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New Neutrosophic Scale System Framework

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Abstract: Scaling system can be considered as range-base measurement system, it's a fatal tool used in all human activities in daily-bases, also, all business domains and sectors heavily use scaling systems in all business process specially in decisions-making as one of the main critical business activities, despite the fact that, there is no scientific base for calculate an unified scale system ranges, all provided scales or ranges are determined based on expert opinions', an enhanced scale system using single-valued neutrosophic set SVNS is offered that suggest a scientific methods for defining ranges in scaling systems, in addition, a new crisp value functions "De-neutrosophication" for converting both Simplified Neutrosophic Number SNN, and SVNS to them equivalent crisp values using distance measure based on Euclidean space are proposed, Finally, the offered framework and methods are implemented with numerical examples for best prove and validate of the framework and proposed methods.

Keywords: Neutrosophic; De-neutrosophic; Single-valued Neutrosophic Set SVNS; Scale system, Scoring System; Decisions-making

1. Introduction

Smarandache presented Neutrosophic Logic as a generalization of fuzzy logic considering Neutrosophic Set NS is a generalization of the intuitionistic set, classical set, and fuzzy set, where Neutrosophic uses every entity $< X >$ and its opposite or negation $< antiX >$ together with their neutralities $< neutX >$ in between them, therefore, the $< neutX >$ & $< antiX >$ together will considered as $< nonX >$, in neutrosophic logic a proposition has a degrees of truth (T), indeterminacy (I), falsity (F), where $(T), (I), (F)$ are standard or non-standard subsets of $]^{-}0,1[^{+}$ [1].

The Neutrosophic logic best fit in decision-making where its process mostly has a lot of vagueness, indeterminacies which is the typical case in real life decision-making process, therefore, using neutrosophic in decision-making activities provides decision-makers with a great flexibility to deal with indeterminacy and uncertainty, in addition, neutrosophic logic and its subfields has a lot of scientific implementations in numerous fields using the three neutrosophic logic's membership degrees (T) truth, (I) indeterminacy $and (F)$ falsity degree to express any system inputs' values in detailed way specially when the system inputs' values characterized with indeterminacy and uncertainty.

Measurement systems is a method of defining a measurement unit for best unify the scales, scaling systems is range-base measurement system, it's a critical tool used to classify measured items into ranges of values, each range has an equivalent qualitative values "Linguistic terms", though, there is no standard way for defining the ranges as ranges are determined based on expert opinions' such as, National Institute of Standards and Technology NIST [2], when performing risk assessments they uses five level scale, first level starting from 0% to 4% and name it "very low", the second level started from 5% to 20% and name it "low", the third level 21% to 79% as "moderate", forth from 80% to 95% as "high", and lastly from 96% to 100% as "very high", while NIST uses different ranges in "Common Vulnerability Scoring System" [3] which firstly uses 10-base scale instead of 100-base scale, also uses different ranges, it was "very low" name it as "none" 0 %, "low" 1-39%, "moderate" 40-69%, "high" 70-89%, and lastly "very high" name it as "Critical" 90-100%, which clearly presenting same scale levels with different ranges, This research paper offers a scientific methods for defining ranges in scaling systems.

Many efforts done for calculate de-neutrosophication for SVNS using Entropy, cross-entropy, distance, similarity, score and accuracy functions which are very important in uncertainty environment while ranking neutrosophic sets and numbers, since entropy is typically developed to determining uncertain degree of information. Distance, similarity, score, accuracy and cross-entropy are mostly applied to calculate the level of similarity among two elements. The importance of these functions manifested of comparing or converting neutrosophic numbers and sets into a comparable crisp value, these functions are completely calculated based on the value of truth, falsity, and indeterminacy memberships [4].

Researchers made an attempt to present a neutrosophic 3D visualization for both SNN and SVNS using Euclidean space, in addition, new crisp value functions "De-neutrosophication" for converting both Simplified Neutrosophic Number SNN, and SVNS to them equivalent crisp values using similarity measure based on Euclidean distance are proposed, also the researchers propose a new Neutrosophic Scaling System algorithm, Finally, the proposed Neutrosophic Scaling System is applied to risk assessment case study.

The remining sections in this paper organized as follows: section two, represent a literature review about scaling system and some neutrosophic concepts used in the paper; Section three, contains some neutrosophic basic definitions are outlined; a proposed neutrosophic scaling system algorithm presented and two illustrative numerical examples are presented in section four; section five contains a conclusion followed by references.

2. Literature review

An overview of neutrosophic logic, Simplified Neutrosophic Number SNN, Single-Valued Neutrosophic Set SVNS, are discussed, in addition to evaluate some de-neutrosophication methods such as distance and similarity, also, concept of scale system is discussed.

Smarandache extend Neutrosophic logic as a branch of philosophy [5] that reviews the basis and scope of neutrality, neutrosophic was discussed by a lot of researchers and applied in a variety of businesses assisting in solving many challenges as a powerful scale in the selection [6], Multi-criteria decision making MCDM [7] [8] [9], achieving PERT in project management [10], exploring the influence of Internet of Things (IoT) and how IOT influence supply chain [11], a lot of studies propose an enhanced variety of aggregation operators [12]. Wen, et al, (2017) [13] offered a novel method to calculate the similarity between SVNSs, plus Jun and Shigui (2017) [14] offer distances, similarity and entropy methods for IVNS, Surapati and Kalyan (2015) [15] explain a rough cosine similarity calculation among two rough NS., said and Florentin (2014) [16] offer a novel cosine similarity among two IVNS based on Bhattacharya's distance, Ye (2014) [17] suggest a few of aggregation operators, as well as a simplified neutrosophic weighted arithmetic average operator and a simplified neutrosophic weighted geometric average operator.

National Institute of Standards and Technology used five level Risk Assessment Scale in its special publication 800-30 "Guide for Conducting Risk Assessments" as standard scale where the percentages from 0% up to 4% refers to the linguistic scale of "Very Low" or lowest scale level, on the other hand they used the percentages from 96% up to 100% to refer to linguistic scale of "Very High" or highest scale level, all five levels of the qualitative risk scale values and its equivalent percentage ranges as proposed by NIST, nevertheless, NIST didn't explain the scientific base for selecting this specific ranges for each Qualitative Values [2]

3. Preliminaries

In this section, the basic definitions related to NS, SVNS, absolute and empty NS, Simplified Neutrosophic Set SNS, SNN and them operations are outlined, in addition de-neutrosophication, score functions, similarity functions, and distance functions are evaluated and enhanced.

Definition 3.1. Neutrosophic Set:

Florentin Smarandache 1998 proposed neutrosophic logic and neutrosophic sets and coined the definition of "Neutrosophic Set" with three principles (membership, indeterminacy, and non-membership) [18], [7] Let $T_A(x), T_A(x), and\ F_A(x)$ be real standard or non-standard Statically subsets (sub) of]-0, 1+[, Let X is a universe of discourse, and M a set included in X, and x is an element from X is described with respect to the set A as $x(T_A(x), T_A(x), F_A(x))$ and belongs to A where x is $(t\%\ true)$ in the set, $(i\%\ indeterminate)$ or undefined in the set, and $(f\%\ false)$, considering that (t) changes in $T_A(x): X \to\ ]^-0, 1^+[$, (i) changes in $I_A(x): X \to\ ]^-0, 1^+[$, (f) changes in $F_A(x): X \to\ ]^-0, 1^+[$, without restriction in the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, and meets the condition of summation: $(^-0 \le \mathrm{supT_A(x)} + \mathrm{supI_A(x)} + \mathrm{supF_A(x)} \le 3^+)$

$$NS(A) = \{\langle x,\ T_A(x),\ I_A(x),\ F_A(x)\rangle | x \in X,\ T_A(x), I_A(x),\ F_A(x) \in\ ]^-0, 1^+[\ \} \tag{1}$$

