New Types of Neutrosophic Crisp Closed Sets

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Abstract: The neutrosophic sets were known since 1999, and because of their wide applications and their great flexibility to solve the problems, we used these the concepts to define a new types of neutrosophic crisp closed sets and limit points in neutrosophic crisp topological space, namely [neutrosophic crisp Gem sets and neutrosophic crisp Turig points] respectively, we study their properties in details and join it with topological concepts. Finally we used [neutrosophic crisp Gem sets and neutrosophic crisp Turig points] to introduce of topological concepts as: neutrosophic crisp closed (open) sets, neutrosophic crisp closure, neutrosophic crisp interior, neutrosophic crisp exterior, and neutrosophic crisp boundary which are fundamental for further research on neutrosophic crisp topology and will strengthen the foundations of theory of neutrosophic topological spaces.

Keywords: Neutrosophic crisp set, Neutrosophic crisp topology, Neutrosophic crisp closed set.

1. Introduction

In 1999, Smarandache firstly proposed the theory of neutrosophic set [1] which is the generalization of the class sets, conventional fuzzy set [2] and intuitionistic set fuzzy [3]. After Smarandache, neutrosophic sets have been successfully applied to many fields such as; topology, control theory, databases, medical diagnosis problem, decision making problem and so on, [4-37].

A.A. Salama, et al. [38] proposed a new mathematical model called "Neutrosophic crisp sets and Neutrosophic crisp topological spaces".

The idea of "Gem-Set", which is a characterization of the concept of closure is introduced by AL-Nafee, Al-Swidi [39]. After AL-Nafee, the idea of "Gem-Set has been successfully using to many topological concepts such as; interior, exterior, boundary, separation axioms, continuous functions, bitopological spaces, compactness, soft topological spaces, and so on, [40,41,42,43,44,45,46,47,48].

The idea of "controlling soft Gem-Set" and join it with topological concepts in soft topological space is introduced by [49]. The concept of the soft Turing point and used it with separation axioms in soft topological space is introduced by [50,51].

The goal of this research is to combine the concept of "Gem-Set" and Turing point with neutrosophic crisp set to define a new types of neutrosophic crisp closed sets and limit points in neutrosophic crisp topological space, namely [neutrosophic crisp Gem sets and neutrosophic crisp Turig points] respectively, we study their properties in details and we also use it to introduce the some of topological concepts as: neutrosophic crisp closed (open) sets, neutrosophic crisp closure, neutrosophic crisp interior, neutrosophic crisp exterior, and neutrosophic crisp boundary which are...
fundamental for further research on neutrosophic crisp topology and will strengthen the foundations of theory of neutrosophic topological spaces.

The paper is structured as follows; In section 2, we first recall the necessary background on neutrosophic and neutrosophic crisp points [NCPn for short]. In section 3, a neutrosophic crisp Turing points properties are introduced with their properties. In section 4, the concept of neutrosophic crisp Gem sets are introduced and studied their properties.

Throughout this paper, NCTS means a neutrosophic crisp topological space, also we write (H) by H (for short), the collection of all neutrosophic crisp sets on H will be denoted by N(H).

2. Preliminaries

2.1. Definition [52]

Let H be a non-empty fixed set, a neutrosophic crisp set (for short NCS) D is an object having the form \( D = < D_1, D_2, D_3 > \) where \( D_1, D_2 \) and \( D_3 \) are subsets of H.

We will exhibit the basic neutrosophic operations definitions (union, intersection and complement). Since there are different definitions of neutrosophic operations, we will organize the existing definitions into two types in each type these operations will be consistent and functional. In this work we will use one Type of neutrosophic crisp sets operations.

2.2. Definition [52]

A neutrosophic crisp topology (NCTS) on an non-empty set H is a family T of neutrosophic crisp subsets in H satisfying the following conditions:
\[
\emptyset, H \in T,\ 
\forall C, D \in T \implies C \cap D \in T
\]

The union of any number of set in T belongs to T.

