Neutrosophic modifications of Simplified TOPSIS for Imperfect Information (nS-TOPSIS)

Azeddine Elhassouny¹, Florentin Smarandache ²

¹Rabat IT Center, ENSIAS, Mohammed V University In Rabat, Rabat, Morocco.
E-mail: azeddine.elhassouny@um5.ac.ma

²University of New, 705 Gurley Ave., Gallup, New Mexico 87301, USA.
E-mail: smarand@unm.edu

Abstract: Technique for order performance by similarity to ideal solution (TOPSIS) is a Multi-Criteria Decision-Making method (MCDM), that consists on handling real complex problems of decision-making. However, real MCDM problems are often involves imperfect information such as uncertainty and inconsistency. The imperfect information is often manipulated through Neutrosophics theory, using certain degree of truth (T), falsity degree (F) and indeterminacy degree(I). and thus single-valued neutrosophic set (SVNs) had prodded a strong capacity to model such complex information. To overcome that kind of problems, In this paper, first, the authors simplify the popular TOPSIS method to a lite TOPSIS (S-TOPSIS), that gives the same result as standard version. Second, mapping S-TOPSIS to Neutrosophics Environment, investigating SVNS, called nS-TOPSIS, to deal with imperfect information in the real decision-making problems. Numerical examples show the contributions of proposed S-TOPSIS method to get the same results with standard TOPSIS with simple way of calculus, and how Neutrosophic environment manage the uncertain information using SVN.

Keywords: Technique for order performance by similarity to ideal solution (TOPSIS), MCDM, Single-Valued Neutrosophic set (SVNs), Neutrosophic Simplified TOPSIS (nS-TOPSIS).

1 Introduction

Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is a popular Multicriteria Decision Making (MCDM). TOPSIS was first introduced by Hwang and Yoon ([1]) to deal with structuring Multicriteria issues with crisp numerical values in real situation. However, real MCDM problems are often formulated under as set of indeterminate or inconsistent information. Thus, TOPSIS consists on many complicate steps of calculation. To deal with thoses problems, First, we introduce a lite version of TOPSIS method (S-TOPSIS) with guaranty of obtention of the same results simplifying many complicated steps of calculation. Thus, we introduce single valued neutrosophic set (SVNs) modifications of Simplified TOPSIS (nS-TOPSIS).

To manage information outcome from real problem, that are usually endowed with imperfection such as uncertainty, fuzziness and inconsistency, Smarandache ([2,3]) initiated a new notion, which is a generalization of the Intuitionistic Fuzzy Set (IFS), called Neutrosophics Set (NS), which based on three values ( truth (T), indeterminacy (I), and falsity (F) membership degrees). The main propriety of NS is that the sum

A.Elhassouny, F. Smarandache, Neutrosophic modifications of Simplified TOPSIS for Imperfect Information (nS-TOPSIS).
of three values is 3 instead of 1 in the case of IFS. Although, the NS as introduced by Smarandache was a philosophical concept, unable to be used in real study cases. Many researchers are working on to produce mathematical property, theories, Arithmetic Operations, etc. On the one hand, Wang and al. ([5]) embodied Neutrosophic concept in a metric, called single-valued neutrosophic set (SVNs) as three values in one (truth−membership degree, indeterminacy−membership degree, and falsity−membership degree). In addition, Broumi and al. ([4, 6, 7]) defined, in Neutrosophic space, similarity measure and distances metric between SVNS values. the defined SVNS show strong power to modelize imperfect information, such as uncertainty, imprecise, incomplete, and inconsistent information.

On the other hand, Other researchers are working on deploying Neutrosophic in MCDM field. Biswas ([8]) proposed extended TOPSIS Method to deal with real MCDM problems based on weighted Neutrosophic and aggregated SVNS operators

Ye [9, 10] introduced two concepts, single valued neutrosophic cross-entropy of single valued neutrosophic and weighted correlation coefficient of SVNS into multicriteria decision-making problems. Deli et al. [11] studied deploying Bipolar Neutrosophic Sets in Multi-Criteria Decision Making field

The remainder of the paper presents the preliminaries to build our Method, TOPSIS method and single valued neutrosophic set (SVNs). next Simplified-TOPSIS as first contribution was introduced. Then, hybrid methods Neutrosophic-TOPSIS and Neutrosophic-Simplified-TOPSIS are proposed to deal with real example. Results and discussions are presented at the end of this paper.

