



Ngpr Homeomorphism in Neutrosophic Topological Spaces

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Abstract: As a generalization of Fuzzy sets introduced by Zadeh [21] in 1965 and Intuitionistic Fuzzy sets introduced by Atanassav [8] in 1983, the Neutrosophic set had been introduced and developed by Smarandache. A Neutrosophic set is characterized by a truth value (membership), an indeterminacy value and a falsity value (non-membership). Salama and Alblowi [17] introduced the new concept of neutrosophic topological space (NTS) in 2012, which had been investigated recently. In 2018, Parimala M et al. introduced and studied the concept of Neutrosophic homeomorphism and Neutrosophic $\alpha\psi$ homeomorphism in Neutrosophic topological spaces. The impact of this article is to introduce and study the concepts of Ngpr homeomorphism and Nigpr homeomorphism in Neutrosophic topological space. Further, the work is extended to Ngpr open mappings, Ngpr closed mappings, Nigpr closed mappings and some of their properties are explored in Neutrosophic topological space.

Keywords: Neutrosophic generalized pre regular closed set, Ngpr open mappings, Ngpr closed mappings, Ngpr homeomorphism and Nigpr homeomorphism.

1. Introduction

Zadeh [21] introduced the concept of fuzzy set in 1965 and Chang C. L. [9] introduced fuzzy topological spaces in 1968. Later, Atanassov [8] proposed the concept of intuitionistic fuzzy sets in 1986, where the degree of membership and degree of non-membership are discussed. Intuitionistic fuzzy topological spaces was introduced by Coker [10] in 1997 using intuitionistic fuzzy sets. As a generalization of Fuzzy sets and Intuitionistic Fuzzy sets, Neutrosophic set have been introduced and developed by Florentin Smarandache [12]. He also defined the Neutrosophic set on three components, namely Truth (membership) (T), Indeterminacy (I) and Falsehood (non-membership) (F).

Neutrosophic concept has wide range of real time applications in the fields of [1 - 6] Information Systems, Computer Science, Artificial Intelligence, Applied Mathematics and Decision Making, Uncertainty assessments of linear time-cost tradeoffs and solving the supply chain problem.

In 2012, Salama A. A and Alblowi [17] introduced the concept of Neutrosophic topological space by using Neutrosophic sets. Salama A. A. [18] introduced Neutrosophic closed set and Neutrosophic continuous function in Neutrosophic topological spaces and their properties are studied by various authors [7 & 11]. Since, Neutrosophic homeomorphism plays an important role in Neutrosophic topology. Parimala M et al. [14] introduced and studied the concept of Neutrosophic homeomorphism and Neutrosophic $\alpha\psi$ homeomorphism in Neutrosophic topological spaces. In this article, introduce and study few properties of Ngpr open mappings, Ngpr closed mappings, Nigpr closed mappings, Ngpr homeomorphism and Nigpr homeomorphism in Neutrosophic topological space. The present study demonstrates some of the related theorems, results and properties.

2. Preliminaries

2.1. Definition: [17] Let X be a non-empty fixed set. A Neutrosophic set (NS for short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ where the functions $\mu_A(x)$, $\sigma_A(x)$ and $\nu_A(x)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element $x \in X$ to the set A .

2.2 Remark: [17] A Neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ can be identified to an ordered triple $A = \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ in non-standard unit interval $]0, 1^+[$ on X .

2.3 Remark: [17] For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle$ for the neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$.

2.4 Example: [17] Every IFS A is a non-empty set in X is obviously on NS having the form $A = \{\langle x, \mu_A(x), 1 - (\mu_A(x) + \nu_A(x)), \nu_A(x) \rangle : x \in X\}$. Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topology, we must introduce the NS 0_N and 1_N in X as follows:

0_N may be defined as:

$$(0_1) 0_N = \{\langle x, 0, 0, 1 \rangle : x \in X\}$$

$$(0_2) 0_N = \{\langle x, 0, 1, 1 \rangle : x \in X\}$$

$$(0_3) 0_N = \{\langle x, 0, 1, 0 \rangle : x \in X\}$$

$$(0_4) 0_N = \{\langle x, 0, 0, 0 \rangle : x \in X\}$$

1_N may be defined as:

$$(1_1) 1_N = \{\langle x, 1, 0, 0 \rangle : x \in X\}$$

$$(1_2) 1_N = \{\langle x, 1, 0, 1 \rangle : x \in X\}$$

$$(1_3) 1_N = \{\langle x, 1, 1, 0 \rangle : x \in X\}$$

$$(1_4) 1_N = \{\langle x, 1, 1, 1 \rangle : x \in X\}$$

2.5 Definition: [17] Let $A = \langle \mu_A, \sigma_A, \nu_A \rangle$ be a NS on X , then the complement of the set A [$C(A)$ for short] may be defined as three kind of complements:

$$(C_1) C(A) = \{\langle x, 1 - \mu_A(x), 1 - \sigma_A(x), 1 - \nu_A(x) \rangle : x \in X\}$$

$$(C_2) C(A) = \{\langle x, \nu_A(x), \sigma_A(x), \mu_A(x) \rangle : x \in X\}$$

$$(C_3) C(A) = \{\langle x, \nu_A(x), 1 - \sigma_A(x), \mu_A(x) \rangle : x \in X\}$$

2.6 Definition: [17] Let X be a non-empty set and Neutrosophic sets A and B in the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x) \rangle : x \in X\}$. Then we may consider two possible definitions for subsets ($A \subseteq B$).

$$(1) A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x) \text{ and } \mu_A(x) \geq \mu_B(x) \quad \forall x \in X$$

$$(2) A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \geq \sigma_B(x) \text{ and } \mu_A(x) \geq \mu_B(x) \quad \forall x \in X$$

2.7 Proposition: [17] For any Neutrosophic set A , the following conditions hold:

$$0_N \subseteq A, 0_N \subseteq 0_N$$

$$A \subseteq 1_N, 1_N \subseteq 1_N$$

2.8 Definition: [17] Let X be a non-empty set and $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x) \rangle : x \in X\}$ are NSs. Then $A \cap B$ may be defined as:

$$(I_1) A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x) \text{ and } \nu_A(x) \vee \nu_B(x) \rangle$$

$$(I_2) A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x) \text{ and } \nu_A(x) \vee \nu_B(x) \rangle$$

$A \cup B$ may be defined as:

$$(U_1) A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x) \text{ and } \nu_A(x) \wedge \nu_B(x) \rangle$$

$$(U_2) A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x) \text{ and } \nu_A(x) \wedge \nu_B(x) \rangle$$

2.9 Definition: [17] A Neutrosophic topology [NT for short] is a non-empty set X is a family τ of Neutrosophic subsets in X satisfying the following axioms:

$$(NT_1) \quad 0_N, 1_N \in \tau,$$

$$(NT_2) \quad G_1 \cap G_2 \in \tau \text{ for any } G_1, G_2 \in \tau,$$

$$(NT_3) \quad \cup G_i \in \tau \text{ for every } \{G_i : i \in J\} \subseteq \tau.$$

Throughout this paper, the pair (X, τ) is called a Neutrosophic topological space (NTS for short). The elements of τ are called Neutrosophic open sets [NOS for short]. A complement $C(A)$ of a NOS A in NTS (X, τ) is called a Neutrosophic closed set [NCS for short] in X .