Definition 3.2. Single-Valued Neutrosophic set (SVNS)

Wang et al. [19], presented "Single Valued Neutrosophic Set" (SVNS), as a subclass of the NS. which defined in Definition 3.1 and Simplified Neutrosophic Set SNS which defined in Definition 3.4 below, in consequence of that, SVNS is an instance of NS that can implemented in our life applications [20], [21], Let X be a universe of discourse, a SVNS A over X is an object with the form of $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\}$, for the intervals $T_A(x)$, $I_A(x)$ and $F_A(x)$ refer to truth, indeterminacy, and falsity memberships degrees respectively of x to A, also, $T_A(x) \in [1,0]$, $I_A(x) \in [1,0]$ and $F_A(x) \in [1,0]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ for all $x \in X$, for X is discrete, a $SVNS\ A$ will stated as shown in formula (2), while X is continuous, a $SVNS\ A$ will stated as shown in formula (3).

$$\mathrm{SVNS}\ (A) = \sum_{i=1}^{n} \frac{\langle T_A(x_i), I_A(x_i), F_A(x_i)\rangle}{x_i}\ |\ x_i \in X \tag{2}$$

$$SVNS\ (A) = \int_X \frac{\langle T_A(x), I_A(x), F_A(x)\rangle}{x}\ |\ x \in \mathrm{X} \tag{3}$$

Definition 3.3. Absolute and Empty Neutrosophic Set

Gayyar (2016) [22] defined two special cases for neutrosophic set which are the Null (Empty) neutrosophic set (0_N) and the absolute (universe) neutrosophic set (1_N) , where Empty Neutrosophic Set has two forms $(0_N) =< x, 0,0,1 >\ x \in X$ and $(0_N) =< x, 0,1,1 >\ x \in X$, also the absolute neutrosophic set has two forms $(1_N) =< x, 1,1,0 >\ x \in X$, and $(1_N) =< x, 1,0,0 >\ x \in X$, which is not accepted where $< x, 0,0,1 >$ is not equal to $< x, 0,1,1 >$ and $< x, 0,1,1 >$ is not empty, on the other hand the $< x, 1,1,0 >$ is not equal to $< x, 1,0,0 >$ and $< x, 1,1,0 >$ is not universal set, Therefore, we propose that, "Empty Simplified Neutrosophic Number" can denoted by one form as shown in formula (4), and , "Absolute Simplified Neutrosophic Number" can denoted by one form as shown in formula (5) only.

$$0_N =< 0,0,1 >\ x \in X \tag{4}$$

$$1_N =< 1,0,0 >\ x \in X \tag{5}$$

Definition 3.4. Simplified Neutrosophic Set (SNS):

Ye, (2014) [17], SNS is an special case of NS, where the functions $T_A(x)$, $I_A(x), and\ F_A(x)$ represented as single points in the real standard [0,1] instead of subintervals / subsets in the real standard [0,1], that is $T_A(x) \in [1,0]$, $I_A(x) \in [1,0]$, and $F_A(x) \in [1,0]$. Therefore, $SNS\ A$ is represented by formula (6), with no limitation on the sum of $T_A(x), I_A(x),\ and\ F_A(x)$, satisfies the condition of: $0 \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3$.

$$SNS\ (A) = \{\langle x,\ T_A(x), I_A(x),\ F_A(x)\rangle | x \in X,\ T_A(x), I_A(x),\ F_A(x) \in]0,1[\ \} \quad (6)$$

Definition 3.5. Simplified Neutrosophic Number (SNN)

Considering SNS is a subclass of NS, Ye, (2014) [17] offer Simplified Neutrosophic Number (SNN) as a special case of SNS, in specific when X consist of one object of A, where $A = \{\langle\ T_A(x), I_A(x), F_A(x)\rangle | x \in X\}$ it named as SNN, for ease, SNN is presented as shown in formula (7),.

$$\text{SNN}\ (A) = \langle T_A, I_A, F_A\rangle \quad (7)$$

Definition 3.6. Cosine Similarity

Ye, (2014) [17], proposed a method to compare any SVNS with absolute SVNS built on the cosine similarity measure as shown in formula (8), that can be extended to SNN $x = (T, I, F)$ considering the absolute SNN = $(1,0,0)$ as defined in formula (5),

$$COS(x) = \frac{T_x}{\sqrt{{T_x}^2 + {I_x}^2 + {F_x}^2}} \quad (8)$$

However, in some cases the formula (8) didn't represent the correct similarity for example: for $A = (0.1{,}0.1{,}0.1)$, $B = (0.9{,}0.9{,}0.9)$ and $K = (k, k, k)|1 \geq k > 0]$ where the three memberships has the same value then $COS(A) = COS(B) = COS(K) = 0.577350269$ using formula (8), also when falsity membership and indeterminacy membership are equal to zero formula (8) returns the similarity value of 1 regardless truth membership value, for $A = (0.1{,}0{,}0)$, $B = (0.9{,}0{,}0)$ and $K = (z, k, k)|z \in [0,1], k = 0]$, then $COS(A) = COS(B) = COS(K) = 1$, which is not accepted.

Definition 3.7. Kanika's similarity measure

Kanika, (2020) [23] propose a similarity measure $S_1(A, B)$ for SVNS, for $A = \langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle$, $B = \langle x_i, T_B(x_i), I_B(x_i), F_B(x_i)\rangle$ where $T_A(x_i)$, $I_A(x_i)$, $F_A(x_i)$, $T_B(x_i)$, $I_B(x_i)$, $F_B(x_i) \in [0,1]$, $x_i\ (i = 1, 2, \dots, n)$ as shown in formula (9).

$$S_1(A, B) = 1 - \frac{1}{2n} \times \sum_{i}^{n} [|T_A(x_i) - T_B(x_i)| + Max\{|I_A(x_i) - I_B(x_i)|, |F_A(x_i) - F_B(x_i)|\ \}] \quad (9)$$

However, in some cases the formula (9) didn't return correct similarity value, for example: when using formula (9) to calculating the similarity between absolute SVNS $1_N = (1{,}0{,}0)$ and both $A = (0.5{,}0{,}0.2)$, $B = (0.4{,}0{,}0.1)$, $S_1(1_N, A) = S_1(1_N, B)$ =0.65, also for $A = (0.5{,}0.2{,}0.6)$, $B = (0.2{,}0.2{,}0.3)$, $S_1(1_N, A) = S_1(1_N, B) = 0.45$ which is not accepted.

Definition 3.8. Score Function

Nancy, et al (2016) [24] propose a score function $S_2(1_N, A)$ shown in formula (10), as an enhancement for $S_3(1_N, A)$ shown in formula (11) proposed by Şahin, (2014) [25], for $A = \langle x_i,\ T_A(x_i), I_A(x_i), F_A(x_i)\rangle$ where $T_A(x_i)$, $I_A(x_i)$, $F_A(x_i) \in [0,1]$, $x_i\ (i = 1, 2, \dots, n)$, in case $T_A(x_i) + F_A(x_i) = 1$, Nancy, et al propose to use $S_3(1_N, A)$ shown in formula (11).

$$S_2(1_N, A) = \frac{1 + (T_A(x_i) - 2I_A(x_i) - F_A(x_i))(2 - T_A(x_i) - F_A(x_i))}{2} \quad (10)$$

$$S_3(1_N, A) = \frac{1 + T_A(x_i) - 2I_A(x_i) - F_A(x_i)}{2} \quad (11)$$

On the other hand, Both formulas (10) and (11) have some limitation in some cases for example: for $A = (0.4,0.9,0.5)$ both formulas return a negative similarity $= -0.545, -0.45$ respectively, in case of $T_A(x_i) + F_A(x_i) = 1, T_A(x_i) = 0, F_A(x_i) = 1$ both formulas return a negative similarity also, in addition, for $A = (0.4,0.4,0.4)$, formulas return $0.02, 0.1$, also for $A = (0.9,0.9,0.9)$ the formulas return $0.32, -0.4$ respectively, which are not accepted.

