A Note on μ_N P Spaces

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Abstract: In this article we introduce a new concept called μ_N D Baire spaces and μ_N P spaces, their properties were contemplated.

Keywords: μ_N D Baire space, D/μ_N space, μ_N P space, $\mu_N F_\sigma$ set, $\mu_N G_\delta$ set.

1.Introduction

The concept fuzziness had a great impact in all branches of mathematics which was put forth by Zadeh [16]. Later on the idea of fuzziness and topological spaces were put together by C.L.Chang[3] and laid a foundation to the theory of fuzzy topological spaces. By focussing the membership and non-membership of the elements, K.T.Attanasov[1] made out intuitionistic fuzzy sets and he extended his research towards and gave out a generalization to intuitionistic L-fuzzy sets with his friend Stoeva. F.Smarandache[6,7,8] put his thoughts towards the degree of indeterminacy and bring forth the neutrosophic sets. Subsequently, the neutrosophic topological spaces with the help of neutrosophic sets were found out by A.A.Salama and S.A.Alblowi[11,12,13]. By making all the works together as inspiration, we[9] made Generalized topological spaces via neutrosophic sets and named it as μ_N topological space (μ_NTS). The μ_N nowhere dense sets in μ_NTS were put forth by us [10]. Here by making use of the concepts of μ_N nowhere dense sets, in this paper we introduce a new concept called μ_N D Baire Spaces and μ_N P Spaces, their properties were contemplated.

2.Necessities

Definition 2.1[13] Let X be a non-empty fixed set. A neutrosophic set [NS for short] A is an object having the form$A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function, the degree of indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set A.

Remark 2.4.[13] Every intuitionistic fuzzy set A is a non empty set in X is obviously on neutrosophic sets having the form $A = \{\langle \mu_A(x), 1 - \mu_A(x) + \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$. Since our main purpose is to construct the tools for developing neutrosophic set and neutrosophic topology, we must introduce the neutrosophic sets 0_N and 1_N in X as follows:

0_N may be defined as follows

$0_N = \{\langle x, 0,1,1\rangle : x\epsilon X\}$

1_N may be defined as follows

$1_N = \{\langle x, 1,0,0\rangle : x\epsilon X\}$

Definition 2.5.[13] Let $A = \{\langle \mu_A, \sigma_A, \gamma_A\rangle\}$ be a neutrosophic set on X, then the complement of the set A [C(A) for short] may be defined and denoted by C(A) or $\bar{A}$

$C(A) = \{\langle x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x)\rangle : x \in X\}$

Definition 2.6.[13] Let X be a non-empty set and the neutrosophic sets A and B are in the form of $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$.
$A \subseteq B$ may be defined as:
$(A \subseteq B) \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \geq \sigma_B(x), \gamma_A(x) \geq \gamma_{\boldsymbol{B}}(x)\ \forall x \in X$
Proposition 2.7. [13] For any neutrosophic set A, the following conditions holds:
$0_N \subseteq$ A, $A \subseteq 1_N$.
Definition 2.8. [13] Let X be a non empty set and $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$
$B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$ are neutrosophic sets. Then $A \cap B$ may be defined as:
$A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle$
$A \cup B$ may be defined as:
$A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle$
Definition 2.9[12]. A μ_N topology on a non - empty set X is a family of neutrosophic subsets in X $satisfying\ the\ following\ axioms$:
$(\mu_{N_1}) 0_N \in \mu_N$
$(\mu_{N_2})\ G_1 \cup G_2 \in \mu_N$ for any $G_1, G_2 \in \mu_N$.
Throughout this paper, the pair of (X, μ_N) is known as μ_N topological space (μ_N TS)
Remark 2.10.[12] The elements of μ_N are μ_N open sets and their complement of μ_N open sets are called μ_N closed sets.
Definition 2.11.[12] The $\mu_N -$ Closure of A is the intersection of all μ_N closed sets containing A.
Definition 2.12.[12] The $\mu_N -$ Interior of A is the union of all μ_N open sets contained in A.
Definition 2.13.[11]. A neutrosophic set A in neutrosophic topological space is called neutrosophic dense if there exists no neutrosophic closed sets B in (X, T) such that $A \subset B \subset 1_N$.
Definition 2.14.[10]. The neutrosophic topological spaces is said to be μ_N Baire space if $N\ Int(\cup_{i=1}^{\infty} G_i) = 0_N$ where $G_i's$ are neutrosophic nowhere dense set in (X, T).
Theorem 2.15.[10]: Let (X, μ_N) be a μ_NTS. Then the following are equivalent.

(i) (X, μ_N) is μ_N Baire space.
(ii) $\mu_N\ Int(A) = 0_N$, for all μ_N first category set in (X, μ_N).
(iii) $\mu_N\ Cl(A) = 1_N$, μ_N Residual set in (X, μ_N).

3. μ_N D Baire spaces

Proposition 3.1: If $\wp$ is a μ_N first category set in a μ_NTS (X, μ_N) such that $\mu_N\ Int(\mu_N Cl\ \wp) = 0_N$,then (X, μ_N) is a μ_N Baire space.