2 TOPSIS method

Consider a multi-attribute decision making problem that could be formulated as follow, \( A = \{ A_1, A_2, \ldots, A_n \} \) a set of \( m \) preferences, and \( C = \{ C_1, C_2, \ldots, C_n \} \) a set of \( n \) criteria. The relationships between preferences \( A_i \) and criteria \( C_j \) quantified by rating \( a_{ij} \) provided by decision maker. Weight vector \( W \) is a set of weights \( \omega_i \) associated to criteria \( C_j \). The all details described above could be reshaped on decision matrix below, denoted by \( D \).

\[
D = (a_{ij})_{m \times n} = \begin{pmatrix}
a_{11} & \cdots & a_{1m} \\
\vdots & \ddots & \vdots \\
a_{n1} & \cdots & a_{nm}
\end{pmatrix} \quad \text{(Decision Matrix)} \quad \text{(2.1)}
\]

Technique for order performance by similarity to ideal solution (TOPSIS) method summarized as follow:

**Step 1:** Calculate normalized form of decision matrix \( r_{ij} \) dividing each element \( a_{ij} \) on the sum of whole column.

\[
r_{ij} = a_{ij} / \left( \sum_{i=1}^{m} a_{ij}^2 \right)^{0.5}; \quad j = 1, 2, \ldots, n; \quad i = 1, 2 \ldots, m
\]

**Step 2:** Calculate also weighted form \( v_{ij} \) of matrix \( r_{ij} \) obtained from previous step, multiplying each element \( r_{ij} \) by its associated weight \( w_j \).

\[
v_{ij} = w_j r_{ij}; \quad j = 1, 2, \ldots, n; \quad i = 1, 2 \ldots, m
\]

**Step 3:** Based on the weighted decision matrix, we calculate positive ideal solution (POS) and negative ideal solution (NIS).
\[ A^+ = (v^+_1, v^+_2, \ldots, v^+_n) = \left\{ \begin{array}{l} \max_i \{v_{ij} | j \in B\} , \\ \min_i \{v_{ij} | j \in C\} \end{array} \right\} \]  \hspace{1cm} (2.4)

\[ A^- = (v^-_1, v^-_2, \ldots, v^-_n) = \left\{ \begin{array}{l} \min_i \{v_{ij} | j \in B\} , \\ \max_i \{v_{ij} | j \in C\} \end{array} \right\} \]  \hspace{1cm} (2.5)

\( B \) quantifies the benefit set, and \( C \) is the cost attribute set. \textbf{Step 4:} By subtracting each weighted element \( v_{ij} \) from POS and NIS, we get two vectors of separation measures cited below.

\[ S^+_i = \left\{ \sum_{j=1}^{n} (v_{ij} - v^+_j)^2 \right\}^{0.5} ; i = 1, 2 \cdots , m \]  \hspace{1cm} (2.6)

\[ S^-_i = \left\{ \sum_{j=1}^{n} (v_{ij} - v^-_j)^2 \right\}^{0.5} ; i = 1, 2 \cdots , m \]  \hspace{1cm} (2.7)

\textbf{Step 5:} Using the both measures calculated in the previous step, we calculate the rating metric.

\[ T_i = \frac{S^-_i}{(S^+_i + S^-_i)}; i = 1, 2 \cdots , m \]  \hspace{1cm} (2.8)

Once we calculate \( T_i \) that will be used to rank set of alternatives \( A_i \).

\section{2.1 Numerical example}

Let consider the numerical example summarized by table Table-1. below, that contains alternatives with respect of criteria weights.

<table>
<thead>
<tr>
<th>( a_{ij} )</th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_i )</td>
<td>12/16</td>
<td>3/16</td>
<td>1/16</td>
</tr>
<tr>
<td>( A_1 )</td>
<td>7</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>8</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>9</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>( A_4 )</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 1: Decision Matrix.

Table Table-2. is result of application of this formula \( \sum_{i=1}^{n} a_{ij} \) on each column.

To determine Normalized matrix \( r_{ij} \) Table-3. each value is divide by \( (\sum_{i=1}^{n} a_{ij}^2)^{1/2} \):

Weighted Decision matrix \( v_{ij} \) Table-4. is the multiplication of each column by \( w_j \).

The table Table-5. below figure out the solution of the above MCDM problem listing furthermore, final rankings for decision matrix, separation metric from POS and NIS.