Definition 3.9. Euclidean-base similarity

Majumdar and Samanta (2014) [26], offer SVNS similarity formula for $A = \langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle$, $B = \langle x_i, T_B(x_i), I_B(x_i), F_B(x_i)\rangle$ where $T_A(x_i)$, $I_A(x_i)$, $F_A(x_i)$, $T_B(x_i)$, $I_B(x_i)$, $F_B(x_i) \in [0,1]$, x_i $(i = 1, 2, \dots, n)$ as shown in formula (12).

$$S_4(A, B) = 1 - \tfrac{1}{3}(|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|) \quad (12)$$

Formula (12), has some drawbacks, such as for two SVNS $A = (0.5,0.2,0.6)$, $B = (0.2,0.2,0.3)$ which are two different SVNS but $S_4(1_N, A) = S_4(1_N, B) = 0.566666667$, which is not accepted for totally different SVNS, also for $A = (0.1,0,0)$, then $S_4(1_N, A) = 0.7$ which is not sound logical similarity value.

Ye, (2014) [27], extend the Euclidean distance measure by adding a weight for his method when measuring distance and similarity between SVNSs, for $A\ and\ B$, two SVNSs giving $SVNS\ (A) = \langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle$, $SVNS\ (B) = \langle x_i, T_B(x_i), I_B(x_i), F_B(x_i)\rangle$ where $T_A(x_i), I_A(x_i), F_A(x_i)$, $T_B(x_i)$, $I_B(x_i)$, $F_B(x_i) \in [0,1]$, consider the weight w_i $(i = 1, 2, \dots, n)$ of an object for x_i $(i = 1, 2, \dots, n)$, for $w_i \geq 0$ $(i = 1, 2, \dots, n)$ and $\sum_i^n w_i = 1$, single-valued neutrosophic weighted distance measure between $A, and\ B$ defined as shown in formula (13), which considered as a generic formula for calculating the distance using both Hamming and Euclidean distance methods, where, $p = 1$ in case of using Hamming distance and $p = 2$ in case of using Euclidean distance, also Ye, (2014) prove the relation distance and similarity are complementary where similarity $S_1(A, B) = 1 - d_p(A, B)$ and vice versa as shown in formula (14) [27].

$$d_p(A, B) = \sqrt[p]{\frac{\sum_i^n w_i([T_A(x_i) - T_B(x_i)]^P + [I_A(x_i) - I_B(x_i)]^P + [F_A(x_i) - F_B(x_i)]^P)}{3}}\ |p > 0 \quad (13)$$

$$S_1(A, B) = 1 - d_p(A, B) = 1 - \sqrt[p]{\frac{\sum_i^n w_i([T_A(x_i) - T_B(x_i)]^P + [I_A(x_i) - I_B(x_i)]^P + [F_A(x_i) - F_B(x_i)]^P)}{3}}\ |\ p > 0 \quad (14)$$

considering that distance $d_p(A, B)$ for $p > 0$ satisfies four properties first: $0 \leq d_p(A, B) \leq 1$; second: $d_p(A, B) = 0$ if and only if $A = B$; third: $d_p(A, B) = d_p(B, A)$; and forth property is: $If A \subseteq B \subseteq C$, for C is an SVNS in X, then $d_p(A, C) \geq d_p(A, B)\ and\ d_p(A, C) \geq d_p(B, C)$ [27], but formulas (13) and (14) have some limitation in some cases such as for when applying formula (14) for SVNS $A(x) = \{x, (0.40,0.65,0.60), (0.50,0.50,0.50), (0.40,0.65,0.60)\}$ it return = -0.005816418 which is not accepted, the proposed formula below overcome that shortage.

Definition 3.10. SNN and SVNS 3D visualization

Few effort paid in visualizing neutrosophic sets and numbers, Smarandache, el at (2019) and others [28] use Figure 1 to demonstrate the graphical visualization for neutrosophic environment, also this graph used as a part from Neutrosophic Sets and Systems journal's cover page.

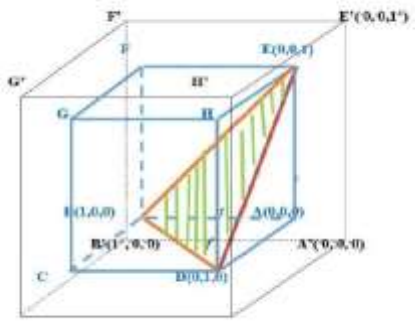

Figure 1 – neutrosophic graphical visualization [28]

Also, Garai et al, 2020 [29], use graph presentation shown in Figure 2 to represent for example SNVN $A = \langle((1,3,5,8),0.9),((1,2,6,8),0.3),((1,3,5,8),0.5)\rangle$.

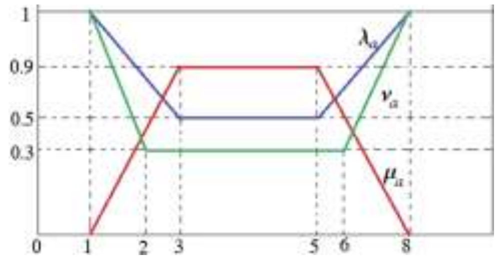

Figure 2 – Single-Valued neutrosophic number [29]

Meanwhile, Karaaslan & Hunu (2020) [30] present SVNN graphically as shown in Figure 3 which represent each truth, indeterminacy, and falsity memberships separately.

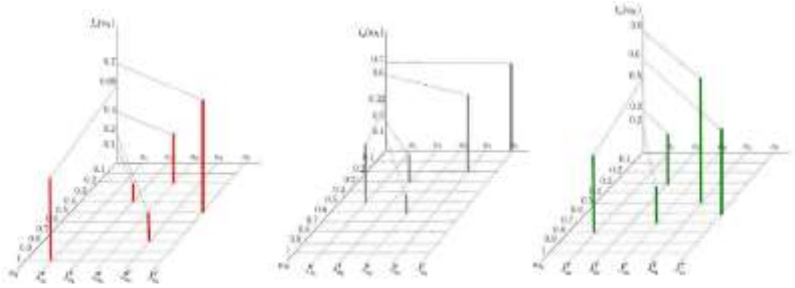

Figure 3 – type 2 SVNS graphical representation [30]

The researchers offer a graphical representation for simplified neutrosophic number SNN and single-valued neutrosophic set SVNS using 3-Dimentional Euclidean space as shown in Figure 4 below, where the empty SNN $0_N = (0,0,1)$ located in the origin point and the absolute SNN $1_N = (1,0,0)$ located in the top $T(x)$ axis, the SNN $A = (0.5,0.3,0.6)$ "an example" which presented in the graph with a "Red Point" using $T(x) = 0.5,\ I(x) = 0.3,\ F(x) = 0.6$.

