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# New Notions On Neutrosophic Random Variables

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**Abstract**. In this paper, new general definitions of neutrosophic random variables are introduced and their properties are studied. Notions of neutrosophic random vector, joint probability function, joint distribution function, neutrosophic random vector expected value, neutrosophic random vector variance, neutrosophic random vector covariance and some examples supported our results are presented which show the power of the study.

**Keywords:** Neutrosophic random vector; Neutrosophic random vector expected Value; Neutrosophic random vector Variance; joint probability function; joint distribution function; Neutrosophic Logic.

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## 1. **Introduction and Preliminaries**

Neutrosophic logic is an extension of intuitionistic fuzzy logic by adding indeterminacy component ($I$) where $I^2 = I, ..., I^n = I, 0.I = 0; n \in \mathbb{N}$ and $I^{-1}$ is undefined [20], [32]. Neutrosophic logic has a huge brand of applications in many fields including decision making [27], [19], [25], machine learning [7], [28], intelligent disease diagnosis [30], [12], communication services [9], pattern recognition [29], social network analysis and e-learning systems [21], physics [34], sequences spaces [14] and so on.

In probability theory, F. Smarandache defined the neutrosophic probability measure as a mapping $NP : X \to [0, 1]^3$ where $X$ is a neutrosophic sample space, and defined the probability function to take the form $NP(A) = (ch(A), ch(neutA), ch(antiA)) = (\alpha, \beta, \gamma)$ where $0 \leq \alpha, \beta, \gamma \leq 1$ and $0 \leq \alpha + \beta + \gamma \leq 3$ [33]. Besides, many researchers have introduced many neutrosophic probability distributions like Poisson, exponential, binomial, normal, uniform, Weibull and so on. [32], [4], [18], [26]. Additionally, researchers have presented the concept of neutrosophic queueing theory in [35], [36] that is one branch of neutrosophic stochastic modelling. Furthermore, researchers have studied neutrosophic time series prediction and modelling in many cases like neutrosophic moving averages, neutrosophic logarithmic models, neutrosophic linear models and so on. [2], [3], [13].

On the other hand, neutrosophic logic has solved many decision-making problems efficiently like evaluating green credit rating, personnel selection, etc. [22], [23], [24], [1].

In this paper we will introduce a generalization to classical random vector to deal with imprecise, uncertainty, ambiguity, vagueness, enigmatic adding the indeterminacy part to its form, then we will show and prove several characteristics of this neutrosophic random vector including expected value, variance, covariance, joint function probability, joint distribution function and study its properties. This extension lets us build and study many stochastic models in the future that help us in modelling, simulation, decision making, prediction and classification specially in the cases of incomplete data and indeterminacy. For more notions associated to neutrosophic theory, we refer the reader to [10, 11, 14–17].

Now, we show some well-known definitions and properties of neutrosophic logic and neutrosophic probability which are useful for the developing of this paper.

**Definition 1.1.** (see [31]) Let $X$ be a non-empty fixed set. A neutrosophic set $A$ is an object having the form $\{x, (\mu A(x), \delta A(x), \gamma A(x)) : x \in X\}$, where $\mu A(x), \delta A(x)$ and $\gamma A(x)$ represent the degree of membership, the degree of indeterminacy , and the degree of non-membership respectively of each element $x \in X$to the set $A$.

**Definition 1.2.** (see [6]) Let $K$ be a field, the neutrosophic filed generated by $K$ and $I$ is denoted by $\langle K \cup I \rangle$ under the operations of $K$, where $I$ is the neutrosophic element with the property $I^2 = I$.

**Definition 1.3.** (see [32]) Classical neutrosophic number has the form $a + bI$ where $a, b$ are real or complex numbers and $I$ is the indeterminacy such that $0.I = 0$ and $I^2 = I$ which results that $I^n = I$ for all positive integers $n$.

**Definition 1.4.** (see [33]) The neutrosophic probability of event $A$ occurrence is $NP(A) = (ch(A), ch(neutA), ch(antiA)) = (T, I, F)$ where $T, I, F$ are standard or non-standard subsets of the non-standard unitary interval $]^{-}0, 1^{+}[$.

Recently, Bisher and Hatip [8] introduced and studied the notions of neutrosophic random variables by using the concepts presented by [33], these notions were defined as follows:

**Definition 1.5.** Consider the real valued crisp random variable $X$ which is defined as follows:

$$X : \Omega \to \mathbb{R}$$

where $\Omega$ is the events space. Now, they defined a neutrosophic random variable $X_N$ as follows:

$$X_N : \Omega \to \mathbb{R}(I)$$

and

$$X_N = X + I$$

where $I$ is indeterminacy.