Proof: Let $\wp$ be a μ_N first category set in μ_NTS (X, μ_N) that implies $\wp = \vee_{i=1}^{\infty} \wp_i$ where $\wp_i's$ are μ_N nowhere dense sets in (X, μ_N). We know that $\mu_N Int(\mu_N Cl\ \wp) = 0_N$. Also, $\mu_N Int\ \wp \subseteq \mu_N Int\ (\mu_N\ Cl\ \wp)$ that entails us $\mu_N Int(\wp) = 0_N \Rightarrow \mu_N Int(\cup_{i=1}^{\infty} \wp_i) = 0_N$, $\wp_i's$ are μ_N nowhere dense sets in $(X, \mu_N) \Rightarrow (X, \mu_N)$ is a μ_N Baire space.

Proposition 3.2: If a μ_N first category set η in a μ_N Baire space (X, μ_N) is a μ_N closed set, then $\mu_N Int(\mu_N Cl\ \eta) = 0_N$ in (X, μ_N).

Proof: Let η be a μ_N first category set in $\mu_N TS$. Owing to the fact that (X, μ_N) is a μ_N Baire space, we have that for every μ_N first category set η in (X, μ_N), $\mu_N Int(\eta) = 0_N$. Now, η is μ_N closed in (X, μ_N) that implies us that $\mu_N\ Cl(\eta) = \eta$. Now we have that $\mu_N Int(\mu_N Cl\ \eta) = \mu_N Int \eta = 0_N \Rightarrow \mu_N Int(\mu_N Cl\ \eta) = 0_N$.

Definition 3.3: A $\mu_N TS$ is called μ_N D Baire space if every μ_N first category set in (X, μ_N) is a μ_N nowhere dense set in (X, μ_N).

Example 3.4: Let $X=\{a,b\}$ and $0_N=\{\langle 0,1,1\rangle\langle 0,1,1\rangle\}, A=\{\langle 0.6,0.4,0.8\rangle\langle 0.8,0.6,0.9\rangle\}, B=\{\langle 0.6,0.3,0.8\rangle\langle 0.9,0.2,0.7\rangle\}$, $C=\{\langle 0.5,0.4,0.9\rangle\langle 0.7,0.8,0.9\rangle\}$, $D=\{\langle 0.4,0.6,0.9\rangle\langle 0.6,0.8,0.9\rangle\}, E=\{\langle 0.3,0.7,0.9\rangle\langle 0.5,0.9,0.9\rangle\}$, $1_N=\{\langle 1,0,0\rangle\langle 1,0,0\rangle\}$. We define a $\mu_N\, TS$ by $\{0_N,A,B,C,D\}$. The μ_N closed sets are $\{\bar{A},\bar{B},\bar{C},\bar{D},1_N\}$. Here, 0_N and $\bar{B}=\{\langle 0.8,0.7,0.6\rangle\langle 0.7,0.8,0.9\rangle\}$ are μ_N first category sets and $0_N,E,\bar{B}$ are μ_N nowhere dense. Hence, every μ_N first category is μ_N nowhere dense. Thus, (X,μ_N) is μ_N D Baire space.

Proposition 3.5: If (X,μ_N) is $\mu_N\, D$ Baire Space, then (X,μ_N) is a μ_N Baire space.

Proof: Let ζ be a μ_N first category set in $\mu_N\, D$ Baire space (X,μ_N). Then $\zeta=\bigcup_{i=1}^{\infty}\zeta_i$ where $\zeta_i's$ are μ_N nowhere dense sets and ζ is a μ_N nowhere dense set in (X,μ_N). Thereupon, we obtain that $\mu_N\, Int\,(\mu_N Cl\,\zeta)=0_N$. Hence, $\mu_N\, Int\,\zeta\subseteq \mu_N\, Int\,(\mu_N Cl\,\zeta)\Rightarrow \mu_N\, Int\,\zeta=0_N$ which entails us that $\mu_N\, Int(\bigcup_{i=1}^{\infty}\zeta_i)=0_N$ where $\zeta_i's$ are μ_N nowhere dense sets in (X,μ_N). Hence (X,μ_N) is a μ_N Baire space.

Remark 3.6: Converse of the above proposition need not be true. Every μ_N Baire space need not be $\mu_N\, D$ Baire space.This can be explained in the following example.

Example 3.7: Let $X=\{a\}, 0_N=\{\langle 0,1,1\rangle\}, A=\{\langle 0.3,0.3,0.5\rangle\}, B=\{\langle 0.1,0.2,0.3\rangle\}, C=\{\langle 0.3,0.2,0.3\rangle\}, D=\{\langle 0.3,0.6,0.2\rangle\}, E=\{\langle 0.3,0.8,0.5\rangle\}, 1_N=\{\langle 1,0,0\rangle\}$ and we define a μ_NTS as $\{0_N,A,B,C\}$.Here (X,μ_N) is a μ_N Baire space. The μ_N first category sets are 0_N and $\bar{E}$ and the μ_N nowhere dense sets are $0_N,E,\bar{A},\bar{B},\bar{C},\bar{D}$ .Here $\bar{E}$ is μ_N first category set but not μ_N Nowhere dense set. Hence (X,μ_N) is not a $\mu_N\, D$ Baire space.