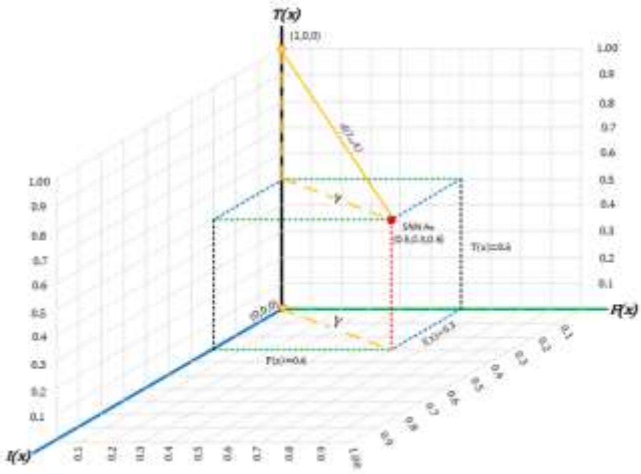

Figure 4 – SNN in 3-Dimentional Euclidean space

Extending the in 3-Dimentional visualization for SNN, Figure 5 below shows in 3-D visualization for two discrete SVNSs $A_i = \langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle$ and $B_i = \langle x_i, T_B(x_i), I_B(x_i), F_B(x_i)\rangle$ where $T_A(x_i)$, $I_A(x_i)$, $F_A(x_i)$, $T_B(x_i)$, $I_B(x_i)$, $F_B(x_i) \in [0,1]$, and x_i $(i = 1,2,3,4)$ giving the value of each element in both SVNSs as the following, for A_i elements $A_1 = (0.8,0.3,0.8)$, $A_2 = (0.2,0.2,0.2)$, $A_3 = (0.5,0.3,0.5)$, $A_4 = (0.8,0.2,0.8)$ and for B_i elements $B_1 = (0.5,0.9,0.1)$, $B_2 = (0.7,0.7,0.4)$, $B_3 = (0.3,0.7,0.5)$, $B_4 = (0.2,1,1)$.

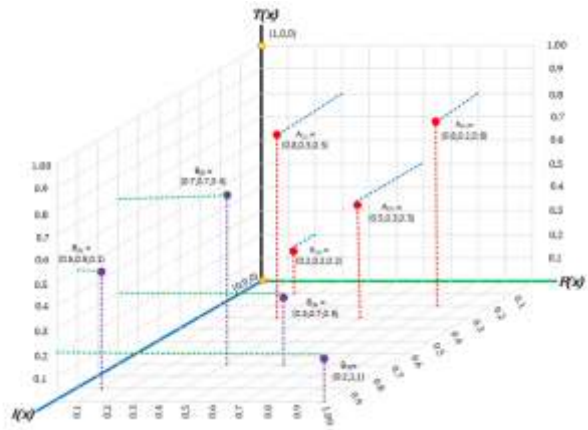

Figure 5 - Two SVNS in 3-Dimentional Euclidean space

Definition 3.11. SNN Euclidean distance

"Euclidean distance" or commanlly named as "Pythagorean distance" which is purely the straight-line distance between two points in the Euclidean space as shown in Figure 4 above, fomula (15) represent the Euclidean distance for $SNN\ (A)$ which refare to the straight-line distance between absolute $SNN\ 1_N = (1,0,0)$ and $SNN\ (A)$, where $d_5(1_n, A) = 0.931149915$ as a pure distance considring $0 \le d_5(1_n, A) \le \sqrt{3}$.

$$d_5(1_n, A) = \sqrt{T_A(x)^2 + I_A(x)^2 + F_A(x)^2} \quad (15)$$

Definition 3.12. Two SNN Euclidean distance

For generalization, it's clear from Figure 6 that, the Euclidean distance $d_6(A, B)$ between two SNNs A and B can be calculated using formula (16) considering that $0 \le d_6(A, B) \le \sqrt{3}$, for seek of normalizations formula (17) provided normalized Euclidean distance $d_7(A, B)$ between the SNN A and SNN B considering that $0 \le d_7(A, B) \le 1$, considering that Ye, (2014) prove the relation distance and similarity are complementary, therefore the normalized similarity $S_7(A,B)$ for normalized Euclidean distance $S_7(A,B) = \left(1 - d_7(A,B)\right) \times 100$ as shown in formula (18).

$$d_6(A,B) = \sqrt{|T_A(x_i) - T_B(x_i)|^2 + |I_A(x_i) - I_B(x_i)|^2 + |F_A(x_i) - F_B(x_i)|^2} \tag{16}$$

$$d_7(A,B) = \sqrt{\frac{|T_A(x_i) - T_B(x_i)|^2 + |I_A(x_i) - I_B(x_i)|^2 + |F_A(x_i) - F_B(x_i)|^2}{3}} \tag{17}$$

$$S_7(A,B) = \left(1 - \sqrt{\frac{|T_A(x_i) - T_B(x_i)|^2 + |I_A(x_i) - I_B(x_i)|^2 + |F_A(x_i) - F_B(x_i)|^2}{3}}\right) \times 100 \tag{18}$$

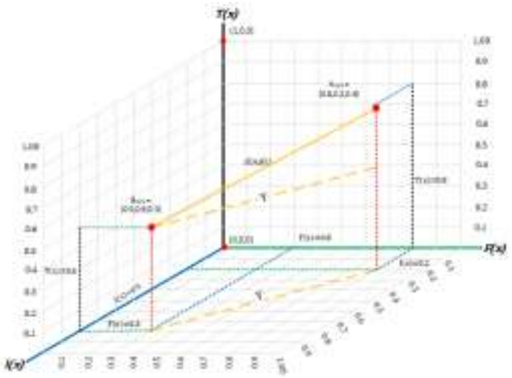

Figure 6 - Two SNN in 3-Dimentional Euclidean space

Definition 3.13. New SVNS distance and similarity measures

Figure 7 represents the Euclidean distance $d(A,B)$, between SVNSs A and B where $d_i(A_i,B_i) \mid (i = 1,2,3,4)$ represents the distance between each two elements in SVNS A_i and SVNS B_i, $d_8(A,B)$, formula (19) represents the Euclidean distance between SVNS A_i and SVNS B_i which extended from formula (16), where $0 \le d_8(A,B) \le \sqrt{3}$, for reaching normalizations, formula (21) provided normalized Euclidean distance $d_9(A,B)$ between SVNS A_i and SVNS B_i considring that $0 \le d_9(A,B) \le 1$, [27] where similarity equal 1- distance and vice versa so, $S_8(A,B) = \left(1 - d_8(A,B)\right) \times 100$ and $S_9(A,B) = \left(1 - d_9(A,B)\right) \times 100$ as shown in formulas (20) and (22) respectively.

$$d_8(A,B) = \sqrt{\textstyle\sum_i^n \left(|T_A(x_i) - T_B(x_i)|^2 + |I_A(x_i) - I_B(x_i)|^2 + |F_A(x_i) - F_B(x_i)|^2\right)} \tag{19}$$

$$S_8(A,B) = \left(1 - \sqrt{\sum_i^n \left(|T_A(x_i) - T_B(x_i)|^2 + |I_A(x_i) - I_B(x_i)|^2 + |F_A(x_i) - F_B(x_i)|^2\right)}\right) \times 100 \tag{20}$$

$$d_9(A,B) = \frac{\sum_i^n \sqrt{|T_A(x_i) - T_B(x_i)|^2 + |I_A(x_i) - I_B(x_i)|^2 + |F_A(x_i) - F_B(x_i)|^2}}{n\sqrt{3}} \tag{21}$$

$$S_9(A,B) = \left(1 - \frac{\sum_i^n \sqrt{|T_A(x_i) - T_B(x_i)|^2 + |I_A(x_i) - I_B(x_i)|^2 + |F_A(x_i) - F_B(x_i)|^2}}{n\sqrt{3}}\right) \times 100 \tag{22}$$