2.10 Definition: [17] Let (X, τ) be NTS and $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ be a NS in X . Then the Neutrosophic closure and Neutrosophic interior of A are defined by

$$NCl(A) = \cap \{K : K \text{ is a NCS in } X \text{ and } A \subseteq K\}$$

$$NInt(A) = \cup \{G : G \text{ is a NOS in } X \text{ and } G \subseteq A\}$$

It can be also shown that $NCl(A)$ is NCS and $NInt(A)$ is a NOS in X .

- a) A is NOS if and only if $A = NInt(A)$,
- b) A is NCS if and only if $A = NCl(A)$.

2.11 Definition: [13] A NS $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ in a NTS (X, τ) is said to be

- (i) Neutrosophic regular closed set (NRCS for short) if $A = NCl(NInt(A))$,
- (ii) Neutrosophic regular open set (NROS for short) if $A = NInt(NCl(A))$,
- (iii) Neutrosophic pre closed set (NPCS for short) if $NCl(NInt(A)) \subseteq A$,
- (iv) Neutrosophic pre open set (NPOS for short) if $A \subseteq NInt(NCl(A))$,
- (v) Neutrosophic α - closed set (NSCS for short) if $NCl(NInt(NCl(A))) \subseteq A$,
- (vi) Neutrosophic α - open set (NSOS for short) if $A \subseteq NInt(NCl(NInt(A)))$.

2.12 Definition: [19] Let (X, τ) be NTS and $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ be a NS in X . Then the Neutrosophic pre closure and Neutrosophic pre interior of A are defined by

$$NPcl(A) = \cap \{K : K \text{ is a NPCS in } X \text{ and } A \subseteq K\},$$

$$NPInt(A) = \cup \{G : G \text{ is a NPOS in } X \text{ and } G \subseteq A\}.$$

2.13 Definition: [15] A NS $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ in a NTS (X, τ) is said to be a Neutrosophic generalized closed set (NGCS for short) if $NCl(A) \subseteq U$ whenever $A \subseteq U$ and U is a NOS in (X, τ) . A NS A of a NTS (X, τ) is called a Neutrosophic generalized open set (NGOS for short) if $C(A)$ is a NGCS in (X, τ) .

2.14 Definition: [20] A NS $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ in a NTS (X, τ) is said to be a Neutrosophic generalized pre closed set (NGPCS for short) if $NPcl(A) \subseteq U$ whenever $A \subseteq U$ and U is a NOS in (X, τ) . A NS A of a NTS (X, τ) is called a Neutrosophic generalized pre open set (NGPOS for short) if $C(A)$ is a NGPCS in (X, τ) .

2.15 Definition: [13] A NS $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ in a NTS (X, τ) is said to be a Neutrosophic generalized pre regular closed set (NGPRCS for short) if $NPcl(A) \subseteq U$ whenever $A \subseteq U$ and U is a NROS in (X, τ) . The family of all NGPRCSs of a NTS (X, τ) is denoted by $NGPRC(X)$. A NS A of a NTS (X, τ) is called a Neutrosophic generalized pre regular open set (NGPROS for short) if $C(A)$ is a NGPRCS in (X, τ) .

Every NRCS, NCS, NWCS, $N\alpha$ CS, NGCS, NPCS, $N\alpha$ GCS, NGPCS, $NR\alpha$ GCS, NRGCS is an NGPRCS but the converses are not true in general.

2.16 Definition: [13] A Neutrosophic topological space (X, τ) is called a Neutrosophic pre regular $T_{1/2}$ (NPRT $_{1/2}$ for short) space if every NGPRCS in (X, τ) is NPCS in (X, τ) .

2.17 Definition: [13] A Neutrosophic topological space (X, τ) is called a Neutrosophic pre regular $T^*_{1/2}$ (NPRT $^*_{1/2}$ for short) space if every NGPRCS in (X, τ) is NCS in (X, τ) .

2.18 Definition: [16] Let (X, τ) and (Y, σ) be two NTSs. A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called Ngpr continuous (resp. NG continuous, NGP continuous) mapping if $f^{-1}(B)$ is NGPRCS (resp. NGCS, NGPCS) in (X, τ) for every NCS B of (Y, σ) .

Every Neutrosophic continuous, NG continuous, NGP continuous is a Ngpr continuous mapping but the converses are not true in general.

2.19 Definition: [16] Let (X, τ) and (Y, σ) be two NTSs. A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called Ngpr irresolute mapping if $f^{-1}(A)$ is NGPRCS in (X, τ) for every NGPRCS A of (Y, σ) .

2.20 Definition: [14] Let (X, τ) and (Y, σ) be two NTSs. A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called Neutrosophic closed mapping (resp. Neutrosophic open mapping) (NCM (resp. NOM) for short) if the image of every Neutrosophic closed set (resp. Neutrosophic open set) in (X, τ) is a Neutrosophic closed set (resp. Neutrosophic open set) in (Y, σ) .

2.21 Definition: [14] Let (X, τ) and (Y, σ) be two NTSs. A bijection $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a Neutrosophic homeomorphism if f and f^{-1} are Neutrosophic continuous mapping.

3. Ngpr open mappings and Ngpr closed mappings

In this section introduce Ngpr open mapping, Ngpr closed mapping and Nigpr closed mapping in the Neutrosophic topological space and study some of their properties. Also established the relation between the newly introduced mappings and already existing mappings.

3.1 Definition: Let (X, τ) and (Y, σ) be two NTSs. A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called

- (i) Neutrosophic generalized open mapping (NGOM for short) if $f(A)$ is NGOS in (Y, σ) for every NOS A of (X, τ) .
- (ii) Neutrosophic α open mapping (N α OM for short) if $f(A)$ is N α OS in (Y, σ) for every NOS A of (X, τ) .
- (iii) Neutrosophic pre-open mapping (NPOM for short) if $f(A)$ is NPOS in (Y, σ) for every NOS A of (X, τ) .
- (iv) Neutrosophic generalized pre-open mapping (NGPOM for short) if $f(A)$ is NGPOS in (Y, σ) for every NOS A of (X, τ) .

3.2 Definition: Let (X, τ) and (Y, σ) be two NTSs. A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called Ngpr open mapping (NGPROM for short) if $f(A)$ is NGPROS in (Y, σ) for every NOS A of (X, τ) .

3.3 Definition: Let (X, τ) and (Y, σ) be two NTSs. A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called

- (i) Neutrosophic generalized closed mapping (NGCM for short) if $f(A)$ is NGCS in (Y, σ) for every NCS A of (X, τ) .
- (ii) Neutrosophic α closed mapping (N α CM for short) if $f(A)$ is N α CS in (Y, σ) for every NCS A of (X, τ) .
- (iii) Neutrosophic pre-closed mapping (NPCM for short) if $f(A)$ is NPCS in (Y, σ) for every NCS A of (X, τ) .
- (iv) Neutrosophic generalized pre-closed mapping (NGPCM for short) if $f(A)$ is NGPCS in (Y, σ) for every NCS A of (X, τ) .

