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Novel Concept of Interval-Valued Neutrosophic Incidence Graphs with Application

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Abstract: Neutrosophic set (NS) is a framework used when the imprecision and uncertainty of an event are described based on three possible aspects, i.e., the membership degree, neutral membership degree and non-membership degree. On the other hand, neutrosophic graphs (NG) are applicable to deal with bulk information events. Furthermore, the incidence graph concept in neutrosophic sets contains a handful of problems like decision-making as well as, social and communication networks. This paper aims to propose the interval-valued neutrosophic sets to incidence graph and represent a new concept, namely interval-valued neutrosophic incidence graph (IVNIG). An IVNIG is a generalization of the concept of single-valued neutrosophic incidence graph (SVNIG). Moreover, some properties related to IVNIG, such as strong edge, strong pair, strong cut pair and neutrosophic incidence cut pair, are also discussed using suitable examples. The defined new concept of IVNIG is applied and investigated on a practical problem of safe root travelling.

Keywords: bridge; cut pair; interval-valued neutrosophic incidence graphs; strong edge; strong pair

1. Introduction

The concept of fuzzy sets was pioneered by Zadeh [1]. Later, Zadeh introduced the notion of interval-valued fuzzy sets as an extension of fuzzy sets [2], where the membership degrees' values are intervals of numbers instead of numbers itself. Interval-valued fuzzy sets provide a more comprehensive overview of uncertainty than traditional fuzzy sets. However, these may not be enough in modelling indeterminate and inconsistent information to deal with in the real world. Therefore, to encounter this problem, Smarandache [3] proposed the notion of neutrosophic sets by combining non-standard analysis. In a neutrosophic set, the membership value is associated with three components which are truth membership (t), indeterminacy membership (i) and falsity membership (f), where each membership value is a real standard or non-standard subset of the non-standard unit interval $]0^-,1^+[$ with no restriction on their sum.

Later, Smarandache and Wang et al. [4] introduced the single-valued neutrosophic sets (SVNS) to apply neutrosophic sets in real-life problems. Also, Wang et al. [5] presented the concept of an interval-valued neutrosophic set (IVNS). Representation of graphs using IVNS is more precise and more flexible than the SVNS. An IVNS is a generalization of the concept of SVNS. Three membership functions (t, i, f) are independent and their values belong to the unit interval [0, 1].

Graph theory has become a significant area of applied mathematics, commonly considered a combinatorics area. The graph is a widely used tool for solving combinatorial problems in different areas such as computer science, optimization, topology, number theory, algebra and geometry. It should be pointed out that, when there is uncertainty regarding either the set of vertices or edges, or both, the model becomes a fuzzy graph [6]. In recent years, there has been an increasing amount of literature performed on the fuzzy graph [7–11], intuitionistic fuzzy graphs [12–18] and interval-valued intuitionistic fuzzy graphs [19–22]. All of them have considered the vertex and edge sets as fuzzy and/or intuitionistic fuzzy sets. However, when the relations between nodes (or vertices) are inconsistent, the fuzzy graphs and intuitionistic fuzzy graphs fail to work.

Therefore, Smarandache [23–25] defined four main categories of neutrosophic graphs. These are based on literal indeterminacy (I), which are I-edge neutrosophic graph and I-vertex neutrosophic graph. These concepts were studied extensively and gained much attention among the researchers due to their applications in real-world problems [26–27]. The two other graph categories are based on (t, i, f) components, called (t, i, f)-edge neutrosophic graph and (t, i, f)-vertex neutrosophic graph. However, these two categories were not developed at all. Later on, Broumi et al. [28] introduced and investigated a new neutrosophic graph model called single-valued neutrosophic graph (SVNG). This model allows attaching the membership (t), indeterminacy (i) and non-membership (f) degrees to both vertices and edges. The SVNG is a generalization of fuzzy graph (FG) and intuitionistic fuzzy graph (IFG).

The same authors, Broumi et al. [29–30] introduced the concept of an interval-valued neutrosophic graph (IVNG) as a generalization of the SVNG. The properties were discussed using proof and examples. Later on, Akram and Nasir [31] showed some flaws in Broumi's definition, which cannot be applied in network models. The authors then modified the definition of an IVNG, discussing some operations involved. Using this approach, Akram and Sitara [32] introduced IVNG structure and several concepts on interval-valued neutrosophic competition graphs were presented in [33].

Dinesh [34] first introduced the concept of unordered pairs of vertices, which are not incident with end vertices. The fuzzy incidence shows the relations between vertices and provides information about the influence of a vertex on the edge. Later, the idea of the fuzzy incidence graph was extended by Dinesh [35], and the author introduces new concepts in this regard. Moreover, Mathew and Mordeson [36] discussed the connectivity concepts in fuzzy incidence graphs. These are important in interconnection networks with influenced flows. Therefore, it is crucial to analyze their connectivity properties. Next, the fuzzy incidence graph was studied by Malik et al. [37]. The authors applied the notion of the fuzzy incidence graph in problems involving human trafficking. They discussed the role played by the vulnerability of countries and their government's response to human trafficking. Other than that, Mathew et al. [38] studied some properties of incidence cuts and connectivity in fuzzy incidence graphs. The incidence is used to model flows in human trafficking networks. Fuzzy incidence block was defined by Mathew and Moderson [39], discussing their applications in illegal migration problems. They used fuzzy incidence graphs as a non-deterministic network model with supporting links by applying fuzzy incidence blocks to avoid the network's vulnerable links.

In view of all that has been mentioned so far, Akram et al. [40] investigated the extension of the fuzzy incidence graph in the form of the neutrosophic environment. The authors introduced the notion of a single-valued neutrosophic incidence graph (SVNIG) and discussed the connectivity in this regard. Later, Akram et al. [41] studied the idea of bipolar neutrosophic sets to incidence graphs, and some related properties were defined. Recently, Hussain et al. [42] have presented the neutrosophic vague incidence graph and defined the edge connectivity, the vertex connectivity, and pair connectivity in neutrosophic vague incidence graph. A summary of the author's contribution toward the incidence graph is presented in Table 1.

Based on the idea of SNVIG, in this paper we propose the interval-valued neutrosophic sets with incidence graph, representing a new concept, namely interval-valued neutrosophic incidence graphs (IVNIG). The properties related to IVNIG, such as strong edge, strong pair, strong cut pair and neutrosophic incidence cut pair are also discussed with suitable examples. The rest of this paper is instructed as follows: Part 2 contains a brief background about graphs and neutrosophic set applied later. We then introduce the concept of IVNIG graph and investigate its properties in Part 3. In Part 4, we apply the proposed method in application of finding the best route. In Part 5, illustrate the comparative study and advantages of the proposed method. Finally, Part 6 outlines the conclusion together with limitations of the study and suggest an open problem for future research.

Table 1. Contribution of authors to incidence graphs

Authors	Year	Contributions
Dinesh [34]	2012	fuzzy incidence graph
Dinesh [35]	2016	extended of fuzzy incidence graph
Mathew and Mordeson [36]	2017	connectivity concepts in fuzzy incidence graph
Mathew and Mordeson [39]	2017	fuzzy incidence block
Malik et al. [37]	2018	fuzzy incidence graph in human trafficking
Akram et al. [40]	2018	single-valued neutrosophic incidence graph
Mathew et al. [38]	2019	incidence cuts and connectivity in fuzzy incidence graph
Akram et al. [41]	2019	bipolar neutrosophic incidence graph
Hussain et al. [42]	2020	neutrosophic vague incidence graph

2. Preliminaries

In this part, some basic concepts related to neutrosophic sets, single-valued neutrosophic sets, interval-valued neutrosophic sets, fuzzy graph, single-valued neutrosophic graphs and interval-valued neutrosophic graphs are presented and used in the next parts.

Definition 2.1 [43]

A **fuzzy graph** is a pair of functions $\mathrm{G}=(\sigma,\mu)$, where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ, .i.e., $\sigma:\mathrm{V}\rightarrow[0,1]$ and $\mu:\mathrm{V}\times\mathrm{V}\rightarrow[0,1]$ such that $\mu(\mathrm{uv})\leq\sigma(\mathrm{u})\wedge\sigma(\mathrm{v})$ for all $\mathrm{u},\mathrm{v}\in\mathrm{V}$. Here, uv denotes the edge between u and v while $\sigma(\mathrm{u})\wedge\sigma(\mathrm{v})$ denotes the minimum of $\sigma(\mathrm{u})$ and $\sigma(\mathrm{v})$. σ is called the fuzzy vertex set of V, while μ is called the fuzzy edge set of E.

Definition 2.2 [36]

Let $\mathrm{G}=(\mathrm{V},\mathrm{E})$ be a graph, σ be a fuzzy subset of V, μ be a fuzzy subset of E and ψ be a fuzzy subset of $\mathrm{V}\times\mathrm{E}$. If $\psi=(\mathrm{v},\mathrm{e})\leq\min(\sigma(\mathrm{v}),\mu(\mathrm{e}))$ for all $\mathrm{v}\in\mathrm{V}$ and $\mathrm{e}\in\mathrm{E}$, then ψ is called a **fuzzy incidence** of G.

Definition 2.3 [36]

Let $\mathrm{G}=(\mathrm{V},\mathrm{E})$ be a graph and (σ,μ) be a fuzzy subgraph of G. If ψ is a fuzzy incidence of G, then $\tilde{\mathrm{G}}=(\sigma,\mu,\psi)$ is called a **fuzzy incidence graph** of G. Any $\mathrm{x}\in\mathrm{V}$ is said to be in support of σ if $\sigma(\mathrm{x})>0,\mathrm{xy}\in\mathrm{V}\times\mathrm{V}$ is said to be in support of μ if $\mu(\mathrm{xy})>0$, and $(\mathrm{x},\mathrm{yz})\in\mathrm{V}\times\mathrm{E}$ is said to be in support of ψ if $\psi(\mathrm{x},\mathrm{yz})>0$. The supports of $\sigma,\mu,$ and ψ are denoted as

σ^*, μ^*, and ψ^*, respectively. Let $xy \in Supp(\mu)$. Then, xy is an edge of the fuzzy incidence graph $\tilde{G} = (\sigma, \mu, \psi)$ and if $(x,xy),(y,xy) \in Supp(\psi)$, then (x,xy) and (y,xy) are called **pairs**. Two vertices v_i and v_j joined by a path in a fuzzy incidence graph are said to be **connected**. The **incidence strength** of a fuzzy incidence graph $\tilde{G} = (\sigma, \mu, \psi)$ is defined to be $\min\{\psi(v,e) \mid (v,e) \in Supp(\psi)\}$.

