

# New Types of Topologies and Neutrosophic Topologies (Improved Version)

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**Abstract:** In this paper we recall the six new types of topologies, and their corresponding topological spaces, that we introduced in the last years (2019-20223), such as: NeutroTopology, AntiTopology, Refined Neutrosophic Topology, Refined Neutrosophic Crisp Topology, SuperHyperTopology, and Neutrosophic SuperHyperTopology.

The n<sup>th</sup>-PowerSets  $P^n(H)$  and  $P^n_*(H)$ , that the SuperHyperTopology and respectively Neutrosophic SuperHyperTopology are based on, better describe our real world, since a system H(that may be a set, company, institution, country, region, etc.) is organized in sub-systems, which in their turn are organized each in sub-systems, and so on.

**Keywords:** Classical Topology; Topological Space; NeutroTopology; AntiTopology; Refined Neutrosophic Topology; Refined Neutrosophic Crisp Topology; SuperHyperTopology; Neutrosophic SuperHyperTopology.

# 1. Introduction

We recall the classical definition of Topology, then the procedures of NeutroSophication and respectively AntiSophication of it, that result in adding in two new types of topologies: NeutroTopology and respectively AntiTopology.

Then we define topology on Refined Neutrosophic Set (2013), Refined Neutrosophic Crisp Set [3]. Afterwards, we extend the topology on the framework of SuperHyperAlgebra [6].

The corresponding neutrosophic topological spaces are presented.

This research is an improvement of paper [7].

2. Classical Topology

Let  $\mathcal{U}$  be a non-empty set, and  $P(\mathcal{U})$  the power set of  $\mathcal{U}$ .

Let  $\tau \subseteq P(\mathcal{U})$  be a family of subsets of  $\mathcal{U}$ .

Then  $\tau$  is called a Classical Topology on  $\mathcal{U}$  if it satisfies the following axioms:

(CT-1)  $\phi$  and  $\mathcal{U}$  belong to  $\tau$ .

(CT-2) The intersection of any finite number of elements in  $\tau$  is in  $\tau$ .

(CT-3) The union of any finite or infinite number of elements in  $\tau$  is in  $\tau$ .

All three axioms are totally (100%) true (or T = 1, I = 0, F = 0). We simply call them (classical) *Axioms*. Then ( $\mathcal{U}$ ,  $\tau$ ) is called a *Classical Topological Space* on  $\mathcal{U}$ .

#### 3. <u>NeutroSophication of the Topological Axioms</u>

*NeutroSophication of the topological axioms* means that the axioms become partially true, partially indeterminate, and partially false. They are called *NeutroAxioms*.

(NCT-1) Either { $\phi \notin \tau$  and  $\mathcal{U} \in \tau$ }, or { $\phi \in \tau$  and  $\mathcal{U} \notin \tau$ }.

(NCT-2) There exist a finite number of elements in  $\tau$  whose intersection belong to  $\tau$  (degree of truth *T*); and a finite number of elements in  $\tau$  whose intersection is indeterminate (degree of indeterminacy *I*); and a finite number of elements in  $\tau$  whose intersection does not belong to  $\tau$  (degree of falsehood *F*); where (*T*, *I*, *F*)  $\notin$  {(1, 0, 0), (0, 0, 1)} since (1, 0, 0) represents the above Classical Topology, while (0, 0, 1) the below AntiTopology.

(NCT-3) There exist a finite or infinite number of elements in  $\tau$  whose union belongs to  $\tau$  (degree of truth *T*); and a finite or infinite number of elements in  $\tau$  whose union is indeterminate (degree of indeterminacy *I*); and a finite or infinite number of elements in  $\tau$  whose union does not belong to  $\tau$  (degree of falsehood *F*); where of course (*T*, *I*, *F*)  $\notin$  {(1, 0, 0), (0, 0, 1)}.

## 4. AntiSophication of the Topological Axioms

AntiSophication of the topological axioms means to negate (anti) the axioms, the axioms become totally (100%) false (or T = 0, I = 0, F = 1). They are called *AntiAxioms*.

(ACT-1)  $\phi \notin \tau$  and  $\mathcal{U} \notin \tau$ .

(ACT-2) The intersection of any finite number ( $n \ge 2$ ) of elements in  $\tau$  is not in  $\tau$ .

(ACT-3) The union of any finite or infinite number ( $n \ge 2$ ) of elements in  $\tau$  is not in  $\tau$ .