**Theorem 1.6.** *Consider the neutrosophic random variable $X_N = X + I$ where cumulative distribution function of $X$ is $F_X(x) = P(X \leq x)$. Then, the following statements hold:*

(1) $F_{X_N}(x) = F_X(x - I)$,
(2) $f_{X_N}(x) = f_X(x - I)$.

*Where $F_{X_N}$ and $f_{X_N}$ are cumulative distribution function and probability density function of $X_N$, respectively.*

**Theorem 1.7.** *Consider the neutrosophic random variable $X_N = X + I$, expected value can be found as follows:*

$$E(X_N) = E(X) + I.$$

**Proposition 1.8** (Properties of expected value of a neutrosophic random variable)**.** *Let $X_N$ and $Y_N$ be neutrosophic random variables, then the following properties holds:*

(1) $E(aX_N + b + cI) = aE(X_N) + b + cI; a, b, c \in \mathbb{R}$,
(2) *If $X_N$ and $Y_N$ are neutrosophic random variables, then $E(X_N \pm E_(Y_N) = E(X_N) \pm E(Y_N)$,*
(3) $E[(a + bI)X_N] = aE(X_N) + bIE(X_N); a, b \in \mathbb{R}$,
(4) $|E(X_N)| \leq E|X_N|$.

**Theorem 1.9.** *Consider the neutrosophic random variable $X_N = X + I$, variance of $X_N$ is equal to variance of $X$, i.e. $V(X_N) = V(X)$.*

For supporting above definitions and their implications, we present some examples by using exponential distribution on a neuttrosophic random variable which show how importance neutrosophic random variable is in neutrosophic probability theory. For more examples on neutrosophic random variable and its difference between classical random variable see [5].

**Example 1.10.** Let $X_N$ be a neutrosophic continuous random variable which has an exponential distribution with parameter $\lambda > 0$, and we will denote this as $X_N \sim exp(\lambda)$, its density function is defined as follows:

$$f_{X_N}(x) = f_X(x - I) = \begin{cases} \lambda e^{-\lambda(x-I)} & if\ x > I, \\ \\ 0 & if\ x \leq I. \end{cases}$$

Applying Theorems 1.7 and 1.9, we can check that $E(X_N) = 1/\lambda + I$ and $V(X_N) = 1/\lambda^2$.

**Example 1.11.** Suppose that the time in minutes that a user spends checking his email follows an exponential parameter distribution $\lambda = 2$. Calculate the probability that the user stay connected to the mail server for less than a minute with an indeterminacy $I$.

**Solution**: Let $X_N$ the connection time to the mail server, by example 1.10 we have

$$P(X_N < 1) = P(X + I < 1) = P(X < 1 - I) = \int_0^{1-I} 2e^{-2(x-I)}dx = e^{2I} - e^{-2(1-2I)},$$

if we take $I = 0.5$, we have $\int_0^{0.5} 2e^{-2(x-0.5)}dx = e - 1 \simeq 0.368$

**Example 1.12.** Let $X_N$ be a neutrosophic random variable with exponential distribution $exp(\lambda)$. We will show that the function generating moments of $X_N$ is the function $M_{X_N}$ that appears below.

$$M_{X_N}(t) = \frac{\lambda}{\lambda - t}e^{tI}, \text{ for all } t < \lambda.$$

Besides, we will find its expected.

**Solution**:

$M_{X_N} = E(e^{tX_N}) = E(e^{t(X+I)}) = E(e^{tx+tI}) = e^{tI}E(e^{tX}) = e^{tI}\int_0^{\infty} e^{tx}\lambda e^{-\lambda x}dx = e^{tI}\frac{\lambda}{\lambda - t}$, if $t < \lambda$.

Now, we will find its expected, Bisher and Hatip [8] proved $\frac{d_{M_{X_N}}(0)}{dt} = E(X_N)$. Therefore we have

$$\frac{d_{M_{X_N}}(t)}{dt} = e^{tI}\frac{\lambda}{(\lambda - t)^2} + \frac{\lambda}{\lambda - t}Ie^{tI},$$

if we take $t = 0$, we obtain

$$\frac{d_{M_{X_N}}(0)}{dt} = \frac{\lambda}{\lambda^2} + \frac{\lambda}{\lambda}I = \frac{1}{\lambda} + I = E(X_N)$$

as was shown in example 1.10.