Definition 2.4 [44]

A **neutrosophic graph** is defined as a pair $G(V,E)$ where

i. $V = \{v_1, v_2, \ldots, v_n\}$ such that $T_1 : V \to [0,1], I_1 : V \to [0,1]$ and $F_1 : V \to [0,1]$ denote the degree of truth-membership function, indeterminacy function and falsity-membership function respectively, where $0 \le T_1(v) + I_1(v) + F_1(v) \le 3$

ii. $E \subseteq V \times V \to [0,1]$ where E is relation on V such that

$$T_2(uv) \le \min\{T_1(u), T_1(v)\},$$
$$I_2(uv) \le \min\{I_1(u), I_1(v)\},$$
$$F_2(uv) \le \max\{F_1(u), F_1(v)\},$$
$$\text{and } 0 \le T_2(uv) + I_2(uv) + F_2(uv) \le 3, \forall uv \in E$$

Definition 2.5 [40]

Let $G' = (V,E,I)$ be an incidence graph, where V is a vertex set of G, E is edge set of G and I is incidence of G, then a single-valued neutrosophic incidence graph is an ordered-triplet, $\tilde{G} = (A,B,C)$ such that

1. A is a single-valued neutrosophic set on V
2. B is a single-valued neutrosophic relation on V
3. C is a single-valued neutrosophic subset of $V \times E$ such that

$$T_C(x,xy) \le \min\{T_A(x), T_B(xy)\},$$
$$I_C(x,xy) \le \min\{I_A(x), I_B(xy)\},$$
$$F_C(x,xy) \le \max\{F_A(x), F_B(xy)\}, \forall x \in V, xy \in E$$

Definition 2.6 [45][46]

The interval-valued neutrosophic set A in X is defined by

$$A = \left\{\left(x, \left[t_A^l(x), t_A^u(x)\right], \left[i_A^l(x), i_A^u(x)\right], \left[f_A^l(x), f_A^u(x)\right]\right) : x \in X\right\},$$

where $t_A^l(x), t_A^u(x), i_A^l(x), i_A^u(x), f_A^l(x)$ and $f_A^u(x)$ are neutrosophic subsets of X such that

$$t_A^l(x) \le t_A^u(x),$$
$$i_A^l(x) \le i_A^u(x), \text{ and}$$
$$f_A^l(x) \le f_A^u(x), \forall x \in X$$

For any two interval-valued neutrosophic sets:

$$A = \left\{\left(x, \left[t_A^l(x), t_A^u(x)\right], \left[i_A^l(x), i_A^u(x)\right], \left[f_A^l(x), f_A^u(x)\right]\right) : x \in X\right\}, \text{ and}$$
$$B = \left\{\left(x, \left[t_B^l(x), t_B^u(x)\right], \left[i_B^l(x), i_B^u(x)\right], \left[f_B^l(x), f_B^u(x)\right]\right) : x \in X\right\},$$

define that,

$$A\cup B=\left\{\begin{pmatrix}x,\max\left(t_A^l(x),t_B^l(x)\right),\max\left(t_A^u(x),t_B^u(x)\right),\max\left(i_A^l(x),i_B^l(x)\right),\\ \max\left(i_A^u(x),i_B^u(x)\right),\min\left(f_A^l(x),f_B^l(x)\right),\min\left(f_A^u(x),f_B^u(x)\right)\end{pmatrix}:x\in X\right\}$$

$$A\cap B=\left\{\begin{pmatrix}x,\min\left(t_A^l(x),t_B^l(x)\right),\min\left(t_A^u(x),t_B^u(x)\right),\min\left(i_A^l(x),i_B^l(x)\right),\\ \min\left(i_A^u(x),i_B^u(x)\right),\max\left(f_A^l(x),f_B^l(x)\right),\max\left(f_A^u(x),f_B^u(x)\right)\end{pmatrix}:x\in X\right\}$$

Definition 2.7 [31]

An interval-valued neutrosophic graph on a nonempty set X is a pair $G=(A,B)$, where A is an interval-valued neutrosophic set on X and B is an interval-valued neutrosophic relation on X such that

$$t_B^l(xy)\le\min\left(t_A^l(x),t_A^l(y)\right),\ t_B^u(xy)\le\min\left(t_A^u(x),t_A^u(y)\right),$$
$$i_B^l(xy)\le\min\left(i_A^l(x),i_A^l(y)\right),\ i_B^u(xy)\le\min\left(i_A^u(x),i_A^u(y)\right),$$
$$f_B^l(xy)\le\min\left(f_A^l(x),f_A^l(y)\right),\ f_B^u(xy)\le\min\left(f_A^u(x),f_A^u(y)\right),\forall x,y\in X.$$

Note that B is called symmetric relation on A.

3. Interval-Valued Neutrosophic Incidence Graphs

Definition 1

An interval-valued neutrosophic incidence graph (IVNIG) of an incidence graph $G=(V,E,I)$ is an ordered-triplet, $\hat{G}=(\hat{X},\hat{Y},\hat{Z})$, such that

1. $\hat{X}$ is an interval-valued neutrosophic set on V
2. $\hat{Y}$ is an interval-valued neutrosophic relation on V
3. $\hat{Z}$ is an interval-valued neutrosophic subset of $V\times E$ such that

$$T_{\hat{Z}}^L(v,vw)\le\min\left\{T_{\hat{X}}^L(v),T_{\hat{Y}}^L(vw)\right\},$$
$$T_{\hat{Z}}^U(v,vw)\le\min\left\{T_{\hat{X}}^U(v),T_{\hat{Y}}^U(vw)\right\},$$
$$I_{\hat{Z}}^L(v,vw)\le\min\left\{I_{\hat{X}}^L(v),I_{\hat{Y}}^L(vw)\right\},$$
$$I_{\hat{Z}}^U(v,vw)\le\min\left\{I_{\hat{X}}^U(v),I_{\hat{Y}}^U(vw)\right\},$$
$$F_{\hat{Z}}^L(v,vw)\le\max\left\{F_{\hat{X}}^L(v),F_{\hat{Y}}^L(vw)\right\},$$
$$F_{\hat{Z}}^U(v,vw)\le\max\left\{F_{\hat{X}}^U(v),F_{\hat{Y}}^U(vw)\right\},\forall v\in V,vw\in E.$$

We now discuss an example of an IVNIG.

Example 1

Consider an incidence graph, $G=(V,E,I)$ such that $V=\{a,b,c,d\},E=\{ab,ac,bc,cd,ad\}$ and $I=\{(a,ab),(b,ab),(a,ac),(c,ac),(b,bc),(c,bc),(c,cd),(d,cd),(a,ad),(d,ad)\}$, as shown in Figure 1.

Let $\hat{G}=(\hat{X},\hat{Y},\hat{Z})$ be an IVNIG associated with G, as shown in Tables 2 – 4 and Figure 2. Also, let $\hat{X}$ be an interval-valued neutrosophic incidence set on V given as:

$$\hat{X}=\left\{\begin{matrix}(a,[0.1,0.4],[0.2,0.5],[0.3,0.7]),(b,[0.3,0.5],[0.2,0.6],[0.1,0.7]),\\ (c,[0.3,0.8],[0.4,0.9],[0.5,0.9]),(d,[0.4,0.7],[0.3,0.8],[0.4,0.9])\end{matrix}\right\},$$

$\hat{Y}$ be an interval-valued neutrosophic incidence relation on V given as:

$$\hat{Y}=\left\{\begin{array}{l}(ab,[0.1,0.4],[0.2,0.5],[0.3,0.7]),(ac,[0.1,0.4],[0.2,0.5],[0.4,0.8]),(bc,[0.3,0.5],[0.2,0.6],[0.5,0.8]),\\(cd,[0.3,0.7],[0.3,0.8],[0.5,0.8]),(ad,[0.1,0.4],[0.2,0.5],[0.4,0.8])\end{array}\right\},$$

$\hat{Z}$ be an interval-valued neutrosophic incidence set on $V\times E$ given as:

$$\hat{Z}=\left\{\begin{array}{l}((a,ab),[0.1,0.3],[0.1,0.4],[0.3,0.7]),((b,ab),[0.1,0.3],[0.1,0.5],[0.2,0.6]),\\((a,ac),[0.1,0.3],[0.1,0.5],[0.3,0.8]),((c,ac),[0.1,0.4],[0.1,0.4],[0.4,0.8]),\\((b,bc),[0.2,0.5],[0.1,0.5],[0.4,0.7]),((c,bc),[0.3,0.5],[0.2,0.6],[0.4,0.8]),\\((c,cd),[0.3,0.7],[0.3,0.8],[0.4,0.9]),((d,cd),[0.3,0.6],[0.2,0.7],[0.3,0.8]),\\((a,ad),[0.1,0.4],[0.2,0.4],[0.3,0.7]),((d,ad),[0.1,0.4],[0.2,0.5],[0.4,0.8])\end{array}\right\}.$$

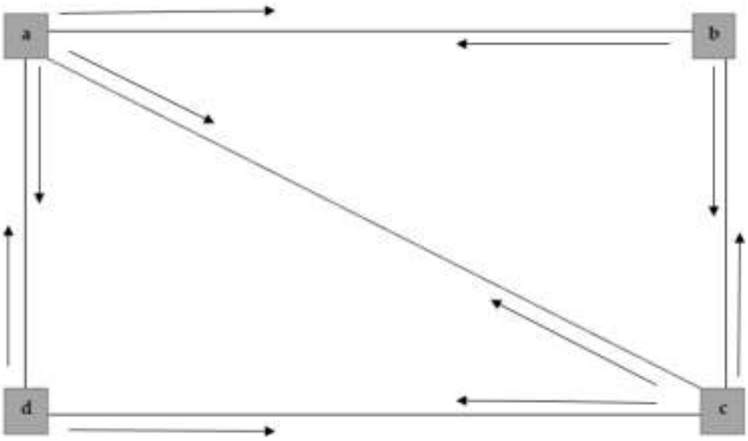

Figure 1. Incidence graph

Table 2. IVNIG set on V

	a	b	c	d
$t_{\hat{X}}$	$[0.1,0.4]$	$[0.3,0.5]$	$[0.3,0.8]$	$[0.4,0.7]$
$i_{\hat{X}}$	$[0.2,0.5]$	$[0.2,0.6]$	$[0.4,0.9]$	$[0.3,0.8]$
$f_{\hat{X}}$	$[0.3,0.7]$	$[0.1,0.7]$	$[0.5,0.9]$	$[0.4,0.9]$