5. Neutrosophic Triplets related to Topology

As such, we have a neutrosophic triplet of the form:

<Axiom(1, 0, 0), NeutroAxiom(*T*, *I*, *F*), AntiAxiom(0, 0, 1)>,

where  $(T, I, F) \neq (1, 0, 0)$  and  $(T, I, F) \neq (0, 0, 1)$ .

Correspondingly, one has:

<Topology, NeutroTopology, AntiTopology>.

Therefore, in general:

(*Classical*) *Topology* is a topology that has all axioms totally true. We simply call them *Axioms*. *NeutroTopology* is a topology that has at least one *NeutroAxiom* and the others are all *classical Axioms* [therefore, no AntiAxiom].

*AntiTopology* is a topology that has one or more *AntiAxioms*, no matter what the others are (*classical Axioms*, or *NeutroAxioms*).

## 6. Theorem on the number of Structures/NeutroStructures/AntiStructures

If a Structure has *m* axioms, with  $m \ge 1$ , then after NeutroSophication and AntiSophication one obtains  $3^m$  types of structures, categorized as follows: 1 *Classical Structure* +  $(2^m - 1)$  *NeutroStructures* +  $(3^m - 2^m)$  *AntiStructures*.

## 7. Consequence on the number of Topologies/NeutroTopologies/AntiTopologies

As a particular case of the previous theorem, from a Topology which has m = 3 axioms, one makes, after NeutrosSophication and AntiSophication,  $3^3 = 27$  types of structures, as follows: 1 classical Topology,  $2^3 - 1 = 7$  NeutroTopologies, and  $3^3 - 2^2 = 19$  AntiTopologies.

1 Classical Topology + 7 NeutroTopologies + 19 AntiTopologies

are presented below:

There is 1 (one) type of Classical Topology, whose axioms are listed below: 1 *Classical Topology* 

$$\begin{pmatrix} CT-1\\ CT-2\\ CT-3 \end{pmatrix}$$

# 8. Definition of NeutroTopology [4, 5]

It is a topology that has at least one topological axiom which is partially true, partially indeterminate, and partially false,

or (*T*, *I*, *F*), where T = True, *I* = Indeterminacy, *F* = False,

and no topological axiom is totally false,

in other words:  $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$ , where (1, 0, 0) represents the classical Topology, while (0, 0, 1) represents the below AntiTopology.

Therefore, the NeutroTopology is a topology in between the classical Topology and the AntiTopology.

There are 7 types of different NeutroTopologies, whose axioms, for each type, are listed below:

7 NeutroTopologies

$$\begin{pmatrix} NCT-1\\ CT-2\\ CT-3 \end{pmatrix}, \begin{pmatrix} CT-1\\ NCT-2\\ CT-3 \end{pmatrix}, \begin{pmatrix} CT-1\\ CT-2\\ NCT-3 \end{pmatrix}, \begin{pmatrix} CT-1\\ CT-2\\ NCT-3 \end{pmatrix}, \begin{pmatrix} NCT-1\\ NCT-2\\ NCT-3 \end{pmatrix}, \begin{pmatrix} NCT-1\\ CT-2\\ NCT-3 \end{pmatrix}, \begin{pmatrix} NCT-1\\ NCT-2\\ NCT-2 \end{pmatrix}, \begin{pmatrix} NCT-1\\ NCT-2 \end{pmatrix}, \begin{pmatrix} NCT-2\\ NCT-2 \end{pmatrix}, \begin{pmatrix} NCT-2\\ NCT-2 \end{pmatrix}, \begin{pmatrix} N$$

# 9. Definition of AntiTopology [4, 5]

It is a topology that has at least one topological axiom that is 100% false (T, I, F) = (0, 0, 1). The NeutroTopology and AntiTopology are particular cases of NeutroAlgebra and AntiAlgebra [4] and, in general, they all are particular cases of the NeutroStructure and AntiStructure respectively, since we consider "Structure" in any field of knowledge [5].