## 2. Main Results

In [33] Smarandache presented the neutrosophic random variable that it is a variable that may have and indeterminate outcome, and later [8] Bisher and Hatip represented that indeterminacy by mathematical formula on neutrosophic random variables. Now, we are going to find the properties of joint neutrosophic random variable by using notions mentioned above, with this we proved that covariance of neutrosophic random variables $X_N$ and $Y_N$ is equal to covariance of $X$ and $Y$ as can be seen in Theorem 2.15. But first, we have to introduce and study the following definitions:

**Definition 2.1.** A neutrosophic random vector of two dimension is a vector $(X_N, Y_N)$ in which each coordinate is a neutrosophic random variable. Analogously, we can define a neutrosophic random vector multidimensional as follows $(X_{N_1}, X_{N_2}, ..., X_{N_n})$ in which $X_{N_1}, X_{N_2}, ..., X_{N_n}$ are neutrosophic random variables for each $n = 1, 2, ....$

**Definition 2.2.** Let $(X_N, Y_N)$ be a neutrosophic random vector in which $X_N$ takes the value $x_1, x_2, ...$ and $Y_N$ takes the value $y_1, y_2, ....$ Then, joint probability function of a neutrosophic discrete random vector $(X_N, Y_N)$ $f_N(x, y) : \mathbb{R}^2 \to [0, 1]$ and any $(x, y) \in \mathbb{R}^2$, it is defined as follows

$$f_{(X_N,Y_N)}(x,y) = f_{(X,Y)}(x-I, y-I) = \begin{cases} P(X = x-I, Y = y-I) = \displaystyle\sum_{u \le x-I}\sum_{v \le y-I} f_{(X_N,Y_N)}(u,v) \\ \qquad if\ (x-I, y-I) \in \{x_1 - I, x_2 - I, ...\} \times \{y_1 - I, y_2 - I, ...\} \\ 0 \qquad otherwise. \end{cases}$$

**Definition 2.3.** Let $(X_N, Y_N)$ be a neutrosophic random vector, we define probability function of a neutrosophic continuous random vector $(X_N, Y_N)$. Then, joint probability neutrosophic function of a discrete random vector $(X_N, Y_N)$ $f_N(x, y) : \mathbb{R}^2 \to [0, \infty)$ in which is non-negative and integrable, and for any $(x, y) \in \mathbb{R}^2$ , it is defined as follows

$$P(X_N \le x, Y_N \le y) = P(X \le x - I, Y \le y - I) = \int_{-\infty}^{y-I}\int_{-\infty}^{x-I} f_{(X_N,Y_N)}(u,v)dvdu$$

**Example 2.4.** We will show that $g : \mathbb{R}^2 \to [0, 1]$ is a joint probability neutrosophic function where.

$$g(x,y) = \begin{cases} 4xy & if \quad 0 < x < 1, 0 < y < 1 \\ 0 & otherwise. \end{cases}$$

Taking into account that $g_N(x, y) = g(x - I, y - I)$,

$$g_{(X_N,Y_N)}(x,y) = g(x-I, y-I) = \begin{cases} 4(x-I)(y-I) & if \quad I < x < 1+I, I < y < 1+I \\ 0 & otherwise. \end{cases}$$

We can see that $g(x, y) \ge 0$ for any $(x, y) \in \mathbb{R}^2$. Now, $\int_I^{1+I}\int_I^{1+I} 4(x-I)(y-I)dxdy = \int_I^{1+I} 2(y-I)[\int_I^{1+I} 2(x-I)dx]dy = \int_I^{1+I} 2(y-I)dy = 1.$

**Definition 2.5.** Let $(X_N, Y_N)$ be a neutrosophic random vetor, we define neutrosophic joint distribution function which will be denoted by $F_{(X_N,Y_N)}(x, y) = P(X_N \le x, Y_N \le y) = P(X \le x - I, Y \le y - I)$.

**Remark 2.6.** The little comma which appears in the right means intersection of $(X_N \leq x)$ and $(Y_N \leq y)$, i.e. $F_{(X_N,Y_N)}(x,y)$ is the probability of $(X_N \leq x) \cap (Y_N \leq y) = (X \leq x - I) \cap (Y \leq y - I)$.