Table 3. IVNIG relation on V

	ab	ac	bc	cd	ad
$t_{\hat{Y}}$	$[0.1,0.4]$	$[0.1,0.4]$	$[0.3,0.5]$	$[0.3,0.7]$	$[0.1,0.4]$
$i_{\hat{Y}}$	$[0.2,0.5]$	$[0.2,0.5]$	$[0.2,0.6]$	$[0.3,0.8]$	$[0.2,0.5]$
$f_{\hat{Y}}$	$[0.3,0.7]$	$[0.4,0.8]$	$[0.5,0.8]$	$[0.5,0.8]$	$[0.4,0.8]$

Table 4. IVNIG set on $V\times E$

	(a,ab)	(b,ab)	(a,ac)	(c,ac)	(b,bc)	(c,bc)	(c,cd)	(d,cd)	(a,ad)	(d,ad)
$t_{\hat{Z}}$	$[0.1,0.3]$	$[0.1,0.3]$	$[0.1,0.3]$	$[0.1,0.4]$	$[0.2,0.5]$	$[0.3,0.5]$	$[0.3,0.7]$	$[0.3,0.6]$	$[0.1,0.4]$	$[0.1,0.4]$
$i_{\hat{Z}}$	$[0.1,0.4]$	$[0.1,0.5]$	$[0.1,0.5]$	$[0.1,0.4]$	$[0.1,0.5]$	$[0.2,0.6]$	$[0.3,0.8]$	$[0.2,0.7]$	$[0.2,0.4]$	$[0.2,0.5]$
$f_{\hat{Z}}$	$[0.3,0.7]$	$[0.2,0.6]$	$[0.3,0.8]$	$[0.4,0.8]$	$[0.4,0.7]$	$[0.4,0.8]$	$[0.4,0.9]$	$[0.3,0.8]$	$[0.3,0.7]$	$[0.4,0.8]$

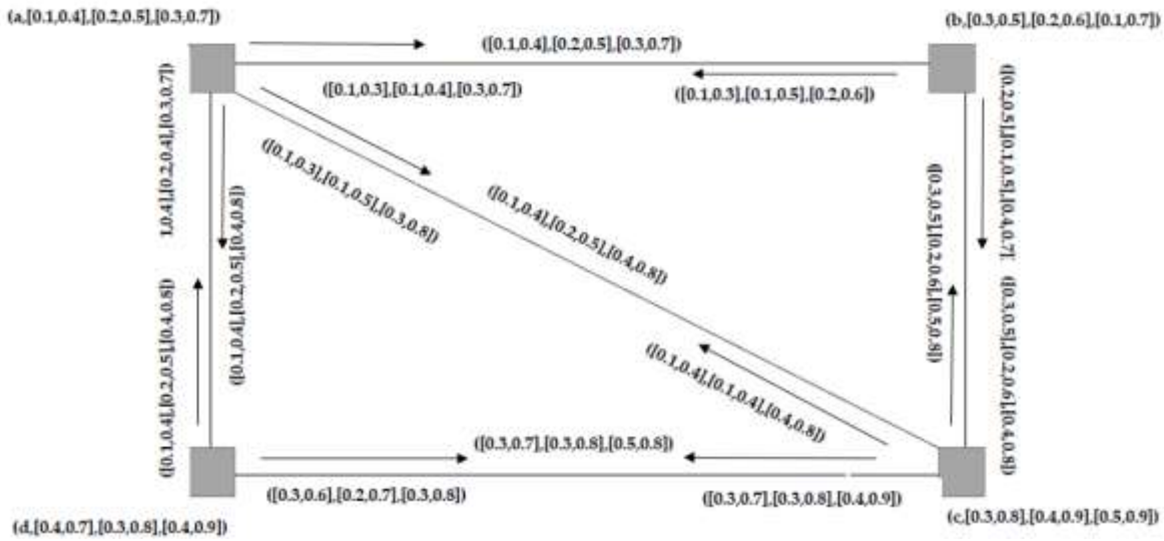

Figure 2 Interval-valued neutrosophic incidence graph

Definition 2

The support of an IVNIG $\hat{G}=(\hat{X},\hat{Y},\hat{Z})$ is denoted by $G^*=(X^*,Y^*,Z^*)$ where

$$X^*=\mathrm{supp}(\hat{X})=\{v\in V:[T_A^L(v),T_A^U(v)]>0,[I_A^L(v),I_A^U(v)]>0,[F_A^L(v),F_A^U(v)]>0\}$$

$$Y^*=\mathrm{supp}(\hat{Y})=\{vw\in E:[T_B^L(vw),T_B^U(vw)]>0,[I_B^L(vw),I_B^U(vw)]>0,[F_B^L(vw),F_B^U(vw)]>0\}$$

$$Z^*=\mathrm{supp}(\hat{Z})=\left\{\begin{matrix}(v,vw)\in I:[T_C^L(v,vw),T_C^U(v,vw)]>0,[I_C^L(v,vw),I_C^U(v,vw)]>0,\\ [F_C^L(v,vw),F_C^U(v,vw)]>0\end{matrix}\right\}$$

Definition 3

If $vw\in Y^*$, then vw is an edge of the IVNIG $\hat{G}=(\hat{X},\hat{Y},\hat{Z})$, while if $(v,vw),(w,vw)\in Z^*$, then (v,vw) and (w,vw) are called pairs of $\hat{G}=(\hat{X},\hat{Y},\hat{Z})$.

Definition 4

A sequence

$$Q: z_0,(z_0,z_0z_1),z_0z_1,(z_1,z_0z_1),z_1,(z_1,z_1z_2),z_1z_2,(z_2,z_1z_2),z_2,\ldots,$$
$$z_{n-1},(z_{n-1},z_{n-1}z_n),z_{n-1}z_n,(z_n,z_{n-1}z_n),z_n$$

of vertices, edges and pairs in $\hat{G}$ is known as a walk. It is a closed walk if $z_0=z_n$. In contrast, if all edges are distinct, it is a trail, while if the pairs are distinct, then it is an incidence trail. Q is called a path if the vertices are distinct. A path is called a cycle if the initial and end vertices of the path are the same. Any two vertices of $\hat{G}$ are said to be connected if a path joins them.

Example 2

In Example 1 presented earlier

$$Q_1: a,(a,ac),ac,(c,ac),c,(c,cd),cd,(d,cd),d,(d,da),da,(a,da),a$$

is known as a walk. In fact, it is a closed walk since the initial and final vertices are the same. It is not a path, but it is a trail and an incidence trail.

$$Q_2: a,(a,ac),ac,(c,ac),c,(c,cd),cd,(d,cd),d$$

On the other hand, Q_2 -is a walk, path, trail and an incidence trail.

Definition 5

Let $\hat{G}=(\hat{X},\hat{Y},\hat{Z})$ be an IVNIG. Then, $\hat{H}=(\hat{L},\hat{M},\hat{N})$ is an interval-valued neutrosophic incidence subgraph of $\hat{G}$ if $\hat{L}\subseteq\hat{X},\hat{M}\subseteq\hat{Y}$ and $\hat{N}\subseteq\hat{Z}$. $\hat{H}$ is an interval-valued neutrosophic incidence spanning subgraph of $\hat{G}$ if $L^*=X^*$.

Definition 6

In an IVNIG, the strength of a path, $\bar{P}$ is an ordered triplet denoted by $\bar{S}(\bar{P})=(\bar{s}_1,\bar{s}_2,\bar{s}_3)$, where

$$\bar{s}_1=\min\left\{\left[T_Y^L(xy),T_Y^U(xy)\right]:xy\in\bar{P}\right\}$$
$$\bar{s}_2=\min\left\{\left[I_Y^L(xy),I_Y^U(xy)\right]:xy\in\bar{P}\right\}$$
$$\bar{s}_3=\max\left\{\left[F_Y^L(xy),F_Y^U(xy)\right]:xy\in\bar{P}\right\}$$

Similarly, the incidence strength of a path, $\bar{P}$ in an IVNIG is denoted by $I\bar{S}(\bar{P})=(i\bar{s}_1,i\bar{s}_2,i\bar{s}_3)$, where

$$i\bar{s}_1=\min\left\{\left[T_Z^L(x,xy),T_Z^U(x,xy)\right]:(x,xy)\in\bar{P}\right\}$$
$$i\bar{s}_2=\min\left\{\left[I_Z^L(x,xy),I_Z^U(x,xy)\right]:(x,xy)\in\bar{P}\right\}$$
$$i\bar{s}_3=\max\left\{\left[F_Z^L(x,xy),F_Z^U(x,xy)\right]:(x,xy)\in\bar{P}\right\}$$

Example 3

Let $G=(V,E,I)$ be an incidence graph and $\hat{G}=(\hat{X},\hat{Y},\hat{Z})$ is an IVNIG associated with G, which is shown in Tables 2 – 4. Clearly, $\bar{P}_1: a,(a,ac),ac,(c,ac),c,(c,cd),cd,(d,cd),d$ is a path in $\hat{G}$.

The strength of the path $\bar{P}_1$ is $S(\bar{P}_1)=([0.1,0.4],[0.2,0.5],[0.5,0.8])$ while the incidence strength of $\bar{P}_1$ is $IS(\bar{P}_1)=([0.1,0.3],[0.1,0.4],[0.4,0.9])$.

Definition 7

In an IVNIG, the greatest strength of the path from m to n, where $m,n\in A^*\cup B^*$ is the maximum strength of all paths from m to n. Moreover, $\bar{S}^\infty(m,n)$ is sometimes called the connectedness between m to n.

$$\begin{aligned}\bar{S}^\infty(m,n)&=\max\left\{\bar{S}(P_1),\bar{S}(P_2),\bar{S}(P_3),\ldots\right\}\\&=\left(\bar{s}_1^\infty,\bar{s}_2^\infty,\bar{s}_3^\infty\right)\\&=\left(\max(\bar{s}_{11},\bar{s}_{12},\bar{s}_{13},\ldots),\max(\bar{s}_{21},\bar{s}_{22},\bar{s}_{23},\ldots),\min(\bar{s}_{31},\bar{s}_{32},\bar{s}_{33},\ldots)\right)\end{aligned}$$

Similarly, the greatest incidence strength of the path from m to n, where $m,n\in A^*\cup B^*$ is the maximum incidence strength of all paths from m to n, given by

$$\begin{aligned}I\bar{S}^\infty(m,n)&=\max\left\{I\bar{S}(P_1),I\bar{S}(P_2),I\bar{S}(P_3),\ldots\right\}\\&=\left(i\bar{s}_1^\infty,i\bar{s}_2^\infty,i\bar{s}_3^\infty\right)\\&=\left(\max(i\bar{s}_{11},i\bar{s}_{12},i\bar{s}_{13},\ldots),\max(i\bar{s}_{21},i\bar{s}_{22},i\bar{s}_{23},\ldots),\min(i\bar{s}_{31},i\bar{s}_{32},i\bar{s}_{33},\ldots)\right)\end{aligned}$$

where $P_i, i=1,2,3,\ldots$ are different paths from m to n. $I\bar{S}^\infty(m,n)$ is sometimes referred to as the incidence connectedness between m to n.