Proposition 3.8: If δ is an arbitrary μ_N first category set in μ_N Baire space and $\delta\ is\ \mu_N\ Closed$ then (X,μ_N) is $\mu_N\, D$ Baire space.

Proof: Let $\delta=\bigcup_{i=1}^{\infty}\delta_i$, where $\delta_i's$ are μ_N nowhere dense sets. From this we obtain that $\mu_N\, Int(\bigcup_{i=1}^{\infty}\delta_i)=0_N$ because of the fact that (X,μ_N) is μ_N Baire Space. Also we have that $\mu_N Cl(\delta)=\delta$ from this we get that $\mu_N\, Int(\mu_N Cl(\delta))=\mu_N\, Int\delta\Rightarrow \mu_N\, Int(\mu_N Cl(\delta))=0_N$. Since δ is μ_N first category set in (X,μ_N). Thus (X,μ_N) is $\mu_N\, D$ Baire space.

Definition 3.9: A neutrosophic set in a μ_NTS (X,μ_N) is called $\mu_N F_\sigma$ set in (X,μ_N) if $\theta=\bigcup_{i=1}^{\infty}\theta_i$, where $\overline{\theta_i}\epsilon\,\mu_N$.

Definition 3.10: A neutrosophic set in a μ_NTS (X,μ_N) is called $\mu_N G_\delta$ set in (X,μ_N) if $\theta=\bigcap_{i=1}^{\infty}\theta_i$, where $\theta_i\epsilon\,\mu_N$.

Proposition 3.11: If α is μ_N dense and $\mu_N G_\delta$ set in (X,μ_N) in a $\mu_N TS$ then $\overline{\alpha}$ is a μ_N first category set in (X,μ_N).

Proof: Since α is $\mu_N G_\delta$ set in (X,μ_N), $\alpha=\bigcap_{i=1}^{\infty}\alpha_i$ where $\alpha_i\ \epsilon\ \mu_N$ and also α is μ_N dense so we get $\mu_N Cl(\alpha)=1_N$. Thereupon we get $\mu_N Cl(\bigcap_{i=1}^{\infty}\alpha_i)=1_N$. But we know that $\mu_N Cl(\bigcap_{i=1}^{\infty}\alpha_i)\subseteq\bigcap_{i=1}^{\infty}(\mu_N Cl(\alpha_i))$. Hence, we retrieve that $1_N\subseteq\bigcap_{i=1}^{\infty}(\mu_N Cl(\alpha_i))\Rightarrow\bigcap_{i=1}^{\infty}(\mu_N Cl(\alpha_i))=1_N$. Thus we have that for each $\alpha_i\ \epsilon\ \mu_N$, $\mu_N Cl(\alpha_i)=1_N$. Now, $\mu_N Cl(\mu_N\ Int\ \alpha_i)=1_N\Rightarrow\overline{\mu_N Cl(\mu_N\ Int\ \alpha_i)}=0_N\Rightarrow\mu_N\ Int(\mu_N Cl(\overline{\alpha_i}))=0_N$ which entails us that $\overline{\alpha_i}$ is μ_N nowhere dense sets in (X,μ_N) that implies us $\overline{\alpha}=\bigcup_{i=1}^{\infty}(\overline{\alpha_i})$ is μ_N nowhere dense sets in $(X,\mu_N)\Rightarrow\overline{\alpha}$ is μ_N first category set in (X,μ_N).

Proposition 3.12: If β is μ_N dense and $\mu_N G_\delta$ set in (X,μ_N) in a $\mu_N TS$ then β is a μ_N residual set in (X,μ_N).

Proof: Owing to the fact that β is μ_N dense and $\mu_N G_\delta$ set in (X,μ_N) by using Proposition 3.11 we obtain that $\bar{\beta}$ is μ_N first category set in (X,μ_N). From this we conclude that β is a μ_N residual set in (X,μ_N).

Proposition 3.13: If ν is both μ_N dense and $\mu_N G_\delta$ set, then $\mu_N Int\overline{\nu_i}=0_N$ where $\nu_i's$ are μ_N nowhere dense sets such that $\overline{\nu}=\bigcup_{i=1}^{\infty}(\overline{\nu_i})$.

Proof: Let ν be a μ_N dense and $\mu_N G_\delta$ set in (X, μ_N). Then by Proposition 3.11, $\bar{\nu}$ is μ_N first category set in (X, μ_N) and $\bar{\nu} = \cup_{i=1}^{\infty}(\bar{\nu_\iota})$ where $\bar{\nu_\iota}$'s are μ_N nowhere dense sets in (X, μ_N). But $\mu_N Int(\bar{\nu}) = \overline{\mu_N Cl\, \nu} = \overline{1_N} = 0_N$. Then $\mu_N Int(\cup_{i=1}^{\infty}(\bar{\nu_\iota})) = \mu_N Int \bar{\nu_\iota} = 0_N \Rightarrow \mu_N Int \bar{\nu_\iota} = 0_N$,where $\bar{\nu_\iota}$'s are μ_N nowhere dense sets in (X, μ_N).

Proposition 3.14: If ξ is μ_N first category set in (X, μ_N) then there is a $\mu_N F_\sigma$ set ℘ in (X, μ_N) such that ξ⊆ ℘.