The pair (H, T) is said to be a neutrosophic crisp topological space (NCTS) in H. Moreover the elements in T are said to be neutrosophic crisp open sets. A neutrosophic crisp set F is closed if its complement (F^C) is an open neutrosophic crisp set.

2.3. Definition [52]

Let NI be a non-null collection of neutrosophic crisp sets over a universe H. Then NI is called neutrosophic crisp ideal on H if:
- \( C \in NI \) and \( D \in NI \) then \( C \cup D \in NI \).
- \( C \in NI \) and \( D \subseteq C \) then \( D \in NI \).

2.4. Definition [52]

Let (H, I) be NCTS, A be a neutrosophic crisp set then: The intersection of any neutrosophic crisp closed sets contained A is called neutrosophic crisp closure of A (for short NC-CL(A)).

2.5. Definition [52]

((neutrosophic crisp sets operations of Type.I))

Let H be a non-empty set and \( C = < C_1, C_2, C_3 > \), \( D = < D_1, D_2, D_3 > \) be two neutrosophic crisp sets, where \( D_1, C_1 \cap D_3, C_2 \) and \( D_3, C_1 \) are subsets of H, such that \( (D_1 \cap D_2) = \emptyset \), \( (D_1 \cap D_3) = \emptyset \), \( (D_2 \cap D_3) = \emptyset \), \( (C_1 \cap C_2) = \emptyset \), \( (C_1 \cap C_3) = \emptyset \), \( (C_2 \cap C_3) = \emptyset \) then:
- \( \emptyset = < \emptyset, \emptyset, H > \) (Neutrosophic empty set).
- \( H = < H, H, H > \) (Neutrosophic universal set).
- \( C \cap D = [ C_1 \cap D_1 ], [ C_2 \cap D_2 ] \) and \( [ C_3 \cap D_3 ] \).
- \( C \cup D = [ C_1 \cup D_1 ], [ C_2 \cup D_2 ] \) and \( [ C_3 \cup D_3 ] \).
- \( C \subseteq D \iff C_1 \subseteq D_1, C_2 \subseteq D_2 \) and \( C_3 \subseteq D_3 \).

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2.6. Definition [53]
((neutrosophic crisp sets operations of Type.2))
Let \( H \) be a non-empty set and \( C = \langle C_1, C_2, C_3 \rangle, D = \langle D_1, D_2, D_3 \rangle \) be two neutrosophic crisp sets, where \( D_1, C_1, D_2, C_2 \) and \( D_3, C_3 \) are subsets of \( H \) then:
- \( \emptyset_N = \langle \emptyset, \emptyset, \emptyset \rangle \) (Neutrosophic empty set).
- \( H_N = \langle H, H, H \rangle \) (Neutrosophic universal set).
- \( C \cap D = \langle [C_1 \cap D_1], [C_2 \cap D_2] \rangle \) and \( [C_3 \cap D_3] \).
- \( C \cup D = \langle [C_1 \cup D_1], [C_2 \cup D_2] \rangle \) and \( [C_3 \cup D_3] \).
- \( C \subseteq D \iff C_1 \subseteq D_1, C_2 \subseteq D_2 \) and \( C_3 \subseteq D_3 \).
- The complement of a NCS \( Y \in H \) may be defined as: \( Y^c = \langle D^c, D^c, D^c \rangle \).
- \( C = D \iff C \subseteq D, D \subseteq C \).

2.7. Definition [53]
For all \( a, b, c \in H \). Then the neutrosophic crisp points related to \( a, b, c \) are defined as follows:
- \( a_N = \langle a, 0, 0 \rangle \) on \( H \).
- \( b_N = \langle 0, b, 0 \rangle \) on \( H \).
- \( c_N = \langle 0, 0, c \rangle \) on \( H \).
(The set of all neutrosophic crisp points \( a_N, b_N, c_N \) is denoted by NCPN).