Preferences, in descending preference order, are ranked as \( A_3 > A_1 > A_4 > A_2 \) as showed in Table-5.
### 3 Simplified-TOPSIS method (our proposed method)

The Simplified-TOPSIS algorithmic consists on steps below:

**Step 1:** Structure the criteria of the decision-making problem under a hierarchy.

Let consider $C = \{C_1, C_2, \cdots, C_n\}$ is a set of Criteria, with $n \geq 2$, $A = \{A_1, A_2, \cdots, A_n\}$ is the set of Preferences (Alternatives), with $m \geq 1$, $a_{ij}$ the score of preference $i$ with respect to criterion $j$, and let $\omega_i$ weight of criteria $C_i$.

$$D = (a_{ij})_{m \times n} = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} \quad \text{(Decision Matrix)} \quad (3.1)$$

**Step 2:** Calculation of the Weighted Decision Matrix $v_{ij}$.

Let $v_{ij}$ Weighted Decision Matrix (WDM) that is obtained by multiplication of each column by its weight.

$$v_{ij} = w_j a_{ij}; \; j = 1, 2, \cdots, n; \; i = 1, 2 \cdots, m \quad (3.2)$$

The difference between proposed method and standard TOPSIS section 2), the normalized step is ignored and WDM $v_{ij}$ is calculated directly without normalization by multiplying $a_{ij}$ with $w_j$.

**Step 3:** Determination of LIS and SIS.

The maximum (largest) ideal solution (LIS), as its name indicate, is the the set of maximums rows and smallest ideal solution (SIS) is the set of minimums rows.

$$A^+ = (v_{11}^+, v_{21}^+, \cdots, v_{n1}^+) = (\max_i \{v_{ij} | j = 1, 2, \cdots, n\}) \quad (3.3)$$
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
$v_{ij}$ & $C_1$ & $C_2$ & $C_3$ \\
\hline
$\omega_i$ & 12/16 & 3/16 & 1/16 \\
$A_1$ & 0.3462 & 0.1151 & 0.0340 \\
$A_2$ & 0.3956 & 0.0895 & 0.0303 \\
$A_3$ & 0.4451 & 0.0767 & 0.0303 \\
$A_4$ & 0.2967 & 0.0895 & 0.0303 \\
$v_{\text{max}}$ & 0.4451 & 0.1151 & 0.0340 \\
$v_{\text{min}}$ & 0.2967 & 0.0767 & 0.0303 \\
\hline
\end{tabular}
\caption{Weighted decision matrix.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Alternative & $S_i^+$ & $S_i^-$ & $T_i$ \\
\hline
$A_1$ & 0.0989 & 0.0627 & 0.3880 \\
$A_2$ & 0.0558 & 0.0997 & 0.6412 \\
$A_3$ & 0.0385 & 0.1484 & 0.7938 \\
$A_4$ & 0.1506 & 0.0128 & 0.0783 \\
\hline
\end{tabular}
\caption{Distance measure and ranking coefficient.}
\end{table}

\[ A^+ = (v_1^-, v_2^-, \cdots , v_m^-) = \left( \min_i \{ v_{ij} | j = 1, 2, \cdots , n \} \right) \] (3.4)

**Step 4:** Calculation of positive and negative solutions.

The positive and negative solution are the entropies of orders two of calculated using the formulas below respectively:

\[ S_i^+ = \left\{ \sum_{j=1}^{n} (v_{ij} - v_{ij}^+)^2 \right\}^{0.5} \quad ; \quad i = 1, 2 \cdots , m \] (3.5)

\[ S_i^- = \left\{ \sum_{j=1}^{n} (v_{ij} - v_{ij}^-)^2 \right\}^{0.5} \quad ; \quad i = 1, 2 \cdots , m \] (3.6)

Arrange preferences (set of alternatives A) based on value of sums of either alternative solutions ($S_i^+$) or ($S_i^-$). The choice of minimum or maximum depend on nature of problem, if the problem to be minimized or maximized

**Step 5 (optional):** Another step is missed in our Simplified TOPSIS is calculation of ranking measure $T_i$ (relative closeness to the ideal solution), because of many reasons: first preferences can classified according to many aggregated measures calculated before, second, it’s a way of normalization that can be changed by any form of normalization dividing by max, or normalized to $[0, 1]$ range, etc.

\[ T_i = \frac{S_i^-}{S_i^+ + S_i^-} \quad ; \quad i = 1, 2 \cdots , m \] (3.7)

A. Elhassouny, F. Smarandache, Neutrosophic modifications of Simplified TOPSIS for Imperfect Information (nS-TOPSIS).
3.1 Numerical example

In order to check the consistency of our proposed method, the Simplified-TOPSIS method is applied on the same example (Decision Matrix presented in Table-1.) as classical TOPSIS.