Proof: Let ξ be a μ_N first category set in (X, μ_N) then ξ= $\cup_{i=1}^{\infty} \xi_i$, where ξ_i's are μ_N nowhere dense sets in (X, μ_N). Now $\overline{\mu_N Cl(\xi)}$ is μ_N open in (X, μ_N) thereupon $\cap_{i=1}^{\infty}(\overline{\mu_N Cl(\xi_\iota)})$ is $\mu_N G_\delta$ set in (X, μ_N). Let κ= $\cap_{i=1}^{\infty}(\overline{\mu_N Cl(\xi_\iota)})$. On considering κ= $\cap_{i=1}^{\infty}(\overline{\mu_N Cl(\xi_\iota)}) = \overline{\cup_{\iota=1}^{\infty} \mu_N Cl(\xi_\iota)} \subseteq \overline{\cup_{\iota=1}^{\infty} \xi_\iota} = \bar{\xi} \Rightarrow \kappa \subseteq \bar{\xi} \Rightarrow \xi \subseteq \bar{\kappa}$. Let ℘= $\bar{\kappa}$. Since κ is $\mu_N G_\delta$ set in (X, μ_N) and ℘ is $\mu_N F_\sigma$ set in (X, μ_N). Thus, we obtain "If ξ is μ_N first category set in (X, μ_N) then there is a $\mu_N F_\sigma$ set ℘ in (X, μ_N) such that ξ⊆ ℘".

Remark 3.15: If $\mu_N\, Int\, (\wp) = 0_N$ in Proposition 3.14 then (X, μ_N) is μ_N Baire space. For $\mu_N Int \xi \subseteq \mu_N\, Int(\wp) = 0_N \Rightarrow \mu_N Int \xi = 0_N \Rightarrow (X, \mu_N)$ is μ_N Baire space.

Proposition 3.16: If $\mu_N Cl(\mu_N\, Int\, \gamma) = 1_N$ for every μ_N *dense* and $\mu_N G_\delta$ set in (X, μ_N) then (X, μ_N) is μ_N D Baire space.

Proof: let γ be a μ_N *dense* and $\mu_N G_\delta$ set in (X, μ_N). Then by Proposition 3.11 we obtain that $\bar{\gamma}$ is a μ_N first category set in (X, μ_N). Now, $\mu_N Cl(\mu_N\, Int\, \gamma) = 1_N \Rightarrow \overline{\mu_N Cl(\mu_N\, Int\, \gamma)} = 0_N \Rightarrow \mu_N\, Int(\mu_N Cl\, \bar{\gamma}) = 0_N$. For the μ_N first category set $\bar{\gamma}$ in (X, μ_N) we have that $\mu_N\, Int(\mu_N Cl\, \bar{\gamma}) = 0_N$. Thus, (X, μ_N) is μ_N D Baire space.

Proposition 3.17: If a $\mu_N\, TS$ (X, μ_N) has a μ_N *dense and* $\mu_N G_\delta$ set in (X, μ_N) then (X, μ_N) is not a μ_N D Baire space.

Proof: Let γ be a μ_N *dense* and $\mu_N G_\delta$ set in (X, μ_N). Then by Proposition 3.11 we obtain that $\bar{\gamma}$ is a μ_N first category set in (X, μ_N). Now, $\overline{\mu_N Cl(\mu_N\, Int\, \gamma)} \supseteq \overline{\mu_N Cl\, \gamma} \supseteq \overline{1_N} = 0_N$. Hence, $\mu_N Int(\mu_N Cl \bar{\gamma}) \supseteq 0_N \neq 0_N$, for the μ_N first category set $\bar{\gamma}$ in (X, μ_N). Clearly we get that (X, μ_N) is not a μ_N D Baire space.

Proposition 3.18: If $\mu_N Cl(\mu_N\, Int\, \vartheta) = 1_N$,for every μ_N residual set ϑ in a $\mu_N TS$ (X, μ_N) then (X, μ_N) is a μ_N D Baire space.

Proof: Let ϑ ba a μ_N residual set in a $\mu_N TS$ (X, μ_N). Thereupon $\bar{\vartheta}$ is a μ_N first category set in (X, μ_N). Now, $\mu_N Cl(\mu_N\, Int\, \vartheta) = 1_N \Rightarrow \overline{\mu_N Cl(\mu_N\, Int\, \vartheta)} = 0_N \Rightarrow$ $\mu_N\, Int\, (\mu_N Cl\, \bar{\vartheta}) = 0_N$ that entails us that for a μ_N first category set $\bar{\vartheta}$ in(X, μ_N), $\mu_N\, Int\, (\mu_N Cl\, \bar{\vartheta}) = 0_N$ which leads us into that (X, μ_N) is a μ_N D Baire space.

Proposition 3.19: If a μ_NTS (X, μ_N) is a μ_N D Baire space then there is no non void μ_N dense set is a μ_N first category set in (X, μ_N).