Example 4

In the IVNIG given in Tables 2 - 4, the total paths from vertex *b* to *d* are given as follows:

$$\bar{P}_1 : b,(b,bc),bc,(c,bc),c,(c,cd),cd,(d,cd),d$$
$$\bar{P}_2 : b,(b,ab),ab,(a,ab),a,(a,ad),ad,(d,ad),d$$
$$\bar{P}_3 : b,(b,bc),bc,(c,bc),c,(c,ac),ac,(a,ac),a,(a,ad),ad,(d,ad),d$$
$$\bar{P}_4 : b,(b,ab),ab,(a,ab),a,(a,ac),ac,(c,ac),c,(c,cd),cd,(d,cd),d$$

The corresponding incidence strengths of each path are

$$IS(\bar{P}_1) = (\bar{s}_{11}, \bar{s}_{21}, \bar{s}_{31}) = ([0.3,0.5],[0.2,0.6],[0.5,0.8])$$
$$IS(\bar{P}_2) = (\bar{s}_{12}, \bar{s}_{22}, \bar{s}_{32}) = ([0.1,0.4],[0.2,0.5],[0.4,0.8])$$
$$IS(\bar{P}_3) = (\bar{s}_{13}, \bar{s}_{23}, \bar{s}_{33}) = ([0.1,0.4],[0.2,0.5],[0.5,0.8])$$
$$IS(\bar{P}_4) = (\bar{s}_{14}, \bar{s}_{24}, \bar{s}_{34}) = ([0.1,0.4],[0.2,0.5],[0.5,0.8])$$

Hence, the greatest incidence strength of the path form is calculated as follows:

$$\begin{aligned}
I\bar{S}^{\infty}(b,d) &= \max\{IS(\bar{P}_1), IS(\bar{P}_2), IS(\bar{P}_3), IS(\bar{P}_4)\} \\
&= \left(\max\{i\bar{s}_{11}, i\bar{s}_{12}, i\bar{s}_{13}, i\bar{s}_{14}\}, \max\{i\bar{s}_{21}, i\bar{s}_{22}, i\bar{s}_{23}, i\bar{s}_{24}\}, \min\{i\bar{s}_{31}, i\bar{s}_{32}, i\bar{s}_{33}, i\bar{s}_{34}\}\right) \\
&= \begin{pmatrix} \max\{[0.3,0.5],[0.1,0.4],[0.1,0.4],[0.1,0.4]\}, \max\{[0.2,0.6],[0.2,0.5],[0.2,0.5],[0.2,0.5]\}, \\ \min\{[0.5,0.8],[0.4,0.8],[0.5,0.8],[0.5,0.8]\} \end{pmatrix} \\
&= ([0.3,0.5],[0.2,0.6],[0.4,0.8]).
\end{aligned}$$

Definition 8

An IVNIG $\hat{G} = (\hat{X}, \hat{Y}, \hat{Z})$ is a cycle if and only if the underlying graph $G^* = (X^*, Y^*, Z^*)$ is a cycle.

Definition 9

The IVNIG $\hat{G} = (\hat{X}, \hat{Y}, \hat{Z})$ is a neutrosophic cycle if and only if, $G^* = (X^*, Y^*, Z^*)$ is a cycle, and there exists no unique edge $vw \in Y^*$, such that

$$T_Y^L(vw) = \min\{T_Y^L(xy) : xy \in Y^*\}, T_Y^U(vw) = \min\{T_Y^U(xy) : xy \in Y^*\},$$
$$I_Y^L(vw) = \min\{I_Y^L(xy) : xy \in Y^*\}, \ I_Y^U(vw) = \min\{I_Y^U(xy) : xy \in Y^*\},$$
$$F_Y^L(vw) = \max\{F_Y^L(xy) : xy \in Y^*\}, F_Y^U(vw) = \max\{F_Y^U(xy) : xy \in Y^*\}.$$

Definition 10

The IVNIG $\hat{G} = (\hat{X}, \hat{Y}, \hat{Z})$ is a neutrosophic incidence cycle if, and only if it is a neutrosophic cycle and there exists no unique pair $(v, vw) \in Z^*$, such that

$$T_Z^L(v,vw) = \min\{T_Z^L(x,xy) : (x,xy) \in Z^*\},$$
$$T_Z^U(v,vw) = \min\{T_Z^U(x,xy) : (x,xy) \in Z^*\},$$
$$I_Z^L(v,vw) = \min\{I_Z^L(x,xy) : (x,xy) \in Z^*\},$$
$$I_Z^U(v,vw) = \min\{I_Z^U(x,xy) : (x,xy) \in Z^*\},$$
$$F_Z^L(v,vw) = \max\{F_Z^L(x,xy) : (x,xy) \in Z^*\},$$
$$F_Z^U(v,vw) = \max\{F_Z^U(x,xy) : (x,xy) \in Z^*\}.$$

Example 5

Let $\hat{G}=(\hat{X},\hat{Y},\hat{Z})$ be an IVNIG. G is a cycle since $G^{*}=(X^{*},Y^{*},Z^{*})$ (support of $\hat{G}$) is a cycle.

$$\hat{X}=\left\{\begin{array}{l}(a,[0.1,0.4],[0.2,0.5],[0.3,0.7]),(b,[0.3,0.5],[0.2,0.6],[0.1,0.7]),\\(c,[0.3,0.8],[0.4,0.9],[0.5,0.8]),(d,[0.4,0.7],[0.3,0.8],[0.4,0.8]),\\(e,[0.7,0.9],[0.5,0.8],[0.3,0.5])\end{array}\right\}$$

$$\hat{Y}=\left\{\begin{array}{l}(ab,[0.2,0.8],[0.2,0.7],[0.3,0.8]),(bc,[0.3,0.8],[0.4,0.7],[0.5,0.8]),\\(cd,[0.4,0.8],[0.4,0.9],[0.3,0.8]),(de,[0.1,0.9],[0.5,0.8],[0.2,0.6]),\\(ea,[0.3,0.6],[0.2,0.5],[0.2,0.5])\end{array}\right\}$$

$$\hat{Z}=\left\{\begin{array}{l}((a,ab),[0.1,0.4],[0.2,0.5],[0.4,0.8]),((b,ab),[0.2,0.5],[0.2,0.6],[0.2,0.8]),\\((b,bc),[0.3,0.4],[0.1,0.5],[0.5,0.7]),((c,bc),[0.2,0.5],[0.2,0.6],[0.5,0.7]),\\((c,cd),[0.3,0.7],[0.3,0.8],[0.5,0.7]),((d,cd),[0.4,0.6],[0.2,0.7],[0.4,0.8]),\\((d,de),[0.1,0.6],[0.3,0.8],[0.4,0.7]),((e,de),[0.1,0.8],[0.4,0.7],[0.3,0.6]),\\((a,ea),[0.1,0.3],[0.1,0.4],[0.3,0.7]),((e,ea),[0.3,0.5],[0.2,0.4],[0.3,0.5])\end{array}\right\},$$

$$\begin{array}{l}T_Y^L(ab)=0.1=\min\{T_Y^L(ab),T_Y^L(bc),T_Y^L(cd),T_Y^L(de),T_Y^L(ea)\},\\T_Y^U(ab)=0.6=\min\{T_Y^U(ab),T_Y^U(bc),T_Y^U(cd),T_Y^U(de),T_Y^U(ea)\},\\I_Y^L(ab)=0.2=\min\{I_Y^L(ab),I_Y^L(bc),I_Y^L(cd),I_Y^L(de),I_Y^L(ea)\},\\I_Y^U(ab)=0.5=\min\{I_Y^U(ab),I_Y^U(bc),I_Y^U(cd),I_Y^U(de),I_Y^U(ea)\},\\F_Y^L(ab)=0.5=\max\{F_Y^L(ab),F_Y^L(bc),F_Y^L(cd),F_Y^L(de),F_Y^L(ea)\},\\F_Y^U(ab)=0.8=\max\{F_Y^U(ab),F_Y^U(bc),F_Y^U(cd),F_Y^U(de),F_Y^U(ea)\}.\end{array}$$

and

$$\begin{array}{l}T_Y^L(bc)=0.1=\min\{T_Y^L(ab),T_Y^L(bc),T_Y^L(cd),T_Y^L(de),T_Y^L(ea)\},\\T_Y^U(bc)=0.6=\min\{T_Y^U(ab),T_Y^U(bc),T_Y^U(cd),T_Y^U(de),T_Y^U(ea)\},\\I_Y^L(bc)=0.2=\min\{I_Y^L(ab),I_Y^L(bc),I_Y^L(cd),I_Y^L(de),I_Y^L(ea)\},\\I_Y^U(bc)=0.5=\min\{I_Y^U(ab),I_Y^U(bc),I_Y^U(cd),I_Y^U(de),I_Y^U(ea)\},\\F_Y^L(bc)=0.5=\max\{F_Y^L(ab),F_Y^L(bc),F_Y^L(cd),F_Y^L(de),F_Y^L(ea)\},\\F_Y^U(bc)=0.8=\max\{F_Y^U(ab),F_Y^U(bc),F_Y^U(cd),F_Y^U(de),F_Y^U(ea)\}.\end{array}$$

Thus, $\hat{G}$ is an interval-valued neutrosophic cycle.