<table>
<thead>
<tr>
<th>$a_{ij}$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_i$</td>
<td>12/16</td>
<td>3/16</td>
<td>1/16</td>
</tr>
<tr>
<td>$A_1$</td>
<td>7</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>$A_2$</td>
<td>8</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>$A_3$</td>
<td>9</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>$A_4$</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 6: Decision matrix.

Weighed Decision Matrix is gotten (Table-2.).

<table>
<thead>
<tr>
<th>$\omega_ja_{ij}$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_i$</td>
<td>12/16</td>
<td>3/16</td>
<td>1/16</td>
</tr>
<tr>
<td>$A_1$</td>
<td>84/16</td>
<td>27/16</td>
<td>9/16</td>
</tr>
<tr>
<td>$A_2$</td>
<td>96/16</td>
<td>21/16</td>
<td>8/16</td>
</tr>
<tr>
<td>$A_3$</td>
<td>108/16</td>
<td>18/16</td>
<td>8/16</td>
</tr>
<tr>
<td>$A_4$</td>
<td>72/16</td>
<td>21/16</td>
<td>8/16</td>
</tr>
</tbody>
</table>

Table 7: Weighted decision matrix.

Next, we calculate the positive and negative solutions as follow:

$S_{1+} = |84/16-108/16| + |27/16-27/16| + |9/16-9/16| = 1.5000$

$S_{2+} = |96/16-108/16| + |21/16-27/16| + |8/16-9/16| = 1.1875$

$S_{3+} = |108/16-108/16| + |18/16-27/16| + |8/16-9/16| = 0.6250$

$S_{4+} = |72/16-108/16| + |21/16-27/16| + |8/16-9/16| = 2.6875$

$S_{1-} = |84/16-72/16| + |27/16-18/16| + |9/16-8/16| = 1.3750$

$S_{2-} = |96/16-72/16| + |21/16-18/16| + |8/16-8/16| = 1.6875$

$S_{3-} = |108/16-72/16| + |18/16-18/16| + |8/16-8/16| = 2.2500$

$S_{4-} = |72/16-72/16| + |21/16-18/16| + |8/16-8/16| = 0.1875$

By the end we got both sets of negative and positive solutions ($S_{3-}, S_{2-}, S_{1-}, S_{4-}$) and ($S_{3+}, S_{2+}, S_{1+}, S_{4+}$), before arranging preferences, we need to determine which solutions to use, that decision tacked based on the nature of problem, if we seek to minimize or maximize. The minimization of the solution, such as cost to pay, consists on the solution closer to the negative solution, while he maximization of the solution, such as price to sale, consists on the solution closer to the positive solution.

The optional ranking measure $T_i$ confirm the same result.

$$T_1 = (S_{1-})/(S_{1-} + S_{1+}) = 0.478261$$

$$T_2 = (S_{2-})/(S_{2-} + S_{2+}) = 0.586957$$

$$T_3 = (S_{3-})/(S_{3-} + S_{3+}) = 0.782609$$

$$T_4 = (S_{4-})/(S_{4-} + S_{4+}) = 0.065217$$
The table (Table-8.) figure out all calculus did before

<table>
<thead>
<tr>
<th>Alternative</th>
<th>$S^+_i$</th>
<th>$S^-_i$</th>
<th>$T_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>1.5000</td>
<td>1.3750</td>
<td>0.478261</td>
</tr>
<tr>
<td>$A_2$</td>
<td>1.1875</td>
<td>1.6875</td>
<td>0.586957</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.6250</td>
<td>2.2500</td>
<td>0.782609</td>
</tr>
<tr>
<td>$A_4$</td>
<td>2.6875</td>
<td>0.1875</td>
<td>0.065217</td>
</tr>
</tbody>
</table>

Table 8: Distance measure and ranking coefficient.

By applying Simplified-TOPSIS, we get for $T_3$ (0.782609), $T_2$ (0.586957), $T_1$ (0.478261) and $T_4$ (0.065217), and we got with classical TOPSIS $T_3$ (0.7938), $T_2$ (0.6412), $T_1$ (0.3880) and $T_4$ (0.0783). Hence the order obtained with our approach simplified-TOPSIS is the same of classical TOPSIS: $T_3$, $T_2$, $T_1$ and $T_4$, with little change in values between both approaches.