Proof: Suppose that ω is a non-void μ_N first category set and μ_N dense set in (X, μ_N). Owing to the fact that (X, μ_N) is a μ_N D Baire space and ω is a μ_N first category set in (X, μ_N). From this we get that $\mu_N\, Int\, (\mu_N Cl\, \omega) = 0_N$. By our assumption we get $\mu_N Cl\, \omega = 1_N$ that implies us $\mu_N\, Int\, (\mu_N Cl\, \omega) = \mu_N\, Int\, (1_N) \neq 1_N$ which is a contradiction to (X, μ_N) is a μ_N D Baire space. Hence, we must have $\mu_N Cl\, \omega \neq 1_N$. Thereupon we get no non zero μ_N dense set is a μ_N first category set in μ_N D Baire space.

4. μ_N *P Spaces*

Definition 4.1: A μ_NTS is called D/μ_N space if for all non-empty neutrosophic set η in (X, μ_N), $\mu_N Cl\, \eta = 1_N$.

Definition 4.2: A μ_NTS is called μ_N P space if the countable intersection of μ_N open sets is μ_N open. That is every non zero $\mu_N G_\delta$ set in (X, μ_N) is a μ_N open set in (X, μ_N).

Example 4.3: Let $X = \{a\}$. We define neutrosophic sets as $A_1 = \{\langle 0.1.0.4,0.6\rangle\}, A_2 = \{\langle 0.2,0.3,0.5\rangle\}$ and we define a μ_NTS as $\{0_N, A_1, A_2\}$. Here the countable intersection of μ_N open sets are μ_N open. Hence, (X, μ_N) is a μ_N P space.

Example 4.4: Let $X = \{a\}$. We define a μ_NTS as $\{0_N, \omega_1, \omega_2, \omega_3\}$ where $\omega_1 = \{\langle 0.3,0.3,0.5\rangle\}, \omega_2 = \{\langle 0.1,0.2,0.3\rangle\}, \omega_3 = \{\langle 0.3,0.2,0.3\rangle\}, \omega_4 = \{\langle 0.3,0.6,0.2\rangle\}, \omega_5 = \{\langle 0.3,0.8,0.5\rangle\}$. Here the countable intersection of μ_N open set is not μ_N open set in (X, μ_N).

Proposition 4.5: If $\wp$ is a non-zero $\mu_N F_\sigma$ set in a μ_N P space (X, μ_N), then $\wp$ is a μ_N closed set in (X, μ_N).

Proof: Since $\wp$ is a non-zero $\mu_N F_\sigma$ set in (X, μ_N), $\wp = \bigcup_{i=1}^{\infty} \wp_i$ where the neutrosophic sets $\wp_i's$ are μ_N closed in (X, μ_N). Then $\overline{\wp} = \overline{\bigcup_{\iota=1}^{\infty} \wp_\iota} = \bigcap_{i=1}^{\infty} \overline{\wp_\iota}$. Now, $\wp_i's$ are μ_N closed in (X, μ_N) that entails $\overline{\wp} = \bigcap_{i=1}^{\infty} \overline{\wp_\iota}$ where $\overline{\wp_\iota} \epsilon \mu_N$.Thereupon $\overline{\wp}$ is a $\mu_N G_\delta$ set in (X, μ_N).Since (X, μ_N) is a μ_N P space, $\overline{\wp}$ is μ_N open. Therefore, $\wp$ is μ_N closed set in (X, μ_N).

Proposition 4.6: If the μ_N TS (X, μ_N) is a μ_N P space and if $\wp$ is a μ_N first category set in (X, μ_N) then $\wp$ is not a μ_N dense.

Proof: Let us assume that the contrary statement. Suppose that $\wp$ is a μ_N first category set in (X, μ_N) such that $\mu_N Cl(\wp) = 1_N$ where $\wp = \bigcup_{i=1}^{\infty} \wp_i$ and $\wp_i's$ are μ_N nowhere dense sets in (X, μ_N). Now, $\overline{\mu_N Cl(\wp_\iota)}$ is μ_N open in (X, μ_N). Let $\xi = \bigcap_{i=1}^{\infty} \overline{\mu_N Cl(\wp_\iota)}$. Thereupon ξ is a non-zero $\mu_N G_\delta$ set in (X, μ_N). Now we have $\bigcap_{i=1}^{\infty} \overline{\mu_N Cl(\wp_\iota)} = \overline{\bigcup_{\iota=1}^{\infty} \mu_N Cl(\wp_\iota)} \subseteq \overline{\bigcup_{\iota=1}^{\infty} \wp_\iota} = \overline{\wp}$. Thus we obtain that $\xi \subseteq \overline{\wp}$. From this we obtain that $\mu_N Int\,(\xi) \subseteq \mu_N Int\,(\overline{\wp}) = \overline{\mu_N Cl\,\wp} = \overline{1_N} = 0_N$. Since (X, μ_N) is a μ_N P spaces, $\mu_N Int\,(\xi) = \xi$ that yields us that $\xi = 0_N$ which is a strict opposite statement to a non- zero $\mu_N G_\delta$ set in μ_N P space (X, μ_N) that implies us that $\mu_N Cl(\wp) \neq 1_N$. Thereupon we conclude that $\wp$ is not a μ_N dense.