Furthermore, $\hat{G}$ is a neutrosophic incidence cycle since there is more than one pair, namely (b,ab) and (d,de) such that

$$\begin{array}{l}T_Z^L(b,ab)=0.1=\min\{T_Z^L(v,vw):(v,vw)\in Z^{*}\},\\T_Z^U(b,ab)=0.3=\min\{T_Z^U(v,vw):(v,vw)\in Z^{*}\},\\I_Z^L(b,ab)=0.1=\min\{I_Z^L(v,vw):(v,vw)\in Z^{*}\},\\I_Z^U(b,ab)=0.4=\min\{I_Z^U(v,vw):(v,vw)\in Z^{*}\},\\F_Z^L(b,ab)=0.5=\max\{F_Z^L(v,vw):(v,vw)\in Z^{*}\},\\F_Z^U(b,ab)=0.8=\max\{F_Z^U(v,vw):(v,vw)\in Z^{*}\}\end{array}$$

and

$$T_Z^L(d,de)=0.1=\min\{T_Z^L(v,vw):(v,vw)\in Z^*\},$$
$$T_Z^U(d,de)=0.3=\min\{T_Z^U(v,vw):(v,vw)\in Z^*\},$$
$$I_Z^L(d,de)=0.1=\min\{I_Z^L(v,vw):(v,vw)\in Z^*\},$$
$$I_Z^U(d,de)=0.4=\min\{I_Z^U(v,vw):(v,vw)\in Z^*\},$$
$$F_Z^L(d,de)=0.5=\max\{F_Z^L(v,vw):(v,vw)\in Z^*\},$$
$$F_Z^U(d,de)=0.8=\max\{F_Z^U(v,vw):(v,vw)\in Z^*\}$$

The concepts of bridges, cut vertices and cut pairs in IVNIG are defined as follows:

Definition 11

Let $\hat{G}=(\hat{X},\hat{Y},\hat{Z})$ be an IVNIG. An edge, vw in $\hat{G}$ is called a bridge if and only if vw is a bridge in $G^*=(X^*,Y^*,Z^*)$ which is the removal of vw disconnects G^*.

An edge vw is called a neutrosophic bridge if

$$\bar{S}'^{\infty}(a,b)<\bar{S}^{\infty}(a,b) \text{ for some } a,b\in X^*$$
$$(\bar{s}_1'^{\infty},\bar{s}_2'^{\infty},\bar{s}_3'^{\infty})<(\bar{s}_1^{\infty},\bar{s}_2^{\infty},\bar{s}_3^{\infty})$$
$$\bar{s}_1'^{\infty}<\bar{s}_1^{\infty},\bar{s}_2'^{\infty}<\bar{s}_2^{\infty},\bar{s}_3'^{\infty}>\bar{s}_3^{\infty}$$

where $\bar{S}'^{\infty}(a,b)$ and $\bar{S}^{\infty}(a,b)$ denote the connectedness between a and b in $\bar{G}=\hat{G}-\{vw\}$ and $\hat{G}$, respectively.

An edge vw is called a neutrosophic incidence bridge if

$$I\bar{S}'^{\infty}(a,b)<I\bar{S}^{\infty}(a,b) \text{ for some } a,b\in X^*$$
$$(i\bar{s}_1'^{\infty},i\bar{s}_2'^{\infty},i\bar{s}_3'^{\infty})<(i\bar{s}_1^{\infty},i\bar{s}_2^{\infty},i\bar{s}_3^{\infty})$$
$$i\bar{s}_1'^{\infty}<i\bar{s}_1^{\infty},i\bar{s}_2'^{\infty}<i\bar{s}_2^{\infty},i\bar{s}_3'^{\infty}>\bar{s}_3^{\infty}$$

where $I\bar{S}'^{\infty}(a,b)$ and $I\bar{S}^{\infty}(a,b)$ denote the incidence connectedness between a and b in $\bar{G}=\hat{G}-\{vw\}$ and $\hat{G}$, respectively.

Definition 12

Let $\hat{G}=(\hat{X},\hat{Y},\hat{Z})$ be an IVNIG. A vertex, v, in $\hat{G}$ is called a cut vertex if and only if it is a cut vertex in $G^*=(X^*,Y^*,Z^*)$ where $G^*-\{v\}$ is the disconnect of G^*.

A vertex, v in an IVNIG is called a neutrosophic cut vertex if the connectedness between any two vertices in $\bar{G}=\hat{G}-\{v\}$ is less than the connectedness between the same vertices in $\hat{G}-$ that is,

$$\bar{S}'^{\infty}(a,b)<\bar{S}^{\infty}(a,b) \text{ for some } a,b\in X^*.$$

A vertex, v in an IVNIG is a neutrosophic incidence cut vertex if for any pair of vertices a and b other than v, the following condition holds:

$$\bar{S}'^{\infty}(a,b)<\bar{S}^{\infty}(a,b),$$

where $I\bar{S}'^{\infty}(a,b)$ and $I\bar{S}^{\infty}(a,b)$ denote the incidence connectedness between a and b in $\bar{G}=\hat{G}-\{vw\}$ and $\hat{G}$, respectively.

Definition 13

Let $\hat{G}=(\hat{X},\hat{Y},\hat{Z})$ be an IVNIG. A pair, (v,vw) in $\hat{G}$ is called a cut pair if and only if (v,vw) is a cut pair in $G^*=(X^*,Y^*,Z^*)$ that is, after removing the pair (v,vw), there is no path between v and vw.

Let $\hat{G}=(\hat{X},\hat{Y},\hat{Z})$ be an IVNIG. A pair, (v,vw) is called a neutrosophic cut pair if deleting the pair (v,vw) reduces the connectedness between $v, vw \in X^* \cup Y^*$, that is,

$$\overline{S}'^{\infty}(v,vw)<\overline{S}^{\infty}(v,vw),$$

where $\overline{S}'^{\infty}(v,vw)$ and $\overline{S}^{\infty}(v,vw)$ denote the connectedness between v and w in $\overline{G}=\hat{G}-\{v,vw\}$ and $\hat{G}$, respectively.

A pair (v,vw) is called neutrosophic incidence cut pair if

$$I\overline{S}'^{\infty}(v,vw)<I\overline{S}^{\infty}(v,vw) \text{ for } v,vw\in X^*\cup Y^*,$$

where $I\overline{S}'^{\infty}(v,vw)$ and $I\overline{S}^{\infty}(v,vw)$ denote the incidence connectedness between v and vw in $\overline{G}=\hat{G}-\{vw\}$ and $\hat{G}$, respectively.

Definition 14

Let $\hat{G}=(\hat{X},\hat{Y},\hat{Z})$ be an IVNIG. An edge, vw of $\hat{G}$ is called a strong edge if

$$\overline{S}'^{\infty}(v,w)\le\left(\left[T_Y^L(vw),T_Y^U(vw)\right],\left[I_Y^L(vw),I_Y^U(vw)\right],\left[F_Y^L(vw),F_Y^U(vw)\right]\right),$$

where $\overline{S}'^{\infty}(v,w)$ represents the connectedness between v and w in $\overline{G}=\hat{G}-\{vw\}$.

In particular, an edge vw is said to be an $\alpha-$strong edge if

$$\overline{S}'^{\infty}(v,w)<\left(\left[T_Y^L(vw),T_Y^U(vw)\right],\left[I_Y^L(vw),I_Y^U(vw)\right],\left[F_Y^L(vw),F_Y^U(vw)\right]\right),$$

and it is called $\beta-$strong edge if

$$\overline{S}'^{\infty}(v,w)=\left(\left[T_Y^L(vw),T_Y^U(vw)\right],\left[I_Y^L(vw),I_Y^U(vw)\right],\left[F_Y^L(vw),F_Y^U(vw)\right]\right).$$

Definition 15

A pair (v,vw) in an IVNIG, $\hat{G}$ is called a strong pair if

$$I\overline{S}'^{\infty}(v,vw)\le\left(\left[T_Z^L(v,vw),T_Z^U(v,vw)\right],\left[I_Z^L(v,vw),I_Z^U(v,vw)\right],\left[F_Z^L(v,vw),F_Z^U(v,vw)\right]\right),$$

where $I\overline{S}'^{\infty}(v,vw)$ represents the incidence connectedness between v and vw in $\overline{G}=\hat{G}-\{(v,vw)\}$.

In particular, an edge (v,vw) is called $\alpha-$strong pair if

$$I\overline{S}'^{\infty}(v,vw)<\left(\left[T_Z^L(v,vw),T_Z^U(v,vw)\right],\left[I_Z^L(v,vw),I_Z^U(v,vw)\right],\left[F_Z^L(v,vw),F_Z^U(v,vw)\right]\right),$$

and it is called $\beta-$strong pair if

$$I\overline{S}'^{\infty}(v,vw)=\left(\left[T_Z^L(v,vw),T_Z^U(v,vw)\right],\left[I_Z^L(v,vw),I_Z^U(v,vw)\right],\left[F_Z^L(v,vw),F_Z^U(v,vw)\right]\right).$$

All edges and pairs do not need to be strong. There exist edges and pairs that are not strong in an IVNIG. Such edges and pairs are given in the following definition.

Definition 16

Let $\hat{G}=(\hat{X},\hat{Y},\hat{Z})$ be an IVNIG. An edge, vw is said to be $\delta-$edge if

$$\overline{S}'^{\infty}(v,w)>\left(\left[T_Y^L(vw),T_Y^U(vw)\right],\left[I_Y^L(vw),I_Y^U(vw)\right],\left[F_Y^L(vw),F_Y^U(vw)\right]\right).$$

Similarly, a pair (v,vw) in $\hat{G}$ is called $\delta-$pair if

$$I\bar{S}^{'\infty}(v,vw) > \left(\left[T_Z^L(v,vw),T_Z^U(v,vw)\right],\left[I_Z^L(v,vw),I_Z^U(v,vw)\right],\left[F_Z^L(v,vw),F_Z^U(v,vw)\right]\right).$$

Theorem 1. Let $\hat{G}=(\hat{X},\hat{Y},\hat{Z})$ be an IVNIG. If vw is a neutrosophic bridge, then vw is a strong edge in any cycle.

Proof. Let vw be a neutrosophic bridge. By contradiction, suppose that vw is not a strong edge of a cycle. Then, in this cycle, we can find an alternative path, P_1 from v to w that contains the edge vw and $S(P_1)$ is less than or equal to $S(P_2)$, where P_2 is the path that does not contain the edge vw. Thus, removing the edge of vw from $\hat{G}$ does not affect the connectedness between v and w, which is a contradiction to our assumption. Hence, vw is a strong edge in any cycle. □

Theorem 2. If (v,vw) is a neutrosophic incidence cut pair, then (v,vw) is a strong pair in any cycle.