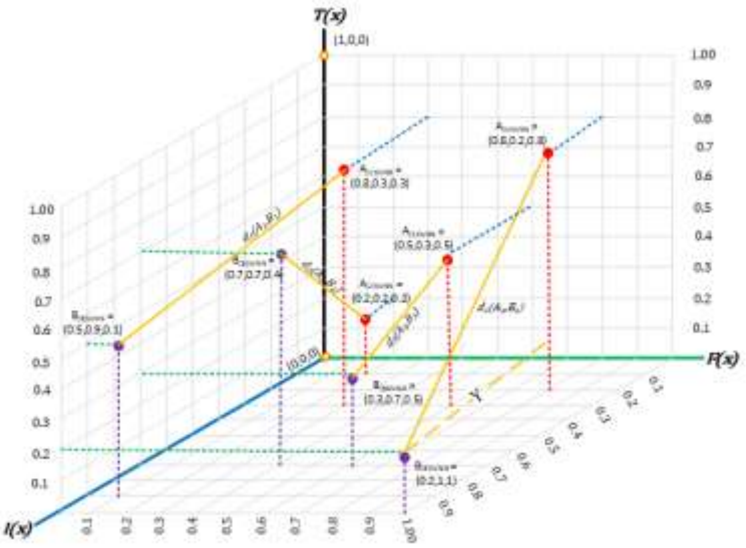

Figure 7 - Euclidean distance between SVNS A_i and SVNS B_i

4. Proposed Framework

in this research paper effort paid off to proposes a scaling system using Simplified Neutrosophic Number or Single-Valued neutrosophic set as elaborated in the below algorithm.

Neutrosophic scaling system algorithm:

Step 1: Create a sorted list of Qualitative terms "Linguistic terms", which will be used as final scaling system outputs, remarking that Linguistic terms shall be sorted either ascending or descending according to the purpose of scaling system, $\mathbb{N}$ represent the number of Linguistic terms as shown in formula (23).

$$\mathbb{N} = \text{Number of language terms} \quad (23)$$

Step 2: Business expert enter the SNN A or SVNS A value for each linguistic term, considering keeping Linguistic terms sorted "bad to good" or "good to bad".

Step 3: Using formula (24) to calculating the equivalent risk crisp value $\mathbb{Q}$ corresponding to each giving SNN using formula (18) similarity $S_7(1_n, A)$ or SVNS using formula (22) similarity $S_9(1_n, A)$ multiplied by number of Linguistic terms $\mathbb{N}$ calculated in formula (23), domain experts can override manually any of calculated equivalent crisp values $\mathbb{Q}$, in this case a modified flag must be added for each override/changed value, keeping in mind that modifying any equivalent crisp values must not changing the order of Linguistic terms.

$$\begin{cases} \mathbb{Q} = S_7(1_n, A) \times \mathbb{N} | \text{A is SNN} \\ \mathbb{Q} = S_9(1_n, A) \times \mathbb{N} | \text{A is SVNS} \end{cases} \quad (24)$$

Step 4: Build 2D Matrix with $\mathbb{N}$ rows and columns specified in Step 1: , then add Linguistic terms in the top row and first column with its corresponding equivalent crisp value $\mathbb{Q}$ and calculate the $matrix\ cells\ values$ by multiple the row value times column value.

Step 5: Convert all $cell\ value$ to $cell\ percentage$ using formula (25) by dividing each matrix cell value by maximum cell value squared, where maximum cell value squared equal $Max(\mathbb{Q})^2$ defined in Step 1: above.

$$cell\ percntage = \frac{cell\ value}{Max(\mathbb{Q})^2} \tag{25}$$

Step 6: Generate Strict risk assessment scale:

1. To determine maximum percentage value for each Linguistic term, look for intersected cells with same Linguistic term, considering these intersected cells as the maximum percentage value for Linguistic terms.
2. To determine minimum percentages values for each Linguistic term, use maximum percentage value for preceding Linguistic terms as minimum percentages values for Linguistic terms.
3. Domain expert can change the range boundary as appropriate.

Step 7: Generate Lenient risk assessment scale:

1. To determine minimum percentage value for each Linguistic term, look for intersected cells with same Linguistic term, considering these intersected cells as the minimum percentage value for Linguistic terms.
2. To determine maximum percentages values for each Linguistic term, use minimum percentage value for following Linguistic term as maximum percentages values for Linguistic terms and add 100% as a maximum for the highest Linguistic term.
3. Domain expert can change the range boundary as appropriate.

Neutrosophic risk assessment scale illustrative numerical example 1:

Step 1: Create a sorted list of qualitative terms "Linguistic terms", as shown in Table 1 $\mathbb{N} = 11$.

Table 1 qualitative value "Linguistic terms"

Linguistic terms	abbreviation
Extremely bad	EB
Very very bad	VVB
Very bad	VB
Bad	B
Medium bad	MB
Medium	M
Medium good	MG
Good	G
Very good	VG
Very very good	VVG
Extremely good	EG

Step 2: Enter the equivalent SNN value provided by business expert for each linguistic term, shown in Table 2

Table 2 Linguistic terms, Equivalent SNN

Linguistic terms bad to good	Linguistic terms good to bad	Equivalent SNN values
Extremely bad	Extremely good	(1,0,0)
Very very bad	Very very good	(0.9, 0.1, 0.1)
Very bad	Very good	(0.8,0.15,0.20)
Bad	Good	(0.70,0.25,0.30)
Medium bad	Medium good	(0.60,0.35,0.40)
Medium	Medium	(0.50,0.50,0.50)

Linguistic terms bad to good	Linguistic terms good to bad	Equivalent SNN values
Medium good	Medium bad	(0.40,0.65,0.60)
Good	Bad	(0.30,0.75,0.70)
Very good	Very bad	(0.20,0.85,0.80)
Very very good	Very very bad	(0.10,0.90,0.90)
Extremely good	Extremely bad	(0,1,1)

Step 3: Calculate the equivalent crisp value $\mathbb{Q}$ corresponding to each giving SNN Using formula (24) as shown in Table 3, noting that the Crisp Values of the linguistic term "Extremely good" was modified from 0 to 0.10 according to expert opinion and modified flag inserted.

Table 3 Linguistic terms, SNN, and its equivalent crisp values $\mathbb{Q}$

Linguistic value bad to good	Linguistic value good to bad	Equivalent SNN values	Calculated Crisp Values	Modified Crisp Values	Modified flag
Extremely bad	Extremely good	(1,0,0)	11	11.00	
Very very bad	Very very good	(0.9, 0.1, 0.1)	9.9	9.90	
Very bad	Very good	(0.8,0.15,0.20)	8.966735	8.97	
Bad	Good	(0.70,0.25,0.30)	7.872568	7.87	
Medium bad	Medium good	(0.60,0.35,0.40)	6.77537	6.78	
Medium	Medium	(0.50,0.50,0.50)	5.5	5.50	
Medium good	Medium bad	(0.40,0.65,0.60)	4.211714	4.21	
Good	Bad	(0.30,0.75,0.70)	3.112404	3.11	
Very good	Very bad	(0.20,0.85,0.80)	2.012926	2.01	
Very very good	Very very bad	(0.10,0.90,0.90)	1.1	1.10	
Extremely good	Extremely bad	(0,1,1)	0	0.10	*

Step 4: Build Two-dimensional Symmetric Matrix with $\mathbb{Q} = 11$ rows and columns then add Linguistic terms in the top row and first column as shown in Table 4 below, then calculate the matrix cells values by multiple the row value times column value, for example: cell(1,8) which are $(row, column)$ reflect is the intersection of row no 1: "EB as Extremely Bad" with the value of (11.00) and column no 8: "B as Bad" with value of (7.87), so the cell(1,8) value equal $7.87 \times 11.0 = 86.60$; Another example: the cell(5,3) which is the intersection of row no:5 "MB" with the value of (6.78) and column no:3 "VG" with value of (2.01), so cell(5.3) value equal $6.78 \times 2.01 = 13.64$, and so on for all matrix cells as shown in Table 4 below.