Proposition 4.7: If λ is a μ_N first category set in μ_N P space such that $\sigma \subseteq \bar{\lambda}$ where σ is a non-zero μ_N dense and $\mu_N G_\delta$ set in (X, μ_N) then λ is a μ_N nowhere dense set in (X, μ_N).

Proof: Let λ be a μ_N first category set in (X, μ_N). Then $\lambda = \bigcup_{i=1}^{\infty} \lambda_i$ where λ_i's are μ_N nowhere dense set in (X, μ_N). Now, $\overline{\mu_N Cl(\lambda_\iota)}$ is μ_N open in (X, μ_N). Let $\sigma = \bigcap_{\iota=1}^{\infty} \overline{\mu_N Cl(\lambda_\iota)} = \overline{\bigcup_{\iota=1}^{\infty} \mu_N Cl(\lambda_\iota)} \subseteq \overline{\bigcup_{\iota=1}^{\infty} \lambda_\iota} = \bar{\lambda}$. Hence, we get that $\sigma \subseteq \bar{\lambda}$. From this we get that $\lambda \subseteq \overline{\sigma}$. Now $\mu_N Int(\mu_N Cl\,\lambda) \subseteq \mu_N Int(\mu_N Cl\,\overline{\sigma})$ which implies us that $\mu_N Int(\mu_N Cl\,\lambda) \subseteq \overline{\mu_N Cl(\mu_N Int\,\sigma)}$. Now owing to the fact that (X, μ_N) is a μ_N P space, the $\mu_N G_\delta$ set σ is $\mu_N$open in (X, μ_N) and μ_N $Int(\sigma) = \sigma$. Therefore we get that $\mu_N Int(\mu_N Cl\,\lambda) \subseteq \overline{\mu_N Cl(\mu_N Int\,\sigma)} = \overline{\mu_N Cl\,\sigma} = \overline{1_N} = 0_N$. Thereupon $\mu_N Int(\mu_N Cl\,\lambda) = 0_N$. Hence λ is μ_N nowhere dense set in (X, μ_N).

Proposition 4.8: If λ is a μ_N first category set in μ_N P space such that $\sigma \subseteq \bar{\lambda}$ where σ is a non-zero μ_N dense and $\mu_N G_\delta$ set in (X, μ_N) then (X, μ_N) is μ_N Baire space.

Proof: Let λ be a μ_N first category set in (X, μ_N). As in the above Proposition 4.7 we have $\mu_N Int(\mu_N Cl\,\lambda) = 0_N$. Thereupon $\mu_N Int\,\lambda \subseteq \mu_N Int(\mu_N Cl\,\lambda)$ which entails us that $\mu_N Int\,\lambda = 0_N$. Thus, we obtain that $\mu_N Int\,\lambda = 0_N$ for every μ_N first category set in (X, μ_N). Hence (X, μ_N) is μ_N Baire space.

Proposition 4.9: If the μ_NTS (X, μ_N) is a μ_N P space and λ is a non-zero μ_N dense and μ_N first category set in (X, μ_N) then there is no non-zero $\mu_N G_\delta$ set in (X, μ_N) such that $\sigma \subseteq \bar{\lambda}$.

Proof: Let λ be a non-zero μ_N first category set in (X, μ_N). Suppose there exists a $\mu_N G_\delta$ set σ in (X, μ_N) such that $\sigma \subseteq \bar{\lambda}$. Thereupon we get $\mu_N Int\ \sigma \subseteq \mu_N Int \bar{\lambda}$ that implies us that $\mu_N Int\ \sigma \subseteq \overline{\mu_N Cl\ \lambda} = 0_N$ *because* λ is μ_N dense. Now we have $\mu_N Int\ \sigma = 0_N$. Since (X, μ_N) is a μ_N P space, $\mu_N Int\ \sigma = \sigma$ and so we obtain $\sigma = 0_N$. Hence we conclude that if λ is μ_N dense and μ_N first category set in (X, μ_N) then there is no non-zero $\mu_N G_\delta$ set in (X, μ_N) such that $\sigma \subseteq \bar{\lambda}$.

Proposition 4.10: If η is a non-empty μ_N residual set in μ_N P space (X, μ_N) then $\mu_N Int\ \eta \neq 0_N$.

Proof: Let η be a non-empty μ_N residual set in μ_N P space (X, μ_N) then $\bar{\eta}$ is a μ_N first category set in (X, μ_N) and hence by proposition 4.6 we obtain that $\bar{\eta}$ is not a μ_N dense set in (X, μ_N). From this we obtain that $\mu_N Cl(\bar{\eta}) \neq 1_N$ which entails us $\overline{\mu_N Int \eta} \neq 1_N \Rightarrow \mu_N Int \eta \neq 0_N$.

Proposition 4.11: If **κ** is a μ_N dense and $\mu_N G_\delta$ set in a μ_N P space (X, μ_N) then $\mu_N Int\ \kappa \neq 0_N$.

Proof: Let κ be a μ_N dense and $\mu_N G_\delta$ set in a μ_N P space (X, μ_N) then by using proposition 3.11 $\bar{\kappa}$ is a μ_N first category set in (X, μ_N). Since (X, μ_N) is μ_N P space by proposition 4.6 $\bar{\kappa}$ is not a μ_N dense set in (X, μ_N) and so $\mu_N Cl(\bar{\kappa}) \neq 1_N \Rightarrow$ that $\overline{\mu_N Int \kappa} \neq 1_N \Rightarrow \mu_N Int \kappa \neq 0_N$.