3.4 Definition: Let (X, τ) and (Y, σ) be two NTSs. A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called Ngpr closed mapping (NGPRCM for short) if $f(A)$ is NGPRCS in (Y, σ) for every NCS A of (X, τ) .

3.5 Example: Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, U, 1_N\}$ and $\sigma = \{0_N, V_1, V_2, 1_N\}$ are Neutrosophic topologies on X and Y respectively, where $U = \langle x, (0.4, 0.4, 0.5), (0.6, 0.3, 0.4) \rangle$ and $V_1 = \langle y, (0.7, 0.5, 0.3), (0.8, 0.4, 0.2) \rangle$ and $V_2 = \langle y, (0.6, 0.4, 0.4), (0.7, 0.3, 0.3) \rangle$. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here the Neutrosophic set $U^c = \langle x, (0.5, 0.6, 0.4), (0.4, 0.7, 0.6) \rangle$ is a Neutrosophic closed set in X . Then $f(U^c) = \langle y, (0.5, 0.6, 0.4), (0.4, 0.7, 0.6) \rangle$ is a NGPRCS in (Y, σ) as $f(U^c) \subseteq 1_N$ implies $Npcl(f(U^c)) = f(U^c) \subseteq 1_N$ where 1_N is a NROS in Y . Therefore f is a Ngpr closed mapping.

3.6 Proposition: Every Neutrosophic closed mapping is Ngpr closed mapping but not conversely in general.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a Neutrosophic closed mapping. Let A be a NCS in X . Then $f(A)$ is a NCS in Y . Since every NCS is a NGPRCS in Y , $f(A)$ is a NGPRCS in Y . Hence f is a Ngpr closed mapping.

3.7 Example: Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, U, 1_N\}$ and $\sigma = \{0_N, V_1, V_2, 1_N\}$ are Neutrosophic topologies on X and Y respectively, where $U = \langle x, (0.4, 0.4, 0.5), (0.6, 0.3, 0.4) \rangle$ and $V_1 = \langle y, (0.7, 0.5, 0.3), (0.8, 0.4, 0.2) \rangle$ and $V_2 = \langle y, (0.6, 0.4, 0.4), (0.7, 0.3, 0.3) \rangle$. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here the Neutrosophic set $U^c = \langle x, (0.5, 0.6, 0.4), (0.4, 0.7, 0.6) \rangle$ is a NCS in X . Then $f(U^c) = \langle y, (0.5, 0.6, 0.4), (0.4, 0.7, 0.6) \rangle$ is a NGPRCS in (Y, σ) as $f(U^c) \subseteq 1_N$ implies $Npcl(f(U^c)) = f(U^c) \subseteq 1_N$ where 1_N is a NROS in Y . Therefore f is a Ngpr closed mapping. But f is not a Neutrosophic closed mapping since U^c is NCS in X but $f(U^c)$ is not a NCS in Y as $Ncl(f(U^c)) = 1_N \neq f(U^c)$.

3.8 Proposition: Every Neutrosophic generalized closed mapping is Ngpr closed mapping but not conversely in general.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a Neutrosophic generalized closed mapping. Let A be a NCS in X . Then $f(A)$ is a NGCS in Y . Since every NGCS is a NGPRCS in Y , $f(A)$ is a NGPRCS in Y . Hence f is a Ngpr closed mapping.

3.9 Example: Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, U, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are Neutrosophic topologies on X and Y respectively, where $U = \langle x, (0.3, 0.5, 0.4), (0.2, 0.5, 0.3) \rangle$ and $V = \langle y, (0.6, 0.5, 0.2), (0.4, 0.5, 0.2) \rangle$. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here the Neutrosophic set $U^c = \langle x, (0.4, 0.5, 0.3), (0.3, 0.5, 0.2) \rangle$ is a NCS in X . Then $f(U^c) = \langle y, (0.4, 0.5, 0.3), (0.3, 0.5, 0.2) \rangle$ is a NGPRCS in (Y, σ) as $f(U^c) \subseteq 1_N$ implies $Npcl(f(U^c)) = f(U^c) \subseteq 1_N$ where 1_N is a NROS in Y . Therefore f is a Ngpr closed mapping. But f is not a Neutrosophic generalized closed mapping since U^c is NCS in X but $f(U^c)$ is not a NGCS in Y as $f(U^c) \subseteq V$ implies $Ncl(f(U^c)) = 1_N \notin V$.

3.10 Proposition: Every Neutrosophic α closed mapping is Ngpr closed mapping but not conversely in general.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a Neutrosophic α closed mapping. Let A be a NCS in X . Then $f(A)$ is a $N\alpha$ CS in Y . Since every $N\alpha$ CS is a NGPRCS in Y , $f(A)$ is a NGPRCS in Y . Hence f is a Ngpr closed mapping.

3.11 Example: Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, U, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are Neutrosophic topologies on X and Y respectively, where $U = \langle x, (0.4, 0.5, 0.4), (0.2, 0.5, 0.3) \rangle$ and $V = \langle y, (0.7, 0.5, 0.2), (0.3, 0.5, 0.2) \rangle$. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here the Neutrosophic set $U^c = \langle x, (0.4, 0.5, 0.4), (0.3, 0.5, 0.2) \rangle$ is a NCS in X . Then $f(U^c) = \langle y, (0.4, 0.5, 0.4), (0.3, 0.5, 0.2) \rangle$ is a NGPRCS in (Y, σ) as $f(U^c) \subseteq 1_N$ implies $Npcl(f(U^c)) = f(U^c) \subseteq 1_N$ where 1_N is a NROS in Y . Therefore f is a Ngpr closed mapping. But f is not a Neutrosophic α closed mapping since U^c is NCS in X but $f(U^c)$ is not a $N\alpha$ CS in Y as $Ncl(Nint(Ncl(f(U^c)))) = 1_N \notin f(U^c)$.

3.12 Proposition: Every Neutrosophic pre-closed mapping is Ngpr closed mapping but not conversely in general.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a Neutrosophic pre-closed mapping. Let A be a NCS in X . Then $f(A)$ is a NPCS in Y . Since every NPCS is a NGPRCS in Y , $f(A)$ is a NGPRCS in Y . Hence f is a Ngpr closed mapping.