Table 4 Two-dimensional Symmetric Matrix value

Row No.	Col No.		1	2	3	4	5	6	7	8	9	10	11
	$\mathbb{Q}$		0.10	1.10	2.01	3.11	4.21	5.50	6.78	7.87	8.97	9.90	11.00
		term Prefix	EG	VVG	VG	G	MG	M	MB	B	VB	VVB	EB
1	11.00	EB	1.10	12.10	22.14	34.24	46.33	60.50	74.53	86.60	98.63	108.90	121.00
2	9.90	VVB	0.99	10.89	19.93	30.81	41.70	54.45	67.08	77.94	88.77	98.01	108.90
3	8.97	VB	0.90	9.86	18.05	27.91	37.77	49.32	60.75	70.59	80.40	88.77	98.63
4	7.87	B	0.79	8.66	15.85	24.50	33.16	43.30	53.34	61.98	70.59	77.94	86.60
5	6.78	MB	0.68	7.45	13.64	21.09	28.54	37.26	45.91	53.34	60.75	67.08	74.53
6	5.50	M	0.55	6.05	11.07	17.12	23.16	30.25	37.26	43.30	49.32	54.45	60.50
7	4.21	MG	0.42	4.63	8.48	13.11	17.74	23.16	28.54	33.16	37.77	41.70	46.33
8	3.11	G	0.31	3.42	6.27	9.69	13.11	17.12	21.09	24.50	27.91	30.81	34.24

Row No.	Col No.		1	2	3	4	5	6	7	8	9	10	11
	ℚ		0.10	1.10	2.01	3.11	4.21	5.50	6.78	7.87	8.97	9.90	11.00
		term Prefix	EG	VVG	VG	G	MG	M	MB	B	VB	VVB	EB
9	2.01	VG	0.20	2.21	4.05	6.27	8.48	11.07	13.64	15.85	18.05	19.93	22.14
10	1.10	VVG	0.11	1.21	2.21	3.42	4.63	6.05	7.45	8.66	9.86	10.89	12.10
11	0.10	EG	0.01	0.11	0.20	0.31	0.42	0.55	0.68	0.79	0.90	0.99	1.10

Step 5: Convert the matrix cells' value to percentage as shown in Table 5 using formula (25) where $\mathbb{Q} = 11$ and maximum cell value is $11^2 = 121$, so for example $cell(5,5)percntage = 28.54/121 = 23.58\%$another example the $cell(2,7)percntage = 67.08/121 = 55.43\%$, and so on for all matrix cells'.

Table 5 Two-dimensional Symmetric Matrix percentage

Row No.	Col No.		1	2	3	4	5	6	7	8	9	10	11
	ℚ		0.10	1.10	2.01	3.11	4.21	5.50	6.78	7.87	8.97	9.90	11.00
		term Prefix	EG	VVG	VG	G	MG	M	MB	B	VB	VVB	EB
1	11.00	EB	0.91%	10.00%	18.30%	28.29%	38.29%	50.00%	61.59%	71.57%	81.52%	90.00%	100%
2	9.90	VVB	0.82%	9.00%	16.47%	25.47%	34.46%	45.00%	55.43%	64.41%	73.36%	81.00%	90.00%
3	8.97	VB	0.74%	8.15%	14.92%	23.06%	31.21%	40.76%	50.21%	58.34%	66.45%	73.36%	81.52%
4	7.87	B	0.65%	7.16%	13.10%	20.25%	27.40%	35.78%	44.08%	51.22%	58.34%	64.41%	71.57%
5	6.78	MB	0.56%	6.16%	11.27%	17.43%	23.58%	30.80%	37.94%	44.08%	50.21%	55.43%	61.59%
6	5.50	M	0.45%	5.00%	9.15%	14.15%	19.14%	25.00%	30.80%	35.78%	40.76%	45.00%	50.00%
7	4.21	MG	0.35%	3.83%	7.01%	10.83%	14.66%	19.14%	23.58%	27.40%	31.21%	34.46%	38.29%
8	3.11	G	0.26%	2.83%	5.18%	8.01%	10.83%	14.15%	17.43%	20.25%	23.06%	25.47%	28.29%
9	2.01	VG	0.17%	1.83%	3.35%	5.18%	7.01%	9.15%	11.27%	13.10%	14.92%	16.47%	18.30%
10	1.10	VVG	0.09%	1.00%	1.83%	2.83%	3.83%	5.00%	6.16%	7.16%	8.15%	9.00%	10.00%
11	0.10	EG	0.01%	0.09%	0.17%	0.26%	0.35%	0.45%	0.56%	0.65%	0.74%	0.82%	0.91%

Step 6: Generate Strict risk assessment scale:

1. To determine maximum percentage value for each Linguistic term, highlight intersected cells with same Linguistic term as shown in Table 6, considering these intersected cells values as the maximum percentage value for Linguistic terms.

Table 6 two-dimensional Symmetric maximum value for category

Ling. Prefix	EG	VVG	VG	G	MG	M	MB	B	VB	VVB	EB
EB											100%
VVB										81.00%	
VB									66.45%		
B								51.22%			
MB							37.94%				
M						25.00%					
MG					14.66%						
G				8.01%							
VG			3.35%								
VVG		1.00%									
EG	0.01%										

2. To determine minimum percentages values for Linguistic terms, use maximum percentage value for preceding Linguistic terms as minimum percentages values for

Linguistic terms From the previous step minimum and maximum percentages values for each qualitative value and qualitative values rang have been determined as shown in Table 7, domain expert can change the range edge as appropriate.

Table 7 **Strict** Linguistic terms rang percentage

Linguistic Terms Good to bad	Linguistic Terms Bad to good	Min	Max	Strict Range
Extremely good	Extremely bad	81.00%	100.0%	>81.0% & <=100%
Very very good	Very very bad	66.45%	81.00%	>66.4% & <=81.0%
Very good	Very bad	51.22%	66.45%	>51.2% & <=66.4%
Good	Bad	37.94%	51.22%	>37.9% & <=51.2%
Medium good	Medium bad	25.00%	37.94%	>25.0% & <=37.9%
Medium	Medium	14.66%	25.00%	>14.7% & <=25.0%
Medium bad	Medium good	8.01%	14.66%	>8.0% & <=14.7%
Bad	Good	3.35%	8.01%	>3.3% & <=8.0%
Very bad	Very good	1.00%	3.35%	>1.0% & <=3.3%
Very very bad	Very very good	0.01%	1.00%	>0.01% & <=1.0%
Extremely bad	Extremely good	0.00%	0.01%	>0% & <=0.01%

Step 7: Generate Lenient risk assessment scale:

1. To determine minimum percentage value foreach Linguistic term, highlight intersected cells with same Linguistic term as shown in Table 6, considering these intersected cells values as the minimum percentage value for Linguistic terms.

Ling. Prefix	EG	VVG	VG	G	MG	M	MB	B	VB	VVB	EB
EB											100%
VVB										81.00%	
VB									66.45%		
B								51.22%			
MB							37.94%				
M						25.00%					
MG					14.66%						
G				8.01%							
VG			3.35%								
VVG		1.00%									
EG	0.01%										

2. To determine maximum percentages values for each Linguistic term, use minimum percentage value for following Linguistic term as maximum percentages values for Linguistic terms and add 100% as a maximum for the highest Linguistic term as shown Table 8.
3. Domain expert can change the range boundary as appropriate

Table 8 **Lenient** qualitative Values rang percentage

Linguistic Terms Good to bad	Linguistic Terms Bad to good	Min	Max	Lenient Range
Extremely good	Extremely bad	100%	100%	>=100.0%

Linguistic Terms Good to bad	Linguistic Terms Bad to good	Min	Max	Lenient Range
Very very good	Very very bad	81.00%	100%	>=81.00% & <100.0%
Very good	Very bad	66.45%	81.00%	>=66.45% & <81.0%
Good	Bad	51.22%	66.45%	>=51.22% & <66.4%
Medium good	Medium bad	37.94%	51.22%	>=37.94% & <51.2%
Medium	Medium	25.00%	37.94%	>=25.00% & <37.9%
Medium bad	Medium good	14.66%	25.00%	>=14.66% & <25.0%
Bad	Good	8.01%	14.66%	>=8.01% & <14.7%
Very bad	Very good	3.35%	8.01%	>=3.35% & <8.0%
Very very bad	Very very good	1.00%	3.35%	>=1.00% & <3.3%
Extremely bad	Extremely good	0.01%	1.00%	>=0% & <1.00%

Calculate risk assessment illustrative numerical example 2:

Step 1: This example aims to calculate risk assessment for a project has four 4 major risk areas named personnel quality, production equipment, work environment, and safety management; these areas contains 23 risk factors, Table 9 below contains list of risk categories and its risk factors.