The both methods our simplified-TOPSIS and Standard TOPSIS produce the same results with the same ranking ($T_3$, $T_2$, $T_1$ and then $T_4$), with a little differences of ranking measures. For example, with Simplified-TOPSIS $T_3$ is 0.782609, and with TOPSIS $T_3$ is 0.7938, the same for all others (Simplified-TOPSIS : $T_2$ (0.586957), $T_1$ (0.478261) and $T_4$ (0.065217) and with TOPSIS : $T_2$ (0.6412), $T_1$ (0.3880) and $T_4$ (0.0783).

## 4 Standard TOPSIS in Neutrosophic [12]

Standard TOPSIS in Neutrosophic procedure can be summarized as follow:

**Step 1:** In order to apply neutrosophic TOPSIS algorithm, crisp number Decision Matrix need to be mapped to single valued neutrosophic environment, then, we got neutrosophic decision matrix

$$D = (d_{ij})_{1 \leq i \leq n, 1 \leq j \leq m} = (T_{ij}, I_{ij}, F_{ij})_{1 \leq i \leq n, 1 \leq j \leq m}$$  \hspace{1cm} (Neutrosophic Decision Matrix) \hspace{1cm} (4.1)

Where $T_{ij}$, $I_{ij}$ and $F_{ij}$ are truth, indeterminacy and falsity membership scores respectively. $i$ refer to preference $A_i$ and $j$ to criterion $C_j$.

And $w = (\omega_1, \omega_2, \cdots, \omega_n)$ with $\omega_i$ a single valued neutrosophic weight of criteria (so $\omega_i = (a, b, c)$).

**Example 1:**

To compare our method Neutrosophic Simplified TOPSIS (nS-TOPSIS : section 5) and standard Neutrosophic TOPSIS proposed by Biswas ([11]), we use Biswas's numerical example.

Let $(DM_1, DM_2, DM_3, DM_4)$ fours decisions makers aims to select an alternative $A_i$ ($A_1, A_2, A_3, A_4$) with respect six criteria ($C_1, C_2, C_3, C_4, C_5, C_6$). The mapped weights of criteria and decision matrix in Neutrosophic environment are presented in tables Table-9. and Table-10. respectively.

<table>
<thead>
<tr>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_i$</td>
<td>(0.755, 0.222, 0.217)</td>
<td>(0.887, 0.113, 0.107)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_i$</td>
<td>(0.692, 0.277, 0.251)</td>
<td>(0.788, 0.200, 0.180)</td>
</tr>
</tbody>
</table>

Table 9: Criteria weights.
Step 2: Weighted decision matrix in neutrosophic is gotten by applying aggregation operator of multiplication i.e. application of generalization of multiplication operator in Neutrosophic space.

\[
D^w = D \otimes W = (d_{ij}^w) \quad 1 \leq i \leq n = (T_{ij}^w, I_{ij}^w, F_{ij}^w) \quad 1 \leq i \leq n, 1 \leq j \leq m
\]  

(4.2)

Table 10: Neutrosophic Decision Matrix.

<table>
<thead>
<tr>
<th></th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>(0.864, 0.136, 0.081)</td>
<td>(0.853, 0.147, 0.902)</td>
<td>(0.800, 0.200, 0.150)</td>
</tr>
<tr>
<td>A₂</td>
<td>(0.667, 0.333, 0.277)</td>
<td>(0.727, 0.273, 0.219)</td>
<td>(0.667, 0.333, 0.277)</td>
</tr>
<tr>
<td>A₃</td>
<td>(0.880, 0.120, 0.067)</td>
<td>(0.887, 0.113, 0.064)</td>
<td>(0.834, 0.166, 0.112)</td>
</tr>
<tr>
<td>A₄</td>
<td>(0.667, 0.333, 0.277)</td>
<td>(0.735, 0.265, 0.195)</td>
<td>(0.768, 0.232, 0.180)</td>
</tr>
</tbody>
</table>

Step 3: Calculate of POS-SVNs (positive ideal solution in SVN) and NIS-SVNs (negative ideal solution in SVNS) measures.

\[
T_{ij}^{w+} = \{ (max_i \{T_{ij}^w | j \in B \}), (min_i \{T_{ij}^w | j \in C \}) \}
\]