There are 19 types of different AntiTopologies, whose axioms, for each type, are listed below:

19 AntiTopologies

$$\begin{pmatrix} ACT - 1 \\ CT - 2 \\ CT - 3 \end{pmatrix}, \begin{pmatrix} CT - 1 \\ ACT - 2 \\ CT - 3 \end{pmatrix}, \begin{pmatrix} CT - 1 \\ CT - 2 \\ ACT - 2 \\ ACT - 3 \end{pmatrix}, \begin{pmatrix} CT - 1 \\ ACT - 2 \\ ACT - 3 \end{pmatrix}, \begin{pmatrix} ACT - 1 \\ CT - 2 \\ ACT - 3 \end{pmatrix}, \begin{pmatrix} ACT - 1 \\ CT - 2 \\ ACT - 3 \end{pmatrix}, \begin{pmatrix} ACT - 1 \\ CT - 2 \\ ACT - 3 \end{pmatrix}, \begin{pmatrix} ACT - 1 \\ CT - 2 \\ ACT - 3 \end{pmatrix}, \begin{pmatrix} NCT - 1 \\ ACT - 2 \\ NCT - 3 \end{pmatrix}, \begin{pmatrix} NCT - 1 \\ ACT - 2 \\ NCT - 3 \end{pmatrix}, \begin{pmatrix} NCT - 1 \\ ACT - 2 \\ ACT - 3 \end{pmatrix}, \begin{pmatrix} NCT - 1 \\ NCT - 2 \\ ACT - 3 \end{pmatrix}, \begin{pmatrix} CT - 1 \\ CT - 2 \\ ACT - 3 \end{pmatrix}, \begin{pmatrix} CT - 1 \\ CT - 2 \\ CT - 3$$

$$\begin{pmatrix} ACT - 1 \\ NCT - 2 \\ CT - 3 \end{pmatrix}, \begin{pmatrix} CT - 1 \\ ACT - 2 \\ NCT - 3 \end{pmatrix}, \begin{pmatrix} NCT - 1 \\ CT - 2 \\ ACT - 3 \end{pmatrix}, \begin{pmatrix} CT - 1 \\ NCT - 2 \\ ACT - 3 \end{pmatrix}, \begin{pmatrix} NCT - 1 \\ ACT - 2 \\ CT - 3 \end{pmatrix}, \begin{pmatrix} CT - 1 \\ ACT - 2 \\ CT - 3 \end{pmatrix}, \begin{pmatrix} NCT - 1 \\ ACT - 2 \\ CT - 3 \end{pmatrix}, \begin{pmatrix} ACT - 1 \\ ACT - 2 \\ ACT - 3 \end{pmatrix}.$$

#### 10. <u>Refined Neutrosophic Set</u>

Let U be a universe of discourse, and a non-empty subset R of it,

$$R = \begin{cases} x \left( T_1(x), T_2(x), \dots, T_p(x) \right); \\ (I_1(x), I_2(x), \dots, I_r(x)); \\ (F_1(x), F_2(x), \dots, F_s(x)); \end{cases}$$

with all  $T_j, I_k, F_l \in [0,1]$ ,  $1 \le j \le p, 1 \le k \le r, 1 \le l \le s$ , and no restriction on their sums  $0 \le T_m + I_m + F_m \le 3$ , with  $1 \le m \le \max\{p, r, s\}$ , where  $p, r, s \ge 0$  are fixed integers, and at least one of them is  $\ge 2$ , in order to ensure the refinement (sub-parts) or multiplicity (multi-parts) – depending on the application, of at least one neutrosophic component amongst T (truth), I (indeterminacy), F (falsehood); and of course  $x \in U$ .

By notation we consider that index zero means the empty-set, i.e.  $T_0 = I_0 = F_0 = \phi$  (or zero), and

the same for the missing sub-parts (or multi-parts).

For example, the below (2,3,1)-Refined Neutrosophic Set is identical to a (3,3,3)-Refined Neutrosophic Set:  $(T_1, T_2; I_1, I_2, I_3; F_1) \equiv (T_1, T_2, 0; I_1, I_2, I_3; F_1, 0, 0)$ ,

where the missing components T<sub>3</sub>, and F<sub>2</sub>, F<sub>3</sub> were replaced each of them by 0 (zero) R is called a (p, r, s)-refined neutrosophic set { or (p, r, s)-RNT }.

The neutrosophic set has been extended to the Refined Neutrosophic Set (Logic, and Probability) by Smarandache [1] in 2013, where there are multiple parts of the neutrosophic components, as such T was split into subcomponents  $T_1, T_2, ..., T_p$ , and I into  $I_1, I_2, ..., I_r$ , and F into  $F_1, F_2, ..., F_s$ , with  $p + r + s = n \ge 2$  and integers p, r,  $s \ge 0$  and at least one of them is  $\ge 2$  in order to ensure the refinement (or multiplicity) of at least one neutrosophic component amongst T, I, and F.