Table 9 –Risks categories and factors

Risk Category	Factors ($\boldsymbol{x_i}$)	Risk Factors (subcategory)
People quality	x_1	Education level
	x_2	Learner's time
	x_3	Age
	x_4	duration of service
	x_5	Worker density
	x_6	Body status
	x_7	Business period
Production equipment	x_8	Restrict dropping devices
	x_9	equipment design dependability
	x_{10}	equipment proper rate
	x_{11}	Protecting equipment dependability
	x_{12}	equipment flexibility
Environment	x_{13}	Heat
	x_{14}	Light
	x_{15}	humidity
	x_{16}	Environmental security dependability
	x_{17}	running surface efficiency
Safety management	x_{18}	Security system
	x_{19}	Safety society
	x_{20}	… feedback
	x_{21}	… assessment
	x_{22}	… cotching
	x_{23}	… checks

Step 2: In this case will use the linguistic terms and its equivalent “strict ranges” and “lenient ranges” previously calculated in Table 7 and Table 8 above, using sorted linguistics terms from “bad to good” as consolidated in Table 10.

Table 10 Linguistic terms, both Strict Range and Lenient Range

Linguistic Terms Bad to good	Strict Ranges	Lenient Ranges
Extremely bad	>81.0% & <=100%	>=100.0%
Very very bad	>66.4% & <=81.0%	>=81.00% & <100.0%
Very bad	>51.2% & <=66.4%	>=66.45% & <81.0%
Bad	>37.9% & <=51.2%	>=51.22% & <66.4%
Medium bad	>25.0% & <=37.9%	>=37.94% & <51.2%
Medium	>14.7% & <=25.0%	>=25.00% & <37.9%
Medium good	>8.0% & <=14.7%	>=14.66% & <25.0%
Good	>3.3% & <=8.0%	>=8.01% & <14.7%
Very good	>1.0% & <=3.3%	>=3.35% & <8.0%
Very very good	>0.01% & <=1.0%	>=1.00% & <3.3%
Extremely good	>0% & <=0.01%	>=0% & <1.00%

Step 3: Each risk factor x_i was evaluated by three experts E_n, each expert used even linguistics terms or SVNS to define the value of risk factors as shown in Table 11.

Table 11 Risk factors evaluation

x_i	E_1	E_2	E_3	SVNS $A(x_i)$
x_1	(0.8,0.15,0.20)	(0.60,0.35,0.40)	Risky	$A(x_1) = \{x_1, (0.8,0.15,0.20), (0.60,0.35,0.40), (0.70,0.25,0.30)\}$
x_2	(0.60,0.35,0.40)	(0.50,0.50,0.50)	(0.70,0.25,0.30)	$A(x_2) = \{x_2, (0.60,0.35,0.40), (0.50,0.50,0.50), (0.70,0.25,0.30)\}$
x_3	(0.40,0.65,0.60)	(0.50,0.50,0.50)	Medium low risky	$A(x_3) = \{x_3, (0.40,0.65,0.60), (0.50,0.50,0.50), (0.40,0.65,0.60)\}$
x_4	(0.30,0.75,0.70)	(0.20,0.85,0.80)	(0.50,0.50,0.50)	$A(x_4) = \{x_4, (0.30,0.75,0.70), (0.20,0.85,0.80), (0.50,0.50,0.50)\}$
x_5	(0.30,0.75,0.70)	Medium low risky	(0.30,0.75,0.70)	$A(x_5) = \{x_5, (0.30,0.75,0.70), (0.40,0.65,0.60), (0.30,0.75,0.70)\}$
x_6	(0.60,0.35,0.40)	(0.8,0.15,0.20)	(0.60,0.35,0.40)	$A(x_6) = \{x_6, (0.60,0.35,0.40), (0.8,0.15,0.20), (0.60,0.35,0.40)\}$
x_7	(0.50,0.50,0.50)	(0.70,0.25,0.30)	(0.50,0.50,0.50)	$A(x_7) = \{x_7, (0.50,0.50,0.50), (0.70,0.25,0.30), (0.50,0.50,0.50)\}$
x_8	(0.40,0.65,0.60)	(0.60,0.35,0.40)	Medium low risky	$A(x_8) = \{x_8, (0.40,0.65,0.60), (0.60,0.35,0.40), (0.40,0.65,0.60)\}$
x_9	(0.30,0.75,0.70)	Medium risky	(0.30,0.75,0.70)	$A(x_9) = \{x_9, (0.30,0.75,0.70), (0.50,0.50,0.50), (0.30,0.75,0.70)\}$
x_{10}	(0.8,0.15,0.20)	Risky	(0.20,0.85,0.80)	$A(x_{10}) = \{x_{10}, (0.8,0.15,0.20), (0.70,0.25,0.30), (0.20,0.85,0.80)\}$
x_{11}	(0.70,0.25,0.30)	(0.60,0.35,0.40)	(0.60,0.35,0.40)	$A(x_{11}) = \{x_{11}, (0.70,0.25,0.30), (0.60,0.35,0.40), (0.60,0.35,0.40)\}$
x_{12}	(0.60,0.35,0.40)	Medium risky	(0.50,0.50,0.50)	$A(x_{12}) = \{x_{12}, (0.60,0.35,0.40), (0.50,0.50,0.50), (0.50,0.50,0.50)\}$
x_{13}	(0.50,0.50,0.50)	(0.40,0.65,0.60)	(0.40,0.65,0.60)	$A(x_{13}) = \{x_{13}, (0.50,0.50,0.50), (0.40,0.65,0.60), (0.40,0.65,0.60)\}$
x_{14}	(0.50,0.50,0.50)	(0.70,0.25,0.30)	(0.30,0.75,0.70)	$A(x_{14}) = \{x_{14}, (0.50,0.50,0.50), (0.70,0.25,0.30), (0.30,0.75,0.70)\}$