(4.3)

\[
Q_{N}^{+} = (d_{1}^{w+}, d_{2}^{w+}, \ldots, d_{n}^{w+})
\]

(4.4)

\[
T_{ij}^{w-} = \{ (max_i \{T_{ij}^w | j \in B \}), (min_i \{T_{ij}^w | j \in C \}) \}
\]

(4.5)

\[
I_{ij}^{w+} = \{ (min_i \{I_{ij}^w | j \in B \}), (max_i \{I_{ij}^w | j \in C \}) \}
\]

(4.6)

\[
F_{ij}^{w+} = \{ (min_i \{F_{ij}^w | j \in B \}), (max_i \{F_{ij}^w | j \in C \}) \}
\]

(4.7)

\[
Q_{N}^{-} = (d_{1}^{w-}, d_{2}^{w-}, \ldots, d_{n}^{w-})
\]

(4.8)

\[
T_{ij}^{w-} = \{ (min_i \{T_{ij}^w | j \in B \}), (max_i \{T_{ij}^w | j \in C \}) \}
\]

(4.9)

\[
I_{ij}^{w-} = \{ (max_i \{I_{ij}^w | j \in B \}), (min_i \{I_{ij}^w | j \in C \}) \}
\]

(4.10)

\[
F_{ij}^{w-} = \{ (max_i \{F_{ij}^w | j \in B \}), (min_i \{F_{ij}^w | j \in C \}) \}
\]

(4.11)

Where B represents the benefit and C quantify the cost.

A.Elhassouny, F. Smarandache, Neutrosophic modifications of Simplified TOPSIS for Imperfect Information (nS-TOPSIS).
Step 4: Calculate length of each alternative from the POS-SVN and NIS-SVN calculated in previous step.

\[
D_{i}^{+} (d_{ij}^{w+}, d_{ij}^{m+}) = \sqrt{\frac{1}{3n} \sum_{j=1}^{n} \left( \frac{(T_{ij}^{w+}(x) - T_{ij}^{m+}(x))^2 + (I_{ij}^{w+}(x) - I_{ij}^{m+}(x))^2 + (F_{ij}^{w+}(x) - F_{ij}^{m+}(x))^2}{3} \right)}
\]

(4.12)

\[
D_{i}^{-} (d_{ij}^{w-}, d_{ij}^{m-}) = \sqrt{\frac{1}{3n} \sum_{j=1}^{n} \left( \frac{(T_{ij}^{w-}(x) - T_{ij}^{m-}(x))^2 + (I_{ij}^{w-}(x) - I_{ij}^{m-}(x))^2 + (F_{ij}^{w-}(x) - F_{ij}^{m-}(x))^2}{3} \right)}
\]

(4.13)

With \( i = 1, 2 \ldots , m \)

Step 5: Calculate the aggregated coefficient of closeness in Neutrosophic.

\[
C_{i}^{*} = \frac{NS_{i}^{-}}{(NS_{i}^{+} + NS_{i}^{-})}; \quad i = 1, 2 \ldots , m
\]

(4.14)

All values of aggregated coefficient of closeness are shown in the table Table-11. below.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>( C_{i}^{*} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{1} )</td>
<td>0.8190</td>
</tr>
<tr>
<td>( A_{2} )</td>
<td>0.1158</td>
</tr>
<tr>
<td>( A_{3} )</td>
<td><strong>0.8605</strong></td>
</tr>
<tr>
<td>( A_{4} )</td>
<td>0.4801</td>
</tr>
</tbody>
</table>

Table 11: Closeness Coefficient.

Using the associate values of aggregated coefficient of closeness \( C_{i}^{*} \) to preference \( A_{i} \), in descending order, to rank alternatives. Hence, preferences could be ordered as follow \( A_{3} > A_{1} > A_{4} > A_{2} \). Then, the alternative \( A_{3} \) is the best solution.

5 Neutrosophic-Simplified-TOPSIS (our proposed method)

Step 1: Construct Neutrosophic decision matrix.

As made for Standard Neutrosophic TOPSIS, let consider neutrosophic decision matrix and SVN weighted criteria.

\[
D = (d_{ij}) \quad 1 \leq i \leq n \quad 1 \leq j \leq m = (T_{ij}, I_{ij}, F_{ij}) \quad 1 \leq i \leq n \quad 1 \leq j \leq m
\]

(5.1)

\[
A_{1} \quad \begin{pmatrix}
C_{1} \\
C_{2} \\
\vdots \\
C_{n}
\end{pmatrix} =
\begin{pmatrix}
d_{11} \\
d_{12} \quad \ldots \quad d_{1n}
\end{pmatrix}
\]

A.

