ.

Proof Let γ be an arbitrarily fixed index and NO(x) be a neutrosophic open orbit set of (X, τ). Let the periodic point x ∈ NO(x), and F = <x,F¹,F²,F³> is neutrosophic clopen chaotic set containing f(NO(x))(x) = f(x). Since f is neutrosophic chaotic continuous there exists a neutrosophic open orbit set NO(x) containing x such that
4.11 Theorem
If a function \( f : (X, \tau) \rightarrow (X, \tau) \) is neutrosophic chaotic continuous then \( P_\lambda f : (X, \tau) \rightarrow (X, \sigma) \) is neutrosophic chaotic continuous for each \( \lambda \in \Lambda \), where \( P_\lambda \) is the projection of \( (X, \sigma) \) onto \( (X, \tau) \).

Proof Let \( F = <x, F_1^1, F_1^2, F_1^2> \) be any neutrosophic chaotic continuous function. Then \( P_\lambda^{-1}(F) \) is a neutrosophic chaotic continuous function in \( (X, \tau) \), and hence \( P_\lambda^{-1}(F) = f^{-1}(F) \) is a neutrosophic open orbit set in \( (X, \tau) \). Therefore, \( P_\lambda \circ f \) is neutrosophic chaotic continuous.

4.12 Theorem
If a function \( f : (X, \tau) \rightarrow (X, \tau) \) is neutrosophic chaotic continuous then \( f_{\alpha} = f : (X, \tau) \rightarrow (X, \sigma) \) is a neutrosophic chaotic continuous function for each \( \alpha \in \Lambda \).

Proof Let \( F = <x, F_1^1, F_1^2, F_1^2> \) be any neutrosophic chaotic continuous function in \( (X, \tau) \). Then \( f_{\alpha}^{-1}(F) \) is a neutrosophic chaotic continuous function in \( (X, \sigma) \).

4.13 Definition
Let \( (X, \tau) \) and \( (X, \sigma) \) be any two neutrosophic chaos spaces. A function \( f : (X, \tau) \rightarrow (X, \sigma) \) is said to be neutrosophic chaotic irresolute if for each neutrosophic chaotic irresolute set \( F = <x, F_1^1, F_1^2, F_1^2> \) in \( (X, \sigma) \), \( f^{-1}(A) \) is a neutrosophic chaotic irresolute set in \( (X, \tau) \).

4.14 Theorem
Let \( (X, \tau) \) and \( (X, \sigma) \) be any two neutrosophic chaos spaces. If \( f : (X, \tau) \rightarrow (X, \sigma) \) is a neutrosophic chaotic continuous function and \( g : (X, \sigma) \rightarrow (X, \gamma) \) is a neutrosophic chaotic irresolute function, then \( g \circ f : (X, \tau) \rightarrow (X, \gamma) \) is neutrosophic chaotic continuous.

Proof Let \( F = <x, F_1^1, F_1^2, F_1^2> \) be a neutrosophic chaotic continuous set in \( (X, \sigma) \). Since \( g \) is neutrosophic chaotic irresolute, \( g^{-1}(F) \) is a neutrosophic chaotic continuous set in \( (X, \sigma) \). Since \( f \) is neutrosophic chaotic continuous, \( f^{-1}(g^{-1}(F)) \) is a neutrosophic chaotic irresolute set in \( (X, \tau) \).

4.15 Definition
Let \( (X, \tau) \) and \( (X, \sigma) \) be any two neutrosophic chaos spaces. A function \( f : (X, \tau) \rightarrow (X, \sigma) \) is said to be neutrosophic orbit irresolute if for each neutrosophic orbit \( \text{orbit set} \ NO_\nu(x) \in (X, \sigma) \), \( f^{-1}(NO_\nu(x)) \) is a neutrosophic open orbit set in \( (X, \tau) \).

4.16 Definition
Let \( (X, \tau) \) and \( (X, \sigma) \) be any two neutrosophic chaos spaces. Let \( f : (X, \tau) \rightarrow (X, \sigma) \) be a function. Then \( f \) is said to be a neutrosophic open orbit function if the image of every neutrosophic open orbit set in \( (X, \tau) \) is neutrosophic open orbit in \( (X, \sigma) \).

4.17 Theorem
Let \( f : (X, \tau) \rightarrow (X, \sigma) \) be neutrosophic orbit irresolute, surjective and neutrosophic open orbit function. Then \( g \circ f : (X, \tau) \rightarrow (X, \nu) \) is neutrosophic chaotic continuous iff \( g : (X, \sigma) \rightarrow (X, \nu) \) is neutrosophic chaotic continuous.

Proof Let \( F = <x, F_1^1, F_1^2, F_1^2> \) be a neutrosophic chaotic continuous function in \( (X, \sigma) \). Since \( g \) is neutrosophic chaotic continuous, \( g^{-1}(F) \) is a neutrosophic chaotic continuous set in \( (X, \sigma) \). Since \( f \) is neutrosophic orbit irresolute, \( f^{-1}(g^{-1}(F)) \) is a neutrosophic orbit irresolute set in \( (X, \tau) \). Hence \( g \circ f \) is neutrosophic chaotic continuous.

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Conclusion

In this paper, characterization of neutrosophic chaotic continuous functions are studied. Some interrelations are discussed with suitable examples. Also, neutrosophic orbit, extremally disconnected spaces and neutrosophic chaotic zero-dimensional spaces has been discussed with some interesting properties. This paper paves way in future to introduce and study the notions of neutrosophic orbit Co-kernal spaces, neutrosophic hardly open orbit spaces, neutrosophic orbit quasi regular spaces and neutrosophic orbit strongly complete spaces, neutrosophic orbit Co-kernal function, neutrosophic hardly open orbit function for which the above discussed set form the basis.

References


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