3. Neutrosophic crisp Turing point
In this work, we will use Type.2 of neutrosophic crisp sets operations, this was necessary to homogeneous suitable results for the upgrade of this research.

3.1. Definition
Let \((H,T)\) be NCTS, \( P \in NCPN \) in \( H \), we define a neutrosophic crisp ideal NI with respect to a neutrosophic crisp point \( P \), as follows:

\[
\ni(P) = \{ D \in T : P \in (D)^c \}
\]

3.2. Definition
Let \((H,T)\) be NCTS, \( P \in NCPN \) in \( H \), \( Y \subseteq H \), we define a neutrosophic crisp ideal \( \forall \ni(P) \) respect to subspace \((Y,T_Y)\), as follows:

\[
\forall \ni(P) = \{ D \in T_Y : P \in (H \setminus D) \}
\]

3.3. Remark
Let \((H,T)\) be NCTS, \( Y \subseteq H \), for each \( D \neq \emptyset_N \) and \( P \in NCPN \) in \( Y \), then;

\[
\forall \ni(P) = \{ D \in T_Y : P \in (H \setminus D) \}
\]

Proof
\[
\forall \ni(P) = \{ D \in T_Y : P \in (H \setminus D) \} = \{ D \in T_Y : P \in (H \setminus D) \}
\]

3.4. Remark
Let \((H,T)\) be NCTS, \( Y \subseteq H \), for each \( D \neq \emptyset_N \) and \( P \in NCPN \) in \( H \), then;

\[
\forall \ni(P) = \{ D \in T_Y : P \in (H \setminus D) \}
\]

3.5. Example
Let \((H,T)\) be NCTS, such that \( H = \{1\} \),
\[T = \{ \emptyset_N, H_N, A, B, C, D, E, F, G \}, \quad P_1 = \langle \emptyset, \{1\}, \emptyset \rangle, \]
\[A = \langle \{1\}, \emptyset, \emptyset \rangle, B = \langle \emptyset, \{1\}, \emptyset \rangle, C = \langle \{1\}, \{1\}, \emptyset \rangle, D = \langle \{1\}, \emptyset, \{1\} \rangle,\]

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E = \{0, \{1\}, \{1\}\}, F = \{0, \emptyset, \{1\}\}, \emptyset = \{1\}, \{1\}\}.

Then, NI(P) =\{\emptyset, A, D, F \}.

3.6. Definition

Let \((H, T)\) be NCTS, \(P \in NCNP\) in \(H\) and NI be a neutrosophic crisp ideal on \((H, T)\), we say that \(p\)

is a neutrosophic crisp turing point of NI if \(D \subseteq NI\) for each \(D \in T\), \(T\) is collection of all

neutrosophic crisp open set of neutrosophic crisp point \(p\).

3.7. Remark

Let \((H, T)\) be NCTS, \(P \in NCNP\) in \(H\) and \(NI(P) =\{D \in T : P \in (D)^C\}\) be a neutrosophic crisp ideal on

\((H, T)\).

Then, \(p\) is a neutrosophic crisp turing point of \(NI(P)\).

3.8. Example

Let \((H, T)\) be NCTS, such that \(H=\{1\},

\(T=\emptyset, H_N, A, B, C, D, E, F, G\), \(P_1 = \{0, \{1\}, \{1\}\}, P_2 = \{\{1\}, \emptyset, \emptyset\}\), such that;

\(A = \{0, \{1\}, \emptyset\}, B = \{\{1\}, \{1\}\}, C = \{\emptyset, \{1\}\}, D = \{\{1\}, \emptyset, \emptyset\}\),

\(E = \{0, \{1\}, \{1\}\}, F = \{\emptyset, \{1\}, \{1\}\}\).

Then, \(P_1\) is a neutrosophic crisp turing point of neutrosophic ideal \(NI(P_1)\), but not \(P_2\).