**Remark 2.7.** If we know joint probability neutrosophic function of a random vector, we can find neutroosphic joint distribution function as

(1) $F_{(X_N,Y_N)}(x,y) = \int_{-\infty}^{y-I} \int_{-\infty}^{x-I} f_{(X_N,Y_N)}(u,v)dvdu$ **(Continuous case)**.

(2) $F_{(X_N,Y_N)}(x,y) = \sum_{u \leq x-I} \sum_{v \leq y-I} f_{(X_N,Y_N)}(u,v)$ **(Discrete case)**.

Besides, if we know neutrosophic joint distribution function, we can find joint probability neutrosophic function as

(3) $f_{(X_N,Y_N)}(x,y) = \frac{\partial^2}{\partial x \partial y} F_{(X_N,Y_N)}(x,y)$ **(continuous case)**.

(4) $f_{(X_N,Y_N)}(x,y) = F_N(x,y) - F_N(x^-,y) - F_N(x,y^-) + F_N(x^-,y^-)$ **(Discrete case)**, where $F_N(x^-,y)$ means the limit of $F_{(X_N,Y_N)}(x,y)$ in the point $(x,y)$ taking into account that $y$ is a constant and we approximate $x$ by left.

Here, we will show a little proof of part four: $(X_N \leq x) = (X \leq x - I)$ can be written as $(X < x-I) \cup (X = x-I)$ where $(X < x-I) \cap (X = x-I) = \emptyset$. Analogously, $(Y_N \leq y) = (Y \leq y - I) = (Y < y - I) \cup (Y = y - I)$. if we write $(X_N \leq x, Y_N \leq y) = (X \leq x - I, Y \leq y - I)$ takes into account mentioned above, the proof follows.

**Proposition 2.8.** *Let $F_{(X_N,Y_N)}(x,y)$ and $G_{(X_N,Y_N)}(x,y)$ be two neutrosophic joint distribution functions . Then, for any $\lambda \in [0,1]$, $(x,y) \to \lambda F_{(X_N,Y_N)}(x,y) + (1-\lambda)G_{(X_N,Y_N)}(x,y)$ is a neutrosophic joint distribution function.*

**Example 2.9.** Let $(X_N, Y_N)$ be a continuous neutrosophic random vector with joint probability neutrosophic function

$$h_{(X_N,Y_N)}(x,y) = \begin{cases} 1 & if \quad 0 < x, y < 1 \\ \\ 0 & otherwise. \end{cases}$$

Taking into account that $h_{(X_N,Y_N)}(x,y) = h(x - I, y - I)$, we have

$$h_{(X_N,Y_N)}(x,y) = h(x - I, y - I) = \begin{cases} 1 & if \quad I < x, y < 1 + I \\ \\ 0 & otherwise. \end{cases}$$

we can see that neutrosophic joint distribution function is determined by

$$F_{(X_N,Y_N)}(x,y) = F_{(X,Y)}(x-I,y-I) = \begin{cases} 0 & if \quad x \leq I \ or \ y \leq I \\ \\ (x-I)(y-I) & if \quad I < x, y < 1+I \\ \\ (x-I) & if \quad I < x < 1+I, y \geq 1+I \\ \\ (y-I) & if \quad I < y < 1+I, x \geq 1+I \\ \\ 1 & if \quad x, y \geq 1+I. \end{cases}$$

**Definition 2.10.** Let $f_{(X_N,Y_N)}(x,y)$ be a joint probability neutrosophic function of a continuous random variable $(X_N,Y_N)$. We define neutrosophic marginal function of $X_N$ as follows:

$$f_{X_N}(x) = \int_{-\infty}^{+\infty} f_{(X_N,Y_N)}(x,y)dy$$

and we define neutrosophic marginal function of $Y_N$ as follows:

$$f_{Y_N}(y) = \int_{-\infty}^{+\infty} f_{(X_N,Y_N)}(x,y)dx$$

**Example 2.11.** Let $(X_N,Y_N)$ be a continuous neutrosophic random vector with joint probability neutrosophic function

$$g_{(X_N,Y_N)}(x,y) = \begin{cases} 4xy & if \quad 0 < x < 1, 0 < y < 1 \\ \\ 0 & otherwise. \end{cases}$$

Then, neutrosophic marginal function of $X_N$ is $f_{X_N}(x) = \int_I^{1+I} 4(x-I)(y-I)dy = 4(x-I)\int_I^{1+I}(y-I)dy = 2(x-I)$. Therefore,

$$f_{X_N}(x,y) = \begin{cases} 2(x-I) \ if & I < x < 1+I \\ \\ 0 & otherwise. \end{cases}$$