3.13 Example: Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, U, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are Neutrosophic topologies on X and Y respectively, where $U = \langle x, (0.4, 0.5, 0.6), (0.2, 0.5, 0.3) \rangle$ and $V = \langle y, (0.3, 0.5, 0.7), (0.3, 0.5, 0.4) \rangle$. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here the Neutrosophic set $U^c = \langle x, (0.6, 0.5, 0.4), (0.3, 0.5, 0.2) \rangle$ is a NCS in X . Then $f(U^c) = \langle y, (0.6, 0.5, 0.4), (0.3, 0.5, 0.2) \rangle$ is a NGPRCS in (Y, σ) as $f(U^c) \subseteq 1_N$ implies $Npcl(f(U^c)) = \langle y, (0.7, 0.5, 0.3), (0.4, 0.5, 0.2) \rangle \subseteq 1_N$ where 1_N is a NROS in Y . Therefore f is a Ngpr closed mapping. But f is not a Neutrosophic pre-closed mapping since U^c is NCS in X but $f(U^c)$ is not a NPCS in Y as $Ncl(Nint(f(U^c))) = V^c \not\subseteq f(U^c)$.

3.14 Proposition: Every Neutrosophic generalized pre-closed mapping is Ngpr closed mapping but not conversely in general.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a Neutrosophic generalized pre-closed mapping. Let A be a NCS in X . Then $f(A)$ is a NGPCS in Y . Since every NGPCS is a NGPRCS in Y , $f(A)$ is a NGPRCS in Y . Hence f is a Ngpr closed mapping.

3.15 Example: Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, U, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are Neutrosophic topologies on X and Y respectively, where $U = \langle x, (0.3, 0.8, 0.5), (0.4, 0.7, 0.6) \rangle$ and $V = \langle y, (0.5, 0.2, 0.3), (0.6, 0.3, 0.4) \rangle$. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here the Neutrosophic set $U^c = \langle x, (0.5, 0.2, 0.3), (0.6, 0.3, 0.4) \rangle$ is a NCS in X . Then $f(U^c) = \langle y, (0.5, 0.2, 0.3), (0.6, 0.3, 0.4) \rangle$ is a NGPRCS in (Y, σ) as $f(U^c) \subseteq 1_N$ implies $Npcl(f(U^c)) = 1_N \subseteq 1_N$ where 1_N is a NROS in Y . Therefore f is a Ngpr closed mapping. But f is not a Neutrosophic generalized pre-closed mapping since U^c is NCS in X but $f(U^c)$ is not a NGPCS in Y as $f(U^c) \subseteq V$ implies $Npcl(f(U^c)) = 1_N \not\subseteq V$ where V is a NOS in Y .

3.16 Definition: Let (X, τ) and (Y, σ) be two NTSs. A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called Nigpr open mapping (NiGPROM for short) if $f(A)$ is NGPROS in (Y, σ) for every NGPROS A of (X, τ) .

3.17 Definition: Let (X, τ) and (Y, σ) be two NTSs. A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called Nigpr closed mapping (NiGPRCM for short) if $f(A)$ is NGPRCS in (Y, σ) for every NGPRCS A of (X, τ) .

3.18 Example: Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, U, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are Neutrosophic topologies on X and Y respectively, where $U = \langle x, (0.5, 0.4, 0.3), (0.7, 0.8, 0.2) \rangle$ and $V = \langle y, (0.7, 0.4, 0.5), (0.8, 0.5, 0.5) \rangle$. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Hence $f(A)$ is NGPRCS in (Y, σ) for every NGPRCS A of (X, τ) . Therefore f is a Nigpr closed mapping.

3.19 Proposition: Every Nigpr closed mapping is Ngpr closed mapping but not conversely in general.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a Nigpr closed mapping. Let A be a NCS in X . Since every NCS is a NGPRCS in X , A is a NGPRCS in X . Then $f(A)$ is a NGPRCS in Y . Hence f is a Ngpr closed mapping.

3.20 Example: Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, U, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are Neutrosophic topologies on X and Y respectively, where $U = \langle x, (0.2, 0.5, 0.7), (0.3, 0.5, 0.6) \rangle$ and $V = \langle y, (0.3, 0.5, 0.6), (0.4, 0.5, 0.5) \rangle$. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here the Neutrosophic set $U^c = \langle x, (0.7, 0.5, 0.2), (0.6, 0.5, 0.3) \rangle$ is a NCS in X . Then $f(U^c) = \langle y, (0.7, 0.5, 0.2), (0.6, 0.5, 0.3) \rangle$ is a NGPRCS in (Y, σ) as $f(U^c) \subseteq 1_N$ implies $Npcl(f(U^c)) = f(U^c) \subseteq 1_N$ where 1_N is a NROS in Y . Therefore f is

a Ngpr closed mapping. But f is not a Nigpr closed mapping since $W = \langle x, (0.3, 0.5, 0.6), (0.4, 0.5, 0.5) \rangle$ is NGPRCS in X but $f(W)$ is not a NGPRCS in Y as $f(W) \subseteq V$ implies $\text{Npcl}(f(W)) = V^c \not\subseteq V$ where V is a NROS in Y . Therefore f is not a Nigpr closed mapping.

The relation between various types of Neutrosophic closed mappings is given by

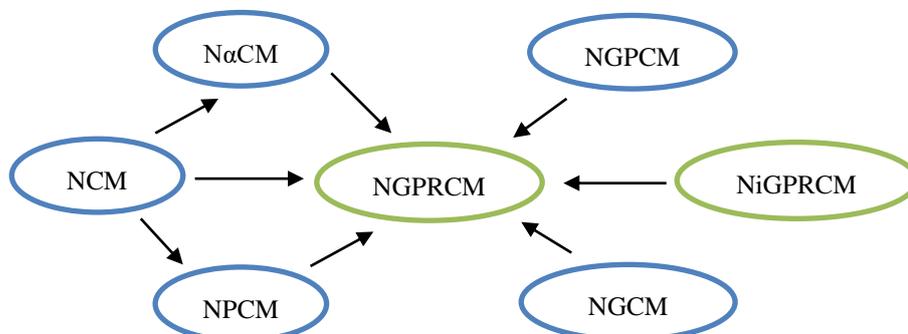


Fig.3.1.1 The reverse implications of Fig.3.1.1 are not true in general in the above diagram.

3.21 Theorem: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is Ngpr closed mapping if and only if $\text{Ngprcl}(f(A)) \subseteq f(\text{Ncl}(A))$.

Proof: Let $A \subseteq X$ and $f: (X, \tau) \rightarrow (Y, \sigma)$ be a Ngpr closed mapping, then $f(\text{Ncl}(A))$ is NGPRCS in Y which implies $\text{Ngprcl}(f(\text{Ncl}(A))) = f(\text{Ncl}(A))$. Since $f(A) \subseteq f(\text{Ncl}(A))$, $\text{Ngprcl}(f(A)) \subseteq \text{Ngprcl}(f(\text{Ncl}(A))) = f(\text{Ncl}(A))$ for every NS A of X .

Conversely, let A be any NCS in (X, τ) . Then $A = \text{Ncl}(A)$ and so $f(A) = f(\text{Ncl}(A)) \supseteq \text{Ngprcl}(f(A))$, by hypothesis. Since $f(A) \subseteq \text{Ngprcl}(f(A))$, therefore $f(A) = \text{Ngprcl}(f(A))$. i.e., $f(A)$ is NGPRCS in Y and hence f is Ngpr closed mapping.