3.9. Theorem

Let \((H, T)\) be NCTS, \(a_{N_1} \neq b_{N_1} \in NCNP\) in \(H\), then, \(<\{1\}, \emptyset, \emptyset >\) is a neutrosophic crisp closed set

if and only if \(a_{N_1}\) is not a neutrosophic crisp turing point of \(NI(b_{N_1})\).

Proof

Let \(a_{N_1} \neq b_{N_1} \in NCNP\) in \(H\). Assume that \(<\{1\}, \emptyset, \emptyset >\) is a neutrosophic crisp closed set, so that

\(<\{1\}, \emptyset, \emptyset > = cl( <\{1\}, \emptyset, \emptyset >)\). But \(a_{N_1} \neq b_{N_1}\) get that \(a_{N_1} \notin cl( <\{1\}, \emptyset, \emptyset >)\). Therefore,

there exists a neutrosophic crisp open set \(U\) such that, \(a_{N_1} \in U, U \cap <\{1\}, \emptyset, \emptyset > =\emptyset, N\). So that \(a_{N_1} \notin U, U \in NI(b_{N_1})\), because if \(U \in NI(b_{N_1})\), then \(<\{1\}, \emptyset, \emptyset > \in U\), that means \(U \cap <\{1\}, \emptyset, \emptyset > =\emptyset, N\), this is a contradiction! Hence \(a_{N_1}\) is not a neutrosophic crisp turing point of \(NI(b_{N_1})\).

Conversely,

Let \(a_{N_1} \neq b_{N_1} \in NCNP\) in \(H\). Since \(a_{N_1}\) is not a neutrosophic crisp turing point of \(NI(b_{N_1})\), then there exists a neutrosophic crisp open set \(U\) such that, \(a_{N_1} \in U, U \notin NI(b_{N_1})\), so \(<\{1\}, \emptyset, \emptyset \notin U\). Thus \(a_{N_1} \in U, U \cap <\{1\}, \emptyset, \emptyset > =\emptyset, N\) implies \(a_{N_1} \notin cl(<\{1\}, \emptyset, \emptyset >)\).

Hence \(<\{1\}, \emptyset, \emptyset > = cl(<\{1\}, \emptyset, \emptyset >)\), thus \(<\{1\}, \emptyset, \emptyset >\) is a neutrosophic crisp closed set in \(H\).

Proof by the same proof of 2.10. Theorem .

4. Neutrosophic crisp Gem set

4.1. Definition

Let \((H, T)\) be NCTS, \(P \in NCNP\) in \(H\), \(NI(P)\) be a neutrosophic crisp ideal on \((H, T)\) and \(D \subseteq (H, T)\),

we defined the neutrosophic crisp set \(ND^p\) with respect to space \((H, T)\) as follows:

\(ND^p = \{P_1 \in NCNP : F \in D \subseteq NI(P)\}\), for each \(F \in T, T, P_1\) is collection of all neutrosophic crisp open

set of neutrosophic crisp point \(P_1\). The neutrosophic crisp set \(ND^p\) is called neutrosophic crisp

Gem-Set.

4.2. Example

Let \((H, T)\) be NCTS, such that \(H=\{1, 2, 3\},

\(T=\emptyset, H, H_N, A, B, C, D, E, F, G\), \(P_1 = \{0, \{1\}, \emptyset\}, P_2 = \{\{1\}, \emptyset, \emptyset\}\), such that;

\(A = \{0, \{1\}, \emptyset\}, B = \{0, \{2\}, \emptyset\}, C = \{\emptyset, \{3\}, \emptyset\}, D = \{\{1\}, \emptyset, \emptyset\}\).
\[ E = \{ \emptyset, \{1,3\}, \emptyset, \emptyset \} , F = \{ \emptyset, \{2,3\}, \emptyset \} , G = \{ \emptyset, \{1,2,3\}, \emptyset \} . \]

Then, \( \text{NI}(P) = \{ \emptyset, N, B, C, F \} \) and \( \text{ND}^{\tau} = \{ \{1\}, \emptyset \} \).