Analogously, we can show that

$$f_{Y_N}(x,y) = \begin{cases} 2(y-I) \ if & I < y < 1+I \\ \\ 0 & otherwise. \end{cases}$$

**Definition 2.12.** Let $f_{(X_N,Y_N)}(x,y)$ be a joint probability neutrosophic function of a discrete random variable $(X_N,Y_N)$. We define neutrosophic marginal function of $X_N$ as follows:

$$f_{X_N}(x) = \sum_y f_{(X_N,Y_N)}(x,y)$$

and we define neutrosophic marginal function of $Y_N$ as follows:

$$f_{Y_N}(y) = \sum_x f_{(X_N,Y_N)}(x,y)$$

**Example 2.13.** Let $(X_N, Y_N)$ be a discrete random variable with joint probability neutrosophic function

$$f_{(X_N,Y_N)}(x,y) = \begin{cases} \frac{x+2y}{30}\ if \quad (x,y) \in \{1,2,3\} \times \{1,2\} \\ 0 \quad otherwise. \end{cases}$$

We can see that $f_{(X_N,Y_N)}(x,y)$ is a joint probability neutrosophic function, we show a little proof,

Since $f_{(X_N,Y_N)}(x,y) = f_{(X,Y)}(x-I, y-I)$, we have

$$f_{X_N}(x,y) = \sum_{y=1+I}^{2+I} f_{(X_N,Y_N)}(x,y) = \sum_{y=1+I}^{2+I} \frac{x+2y-3I}{30} = \frac{x+3-I}{15}$$

for $x \in \{1+I, 2+I, 3+I\}$ and 0 otherwise. In the same way

$$f_{Y_N}(x,y) = \sum_{x=1+I}^{3+I} f_{(X_N,Y_N)}(x,y) = \sum_{x=1+I}^{3+I} \frac{x+2y-3I}{30} = \frac{1+y-I}{5}$$

for $y \in \{1+I, 2+I\}$ and 0 otherwise.

Now, the neutrosophic marginal functions $f_{X_N}(x)$ and $f_{Y_N}(y)$ are

$$f_{X_N}(x,y) = \sum_{y=1+I}^{2+I} = \begin{cases} 8/30\ if \quad x = 1+I \\ 10/30 \quad if\ x = 2+I \\ 12/30 \quad if\ x = 3+I \\ 0 \quad otherwise. \end{cases}$$

and

$$f_{Y_N}(x,y) = \sum_{x=1+I}^{3+I} = \begin{cases} 12/30\ if \quad y = 1+I \\ 18/30 \quad if\ y = 2+I \\ 0 \quad otherwise. \end{cases}$$

**Definition 2.14.** Expected of a neutrosophic random vector $(X_N, Y_N)$ in which expected of $X_N$ and $Y_N$ exist, we define $E(X_N, Y_N) = (E(X_N), E(Y_N))$.

**Theorem 2.15.** *Consider the neutrosophic random variables $X_N = X + I$ and $Y_N = Y + I$, covariance of $(X_N, Y_N)$ is equal to covariance of $(X, Y)$, i.e. $Cov(X_N, Y_N) = Cov(X, Y)$.*

**Proof**: Let $Cov(X_N, Y_N) = E[(X_N - E(X_N)(Y_N - E(Y_N)]$,

$Cov(X_N, Y_N) = E[(X_N - E(X_N)(Y_N - E(Y_N)]$

$= E[(X + I - E(X) - I)(Y + I - E(Y) - I)]$

$E[(X-E(X))(Y-E(Y))]$
$=Cov(X,Y)$.

**Example 2.16.** Let $(X_N,Y_N)$ be a continuous neutrosophic random vector with joint probability neutrosophic function

$$g_{(X_N,Y_N)}(x,y)=\begin{cases} 4xy \qquad if \qquad 0<x<1, 0<y<1 \\ \\ 0 \qquad otherwise. \end{cases}$$

Then, covariance of $(X_N,Y_N)$ is calculated as

$$Cov(X_N,Y_N)=Cov(X,Y)=\int_0^1\int_0^1(x-\frac{2}{3})(y-\frac{2}{3})4xydxdy=$$
$$\int_0^1 4y(y-\frac{2}{3})[\int_0^1(x^2-\frac{2}{3}x)dx]dy=0.$$