3.22 Theorem: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is Ngpr open mapping iff for every NS A of (X, τ) , $f(\text{Nint}(A)) \subseteq \text{Ngprint}(f(A))$.

Proof: Necessity: Let A be a NOS in X and $f: (X, \tau) \rightarrow (Y, \sigma)$ be a Ngpr open mapping then $f(\text{Nint}(A))$ is NGPROS in Y . Since $f(\text{Nint}(A)) \subseteq f(A)$ which implies $\text{Ngprint}(f(\text{Nint}(A))) \subseteq \text{Ngprint}(f(A))$. Since $f(\text{Nint}(A))$ is NGPROS in Y , we have $f(\text{Nint}(A)) \subseteq \text{Ngprint}(f(A))$.

Sufficiency: Assume A is a NOS of (X, τ) . Then $f(A) = f(\text{Nint}(A)) \subseteq \text{Ngprint}(f(A))$. But $\text{Ngprint}(f(A)) \subseteq f(A)$. So $f(A) = \text{Ngprint}(f(A))$ which implies $f(A)$ is a NGPROS in (Y, σ) and hence f is a Ngpr open mapping.

3.23 Theorem: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a Ngpr open mapping then $\text{Nint}(f^{-1}(A)) \subseteq f^{-1}(\text{Ngprint}(A))$ for every NS A of (Y, σ) .

Proof: Let A be a NS in (Y, σ) . Then $\text{Nint}(f^{-1}(A))$ is a NOS of (X, τ) . Since f is Ngpr open mapping which implies $f(\text{Nint}(f^{-1}(A)))$ is Neutrosophic gpr open in (Y, σ) and hence $f(\text{Nint}(f^{-1}(A))) \subseteq \text{Ngprint}(f(f^{-1}(A))) \subseteq \text{Ngprint}(A)$. Thus $\text{Nint}(f^{-1}(A)) \subseteq f^{-1}(\text{Ngprint}(A))$.

3.24 Theorem: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is Ngpr open mapping iff for each NS A of (Y, σ) and for each NCS B of (X, τ) containing $f^{-1}(A)$ there is a NGPRCS C of (Y, σ) such that $A \subseteq C$ and $f^{-1}(C) \subseteq B$.

Proof: Necessity: Assume $f: (X, \tau) \rightarrow (Y, \sigma)$ is Ngpr open mapping. Let A be the NS of (Y, σ) and B be a NCS of (X, τ) such that $f^{-1}(A) \subseteq B$. Then $C = (f(B^c))^c$ is NGPRCS of (Y, σ) such that $f^{-1}(C) \subseteq B$.

Sufficiency: Assume D is a NOS of (X, τ) . Then $f^{-1}((f(D))^c) \subseteq D^c$ and D^c is NCS in (X, τ) . By hypothesis there is a NGPRCS C of (Y, σ) such that $(f(D))^c \subseteq C$ and $f^{-1}(C) \subseteq D^c$. Therefore $D \subseteq (f^{-1}(C))^c$. Hence $C^c \subseteq$

$f(D) \subseteq f((f^{-1}(C))^c) \subseteq C^c$ which implies $f(D) = C^c$. Since C^c is NGPROS of (Y, σ) . Hence $f(D)$ is Neutrosophic gpr open in (Y, σ) and thus f is Ngpr open mapping.

3.25 Theorem: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is Ngpr open mapping iff $f^{-1}(\text{Ngprcl}(A)) \subseteq \text{Ncl}(f^{-1}(A))$ for every NS A of (Y, σ) .

Proof: Necessity: Assume f is a Ngpr open mapping. For any NS A of (Y, σ) , $f^{-1}(A) \subseteq \text{Ncl}(f^{-1}(A))$. Therefore by Theorem 3.24., there exists a NGPRCS C in (Y, σ) such that $A \subseteq C$ and $f^{-1}(C) \subseteq \text{Ncl}(f^{-1}(A))$. Therefore we obtain $f^{-1}(\text{Ngprcl}(A)) \subseteq f^{-1}(C) \subseteq \text{Ncl}(f^{-1}(A))$.

Sufficiency: Assume A is a NS of (Y, σ) and B is a NCS of (X, τ) containing $f^{-1}(A)$. Put $C = \text{Ncl}(A)$, then $A \subseteq C$ and C is NGPRCS, since $f^{-1}(C) \subseteq \text{Ncl}(f^{-1}(A)) \subseteq B$. Then by Theorem 3.24., f is Ngpr open mapping.

3.26 Theorem: If $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be two Neutrosophic mappings and $\text{gof}: (X, \tau) \rightarrow (Z, \eta)$ Ngpr open mapping. If g is Ngpr irresolute mapping then f is Ngpr open mapping.

Proof: Let A be a NOS of (X, τ) . Then $\text{gof}(A)$ is NGPROS in (Z, η) because gof is Ngpr open mapping. Since g is Ngpr irresolute mapping and $\text{gof}(A)$ is NGPROS of (Z, η) therefore $g^{-1}(\text{gof}(A)) = f(A)$ is NGPROS in (Y, σ) . Hence f is Ngpr open mapping.

3.27 Theorem: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is Neutrosophic open mapping and $g: (Y, \sigma) \rightarrow (Z, \eta)$ is Ngpr open mapping then $\text{gof}: (X, \tau) \rightarrow (Z, \eta)$ is Ngpr open mapping.

Proof: Let A be a NOS of (X, τ) . Then $f(A)$ is a NOS in (Y, σ) because f is a Neutrosophic open mapping. Since g is Ngpr open mapping, $g(f(A)) = \text{gof}(A)$ is NGPROS in (Z, η) . Hence gof is Ngpr open mapping.

3.28 Theorem: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a bijective mapping then the following statements are equivalent:

- (i) f is a Ngpr open mapping.
- (ii) f is a Ngpr closed mapping.
- (iii) f^{-1} is Neutrosophic continuous mapping.

Proof: (i) \Rightarrow (ii): Let us assume that f is a Ngpr open mapping. By definition, A is a NOS in (X, τ) , then $f(A)$ is a NGPROS in (Y, σ) . Here A is NCS of (X, τ) , then $X-A$ is a NOS of (X, τ) . By assumption, $f(X-A)$ is a NGPROS in (Y, σ) . Hence, $Y-f(X-A)$ is a NGPRCS in (Y, σ) . Therefore, f is a Ngpr closed mapping.

(ii) \Rightarrow (iii): Let A be a NCS in (X, τ) . By (ii), $f(A)$ is a NGPRCS in (Y, σ) . Hence, $f(A) = (f^{-1})^{-1}(A)$, so f^{-1} is a NGPRCS in (Y, σ) . Therefore, f^{-1} is Neutrosophic continuous mapping.

(iii) \Rightarrow (iv): Let A be a NOS in (X, τ) . By (iii), $(f^{-1})^{-1}(A) = f(A)$ is a Ngpr open mapping.

3.29 Theorem: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping. Then the following statements are equivalent if Y is a NPRT_{1/2} space:

- (i) f is a Ngpr closed mapping.
- (ii) $\text{Npcl}(f(A)) \subseteq f(\text{Ncl}(A))$ for each NS A of X .