Even more: the subcomponents  $T_{i}$ ,  $I_{k}$ , and/or  $F_{1}$  can be countable or uncountable infinite subsets of [0, 1].

This definition also includes the *Refined Fuzzy Set*, when r = s = 0 and  $p \ge 2$ ;

and the definition of the *Refined Intuitionistic Fuzzy Set*, when r = 0, and either  $p \ge 2$  and  $s \ge 1$ , or  $p \ge 1$  and  $s \ge 2$ .

All other fuzzy extension sets (Pythagorean Fuzzy Set, Spherical Fuzzy Set, Fermatean Fuzzy Set, q-Rung Orthopair Fuzzy Set, etc.) can be refined/multiplicated in a similar way.

#### 11. Definition of Refined Neutrosophic Topology

Let  $\mathcal{U}$  be a universe of discourse, and  $\mathcal{P}(\mathcal{U})$  be the family of all (p, r, s)-refined neutrosophic

subsets of  $\mathcal{U}$ .

Let  $\tau_{RNT} \subseteq \mathcal{P}(\mathcal{U})$  be a family of (p, r, s)-refined neutrosophic subsets of  $\mathcal{U}$ .

Then  $\tau_{RNT}$  is called a *Refined Neutrosophic Topology* (*RNT*) if it satisfies the axioms:

(RNT-1)  $\phi$  and  $\mathcal{U}$  belong to  $\tau_{RNT}$ ;

(RNT-2) The intersection of any finite number of elements in  $\tau_{RNT}$  is in  $\tau_{RNT}$ ;

(RNT-3) The union of any finite or infinite number of elements in  $\tau_{RNT}$  is in  $\tau_{RNT}$ .

Then ( $\mathcal{U}, \tau_{RNT}$ ) is called a Refined Neutrosophic Topological Space on  $\mathcal{U}$ .

The Refined Neutrosophic Topology is a topology defined on a Refined Neutrosophic Set.

{Similarly, the Refined Fuzzy Topology is defined on a Refined Fuzzy Set, while the Refined Intuitionistic Fuzzy Topology is defined on a Refined Intuitionistic Fuzzy Set, etc.

And, as a generalization, on any type of fuzzy extension set [such as: Pythagorean Fuzzy Set, Spherical Fuzzy Set, Fermatean Fuzzy Set, q-Rung Orthopair Fuzzy Set, etc.] one can define a corresponding fuzzy extension topology.}

#### 12. Neutrosophic Crisp Set

The *Neutrosophic Crisp Set* was defined by Salama and Smarandache in 2014 and 2015. Let X be a non-empty fixed space. And let *D* be a Neutrosophic Crisp Set [2], where  $D = \langle A, B, C \rangle$ , with *A*, *B*, *C* as subsets of *X*. Depending on the intersections and unions between these three sets *A*, *B*, *C* one gets several: Types of Neutrosophic Crisp Sets [2, 3]

The object having the form  $D = \langle A, B, C \rangle$  is called:

(a) A neutrosophic crisp set of Type 1 (NCS-Type1) if it satisfies:

$$A \cap B = B \cap C = C \cap A = \overset{\text{p}}{=} (\text{empty set}).$$

(b) A neutrosophic crisp set of Type 2 (NCS-Type2) if it satisfies:

$$A \cap B = B \cap C = C \cap A = \overset{\phi}{=} and A \cup B \cup C = X$$

(c) A neutrosophic crisp set of Type 3 (NCS-Type3) if it satisfies:

$$A \cap B \cap C = \overset{\phi}{=} and A \cup B \cup C = X.$$

Of course, more types of Neutrosophic Crisp Sets may be defined by modifying the intersections and unions of the subsets *A*, *B*, and *C*.

## 13. Refined Neutrosophic Crisp Set

The *Refined Neutrosophic Crisp Set* [3] was introduced by Smarandache in 2019, by refining/multiplication of *D* (and denoting it by *RD* = Refined D) by refining/multiplication of its sets *A*, *B*, *C* into sub-subsets/multi-sets as follows:

 $RD = (A_1, ..., A_p; B_1, ..., B_r; C_1, ..., C_s)$ , with  $p, r, s \ge 1$  be positive integers and at least one of them be  $\ge 2$  in order to ensure the refinement/multiplication of at least one component amongs A, B, C, where

$$A = \bigcup_{i=1}^{p} A_i, B = \bigcup_{j=1}^{r} B_j, C = \bigcup_{k=1}^{s} C_k$$

and many types of Refined Neutrosophic Crisp Sets may be defined by modifying the intersections or unions of the subsets/multisets  $A_i, B_j, C_k, 1 \le i \le p, 1 \le j \le r, 1 \le k \le s$ , depending on each application.