### 4.3. Theorem

Let \( (H,T) \) be NCTS, \( P \in \text{NCPN} \) in \( H \), and let \( D, C \) be subsets of \( (H,T) \). Then

1. \( \emptyset_N^{\tau} = \emptyset_N \)
2. \( H_N^{\tau} = H_N \), whenever \( \text{NI}(P) = \emptyset_N \).
3. \( C \subseteq D \rightarrow \text{NC}^{\tau} \subseteq \text{ND}^{\tau}. \)
4. For any points \( P_1, P_2 \in \text{NCPN} \) in \( H \), with \( \text{NI}(P_2) \supseteq \text{NI}(P_1) \), then \( \text{ND}^{\tau} P_2 \subseteq \text{ND}^{\tau} P_1 \).
5. \( P \in D \) if and only if \( P \in \text{ND}^{\tau} \).
6. If \( P \in D \), then \( (\text{ND}^{\tau})^{\tau} = \text{ND}^{\tau} \).
7. If \( P_1 \in D, P_2 \in C \) with \( P_1 \neq P_2, D \cap C = \emptyset_N \), then \( \text{ND}^{\tau} P_1 \cap \text{NC}^{\tau} P_2 = \emptyset_N \).
8. If \( a_N, b_N \in \text{NCPN} \) in \( H \) with \( a_N \neq b_N \), then \( b_N \subseteq (a_N)^{\tau} \) implies \( a_N \subseteq (a_N)^{\tau} b_N \) and \( b_N \subseteq (b_N)^{\tau} a_N \).

### 4.4. Remark

The equality of theorem part (3),(4) does not necessarily hold as shown:

Let \( (H,T) \) be NCTS, such that \( H = \{1,2\}, D = \{ \emptyset, \{2\}, \emptyset \} , C = \{ \emptyset, \{1\}, \emptyset \} , \)
\( T = \{ \emptyset_N , H_N , A , B , C , G \} , P_1 = \{ \emptyset, \{2\}, \emptyset \} , P_2 = \{ \emptyset, \{1\}, \emptyset \} , \)
\( A = \{ \emptyset, \{1\}, \emptyset \} , B = \{ \emptyset, \{2\}, \emptyset \} , G = \{ \emptyset, \{1,2\}, \emptyset \} . \)

Then, \( \text{NI}(P_1) = \{ \emptyset_N, A \} , \text{NI}(P_2) = \{ \emptyset_N, B \} \) and \( \text{ND}^{\tau} P_1 = \{ \emptyset, \{2\}, \emptyset \} , \text{ND}^{\tau} P_2 = \emptyset_N \), \( \text{NC}^{\tau} P_1 = \emptyset_N \).

Note that,

1) \( \text{ND}^{\tau} P_2 \subseteq \text{ND}^{\tau} P_1 \) but \( \text{NI}(P_2) \not\supseteq \text{NI}(P_1) \).
2) \( \text{NC}^{\tau} P_1 \subseteq \text{NC}^{\tau} P_2 \), but \( C \not\subseteq D \).

### 4.5. Theorem

Let \( (H,T) \) be NCTS, \( P_1 \in \text{NCPN} \) in \( H \) and \( D, C \) be subsets of \( (H,T) \). Then \( \text{ND}^{\tau} P_1 \cup \text{NC}^{\tau} P_1 = N(D \cup C)^{\tau} P_1 \).

**Proof**

It is obviously known that \( D \subseteq (D \cup C) \) and \( C \subseteq (D \cup C) \), then from theorem 3.3 part(3) we get, \( \text{ND}^{\tau} P_1 \subseteq N(D \cup C)^{\tau} P_1 \) and \( \text{ND}^{\tau} P_1 \subseteq N(A \cup C)^{\tau} P_1 \), for any \( P_1 \in \text{NCPN} \) in \( H \). Hence
\[ \text{ND}^{\tau} P_1 \cup \text{NC}^{\tau} P_1 \subseteq N(D \cup C)^{\tau} P_1 \]