x_i	E_1	E_2	E_3	SVNS $A(x_i)$
x_{15}	(0.40,0.65,0.60)	(0.60,0.35,0.40)	(0.50,0.50,0.50)	$A(x_{15}) = \{x_{15}, (0.40,0.65,0.60), (0.60,0.35,0.40), (0.50,0.50,0.50)\}$
x_{16}	(0.30,0.75,0.70)	Medium risky	(0.40,0.65,0.60)	$A(x_{16}) = \{x_{16}, (0.30,0.75,0.70), (0.50,0.50,0.50), (0.40,0.65,0.60)\}$
x_{17}	(0.20,0.85,0.80)	(0.40,0.65,0.60)	(0.30,0.75,0.70)	$A(x_{17}) = \{x_{17}, (0.20,0.85,0.80), (0.40,0.65,0.60), (0.30,0.75,0.70)\}$
x_{18}	(0.8,0.15,0.20)	(0.70,0.25,0.30)	(0.20,0.85,0.80)	$A(x_{18}) = \{x_{18}, (0.8,0.15,0.20), (0.70,0.25,0.30), (0.20,0.85,0.80)\}$
x_{19}	(0.70,0.25,0.30)	(0.60,0.35,0.40)	Medium risky	$A(x_{19}) = \{x_{19}, (0.70,0.25,0.30), (0.60,0.35,0.40), (0.50,0.50,0.50)\}$
x_{20}	(0.60,0.35,0.40)	Medium risky	(0.40,0.65,0.60)	$A(x_{20}) = \{x_{20}, (0.60,0.35,0.40), (0.50,0.50,0.50), (0.40,0.65,0.60)\}$
x_{21}	Medium risky	(0.40,0.65,0.60)	(0.30,0.75,0.70)	$A(x_{21}) = \{x_{21}, (0.50,0.50,0.50), (0.40,0.65,0.60), (0.30,0.75,0.70)\}$
x_{22}	(0.20,0.85,0.80)	(0.30,0.75,0.70)	(0.40,0.65,0.60)	$A(x_{22}) = \{x_{22}, (0.20,0.85,0.80), (0.30,0.75,0.70), (0.40,0.65,0.60)\}$
x_{23}	(0.10,0.90,0.90)	(0.20,0.85,0.80)	(0.30,0.75,0.70)	$A(x_{23}) = \{x_{23}, (0.10,0.90,0.90), (0.20,0.85,0.80), (0.30,0.75,0.70)\}$

Step 4: Using formula (22) to calculate the crisp value for SVNS $A(x_i)$, results shown in Table 12, then used both Table 7 and Table 8 above to compare calculated crisp value for each SVNS $A(x_i)$ with risk ranges to select the equivalent risk level, result shown in Table 13 below.

Table 12 Risk factors and its crisp value

SVNS $A(x_i)$	Crisp value
$A(x_1) = \{x_1, (0.8,0.15,0.20), (0.60,0.35,0.40), (0.70,0.25,0.30)\}$	71.56%
$A(x_2) = \{x_2, (0.60,0.35,0.40), (0.50,0.50,0.50), (0.70,0.25,0.30)\}$	61.05%
$A(x_3) = \{x_3, (0.40,0.65,0.60), (0.50,0.50,0.50), (0.40,0.65,0.60)\}$	42.19%
$A(x_4) = \{x_4, (0.30,0.75,0.70), (0.20,0.85,0.80), (0.50,0.50,0.50)\}$	32.20%
$A(x_5) = \{x_5, (0.30,0.75,0.70), (0.40,0.65,0.60), (0.30,0.75,0.70)\}$	31.63%
$A(x_6) = \{x_6, (0.60,0.35,0.40), (0.8,0.15,0.20), (0.60,0.35,0.40)\}$	68.23%
$A(x_7) = \{x_7, (0.50,0.50,0.50), (0.70,0.25,0.30), (0.50,0.50,0.50)\}$	57.19%
$A(x_8) = \{x_8, (0.40,0.65,0.60), (0.60,0.35,0.40), (0.40,0.65,0.60)\}$	46.06%
$A(x_9) = \{x_9, (0.30,0.75,0.70), (0.50,0.50,0.50), (0.30,0.75,0.70)\}$	35.53%
$A(x_{10}) = \{x_{10}, (0.8,0.15,0.20), (0.70,0.25,0.30), (0.20,0.85,0.80)\}$	57.13%
$A(x_{11}) = \{x_{11}, (0.70,0.25,0.30), (0.60,0.35,0.40), (0.60,0.35,0.40)\}$	64.92%
$A(x_{12}) = \{x_{12}, (0.60,0.35,0.40), (0.50,0.50,0.50), (0.50,0.50,0.50)\}$	53.86%
$A(x_{13}) = \{x_{13}, (0.50,0.50,0.50), (0.40,0.65,0.60), (0.40,0.65,0.60)\}$	42.19%
$A(x_{14}) = \{x_{14}, (0.50,0.50,0.50), (0.70,0.25,0.30), (0.30,0.75,0.70)\}$	49.95%
$A(x_{15}) = \{x_{15}, (0.40,0.65,0.60), (0.60,0.35,0.40), (0.50,0.50,0.50)\}$	49.96%
$A(x_{16}) = \{x_{16}, (0.30,0.75,0.70), (0.50,0.50,0.50), (0.40,0.65,0.60)\}$	38.86%
$A(x_{17}) = \{x_{17}, (0.20,0.85,0.80), (0.40,0.65,0.60), (0.30,0.75,0.70)\}$	28.29%
$A(x_{18}) = \{x_{18}, (0.8,0.15,0.20), (0.70,0.25,0.30), (0.20,0.85,0.80)\}$	57.13%
$A(x_{19}) = \{x_{19}, (0.70,0.25,0.30), (0.60,0.35,0.40), (0.50,0.50,0.50)\}$	61.05%
$A(x_{20}) = \{x_{20}, (0.60,0.35,0.40), (0.50,0.50,0.50), (0.40,0.65,0.60)\}$	49.96%
$A(x_{21}) = \{x_{21}, (0.50,0.50,0.50), (0.40,0.65,0.60), (0.30,0.75,0.70)\}$	38.86%
$A(x_{22}) = \{x_{22}, (0.20,0.85,0.80), (0.30,0.75,0.70), (0.40,0.65,0.60)\}$	28.29%
$A(x_{23}) = \{x_{23}, (0.10,0.90,0.90), (0.20,0.85,0.80), (0.30,0.75,0.70)\}$	18.86%

Table 13 Risk factors and its equivalent risk level

$SVNS\ \boldsymbol{A(x_i)}$	Crisp value	Strict risk level	Lenient risk level
$A(x_1)$	71.56%	Very very bad	Very bad
$A(x_2)$	61.05%	Very bad	Bad
$A(x_3)$	42.19%	Bad	Medium bad
$A(x_4)$	32.20%	Medium bad	Medium
$A(x_5)$	31.63%	Medium bad	Medium
$A(x_6)$	68.23%	Very very bad	Very bad
$A(x_7)$	57.19%	Very bad	Bad
$A(x_8)$	46.06%	Bad	Medium bad
$A(x_9)$	35.53%	Medium bad	Medium
$A(x_{10})$	57.13%	Very bad	Bad
$A(x_{11})$	64.92%	Very bad	Bad
$A(x_{12})$	53.86%	Very bad	Bad
$A(x_{13})$	42.19%	Bad	Medium bad
$A(x_{14})$	49.95%	Bad	Medium bad
$A(x_{15})$	49.96%	Bad	Medium bad
$A(x_{16})$	38.86%	Bad	Medium bad
$A(x_{17})$	28.29%	Medium bad	Medium
$A(x_{18})$	57.13%	Very bad	Bad
$A(x_{19})$	61.05%	Very bad	Bad
$A(x_{20})$	49.96%	Bad	Medium bad
$A(x_{21})$	38.86%	Bad	Medium bad
$A(x_{22})$	28.29%	Medium bad	Medium
$A(x_{23})$	18.86%	Medium	Medium good

Step 5: After calculating the crisp values for each risk factor, risk assessment expert shall take the appropriate decisions.

5. Conclusion and future works:

In this research paper a neutrosophic 3D visualization for both SNN and SVNS was presented, in addition, some existing distance and similarity measure are validated and shortcoming are exposed, new crisp value functions "De-neutrosophication" for converting both Simplified Neutrosophic Number SNN, and Single-Valued Neutrosophic set SVNS to them equivalent crisp values using similarity measure based on Euclidean distance are proposed to overcome the exposed shortcoming, also a new Neutrosophic Scaling System algorithm is proposed, Finally, the proposed Neutrosophic Scaling System is applied to risk assessment case study.

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