5. μ_N P space & μ_N Submaximal Space

Proposition 5.1: If each non-zero μ_N G_δ set is a μ_N dense set in a μ_N submaximal space (X, μ_N) then (X, μ_N) is a μ_N P space.

Proof: Let λ be a μ_N G_δ set in a μ_N submaximal space (X, μ_N) then by hypothesis λ is a μ_N dense set in (X, μ_N). Since (X, μ_N) is a μ_N submaximal space, the μ_N dense set λ in (X, μ_N) is μ_N open in (X, μ_N). That is every μ_N G_δ set in (X, μ_N) is μ_N open in (X, μ_N). Thus (X, μ_N) is a μ_N P space.

Proposition 5.2: If μ_N $Int(\lambda) = 0_N$, where λ is a μ_N F_σ set in a μ_N submaximal space (X, μ_N) then (X, μ_N) is a μ_N P space.

Proof: Let λ be a μ_N G_δ set in a μ_N submaximal space (X, μ_N). Then $\bar{\lambda}$ is a μ_N F_σ set in (X, μ_N). By hypothesis μ_N $Int(\bar{\lambda}) = 0_N$, for the μ_N F_σ set $\bar{\lambda}$ in (X, μ_N) which entails us that $\mu_N Cl(\lambda) = 1_N$. Then λ is a μ_N dense set in (X, μ_N). Since (X, μ_N) is a μ_N submaximal space, the μ_N dense set λ in (X, μ_N) is μ_N open in (X, μ_N). Henceforth every μ_N G_δ set in (X, μ_N) is μ_N open in (X, μ_N). Thus we conclude that (X, μ_N) is a μ_N P space.

Proposition 5.3: If each μ_N F_σ set except 1_N is a μ_N nowhere dense set in a μ_N submaximal space (X, μ_N) then (X, μ_N) is a μ_N P space.

Proof: Let λ be a μ_N F_σ set in a μ_N submaximal space (X, μ_N) such that $\mu_N Int(\mu_N Cl\ \lambda) = 0_N$. Then $\mu_N Int(\lambda) \subseteq \mu_N Int(\mu_N Cl\ \lambda) \Rightarrow \mu_N Int(\lambda) = 0_N$. Now, $\mu_N Int(\lambda) = 0_N$ for the μ_N F_σ set λ in μ_N submaximal space (X, μ_N),then by proposition 5.2 (X, μ_N) is a μ_N P space.

Proposition 5.4: If $\mu_N Cl(\mu_N Int\ \lambda) = 1_N$ for each non-empty μ_N G_δ set in a μ_N submaximal space (X, μ_N) then (X, μ_N) is a μ_N P space.

Proof: Let λ be a μ_N F_σ set in a μ_N submaximal space (X, μ_N), then $\bar{\lambda}$ is a μ_N G_δ set in a μ_N submaximal space (X, μ_N). By the given condition $\mu_N Cl(\mu_N Int\ \bar{\lambda}) = 1_N \Rightarrow \overline{\mu_N Cl(\mu_N Int\ \bar{\lambda})} = 0_N$ and hence we retrieve that λ

is a μ_N nowhere dense set in (X, μ_N). Thus the μ_N F_σ set λ is a μ_N nowhere dense set in a μ_N submaximal space (X, μ_N). Hence by proposition 5.3 we derive that (X, μ_N) is a μ_N P space.

Proposition 5.5: If λ is a μ_N residual set in a μ_N submaximal space (X, μ_N) then λ is a μ_N G_δ set in (X, μ_N).

Proof: Let λ be a μ_N residual set in a μ_N submaximal space (X, μ_N) then $\bar{\lambda}$ is a μ_N first category set in (X, μ_N) and so $\bar{\lambda} = \bigcup_{i=1}^{\infty} \lambda_i$,where $\lambda_i's$ are μ_N nowhere dense set in (X, μ_N). By owing to the fact that $\lambda_i's$ are μ_N nowhere dense set in (X, μ_N), $\mu_N Int(\mu_N Cl\, \lambda_i) = 0_N$.Then $\mu_N Int(\lambda_i) \subseteq \mu_N Int(\mu_N Cl\ \lambda_i) \Rightarrow \mu_N Int(\lambda_i) = 0_N \Rightarrow \overline{\mu_N Int(\lambda_\iota)} = 1_N \Rightarrow \mu_N Cl\, \overline{\lambda_\iota} = 1_N \Rightarrow \overline{\lambda_\iota}'s$ are μ_N dense set in (X, μ_N). Since, (X, μ_N) is μ_N submaximal space, $\overline{\lambda_\iota}'s$ are μ_N open in (X, μ_N) that entails us that $\lambda_i's$ are μ_N closed in (X, μ_N). Hence $\bar{\lambda} = \bigcup_{i=1}^{\infty} \lambda_i$,where $\lambda_i's$ are μ_N closed in (X, μ_N). Thus we retrieve that $\bar{\lambda}$ is μ_N F_σ set in (X, μ_N). Thus, λ is a μ_N G_δ set in (X, μ_N).