Proof. Let (v,vw) be a neutrosophic incidence cut pair in $\hat{G}$ and by contradiction, suppose that (v,vw) is not a strong pair of a cycle. Then, we can find an alternative path from v to vw having incidence strength greater than or equal to that of the path involving the pair (v,vw). Thus, removal of the pair (v,vw) does not affect the incidence connectedness between v and vw. This is a contradiction to our assumption that (v,vw) is a neutrosophic incidence cut pair. Hence, (v,vw) is a strong pair in any cycle. □

Theorem 3. Let $\hat{G}=(\hat{X},\hat{Y},\hat{Z})$ be an IVNIG. If vw is a neutrosophic bridge in $\hat{G}$, then

$$\bar{S}^{\infty}(v,w)=(\bar{s}_1^{\infty},\bar{s}_2^{\infty},\bar{s}_3^{\infty})=\left(\left[T_Y^L(vw),T_Y^U(vw)\right],\left[I_Y^L(vw),I_Y^U(vw)\right],\left[F_Y^L(vw),F_Y^U(vw)\right]\right).$$

Proof. Let $\hat{G}$ be an IVNIG and vw is a neutrosophic bridge in $\hat{G}$. By contradiction, suppose that

$$\bar{S}^{\infty}(v,w) > \left(\left[T_Y^L(vw),T_Y^U(vw)\right],\left[I_Y^L(vw),I_Y^U(vw)\right],\left[F_Y^L(vw),F_Y^U(vw)\right]\right).$$

Then, there exists a $v-w$ path, P, with

$$\bar{S}(P) > \left(\left[T_Y^L(vw),T_Y^U(vw)\right],\left[I_Y^L(vw),I_Y^U(vw)\right],\left[F_Y^L(vw),F_Y^U(vw)\right]\right)$$

and

$$\left(\left[T_Y^L(xy),T_Y^U(xy)\right],\left[I_Y^L(xy),I_Y^U(xy)\right],\left[F_Y^L(xy),F_Y^U(xy)\right]\right) >$$
$$\left(\left[T_Y^L(vw),T_Y^U(vw)\right],\left[I_Y^L(vw),I_Y^U(vw)\right],\left[F_Y^L(vw),F_Y^U(vw)\right]\right),$$

for all edges on path P. Now, P together with the edge vw forms a cycle in which vw is the weakest edge, but it is a contradiction to the fact that vw is a neutrosophic bridge. Hence,

$$\bar{S}^{\infty}(v,w)=(\bar{s}_1^{\infty},\bar{s}_2^{\infty},\bar{s}_3^{\infty})=\left(\left[T_Y^L(vw),T_Y^U(vw)\right],\left[I_Y^L(vw),I_Y^U(vw)\right],\left[F_Y^L(vw),F_Y^U(vw)\right]\right).$$

□

Theorem 4. Let $\hat{G}=(\hat{X},\hat{Y},\hat{Z})$ be an IVNIG. If (v,vw) is a neutrosophic incidence cut pair in $\hat{G}$, then

$$\begin{aligned} I\bar{S}^{\infty}(v,vw) &= (i\bar{s}_1^{\infty}, i\bar{s}_2^{\infty}, i\bar{s}_3^{\infty}) \\ &= ([T_Z^L(v,vw), T_Z^U(v,vw)], [I_Z^L(v,vw), I_Z^U(v,vw)], [F_Z^L(v,vw), F_Z^U(v,vw)]). \end{aligned}$$

Proof. Let $\hat{G}$ be an IVNIG and (v,vw) is a neutrosophic incidence cut pair in $\hat{G}$. By contradiction, suppose that

$$I\bar{S}^{\infty}(v,vw) > ([T_Z^L(v,vw), T_Z^U(v,vw)], [I_Z^L(v,vw), I_Z^U(v,vw)], [F_Z^L(v,vw), F_Z^U(v,vw)]).$$

Then, there exists a $v-w$ path, P, with

$$I\bar{S}(P) > ([T_Z^L(v,vw), T_Z^U(v,vw)], [I_Z^L(v,vw), I_Z^U(v,vw)], [F_Z^L(v,vw), F_Z^U(v,vw)])$$

and

$$\begin{aligned} &([T_Z^L(x,xy), T_Z^U(x,xy)], [I_Z^L(x,xy), I_Z^U(x,xy)], [F_Z^L(x,xy), F_Z^U(x,xy)]) > \\ &([T_Z^L(v,vw), T_Z^U(v,vw)], [I_Z^L(v,vw), I_Z^U(v,vw)], [F_Z^L(v,vw), F_Z^U(v,vw)]), \end{aligned}$$

for all pairs on path P. Now, P together with the pair (v,vw) forms a cycle in which (v,vw) is the weakest pair. However, it is a contradiction to the fact that (v,vw) is a neutrosophic incidence cut pair. Hence,

$$\begin{aligned} I\bar{S}^{\infty}(v,vw) &= (i\bar{s}_1^{\infty}, i\bar{s}_2^{\infty}, i\bar{s}_3^{\infty}) \\ &= ([T_Z^L(v,vw), T_Z^U(v,vw)], [I_Z^L(v,vw), I_Z^U(v,vw)], [F_Z^L(v,vw), F_Z^U(v,vw)]). \end{aligned}$$

□

Theorem 5. Every neutrosophic incidence cut pair in IVNIG is a strong cut pair.

Proof. Let $\hat{G}=(\hat{X},\hat{Y},\hat{Z})$ be an IVNIG. Let $(v,vw)\in Z^*$ be a neutrosophic incidence cut pair. Then, by Definition 12, we have

$$I\bar{S}'^{\infty}(v,vw) < I\bar{S}^{\infty}(v,vw).$$

By contradiction, suppose that (v,vw) is not a strong incidence pair. Then it follows that

$$I\bar{S}'^{\infty}(v,vw) > ([T_Z^L(v,vw), T_Z^U(v,vw)], [I_Z^L(v,vw), I_Z^U(v,vw)], [F_Z^L(v,vw), F_Z^U(v,vw)]).$$

Let P be the path from v to vw in $\bar{G}=\hat{G}-\{(v,vw)\}$ with the greatest incidence strength. Then P together with (v,vw) forms a cycle in $\hat{G}$. Now, in this cycle, (v,vw) is the weakest pair. However, based on Theorem 2, this is not possible since (v,vw) is a neutrosophic incidence cut pair. This is a contradiction to our assumption, hence (v,vw) is a strong incidence pair. □

Theorem 6. Let $\hat{G}=(\hat{X},\hat{Y},\hat{Z})$ be an IVNIG. The pair (v,vw) is a neutrosophic incidence cut pair if and only if it is $\alpha-$strong.

Proof. Let (v,vw) be a neutrosophic incidence cut pair in $\hat{G}$. Based on Definition 12,

$$I\bar{S}^{\infty}(v,vw) > I\bar{S}'^{\infty}(v,vw)$$

Then, based on Theorem 4, it follows that

$$\left(\left[T_Z^L(v,vw),T_Z^U(v,vw)\right],\left[I_Z^L(v,vw),I_Z^U(v,vw)\right],\left[F_Z^L(v,vw),F_Z^U(v,vw)\right]\right) > I\bar{S}^{'\infty}(v,vw),$$

which is the definition of $\alpha-$strong. Hence, (v,vw) is an $\alpha-$strong pair in $\hat{G}$.

Conversely, suppose that (v,vw) is an $\alpha-$strong pair in $\hat{G}$. Then, by definition

$$\left(\left[T_Z^L(v,vw),T_Z^U(v,vw)\right],\left[I_Z^L(v,vw),I_Z^U(v,vw)\right],\left[F_Z^L(v,vw),F_Z^U(v,vw)\right]\right) > I\bar{S}^{'\infty}(v,vw).$$

It follows that $P: v,(v,vw),vw$ is the unique strongest incidence path from v to vw. The removal of (v,vw) reduces the incidence strength between v and vw, giving

$$I\bar{S}^{\infty}(v,vw) > I\bar{S}^{'\infty}(v,vw).$$

Hence, (v,vw) is a neutrosophic incidence cut pair. □

4. Application in Finding the Best Route

In this section, the developed approach of IVNIGs is utilized in the safe root problem dealing with the selection of the best route among some routes. Suppose Mr Manapat wants to travel from Thailand to Indonesia following all border lines between Thailand and Indonesia. There are basically three ways of doing so. The first one is a direct way, i.e., Thailand to Indonesia, the second one is Thailand to Malaysia and Malaysia to Indonesia and the last one is Thailand to Singapura, Singapura to Malaysia and Malaysia to Indonesia, as shown in Figure 3.

Let V = {Thailand (THAI), Singapura (SGPR), Malaysia (MAL), Indonesia (IDN)} be the set of countries.

Let E = {(THAI,SGPR),(SGPR,MAL),(THAI,MAL),(MAL,IDN),(THAI,IDN)} a subset of $V \times V$.

Let X be the interval-valued neutrosophic set on V, given as:

$$X = \left\{\begin{matrix}(\text{THAI},[0.1,0.4],[0.2,0.5],[0.3,0.7]),(\text{SGPR},[0.3,0.5],[0.2,0.6],[0.1,0.7]),\\(\text{MAL},[0.3,0.8],[0.4,0.9],[0.5,0.9]),(\text{IDN},[0.4,0.7],[0.3,0.8],[0.4,0.9])\end{matrix}\right\}.$$

Let Y be the interval-valued neutrosophic relation on V, given as:

$$Y = \left\{\begin{matrix}((\text{THAI},\text{SGPR}),[0.1,0.4],[0.2,0.5],[0.3,0.7]),((\text{THAI},\text{MAL}),[0.1,0.4],[0.2,0.5],[0.4,0.8]),\\((\text{SGPR},\text{MAL}),[0.3,0.5],[0.2,0.6],[0.5,0.8]),((\text{MAL},\text{IDN}),[0.3,0.7],[0.3,0.8],[0.5,0.8]),\\((\text{THAI},\text{IDN}),[0.1,0.4],[0.2,0.5],[0.4,0.8])\end{matrix}\right\}.$$