Proof: (i) \Rightarrow (ii): Let A be a NS in X . Then $\text{Ncl}(A)$ is a NCS in X . By (i) implies that $f(\text{Ncl}(A))$ is a NGPRCS in Y . Since Y is a NPRT_{1/2} space, $f(\text{Ncl}(A))$ is a NPCS in Y . Therefore $\text{Npcl}(f(\text{Ncl}(A))) = f(\text{Ncl}(A))$. Now $\text{Npcl}(f(A)) \subseteq \text{Npcl}(f(\text{Ncl}(A))) = f(\text{Ncl}(A))$. Hence $\text{Npcl}(f(A)) \subseteq f(\text{Ncl}(A))$ for each NS A of X .

(ii) \Rightarrow (i): Let A be any NCS in X. Then $Ncl(A) = A$. By (ii) implies that $Npcl(f(A)) \subseteq f(Ncl(A)) = f(A)$. But $f(A) \subseteq Npcl(f(A))$. Therefore $Npcl(f(A)) = f(A)$. This implies $f(A)$ is a NPCS in Y. Since every NPCS is NGPRCS in Y, $f(A)$ is NGPRCS in Y. Hence f is a Ngpr closed mapping.

3.30 Theorem: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a mapping where X and Y are NPRT_{1/2} space. Then the following statements are equivalent:

- (i) f is a Nigpr closed mapping.
- (ii) $f(A)$ is a NGPROS in Y for every NGPROS A in X.
- (iii) $f(Npint(B)) \subseteq Npint(f(B))$ for each NS B of X.
- (iv) $Npcl(f(B)) \subseteq f(Npcl(B))$ for each NS B of X.

Proof: (i) \Rightarrow (ii): is obvious by definition of Nigpr closed mapping.

(ii) \Rightarrow (iii): Let B be any NS in X. Since $Npint(B)$ is a NPOS, it is a NGPROS in X. Then by hypothesis, $f(Npint(B))$ is a NGPROS in Y. Since Y is NPRT_{1/2} space, $f(Npint(B))$ is a NPOS in Y. Therefore, $f(Npint(B)) = Npint(f(Npint(B))) \subseteq Npint(f(B))$.

(iii) \Rightarrow (iv) is obvious by taking complement in (iii).

(iv) \Rightarrow (i) Let B be a NGPRCS in X. By Hypothesis, $Npcl(f(B)) \subseteq f(Npcl(B))$. Since X is a NPRT_{1/2} space, B is a NPCS in X. Therefore, $Npcl(f(B)) \subseteq f(Npcl(B)) = f(B) \subseteq Npcl(f(B))$ implies $f(B)$ is NPCS in Y and hence $f(B)$ is a NGPRCS in Y. Thus f is Nigpr closed mapping.

4. Ngpr homeomorphism and Nigpr homeomorphism

4.1 Definition: A bijection $f: (X, \tau) \rightarrow (Y, \sigma)$ is called Ngpr homeomorphism (resp. NG homeomorphism, NGP homeomorphism) if f and f^{-1} are Ngpr continuous (resp. NG continuous, NGP continuous) mapping.

4.2 Example: Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, U_1, U_2, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are Neutrosophic topologies on X and Y respectively, where $U_1 = \langle x, (0.3, 0.5, 0.6), (0.5, 0.5, 0.5) \rangle$, $U_2 = \langle x, (0.2, 0.4, 0.7), (0.4, 0.5, 0.6) \rangle$ and $V = \langle y, (0.2, 0.4, 0.7), (0.4, 0.3, 0.6) \rangle$. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here $V^c = \langle y, (0.7, 0.6, 0.2), (0.6, 0.7, 0.4) \rangle$ is a Neutrosophic closed set in (Y, σ) . Then $f^{-1}(V^c)$ is a NGPRCS in (X, τ) . Therefore f is Ngpr continuous mapping. Here $U_1^c = \langle x, (0.6, 0.5, 0.3), (0.5, 0.5, 0.5) \rangle$ is a Neutrosophic closed set in (X, τ) . Then $f(U_1^c)$ is a NGPRCS in (Y, σ) . Therefore f^{-1} is a Ngpr continuous mapping. Hence, f and f^{-1} are Ngpr continuous mapping then it is a Ngpr homeomorphism.

4.3 Theorem: Each Neutrosophic homeomorphism is Ngpr homeomorphism but not conversely in general.

Proof: Let a bijection mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ be Neutrosophic homeomorphism, in which f and f^{-1} are Neutrosophic continuous mapping. Since every Neutrosophic continuous mapping is Ngpr continuous mapping. Hence f and f^{-1} are Ngpr continuous mapping. Therefore, f is Ngpr homeomorphism.

4.4 Example: Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, U_1, U_2, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are Neutrosophic topologies on X and Y respectively, where $U_1 = \langle x, (0.2, 0.5, 0.7), (0.5, 0.5, 0.5) \rangle$, $U_2 = \langle x, (0.1, 0.4, 0.7), (0.4, 0.5, 0.6) \rangle$ and $V = \langle y, (0.4, 0.3, 0.5), (0.3, 0.4, 0.7) \rangle$. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here $V^c = \langle y, (0.5, 0.7, 0.4), (0.7, 0.6, 0.3) \rangle$ is a NCS in (Y, σ) . Then $f^{-1}(V^c)$ is a NGPRCS in (X, τ) . Therefore f is Ngpr continuous mapping. Here $U_1^c = \langle x, (0.7, 0.5, 0.2), (0.5, 0.5, 0.5) \rangle$ is a NCS in (X, τ) . Then $f(U_1^c)$ is a NGPRCS in (Y, σ) . Therefore f^{-1} is a Ngpr continuous. Hence, f and f^{-1} are Ngpr continuous mapping then it is a Ngpr homeomorphism. However, here V^c is a NCS in (Y, σ) but it is not a NCS in (X, τ) . Hence, f is not Neutrosophic continuous mapping. Therefore, f is not a Neutrosophic homeomorphism.

4.5 Theorem: Each NG homeomorphism is Ngpr homeomorphism but not conversely in general.

Proof: Let a bijection mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ be NG homeomorphism, in which f and f^{-1} are NG continuous mapping. Since every NG continuous mapping is Ngpr continuous mapping. Hence f and f^{-1} are Ngpr continuous mapping. Therefore, f is Ngpr homeomorphism.