For reverse inclusion, let \( P_2 \notin \text{ND}^{\tau} P_1 \). Then there exists neutrosophic crisp open set \( U \) containing \( P \), with \( D \cap U \subseteq \text{NI}(P_1) \). Similarly, if \( P_2 \notin \text{NC}^{\tau} P_1 \), then there exists neutrosophic crisp open set \( V \) containing \( P \), with \( C \cap V \subseteq \text{NI}(P_1) \). Then by hereditary property of neutrosophic crisp ideal, we get, \( D \cap U \cap V \subseteq \text{NI}(P_1) \) and \( C \cap U \cap V \subseteq \text{NI}(P_1) \). Again by the finite additivity condition of neutrosophic crisp ideal, we get \( (D \cup C) \cap U \cap V \subseteq \text{NI}(P_1) \). Hence \( P_2 \notin N(D \cup C)^{\tau} P_1 \). So,
\[ N(D \cup C)^{\tau} P_1 \subseteq \text{ND}^{\tau} P_1 \cup \text{NC}^{\tau} P_1 \]

From (1) and (2) we get, \( \text{ND}^{\tau} P_1 \cup \text{NC}^{\tau} P_1 = N(D \cup C)^{\tau} P_1 \).

### 4.6. Theorem

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Let \((H, T)\) be NCTS, \(P_1 \in \text{NCPN in } H\) and \(D, C\) be subsets of \((H, T)\). Then \(N(D \cap C)^{P_1} \subseteq ND^{P_1} \cap NC^{P_1}\).

**Proof**

It is known that \(D \cap C \subseteq D\) and \(D \cap C \subseteq C\), then from theorem part (3), \(N(D \cap C)^{P_1} \subseteq ND^{P_1}\) and \(N(D \cap C)^{P_1} \subseteq NC^{P_1}\). Hence \(N(D \cap C)^{P_1} \subseteq ND^{P_1} \cap NC^{P_1}\), for any \(P_1 \in \text{NCPN in } H\).

### 4.7 Theorem

Let \((H, T)\) be NCTS, \(a_{N_1} \in \text{NCPN in } H\), for each neutrosophic crisp open set \(U\) containing \(a_{N_1}\), then \((a_{N_1})^*a_{N_1} \subseteq U\).

**Proof**

Let \(b_{N_1} \in U\), so \(a_{N_1} \neq b_{N_1}\), then we get that \(U \cap (b_{N_1}) = \emptyset \in \text{NI}(a_{N_1})\). That means \((b_{N_1}) \neq (a_{N_1})^*a_{N_1}\). Thus \((a_{N_1})^*a_{N_1} \subseteq U\).

### 4.8 Theorem

Let \((H, T)\) be NCTS, \(P_1 \in \text{NCPN in } H\) and \(D\) be subsets of \((H, T)\). Then

\[
D^{P_1} = \begin{cases} \emptyset_N & \text{if } P_1 \notin D \\ \text{cl}(P_1) & \text{if } P_1 \in D \end{cases}
\]

**Proof**

**Case (1)**

If \(P_1 \notin D\), To prove \(D^{P_1} = \emptyset\). Let \(D^{P_1} \neq \emptyset\), then there exists least one element say \(P_2 \in D^{P_1}\) (by definition of \(D^{P_1}\)), we have \(C_{P_2} \cap D \notin \text{NI}(P_1)\). Hence \(P_1 \notin D \cap C_{P_2}\). So \(P_1 \notin D\), which contradiction! Then \(D^{P_1} = \emptyset_N\).