Let Z be the interval-valued neutrosophic set on $V \times E$, given as:

$$Z = \left\{ \begin{array}{l} ((\mathrm{THAI},(\mathrm{THAI},\mathrm{SGPR})),[0.1,0.3],[0.1,0.4],[0.3,0.7]), \\ ((\mathrm{SGPR},(\mathrm{THAI},\mathrm{SGPR})),[0.1,0.3],[0.1,0.5],[0.2,0.6]), \\ ((\mathrm{THAI},(\mathrm{THAI},\mathrm{MAL})),[0.1,0.3],[0.1,0.5],[0.3,0.8]), \\ ((\mathrm{MAL},(\mathrm{THAI},\mathrm{MAL})),[0.1,0.4],[0.1,0.4],[0.4,0.8]), \\ ((\mathrm{SGPR},(\mathrm{SGPR},\mathrm{MAL})),[0.2,0.5],[0.1,0.5],[0.4,0.7]), \\ ((\mathrm{MAL},(\mathrm{SGPR},\mathrm{MAL})),[0.3,0.5],[0.2,0.6],[0.4,0.8]), \\ ((\mathrm{MAL},(\mathrm{MAL},\mathrm{IDN})),[0.3,0.7],[0.3,0.8],[0.4,0.9]), \\ ((\mathrm{IDN},(\mathrm{MAL},\mathrm{IDN})),[0.3,0.6],[0.2,0.7],[0.3,0.8]), \\ ((\mathrm{THAI},(\mathrm{THAI},\mathrm{IDN})),[0.1,0.4],[0.2,0.4],[0.3,0.7]), \\ ((\mathrm{IDN},(\mathrm{THAI},\mathrm{IDN})),[0.1,0.4],[0.2,0.5],[0.4,0.8]) \end{array} \right\}.$$

Let $\left(T^L_{\bar{P}}(uv), T^U_{\bar{P}}(uv)\right)$ represent the degree of protection for travelling from country u to the country v. There are three paths from THAI to IDN, such that

$\bar{P}_1 : \mathrm{THAI},(\mathrm{THAI},(\mathrm{THAI},\mathrm{IDN}))(\mathrm{THAI},\mathrm{IDN}),(\mathrm{IDN},(\mathrm{THAI},\mathrm{IDN})),\mathrm{IDN}$

$\bar{P}_2 : \mathrm{THAI},(\mathrm{THAI},(\mathrm{THAI},\mathrm{MAL}))(\mathrm{THAI},\mathrm{MAL}),(\mathrm{MAL},(\mathrm{THAI},\mathrm{MAL})),\mathrm{MAL},(\mathrm{MAL},(\mathrm{MAL},\mathrm{IDN})),$
$(\mathrm{MAL},\mathrm{IDN}),(\mathrm{IDN},(\mathrm{MAL},\mathrm{IDN})),\mathrm{IDN}$

$\bar{P}_3 : \mathrm{THAI},(\mathrm{THAI},(\mathrm{THAI},\mathrm{SGPR}))(\mathrm{THAI},\mathrm{SGPR}),(\mathrm{SGPR},(\mathrm{THAI},\mathrm{SGPR})),\mathrm{SGPR},(\mathrm{SGPR},(\mathrm{SGPR},\mathrm{MAL})),$
$(\mathrm{SGPR},\mathrm{MAL}),(\mathrm{MAL},(\mathrm{SGPR},\mathrm{MAL})),\mathrm{MAL},(\mathrm{MAL},(\mathrm{MAL},\mathrm{IDN})),(\mathrm{MAL},\mathrm{IDN}),(\mathrm{IDN},(\mathrm{MAL},\mathrm{IDN})),\mathrm{IDN}$

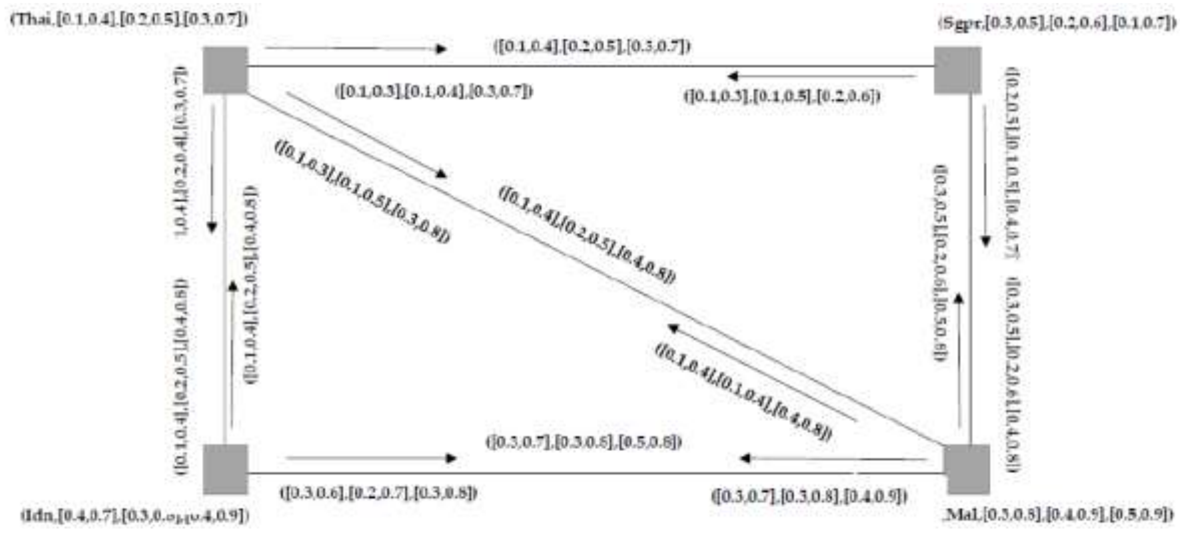

Figure 3 Model of travelling paths from Thailand to Indonesia

$I\bar{S}^{\infty}(THAI, IDN)$ is the greatest incidence strength of the path between THAI and IDN. In other words, this is the safest path between THAI and IDN. To calculate the value of $I\bar{S}^{\infty}(THAI, IDN)$, we need to first calculate the incidence strength of paths $\bar{P}_1, \bar{P}_2$ and $\bar{P}_3$ denoted by $I\bar{S}_{\bar{P}_1}(THAI, IDN)$, $I\bar{S}_{\bar{P}_2}(THAI, IDN)$ and $I\bar{S}_{\bar{P}_3}(THAI, IDN)$, respectively. By calculation, we obtain

$$I\bar{S}_{\bar{P}_1}(THAI, IDN) = ([0.1, 0.4], [0.2, 0.5], [0.4, 0.8])$$
$$I\bar{S}_{\bar{P}_2}(THAI, IDN) = ([0.3, 0.7], [0.3, 0.8], [0.5, 0.9])$$
$$I\bar{S}_{\bar{P}_3}(THAI, IDN) = ([0.2, 0.4], [0.2, 0.5], [0.5, 0.9]).$$

Hence,

$$I\bar{S}^{\infty}(THAI, IDN) = ([0.3, 0.7], [0.2, 0.6], [0.4, 0.8]).$$

We see that $\left(T^L_{I\bar{S}^{\infty}}(THAI, IDN), T^U_{I\bar{S}^{\infty}}(THAI, IDN)\right) = \left(T^L_{I\bar{S}_{\bar{P}2}}(THAI, IDN), T^U_{I\bar{S}_{\bar{P}2}}(THAI, IDN)\right)$.

Therefore, $\bar{P}_2$ is the safest path for travelling. We present the proposed method in the following algorithm.

Algorithm:

1. Input the vertex set $\hat{V}$.
2. Input the edge set $\hat{E} \subseteq \hat{V} \times \hat{V}$.
3. Set up the interval-valued neutrosophic set X on $\hat{V}$.
4. Set up the interval-valued neutrosophic relation Y on $\hat{V}$.
5. Set up the interval-valued neutrosophic set Z on $\hat{V} \times \hat{E}$.
6. Calculate the incidence strength $I\bar{S}(x_i, y_j)$ of all possible paths from x to y such that

$$i\bar{s}_1 = \min\left\{\left[T^L_Z(x_i, x_i x_{i+1}), T^U_Z(x_i, x_i x_{i+1})\right] : (x_i, x_i x_{i+1}) \in I\right\}$$
$$i\bar{s}_2 = \min\left\{\left[I^L_Z(x_i, x_i x_{i+1}), I^U_Z(x_i, x_i x_{i+1})\right] : (x_i, x_i x_{i+1}) \in I\right\}$$
$$i\bar{s}_3 = \min\left\{\left[F^L_Z(x_i, x_i x_{i+1}), F^U_Z(x_i, x_i x_{i+1})\right] : (x_i, x_i x_{i+1}) \in I\right\}$$

7. Calculate the greatest incidence strength $I\bar{S}^{\infty}$ of the path from x to y.
8. The safest path is $S(v_k) = \min\left(T^L_{\bar{P}_i}(xy), T^U_{\bar{P}_i}(xy)\right)$ where $i = 1 \ldots k$
9. If v_k has more than one value then any path can be chosen.

5. Comparative Study and Advantages of the proposed algorithm

In this section, a comparative study based on the results of numerical computation is performed to validate the proposed method. For this purpose, we present a comparative analysis between fuzzy incidence graphs (FIG), single-valued neutrosophic incidence graph (SVNIG) and the proposed method IVNIG as presented in Table 5.

From the safest path column in Table 5, it can be seen that the safest path of our proposed method IVNIG is consistent with the FIG and SVNIG which is P_2. However, as we may notice, FIG just takes into consideration crisp membership values to represent the uncertain data. In this case, the non-membership values are directly complementing to their respective membership values. We can observe that these two elements of membership and non-membership are said to be dependent here. This approach, even though effective in dealing with uncertainty, but still cannot capture some types of uncertainties such as indeterminate and inconsistent information. Therefore, some new theories are required to overcome this problem.

Should be noted that, the proposal of IVNIG is to provide a generalization of the notion of SVNIG. The justification of this generalization lies in the following observation: sometimes it is not appropriate to assume that the degrees of (t, i, f) are exactly defined, therefore we can admit a kind of further uncertainty where the values of these components are not numbers, but interval of numbers. Clearly, SVNIG may be viewed as special cases of IVNIG here if the degrees of (t, i, f) are the only numbers. Furthermore, an IVNIG also can avoid the loss of information. Sometimes, the degree of memberships is not certainly known. Then interval-valued may better represent this kind of information. Similarly, if the three components are dependent, then IVNIG can be reduced to the FIG.

For SVNIG, the sum of the components is; $0 \le t+i+f \le 3$ when all three components are independent; $0 \le t+i+f \le 2$ when two components are dependent, while the third one is independent; $0 \le t+i+f \le 1$ when all three components are dependent. When three or two of the components T, I, F are independent, one leaves room for incomplete information (sum < 1), paraconsistent and contradictory information (sum > 1), or complete information (sum = 1). If all three components T, I, F are dependent, then similarly one leaves room for incomplete information (sum < 1), or complete information (sum = 1).