4.6 Example: Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, U, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are Neutrosophic topologies on X and Y respectively, where $U = \langle x, (0.4, 0.5, 0.6), (0.3, 0.4, 0.5) \rangle$ and $V = \langle y, (0.8, 0.5, 0.2), (0.7, 0.7, 0.3) \rangle$. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here $V^c = \langle y, (0.2, 0.5, 0.8), (0.3, 0.3, 0.7) \rangle$ is a NCS in (Y, σ) . Then $f^{-1}(V^c)$ is a NGPRCS in (X, τ) . Therefore f is Ngpr continuous mapping. Here $U^c = \langle x, (0.6, 0.5, 0.4), (0.5, 0.6, 0.3) \rangle$ is a NCS in (X, τ) . Then $f(U^c)$ is a NGPRCS in (Y, σ) . Therefore f^{-1} is a Ngpr continuous mapping. Hence, f and f^{-1} are Ngpr continuous mapping then it is Ngpr homeomorphism. However, here V^c is a NCS in (Y, σ) but it is not a NGCS in (X, τ) . Hence, f is not Neutrosophic continuous mapping. Therefore, f is not a NG homeomorphism.

4.7 Theorem: Each NGP homeomorphism is a Ngpr homeomorphism but not conversely in general.

Proof: Let a bijection mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ be NGP homeomorphism, in which f and f^{-1} are NGP continuous mapping. Since every NGP continuous mapping is Ngpr continuous mapping. Hence f and f^{-1} are Ngpr continuous mapping. Therefore, f is Ngpr homeomorphism.

4.8 Example: Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, U_1, U_2, U_3, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are Neutrosophic topologies on X and Y respectively, where $U_1 = \langle x, (0.3, 0.5, 0.7), (0.2, 0.5, 0.6) \rangle$, $U_2 = \langle x, (0.6, 0.5, 0.5), (0.7, 0.5, 0.5) \rangle$, $U_3 = \langle x, (0.8, 0.5, 0.2), (0.7, 0.5, 0.1) \rangle$ and $V = \langle y, (0.3, 0.5, 0.7), (0.3, 0.5, 0.7) \rangle$. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here $V^c = \langle y, (0.7, 0.5, 0.3), (0.7, 0.5, 0.3) \rangle$ is a NCS in (Y, σ) . Then $f^{-1}(V^c)$ is a NGPRCS in (X, τ) . Therefore f is Ngpr continuous mapping. Here $U_1^c = \langle x, (0.7, 0.5, 0.3), (0.6, 0.5, 0.2) \rangle$ is a NCS in (X, τ) . Then $f(U_1^c)$ is a NGPRCS in (Y, σ) . Therefore f^{-1} is a Ngpr continuous mapping. Hence, f and f^{-1} are Ngpr continuous mapping then it is a Ngpr homeomorphism. However, here V^c is a NCS in (Y, σ) but it is not a NGPCS in (X, τ) . Hence, it is not NGP continuous mapping. Therefore, it is not a NGP homeomorphism.

The relation between various types of Neutrosophic homeomorphisms is given by

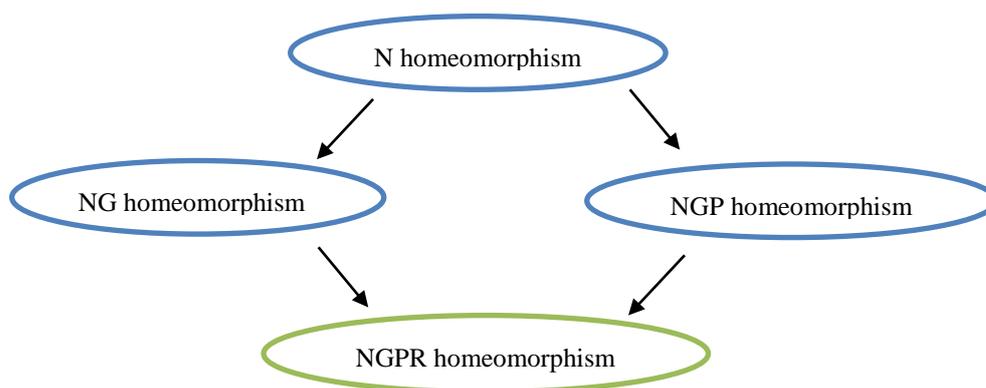


Fig.4.1.1 The reverse implications of Fig.4.1.1 are not true in general in the above diagram.

4.9 Theorem: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a Ngpr homeomorphism, then f is a Neutrosophic homeomorphism if X and Y are $NPRT^*_{1/2}$ space.

Proof: Let A be a NCS in (Y, σ) , then $f^{-1}(A)$ is a NGPRCS in (X, τ) . Since X is $NPRT^*_{1/2}$ space, $f^{-1}(A)$ is a NCS in (X, τ) . Therefore, f is Neutrosophic continuous mapping. By hypothesis, f^{-1} is Ngpr continuous mapping. Let B be a NCS in (X, τ) . Then $(f^{-1})^{-1}(B) = f(B)$ is a NGPRCS in Y . Since Y is $NPRT^*_{1/2}$ space, $f(B)$ is NCS in Y . Hence f^{-1} is Neutrosophic continuous mapping. Hence f is a Neutrosophic homeomorphism.

4.10 Theorem: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a bijective mapping. If f is Ngpr continuous mapping then the following statements are equivalent:

- (i) f is a Ngpr closed mapping.
- (ii) f is a Ngpr open mapping.
- (iii) f is a Ngpr homeomorphism.

Proof: (i) \Rightarrow (ii): Let us assume that f be a bijective mapping and a Ngpr closed mapping. Hence f^{-1} is Ngpr continuous mapping. Since each NOS in (X, τ) is a NGPROS in (Y, σ) . Hence, f is a Ngpr open mapping.

(ii) \Rightarrow (iii): Let f be a bijective mapping and a Ngpr open mapping. Furthermore, f^{-1} is a Ngpr continuous mapping. Hence f and f^{-1} are Ngpr continuous mapping. Therefore, f is a Ngpr homeomorphism.

(iii) \Rightarrow (i): Let f be a Ngpr homeomorphism. Then f and f^{-1} are Ngpr continuous mapping. Since each NCS in (X, τ) is a NGPRCS in (Y, σ) . Hence f is a Ngpr closed mapping.

4.11 Theorem: The composition of two Ngpr homeomorphisms need not be a Ngpr homeomorphism in general.

4.12 Example: Let $X = \{a, b\}$, $Y = \{c, d\}$ and $Z = \{e, f\}$. Then $\tau = \{0_N, U, 1_N\}$, $\sigma = \{0_N, V, 1_N\}$ and $\eta = \{0_N, W, 1_N\}$ are Neutrosophic topologies on X and Y respectively, where $U = \langle x, (0.2, 0.5, 0.8), (0.3, 0.3, 0.7) \rangle$, $V = \langle y, (0.4, 0.5, 0.6), (0.3, 0.4, 0.5) \rangle$, $W = \langle z, (0.8, 0.5, 0.2), (0.7, 0.7, 0.3) \rangle$. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = c$ and $f(b) = d$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ by $g(c) = e$ and $g(d) = f$. Then f and g are Ngpr homeomorphisms but their composition $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is not a Ngpr homeomorphism. Since W^c is NCS in (Z, η) but it is not NGPRCS in (X, τ) .

4.13 Definition: A bijection $f: (X, \tau) \rightarrow (Y, \sigma)$ is called Nigpr homeomorphism if f and f^{-1} are Ngpr irresolute mappings.