**Remark 2.17.** It is clear that $Cov(X_N,Y_N)=Cov(Y_N,X_N)$.

**Theorem 2.18.** *Variance of a neutrosophic random vector $(X_N,Y_N)$ is equal to variance of a random vector $(X,Y)$, i.e. $Var(X_N,Y_N)=Var(X,Y)$. In others words,*

$$Var(X_N,Y_N)=\begin{pmatrix} Var(X_N) & Cov(X_N,Y_N) \\ Cov(Y_N,X_N) & Var(Y_N)\end{pmatrix}=\begin{pmatrix} Var(X) & Cov(X,Y) \\ Cov(Y,X) & Var(Y)\end{pmatrix}=Var(X,Y). \tag{1}$$

**Proof**: The proof is followed by Theorems 1.9 and 2.15.

**Example 2.19.** Let $(X_N,Y_N)$ be a continuous neutrosophic random vector with normal distribution and parameters $(\mu_1,\sigma_1^2,\mu_2,\sigma_2^2,\rho)$. This distribution is defined as follows:

$f_{(X_N,Y_N)}(x,y)=f_{(X,Y)}(x-I,y-I)=\dfrac{1}{2\pi\sigma_1^2\sigma_2^2\sqrt{1-\rho^2}}\exp(-\dfrac{1}{2(1-\rho^2)}[\dfrac{(x-I-\mu_1)^2}{\sigma_1^2}-\dfrac{2\rho}{\sigma_1^2\sigma_2^2}(x-I-\mu_1)(y-I-\mu_2)+\dfrac{(y-I-\mu_2)^2}{\sigma_2^2}])$ where $\mu_1,\mu_2\in\mathbb{R}$ ; $\sigma_1^2,\sigma_2^2>0$ and $-1<\rho<1$ .

When, $\mu_1=\mu_2=0$ and $\sigma_1^2=\sigma_2^2$, we have neutrosophic random vector with standard normal distribution, and $f_{(X_N,Y_N)}(x,y)$ is reduced as:

$f_{(X_N,Y_N)}(x,y)=f_{(X,Y)}(x-I,y-I)=\dfrac{1}{2\pi\sqrt{1-\rho^2}}\exp(-\dfrac{1}{2(1-\rho^2)}((x-I)^2-2\rho(x-I)(y-I)+(y-I)^2))$.

Then, we will show that:

(1) $E(X_N,Y_N)=(\mu_1+I,\mu_2+I)$.
(2) $Cov(X_N,Y_N)=\rho\sigma_1^2\sigma_2^2$.
(3) $Var(X_N,Y_N)=\begin{pmatrix}\sigma_1^2 & \rho\sigma_1^2\sigma_2^2 \\ \rho\sigma_1^2\sigma_2^2 & \sigma_2^2\end{pmatrix}$.

**Solution**:

(1) Since $E(X_N,Y_N)=(E(X_N),E(Y_N))=(E(X)+I,E(Y)+I)=(\mu_1+I,\mu_2+I)$.

(2) $Cov(X_N, Y_N) = Cov(X,Y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xyf_{(X_N,Y_N)}(x,y)dxdy - \mu_1\mu_2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xyf_{(X,Y)}(x,y)dxdy - \mu_1\mu_2 = \mu_1\mu_2 + \rho_{12}^{22} - \mu_1\mu_2 = \rho\sigma_1^2\sigma_2^2$.

(3) By part (2) of this example and takes into account that $Var(X_N) = Var(X)$. We have, $Var(X_N, Y_N) = Var(X,Y) = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1^2\sigma_2^2 \\ \rho\sigma_1^2\sigma_2^2 & \sigma_2^2 \end{pmatrix}$.

## 3. Conclusion

The results that are presented in this paper can be applied to define several notions in neutrosophic probability theory that are not defined and not studied yet including independence random variables, convergence in random variables, stochastic processes, reliability theory models, quality control techniques. where all depend on the concept of neutrosophic random variables and its properties. Besides, these results can be applied in stochastic modelling and random numbers generating which is very important in simulation of probabilistic models.

We are looking forward to studying the properties of joint neutrosophic probability distributions when the distribution of random vector changes $(X_N, Y_N) = (X + I, Y + I)$ i.e., when the random vector contains an indeterminant part so we can model and simulate many stochastic problems.

In this research, we firstly obtained a new general definitions of neutrosophic random vector, concepts of joint probability distribution function and joint distribution function. We focused on the neutrosophic representation and proved some properties. Additionally, we showed various examples in which can help to applied them in several applications problems.

## Conflicts of Interest

The author declares no conflict of interest

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