Proposition 5.6: If λ is μ_N nowhere dense set in a μ_N submaximal space (X, μ_N) then λ is μ_N closed set in (X, μ_N).

Proof: Let λ be a μ_N nowhere dense set in (X, μ_N) where (X, μ_N) is μ_N submaximal space. Thereupon we obtain that $\mu_N Int(\mu_N Cl(\lambda)) = 0_N$ and $\mu_N Int(\lambda) \subseteq \mu_N Int(\mu_N Cl(\lambda))$ which implies us that $\mu_N Int(\lambda) = 0_N$. Hence $\overline{\mu_N Int(\lambda)} = 1_N$ that yields us $\mu_N Cl(\bar{\lambda}) = 1_N \Rightarrow \bar{\lambda}$ is μ_N dense in (X, μ_N). Since (X, μ_N) is μ_N submaximal space, $\bar{\lambda}$ is μ_N open in (X, μ_N) that yields us λ is μ_N closed set in (X, μ_N).

Proposition 5.7: If a μ_N TS (X, μ_N) is μ_N submaximal space and also μ_N Baire space then (X, μ_N) is μ_N D Baire space.

Proof: Let (X, μ_N) be a μ_N submaximal space and μ_N Baire space. Let λ be the μ_N first category set in (X, μ_N). Since (X, μ_N) is a μ_N Baire space, $\mu_N Int(\lambda) = 0_N$. Thereupon $\overline{\mu_N Int(\lambda)} = 1_N$ that yields us $\mu_N Cl(\bar{\lambda}) = 1_N \Rightarrow \bar{\lambda}$ is μ_N dense in (X, μ_N). Since (X, μ_N) is μ_N submaximal space, $\bar{\lambda}$ is μ_N open in (X, μ_N) that yields us λ is μ_N closed set in (X, μ_N). Now $\mu_N Int(\mu_N Cl(\lambda)) = \mu_N Int(\lambda) = 0_N$. Then λ is μ_N nowhere dense set in (X, μ_N). Hence each μ_N first category set in (X, μ_N) is μ_N nowhere dense set in (X, μ_N). Therefore (X, μ_N) is μ_N D Baire space.

Conclusion: In this article we have listed many new aspects of μ_N topological space with respect to μ_N D-Baire space and μ_N space. In future μ_N filter, μ_N-ultrafilter can be implemented and further the applications of μ_N topological space can be found out.

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References

1. Atanassov.K. T, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1986), 87–96.
2. Al-Omeri.W, Smarandache.F, New Neutrosophic Sets via Neutrosophic Topological Spaces. In Neutrosophic Operational Research; Smarandache.F, Pramanik.S, Eds.; Pons Editions: Brussels, Belgium, 2017; Volume, pp. 189-209.
3. Change.C.L, Fuzzy topological spaces, Journal of Mathematical Analysis and Application, 24(1968), 183-190.
4. Dhavaseelan.R and Jafari, Generalized Neutrosophic closed sets, New trends in Neutrosophic theory and applications, 2(2018), 261-273.
5. Doger Coker, An introduction to intuitionistic funny topological spaces, Fuzzy Sets and Systems, 88(1997), 81-89.

6. Floertin Smarandache, Neutrosphy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability and Statistics University of New Mexico, Gallup, NM 87301, USA, 2002.
7. Floretin Smarandache, Neutrosophic Set: - A Generalization of Intuitionistic Fuzzy Set, Journal of Defense Resources Management, 1(2010), 107-116.
8. Floretin Smarandache, A Unifying Field in Logic: Neutrosophic Logic, Neutrosophy, Neutrosophic set, Neutrosophic Probability, American Research Press, Rehoboth, NM, 1999.
9. Raksha Ben.N, Hari Siva Annam.G, Generalized Topological Spaces via Neutrosophic Sets, J.Math.Comput.Sci., 11(2021).
10. Raksha Ben.N, Hari Siva Annam.G, μ_N Dense sets and its nature. (Submitted)
11. Salama.A.A and Alblowi.S.A, Neutrosophic set and Neutrosophic topological space, ISOR J.Mathematics, 3(4)(2012), 31-35.
12. Salama.A.A and Alblowi.S.A, Generalized Neutrosophic Set and Generalized Neutrosophic Topological Spaces, Journal Computer Sci.Engineering, 2(7)(2012), 12-13.
13. Salama.A.A, Florentin Smarandache and Valeri Kroumov, Neutrosophic Closed Set and Neutrosophic Continuous Function, Neutrosophic Sets and Systems, 4(2014), 4-8.
14. Thangaraj.G, C.Anbazhagan, Some remarks on fuzzy P spaces, Gen.Math.Notes, Vol.26, No.1, January 2015, (8-16).
15. Wadel Faris Al-Omeri and Florentin Smarandache, New Neutrosophic Sets via Neutrosophic Topological Spaces, New Trends in Neutrosophic Theory and Applications, Vol (2), June 2016.
16. Zadeh.L.A, Fuzzy set, Inform and Control, 8(1965), 338-353.

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