4.14 Theorem: Each Nigpr homeomorphism is a Ngpr homeomorphism but not conversely in general.

Proof: Let a bijection mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ be Nigpr homeomorphism. Assume that A is a NCS in (Y, σ) implies A is a NGPRCS in (Y, σ) . Since f is Ngpr irresolute mapping, $f^{-1}(A)$ is a NGPRCS in (X, τ) . Hence f is Ngpr continuous mapping. Therefore, f and f^{-1} are Ngpr continuous mapping. Hence, f is Ngpr homeomorphism.

4.15 Example: Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_N, U_1, U_2, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are Neutrosophic topologies on X and Y respectively, where $U_1 = \langle x, (0.2, 0.5, 0.7), (0.4, 0.5, 0.6) \rangle$, $U_2 = \langle x, (0.2, 0.4, 0.8), (0.3, 0.5, 0.7) \rangle$ and $V = \langle y, (0.5, 0.4, 0.5), (0.4, 0.5, 0.6) \rangle$. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here $V^c = \langle y, (0.5, 0.6, 0.5), (0.6, 0.5, 0.4) \rangle$ is a NCS in (Y, σ) . Then $f^{-1}(V^c)$ is a NGPRCS in (X, τ) . Therefore f is Ngpr continuous mapping. Here $U_1^c = \langle x, (0.7, 0.5, 0.2), (0.6, 0.5, 0.4) \rangle$ is a NCS in (X, τ) . Then $f(U_1^c)$ is a NGPRCS in (Y, σ) . Therefore f^{-1} is a Ngpr continuous mapping. Hence, f and f^{-1} are Ngpr continuous mapping then it is a Ngpr homeomorphism. However, here $A = \langle y, (0.2, 0.4, 0.7), (0.3, 0.5, 0.6) \rangle$ is a NGPRCS in (Y, σ) but it is not a NGPRCS in (X, τ) . Hence, f is not Neutrosophic irresolute mapping. Therefore, f is not a Nigpr homeomorphism.

4.16 Theorem: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a Nigpr homeomorphism then $\text{Ngprcl}(f^{-1}(A)) \subseteq f^{-1}(\text{Npcl}(A))$ for each NS A in (Y, σ) .

Proof: Let A be a NS in (Y, σ) . Then $\text{Npcl}(A)$ is NPCCS in (Y, σ) and since every NPCCS is NGPRCS in (Y, σ) . Assuming f is Ngpr irresolute mapping, $f^{-1}(\text{Npcl}(A))$ is a NGPRCS in (X, τ) , then $\text{Ngprcl}(f^{-1}(\text{Npcl}(A))) = f^{-1}(\text{Npcl}(A))$. Here, $\text{Ngprcl}(f^{-1}(A)) \subseteq \text{Ngprcl}(f^{-1}(\text{Npcl}(A))) = f^{-1}(\text{Npcl}(A))$. Therefore, $\text{Ngprcl}(f^{-1}(A)) \subseteq f^{-1}(\text{Npcl}(A))$ for each NS A in (Y, σ) .

4.17 Theorem: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a Nigpr homeomorphism then $\text{Npcl}(f^{-1}(A)) = f^{-1}(\text{Npcl}(A))$ for each NS A in (Y, σ) .

Proof: Given f is a Nigpr homeomorphism, then f is a Ngpr irresolute mapping. Let A be a NS in (Y, σ) . Clearly, $\text{Npcl}(A)$ is a NPCS in (Y, σ) . This shows that $\text{Npcl}(A)$ is a NGPRCS in (Y, σ) . Since $f^{-1}(A) \subseteq f^{-1}(\text{Npcl}(A))$, then $\text{Npcl}(f^{-1}(A)) \subseteq \text{Npcl}(f^{-1}(\text{Npcl}(A))) = f^{-1}(\text{Npcl}(A))$. Therefore, $\text{Npcl}(f^{-1}(A)) \subseteq f^{-1}(\text{Npcl}(A))$.

Let f be a Nigpr homeomorphism, f^{-1} is a Ngpr irresolute mapping. Let us consider NS $f^{-1}(A)$ in (X, τ) , which bring out that $\text{Npcl}(f^{-1}(A))$ is a NGPRCS in (X, τ) . Hence $\text{Ngprcl}(f^{-1}(A))$ is a NGPRCS in (X, τ) . This implies that $(f^{-1})^{-1}(\text{Npcl}(f^{-1}(A))) = f(\text{Npcl}(f^{-1}(A)))$ is a NPCS in (Y, σ) . This proves $A = (f^{-1})^{-1}(f^{-1}(A)) \subseteq (f^{-1})^{-1}(\text{Npcl}(f^{-1}(A))) = f(\text{Npcl}(f^{-1}(A)))$. Therefore, $\text{Npcl}(A) \subseteq \text{Npcl}(f(\text{Npcl}(f^{-1}(A)))) = f(\text{Npcl}(f^{-1}(A)))$, since f^{-1} is a Ngpr irresolute mapping. Hence $f^{-1}(\text{Npcl}(A)) \subseteq f^{-1}(f(\text{Npcl}(f^{-1}(A)))) = \text{Npcl}(f^{-1}(A))$. That is $f^{-1}(\text{Npcl}(A)) \subseteq \text{Npcl}(f^{-1}(A))$. Hence, $\text{Npcl}(f^{-1}(A)) = f^{-1}(\text{Npcl}(A))$.

4.18 Theorem: If $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ are Nigpr homeomorphisms, then the composition $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a Nigpr homeomorphism.

Proof: Let f and g be two Nigpr homeomorphisms. Assume C is a NGPRCS in (Z, η) . Then $g^{-1}(C)$ is a NGPRCS in (Y, σ) . Then by hypothesis, $f^{-1}(g^{-1}(C))$ is a NGPRCS in (X, τ) . Hence $g \circ f$ is a Ngpr irresolute mapping. Now, let A be a NGPRCS in (X, τ) . By assumption, $f(A)$ is a NGPRCS in (Y, σ) . Then by hypothesis, $g(f(A))$ is a NGPRCS in (Z, η) . This implies that $g \circ f$ is a Ngpr irresolute mapping. Hence, $g \circ f$ is a Nigpr homeomorphism.

5. Conclusion

In this article, the new class of Neutrosophic homeomorphism namely, Ngpr homeomorphism and Nigpr homeomorphism was defined and studied some of their properties in Neutrosophic topological spaces. Furthermore, the work was extended as the Ngpr open mappings, Ngpr closed mappings and Nigpr closed mappings and discussed some of their properties. Many results have been established to show how far topological structures are preserved by this Ngpr homeomorphism.

Also, the relation between Ngpr closed mappings and other existed Neutrosophic closed mappings in Neutrosophic topological spaces were established and derived some of their related attributes. Many examples are given to justify the results.

This concept can be used to drive few more new results of Ngpr connectedness and Ngpr compactness in Neutrosophic topological spaces.

Acknowledgements

The author would like to thank the referees for their valuable suggestions to improve the paper.

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Received: Dec 29, 2019. Accepted: Mar 15, 2020.