Elhassouny, F. Smarandache, Neutrosophic modifications of Simplified TOPSIS for Imperfect Information (nS-TOPSIS).
Where $T_{ij}$ denote truth, $I_{ij}$ indeterminacy and $N_{ij}$ falsity membership score of preference $i$ knowing $j$ in neutrosophic environment.

$w = (\omega_1, \omega_2, \cdots, \omega_n)$ with $\omega_i$ a single valued neutrosophic weight of criteria (so $\omega_i = (a_i, b_i, c_i)$).

**Step 2:** Calculate SVNs weighted decision matrix.

$$D^w = D \otimes W = (d_{ij}^w) \quad 1 \leq i \leq n \quad 1 \leq j \leq m$$

$$= \omega_j \otimes d_{ij} = (T_{ij}, I_{ij}, F_{ij}) \quad 1 \leq i \leq n \quad 1 \leq j \leq m$$

$$\omega_j \otimes d_{ij} = (a_j T_{ij}, b_j + I_{ij} - b_j I_{ij}, c_j + F_{ij} - c_j F_{ij})$$

**Step 3:** Calculate LNIS and SNIS metrics.

LNIS and SNIS are maximum (larger) and minimum (smaller) neutrosophic ideal solution respectively.

$$A^+_N = (d^+_1, d^+_2, \cdots, d^+_n)$$

$$d^+_j = (T^+_j, I^+_j, F^+_j)$$

$$T^+_j = \{(max_i \{T_{ij} \mid j = 1, \cdots, n\})\}$$

$$I^+_j = \{(min_i \{I_{ij} \mid j = 1, \cdots, n\})\}$$

$$F^+_j = \{(min_i \{F_{ij} \mid j = 1, \cdots, n\})\}$$

$$A^-_N = (d^-_1, d^-_2, \cdots, d^-_n)$$

$$d^-_j = (T^-_j, I^-_j, F^-_j)$$

$$T^-_j = \{(min_i \{T_{ij} \mid j = 1, \cdots, n\})\}$$

$$I^-_j = \{(max_i \{I_{ij} \mid j = 1, \cdots, n\})\}$$

$$F^-_j = \{(max_i \{F_{ij} \mid j = 1, \cdots, n\})\}$$

**Step 4:** Determination of the distance measure of every alternative from the RNPIS and the RNNIS for SVNSs.

To perform that calculus, we need to introduce a new distance measure, in this paper we mapped Manhattan distance ([13]) to Neutrosophic environment (definition 1). The new proposed distance called Neutrosophic Manhattan distance that perform the difference between two single-valued neutrosophic measures.

**Definition 1.** Let $X_1 = (x_1, y_1, z_1)$ and $X_2 = (x_2, y_2, z_2)$ be a SVN numbers. Then the separation measure between $X_1$ and $X_2$ based on Manhattan distance is defined as follows:

$$D_{Manh} (X_1, X_2) = |x_1 - x_2| + |y_1 - y_2| + |z_1 - z_2|$$

A.Elhassouny, F. Smarandache, Neutrosophic modifications of Simplified TOPSIS for Imperfect Information (nS-TOPSIS).
The application of Neutrosophic Manhattan distance to calculate the separation from the maximum and minimum Neutrosophic ideal solution respectively are:

\[
D_{\text{Manh}}^j \left( d_{ij}^w, d_{ij}^{w+} \right) = \left\{ \begin{array}{c}
|T_{ij}^w(x) - T_{ij}^{w+}(x)| + \\
|I_{ij}^w(x) - I_{ij}^{w+}(x)| + \\
|F_{ij}^w(x) - F_{ij}^{w+}(x)|
\end{array} \right. \tag{5.15}
\]

with \( j = 1, 2 \cdots, n \)

\[
NS^+_i = \sum_{j=1}^n D_{\text{Manh}}^j \left( d_{ij}^w, d_{ij}^{w+} \right)
\tag{5.16}
\]

with \( i = 1, 2 \cdots, m \)