#### 14. Definition of Refined Neutrosophic Crisp Topology

Let  $\mathcal{U}$  be a universe of discourse, and  $\mathcal{P}(\mathcal{U})$  be the family of all (*p*, *r*, *s*)-refined neutrosophic crisp

subsets of  $\mathcal{U}$ .

Let  $\tau_{RNCT} \subseteq \mathcal{P}(\mathcal{U})$  be a family of (p, r, s)-refined neutrosophic crisp subsets of  $\mathcal{U}$ .

Then  $\tau_{RNCT}$  is called a *Refined Neutrosophic Crisp Topology* (*RNCT*) if it satisfies the axioms:

(RNCT-1)  $\phi$  and  $\mathcal{U}$  belong to  $\tau_{RNCT}$ ;

(RNCT-2) The intersection of any finite number of elements in  $\tau_{RNCT}$  is in  $\tau_{RNCT}$ ;

(RNCT-3) The union of any finite or infinite number of elements in  $\tau_{RNCT}$  is in  $\tau_{RNCT}$ .

Then ( $\mathcal{U}, \tau_{RNCT}$ ) is called a Refined Neutrosophic Crisp Topological Space on  $\mathcal{U}$ .

Therefore, the *Refined Neutrosophic Crisp Topology* is a topology defined on the Refined Neutrosophic Crisp Set.

#### 15. SuperHyperOperation

We recall our 2016 concepts of SuperHyperOperation, SuperHyperAxiom, SuperHyperAlgebra, and their corresponding Neutrosophic SuperHyperOperation Neutrosophic SuperHyperAxiom and Neutrosophic SuperHyperAlgebra [6].

Let  $P_*^n(H)$  be the n<sup>th</sup>-powerset of the set *H* such that none of P(H),  $P^2(H)$ , ...,  $P^n(H)$  contain the empty set  $\phi$ .

Also, let  $P_n(H)$  be the n<sup>th</sup>-powerset of the set H such that at least one of the P(H),  $P^2(H)$ , ...,  $P^n(H)$  contain the empty set  $\phi$ . For any subset A, we identify {A} with A.

The SuperHyperOperations are operations whose codomain is either  $P_*^n(H)$  and in this case one has classical-type SuperHyperOperations, or  $P^n(H)$  and in this case one has Neutrosophic SuperHyperOperations, for integer  $n \ge 2$ .

## 16. The nth-PowerSet better describe our real world

The n<sup>th</sup>-PowerSets  $P^n(H)$  and  $P^n_*(H)$ , that the SuperHyperTopology and respectively Neutrosophic SuperHyperTopology are based on, better describe our real world, since a system H(that may be a set, company, institution, country, region, etc.) is organized in sub-systems, which in their turn are organized each in sub-systems, and so on.

## 17. SuperHyperAxiom

A classical-type SuperHyperAxiom or more accurately a (*m*, *n*)-SuperHyperAxiom is an axiom based on classical-type SuperHyperOperations.

Similarly, a Neutrosophic SuperHyperAxiom {or Neutrosphic (m, n)-SuperHyperAxiom} is an axiom based on Neutrosophic SuperHyperOperations.

There are:

- Strong SuperHyperAxioms, when the left-hand side is equal to the right-hand side as in non-hyper axioms,
- and Week SuperHyperAxioms, when the intersection between the left-hand side and the right-hand side is non-empty.

#### 18. SuperHyperAlgebra and SuperHyperStructure

A SuperHyperAlgebra or more accurately (*m*-*n*)-SuperHyperAlgebra is an algebra dealing with SuperHyperOperations and SuperHyperAxioms.

Again, a Neutrosophic SuperHyperAlgebra {or Neutrosphic (m, n)-SuperHyperAlgebra} is an algebra dealing with Neutrosophic SuperHyperOperations and Neutrosophic SuperHyperOperations.