**Case (2)**

If \(P_1 \in D\), to prove \(D^{P_1} = \text{cl}(P_1)\). Let \(P_2 \in D^{P_1}\) implies \(P_1 \in D \cap V_{P_2}\) for each \(V_{P_2} \in T\). It follows \(P_2 \in \text{cl}(P_1)\) then \(D^{P_1} \subseteq \text{cl}(P_1)\) for each \(D\) be subsets of \((H, T)\). Let \(P_2 \in \text{cl}(P_1)\) and \(P_2 \notin D^{P_1}\) then there exists neutrosophic crisp open set \(V_{P_2}\) containing \(P_2\) such that \(D \cap V_{P_2} \notin \text{NI}(P_1)\), which implies \(P_1 \notin D \cap V_{P_2}\) then \(P_1 \notin D\) or \(P_1 \notin V_{P_2}\) which means that \(P_1 \notin D\) or \(P_1 \notin \text{cl}(P_1)\) which contradiction! in two case. Hence \(P_2 \in D^{P_1}\) implies that \(\text{cl}(P_1) \subseteq D^{P_1}\). Therefore, \(D^{P_1} = \text{cl}(P_1)\), if \(P_1 \in D\).

### 4.9 Definition

Let \((H, T)\) be NCTS. Then, the mapping \(f:(H, T) \rightarrow (Y, \delta)\) is called NI*-map if and only if, for every subset \(D\) of \((H, T)\), \(P_1 \in \text{NCPN in } H\), \(f(D^{P_1}) = (f(D))^*P_1\).

### 4.10 Example

Let \((H, T)\) be NCTS, such that \(H = \{1, 2, 3\}\), \(Y = \{a, b, c\}\),

\[
T = \{\emptyset_N, H_N, A, B\}, \quad \delta = \{\emptyset_N, Y_N, G\}, \text{ such that .}
\]

A \(<\{1\}, \emptyset, \emptyset \rangle, B = \langle \{2, 3\}, \emptyset, \emptyset \rangle, G = \langle \{a\}, \emptyset, \emptyset \rangle.\]

Define \(f(2) = f(1) = c\) and \(f(3) = a\). Put \(D = \{3\}\) subset of \((H, T)\). Then \(D^* = B = \langle \{2, 3\}, \emptyset, \emptyset \rangle\), so \(f(D^*) = (f(D))^*P_1 = \langle \{a, c\}, \emptyset, \emptyset \rangle^*a = \langle \{a, b, c\}, \emptyset, \emptyset \rangle\).

### 4.11 Definition

Let \((H, T)\) be NCTS. Then, the mapping \(f:(H, T) \rightarrow (Y, \delta)\) is called NI**-map if and only if, for every subset \(D\) of \((Y, \delta)\), \(P \in \text{NCPN in } Y\), \(f(D)^P) = (f^{-1}(D))^P\).

### 4.12 Example

Let \((H, T)\) be NCTS, such that \(H = \{a, b, c\}\), \(Y = \{1, 2, 3\}\)

\[
T = \{\emptyset_N, H_N, A, B\}, \quad \delta = \{\emptyset_N, Y_N, G\}, \text{ such that .}
\]
A =\{a\}, \emptyset, \emptyset >,  B= \{b, c\}, \emptyset, \emptyset >,  G =\{1\}, \emptyset, \emptyset >.

Define f(b)=f(a)=3 and f(c)=1. Put D=[3] subset of (Y, δ).

Then \(D^* = B = \{2, 3\}, \emptyset, \emptyset >, \) so \(f^i(D^*) = (f^i(D))^c = (\{b, a\}, \emptyset, \emptyset >)^c = \{b, c\}, \emptyset, \emptyset >\).

**Conclusion**

We defined a new types of neutrosophic crisp closed sets and limit points in neutrosophic crisp topological space namely [neutrosophic crisp Gm sets and neutrosophic crisp Turing points] respectively, we study their properties in details and we also use it to introduce the some of topological concepts as: neutrosophic crisp closed (open) sets, neutrosophic crisp closure, neutrosophic crisp interior, neutrosophic crisp exterior, and neutrosophic crisp boundary which are fundamental for further research on neutrosophic crisp topology and will strengthen the foundations of theory of neutrosophic topological spaces.

We expect, this paper will promote the future study on neutrosophic crisp topological spaces and many other general frameworks.

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Received: May 3, 2020. Accepted: September 21, 2020