Similarly, the separation from the minimum neutrosophic ideal solution is:

\[
D_{\text{Manh}}^j \left( d_{ij}^w, d_{ij}^{w-} \right) = \left\{ \begin{array}{c}
|T_{ij}^w(x) - T_{ij}^{w-}(x)| + \\
|I_{ij}^w(x) - I_{ij}^{w-}(x)| + \\
|F_{ij}^w(x) - F_{ij}^{w-}(x)|
\end{array} \right. \tag{5.17}
\]

with \( j = 1, 2 \cdots, n \)

\[
NS^-_i = \sum_{j=1}^n D_{\text{Manh}}^j \left( d_{ij}^w, d_{ij}^{w-} \right)
\tag{5.18}
\]

with \( i = 1, 2 \cdots, m \)

Preferences are ordered regarding to the values of \( NS^-_i \) or according to \( 1/NS^+_i \). In other words, the alternatives with the highest appraisal score is the best solution.

**Step 5:** Rank the alternatives according to Ranking coefficient \( NT_i \).

Ranking coefficient is formulated as:

\[
NT_i = \frac{NS^-_i}{(NS^+_i + NS^-_i)}; \quad i = 1, 2 \cdots, m \tag{5.19}
\]

A set of alternatives can now be ranked according to the descending order of the value of \( NT_i \)

### 5.1 Numerical example

**Step 1.** Formulate the MCDM problem in neutrosophic by building Neutrosophic decision matrix decision matrix and SVNs weights of criteria.

Let \( A_i (A_1, A_2, A_3, A_4) \) a set of alternative and \( C_i (C_1, C_2, C_3, C_4, C_5, C_6) \) a set of criteria. Let considers the following neutrosophic weights of criteria (Table-12.) and neutrosophic decision matrix (Table-13.) respectively (used in above example 1).

**Step 2:** Calculation of SVNs Weighted Decision Matrix

\[
D^w = (d_{ij}^w) \quad 1 \leq i \leq n, \quad 1 \leq j \leq m = (T_{ij}^w, I_{ij}^w, F_{ij}^w) \quad 1 \leq i \leq n, \quad 1 \leq j \leq m \tag{5.20}
\]

A.Elhassouny, F. Smarandache, Neutrosophic modifications of Simplified TOPSIS for Imperfect Information (nS-TOPSIS).
Step 3: Determination of LNIS and SNIS.

A. Elhassouny, F. Smarandache, Neutrosophic modifications of Simplified TOPSIS for Imperfect Information (nSTOPSIS).

Table 12: Criteria neutrosophic weights.

$$d_{ij} = \left( a_j T_{ij}, b_j + I_{ij} - b_j I_{ij}, c_j + F_{ij} - c_j F_{ij} \right)$$

Table 13: Neutrosophic Decision Matrix.

Table 14: Weighted Neutrosophic decision matrix.

SVNs Weighted Decision Matrix is obtained by multiplication of weights of criteria with its associated column of neutrosophic decision matrix:

$$T_{11}^w = 0.864 \times 0.755 = 0.6523$$

$$I_{11}^w = 0.136 + 0.222 - 0.136 \times 0.222 = 0.328$$

$$F_{11}^w = 0.081 + 0.217 - 0.081 \times 0.217 = 0.280$$
Step 4: Calculation of $NS^+_i$ and $NS^-_i$ To calculate $NS^+_i$ and $NS^-_i$, we calculate sum of each line, and then subtracting from the LNIS and from SNIS respectively.

According to the obtained result (Table-17.), alternatives can be ranked as follow $A_3 > A_1 > A_4 > A_2$. Then the best preference is $A_3$. Using the same example, our proposed method neutrosophic-simplified-TOPSIS (nTOPSIS), we get similar result as neutrosophic-TOPSIS.

6 Conclusion

This paper aims to present two new TOPSIS based approaches for MCDM. First one is Simplified TOPSIS (sTOPSIS) that simplify the TOPSIS calculation procedure. Second one, neutrosophic simplified-TOPSIS (nTOPSIS) extend the proposed method to neutrosophic environment, that use, instead of crisp number, the single valued neutrosophic (SVN). To formulate the both proposed method, many measures are defined such as Neutrosophic Manhattan Distance measure, that is used to calculate, distances from Maximum (larger) Neutrosophic Ideal Solution (LNIS) minimum neutrosophic ideal solutions, as two new defined measures.
References


13 Paul E. Black, ”Manhattan distance”, in Dictionary of Algorithms and Data Structures.

Received: Nov 17, 2018. Accepted: March 13, 2019.