In general, we have SuperHyperStructures {or (m-n)-SuperHyperStructures}, and corresponding Neutrosophic SuperHyperStructures.

For example, there are SuperHyperGrupoid, SuperHyperSemigroup, SuperHyperGroup, SuperHyperRing, SuperHyperVectorSpace, etc.

- 19. Distinction between SuperHyperAlgebra vs. Neutrosophic SuperHyperAlgebra
  - i. If none of the power sets  $P^k(H)$ ,  $1 \le k \le n$ , do not include the empty set  $\phi$ , then one has a classical-type SuperHyperAlgebra;
  - ii. If at least one power set,  $P^k(H)$ ,  $1 \le k \le n$ , includes the empty set  $\phi$ , then one has a

Neutrosophic SuperHyperAlgebra.

## 20. Definition of SuperHyperTopology (SHT) [6]

It is a topology designed on the n<sup>th</sup>-PowerSet of a given non-empty set H, that excludes the

empty-set, denoted as  $P_*^n(H)$ , built as follows:

 $P_*(H)$  is the first powerset of the set *H*, and the index \* means without the empty-set ( $\emptyset$ );

 $P_*^2(H) = P_*(P_*(H))$  is the second powerset of *H* (or the powerset of the powerset of *H*), without the

empty-sets; and so on,

the *n*-th powerset of *H*,

 $P_*^n(H) = P_*(P_*^{n-1}(H)) = \underbrace{P_*(P_*(\dots,P_*(H)\dots))}_n, \text{ where } P_* \text{ is repeated } n \text{ time } (n \ge 2), \text{ and without the}$ 

empty-sets.

Let consider  $\tau_{SHT}$  be a family of subsets of  $P_*^n(H)$ .

Then  $\tau_{SHT}$  is called a Neutrosophic SuperHyperTopology on  $P_*^n(H)$ , if it satisfies the following axioms:

(SHT-1)  $\boldsymbol{\phi}$  and  $P^n_*(H)$  belong to  $\tau_{\scriptscriptstyle SHT}$ .

(SHT-2) The intersection of any finite number of elements in  $\tau_{\scriptscriptstyle SHT}$  is in  $\tau_{\scriptscriptstyle SHT}$ .

(SHT-3) The union of any finite or infinite number of elements in  $\tau_{SHT}$  is in  $\tau_{SHT}$ .

Then  $(P_*^n(H), \tau_{SHT})$  is called a SuperHyperTopological Space on  $P_*^n(H)$ .

# 21. Definition of Neutrosophic SuperHyperTopology (NSHT) [6]

It is, similarly, a topology designed on the *n*-th PowerSet of a given non-empty set *H*, but includes the empty-sets [that represent indeterminacies] too.

As such, in the above formulas,  $P_*(H)$  that excludes the empty-set, is replaced by P(H) that

includes the empty-set.

P(H) is the first powerset of the set *H*, including the empty-set ( $\emptyset$ );

 $P^{2}(H) = P(P(H))$  is the second powerset of *H* (or the powerset of the powerset of *H*), that

includes the empty-sets;

and so on, the *n*-th powerset of *H*,

$$P^{n}(H) = P(P^{n-1}(H)) = \underbrace{P(P(...P(H)...))}_{n}$$

where *P* is repeated *n* times ( $n \ge 2$ ), and includes the empty-sets ( $\emptyset$ ).

Let consider  $\tau_{NSHT}$  be a family of subsets of  $P^n(H)$ .

Then  $\tau_{NSHT}$  is called a Neutrosophic SuperHyperTopology on  $P^{n}(H)$ , if it satisfies the following axioms:

axioms:

(NSHT-1)  $\phi$  and  $P^n(H)$  belong to  $\tau_{NSHT}$ .

(NSHT-2) The intersection of any finite number of elements in  $\tau_{\rm NSHT}$  is in  $\tau_{\rm NSHT}$ .

(NSHT-3) The union of any finite or infinite number of elements in  $\tau_{\scriptscriptstyle NSHT}$  is in  $\tau_{\scriptscriptstyle NSHT}$ .

Then  $(P^n(H), \tau_{NSHT})$  is called a Neutrosophic SuperHyperTopological Space on  $P^n(H)$ .

# 22. Conclusion

These six new types of topologies, and their corresponding topological space, were introduced by Smarandache in 2019-2023, but they have not yet been much studied and applied, except the NeutroTopologies and AntiTopologies which got some attention from researchers.

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