Table 5. A comparative study between FIG, SVNIG and IVNIG

	Incidence strength of path	Greatest incidence strength	Safest path
FIG	$I\bar{S}_{\bar{P}_1}=(0.1)$ $I\bar{S}_{\bar{P}_2}=(0.3)$ $I\bar{S}_{\bar{P}_3}=(0.2)$	$I\bar{S}^{\infty}=(0.3)$	$\bar{P}_2$
SVNIG	$I\bar{S}_{\bar{P}_1}=(0.1,0.3,0.4)$ $I\bar{S}_{\bar{P}_2}=(0.3,0.4,0.5)$ $I\bar{S}_{\bar{P}_3}=(0.2,0.3,0.5)$	$I\bar{S}^{\infty}=(0.3,0.2,0.5)$	$\bar{P}_2$
IVNIG (proposed method)	$I\bar{S}_{\bar{P}_1}=([0.1,0.4],[0.2,0.5],[0.4,0.8])$ $I\bar{S}_{\bar{P}_2}=([0.3,0.7],[0.3,0.8],[0.5,0.9])$ $I\bar{S}_{\bar{P}_3}=([0.2,0.4],[0.2,0.5],[0.5,0.9])$	$I\bar{S}^{\infty}=([0.3,0.7],[0.2,0.6],[0.4,0.8])$	$\bar{P}_2$

6. Conclusions

A new IVNIG has been successfully proposed. We constructed a new set for neutrosophic incidence graphs based on the definition from the previous study. An interval-valued neutrosophic set is an extension of an interval-valued fuzzy set combined with a single-valued neutrosophic set, a more powerful model to solve real-life problems. This paper has presented certain properties related to IVNIG such as strong edge, strong pair, strong cut pair and neutrosophic incidence cut pair. Also, in this work, we just limit our attention to the class of standard unit interval [0,1]. The assumption is that this unit interval may be sufficient to be applied in the real-life problems. However, further analysis can be potentially conducted on the non-standard unit interval to generalize the developed concept. Moreover, for future research, another higher order of uncertainty can be proposed to the neutrosophic set or neutrosophic incidence graph, i.e., incorporate the membership function to each element of (t, i, f) instead of interval-valued.

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References

[1] Zadeh, L.A. Fuzzy sets. *Inf Control* **1965**, 8(8), 338-353.

[2] Zadeh, L.A. The Concept of a Linguistic Variable and its Application to Approximate Reasoning-I. *Information Sciences* **1975**, 8(3),199-249.

[3] Smarandache, F. Neutrosophic Set-A Generalization of The Intuitionistic Fuzzy Set. *International Journal of Pure and Applied Mathematics* **2004**,24.

[4] Wang, H., Smarandache, F., Zhang, Y. & Sunderraman, R. Single valued neutrosophic sets. **2010**.

[5] Wang, H., Zhang, Y. & Sunderraman, R. Truth-value based interval neutrosophic sets. *2005 IEEE International Conference on Granular Computing* **2005**,1,274–277.

[6] Rosenfeld, A. Fuzzy graphs,Fuzzy Sets and their Applications to Cognitive and Decision Processes. *Elsevier* **1975**, 77–95.

[7] Kalampakas, A., Spartalis, S., Iliadis, L., & Pimenidis, E. Fuzzy graphs: Algebraic structure and syntactic recognition. *Artif. Intell. Rev.* **2014**, 42(3),479–490.

[8] Bhattacharya, P. Some remarks on fuzzy graphs. *Pattern Recognit. Lett.* **1987**, 6(5), 297–302.

[9] Gani, A. N., & Latha, S. R. On irregular fuzzy graphs. *Applied Mathematical Sciences* **2012**, 6(11), 517-523.

[10] Gani, A. N., & Ahamed, M. B. Order and size in fuzzy graphs. *Bulletin of Pure and Applied Sciences* **2003**, *22*(1), 145-148.

[11] Gani, A. N., & Radha, K. On Regular Fuzzy Graphs. *Journal of Physical Sciences* **2008**, 12, 33-40.

[12] Akram, M., & Davvaz, B. Strong intuitionistic fuzzy graphs. *Filomat* **2012**, 26(1), 177–196.

[13] Atanassov, K., & Shannon, K. On index matrix interpretations of intuitionistic fuzzy graphs. *Notes on Intuitionistic Fuzzy sets* **2002**, 8(4), 73–78.

[14] Atanassov, K., & Shannon, K. On a generalization of intuitionistic fuzzy graphs. *Notes on Intuitionistic Fuzzy sets* **2006**, 12(1), 24–29.

[15] Akram, M., & Parvathi, R. Properties of intuitionistic fuzzy line graphs. *Notes on Intuitionistic Fuzzy Sets* **2012**, 18(3), 52–60.

[16] Parvathi, R., Thilagavathi, S., Thamizhendhi, G., & Karunambigai, M. G. Index matrix representation of intuitionistic fuzzy graphs. *Notes on Intuitionistic Fuzzy Sets* **2014**, *20*(2), 100–108.

[17] Karunambigai, M. G., Parvathi, R., & Buvaneswari, R. Arcs in intuitionistic fuzzy graphs. *Notes on Intuitionistic Fuzzy Sets* **2011**, *17*, 37–47.

[18] Gani, A. N., & Begum, S. S. Degree, order and size in intuitionistic fuzzy graphs. *International journal of algorithms, Computing and Mathematics* **2010**, *3*(3), 11–16.

[19] Mishra, S. N., & Pal, A. Product of interval valued intuitionistic fuzzy graph. *Annals of pure and applied mathematics* **2013**, *5*(1), 37–46.

[20] Ismayil, A. M., & Ali, A. M. On strong interval-valued intuitionistic fuzzy graph. *International Journal of Fuzzy Mathematics and Systems* **2014**, *4*(2), 161–168.

[21] Ismayil, A. M., & Ali, A. M. On Complete interval-valued intuitionistic fuzzy graph. *Advances in fuzzy sets and systems* **2014**, *18*(1), 71–86.

[22] Wei, G. W., Wang, H. J., & Lin, R. Application of correlation coefficient to interval-valued intuitionistic fuzzy multiple attribute decision-making with incomplete weight information. *Knowledge and Information Systems* **2011**, *26*(2), 337–349.

[23] Smarandache, F. *Symbolic Neutrosophic Theory*. Europanova asbl, Brussels, 2015.

[24] Smarandache, F. Refined Literal Indeterminacy and the Multiplication Law of Sub-Indeterminacies. *Neutrosophic Sets and System* **2015**, 9, 58–63.

[25] Smarandache, F. Types of Neutrosophic Graphs and neutrosophic Algebraic Structures together with their Applications in Technology, seminar, Universitatea Transilvania din Brasov, Facultatea de Design de Produs si Mediu, Brasov, Romania 06 June 2015.

[26] Kandasamy, W. V., Ilanthenral, K., & Smarandache, F. *Neutrosophic Graphs: A new dimension to graph theory*. Brussels: EuropaNova, 2015.

[27] Kumar, M., Bhutani, K., & Aggarwal, S. Hybrid model for medical diagnosis using Neutrosophic Cognitive Maps with Genetic Algorithms. *2015 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, 1–7.

[28] Broumi, S., Talea, M., Bakali, A., & Smarandache, F. Single valued neutrosophic graphs. *Journal of New theory* **2016**, (10), 86–101.

[29] Broumi, S., Smarandache, F., Talea, M., & Bakali, A. Operations on interval valued neutrosophic graphs. *New Trends Neutrosophic Theory Appl.*, **2016**, 231–254

[30] Broumi, S., Talea, M., Bakali, A., & Smarandache, F. Interval valued neutrosophic graphs. *Critical Review, XII*, **2016**, 5-33.

[31] Akram, M. & Nasir, M. Concepts of Interval-Valued Neutrosophic Graphs. *Infin. Study* **2017**, 6, 22–41

[32] Akram, M., & Sitara, M. Novel applications of single-valued neutrosophic graph structures in decision-making. *Journal of Applied Mathematics and Computing* **2018**, *56*(1), 501–532.

[33] Akram, M., & Nasir, M. Interval-valued neutrosophic competition graphs. *Ann. Fuzzy Math. Informatics* **2017**, vol. 14, no. 1, pp. 99–120.

[34] Dinesh, T., A study on graph structures, incidence algebras and their fuzzy analogues. Ph.D. thesis, Kannur University, Kannur, India, 2011.

[35] Dinesh, T., Fuzzy incidence graph-an introduction. *Advances in Fuzzy Sets and Systems* **2016**, *21*(1), 33.

[36] Mathew, S., & Mordeson, J. N. Connectivity concepts in fuzzy incidence graphs. *Information Sciences.* **2017**, 382, 326–333.

[37] Malik, D. S., Mathew, S., & Mordeson, J. N. Fuzzy incidence graphs: Applications to human trafficking. *Information Sciences* **2018**, *447*, 244-255.

[38] Mathew, S., Mordeson, J., & Yang, H. L. Incidence cuts and connectivity in fuzzy incidence graphs. *Iranian Journal of Fuzzy Systems* **2019**, *16*(2), 31-43.

[39] Mathew, S., & Mordeson, J. N. Fuzzy incidence blocks and their applications in illegal migration problems. *New Mathematics and Natural Computation* **2017**, *13*(03), 245-260.

[40] Akram, M., Sayed, S., & Smarandache, F. Neutrosophic incidence graphs with application. *Axioms* **2018**, *7*(3), 47.

[41] Akram, M., Ishfaq, N., Smarandache, F., & Broumi, S. Application of Bipolar Neutrosophic sets to Incidence Graphs. *Neutrosophic Sets and Systems* **2019**, 27,180-199.

[42] Hussain, S. S., Hussain, R. J., & Babu, M. V. Neutrosophic Vague Incidence Graph. *International Journal of Neutrosophic Science* **2020**, 12(1), 29–38.

[43] Sunitha, M. S., & Mathew, S. Fuzzy Graph Theory: A Survey. *Ann. Pure Appl. Math.***2013**, 4(1), 92–110.

[44] Akram, M., & Shahzadi, G. Operations on single-valued neutrosophic graphs. *J. Uncertain Syst.* **2017**, 11(3), 176–196.

[45] Wang, H., Zhang, Y. Q. & Sunderraman, R. Truth-value based interval neutrosophic sets. *2005 IEEE Int.*

Conf. Granul. Comput. **2005,** 1, 274–277.

[46] Wang, H., Smarandache, F., Zhang, Y. Q., & Sunderraman, R. *Interval neutrosophic sets and logic: Theory and applications in computing*, Hexis, Phoenix, AZ 2005.

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