On Neutrosophic Triplet quasi–dislocated-b-metric space

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Abstract: The concept of neutrosophic triplet firstly introduced by F. Smarandache and M. Ali [28]. This notion (neutrosophic triplet) is a group of three elements that satisfy certain properties with some binary operation. These neutrosophic triplets highly depends on the proposed binary operation. In this article, we make some observations concerning Neutrosophic triplet metric space (NTMS), Neutrosophic triplet partial metric space (NTPMS), Neutrosophic triplet-b-metric space (NT-b-MS) introduced by Sahin et al. [18-20] and put our observation on the definitions defined in these articles. Moreover, inspired by Ur Rahaman [17] and Sahin et al. [18-20] further we define a new topological construction named as Neutrosophic Triplet quasi–dislocated-b-metric space (NT-qdb-MS) and study some properties of NT-qdb-MS. Furthermore using this construction, we establish some fixed point theorems in the context of NT-qdb-MS using graph. For the validity of our results, we also provide an example.

Keywords: Neutrosophic triplet group, neutrosophic triplet metric space, neutrosophic triplet partial metric space, neutrosophic triplet quasi–dislocated-b-metric space, fixed point, graph.

Abbreviations:

1. NTS – neutrosophic triplet set
2. NTG - neutrosophic triplet group
3. NTMS- neutrosophic triplet metric space
4. NTM- neutrosophic triplet metric
5.NTPMS- neutrosophic triplet partial metric space
6. NTPM- neutrosophic triplet partial metric
7. NT-b-MS- neutrosophic triplet– b-metric space
7. NT-qdb-MS- neutrosophic triplet quasi–dislocated-b-metric space
8. NT-qdb-M- neutrosophic triplet quasi–dislocated-b-metric

1. Introduction and Preliminaries

Concept of fuzzy sets were introduced by Zadeh [29] to deal the problem of uncertainty existing in real-world. Since its initiation, as a generalization of it, interval valued fuzzy set [13] and intuitionistic fuzzy set [24] have come into sight. These extensions can deal with uncertain real-world problems but it does not cope with indeterminate data. Thus, in order to cope with these uncertainties, Smarandache began to use the nonstandard analysis and proposed the term "neutrosophic" which means knowledge of neutral thought, and this neutral represents the main distinction between "fuzzy" and "intuitionistic fuzzy" set. Neutrosophy is a new subsidiary of philosophy which is initiated by Florentin Smarandache [27]. The concept of neutrosophic logic was first studied by Florentin Smarandache in 1995. Neutrosophic set is a stereotype of interval valued fuzzy set [13], intuitionistic fuzzy set [24], fuzzy sets [29] and classical sets which is used to handle problems issues containing inconsistent, indeterminate, falsity and imprecise data.

In the concept of neutrosophic logic and neutrosophic sets, there is T degree of membership, I degree of undeterminacy and F degree of non-membership. These degrees are defined independently of each other in neutrosophic logic and neutrosophic sets whereas these degrees are defined dependently of each other in intuitionistic fuzzy logic and intuitionistic fuzzy set. Thus, neutrosophic set is an extension of fuzzy and intuitionistic fuzzy set. Many authors have worked in neutrosophic theory for more details see [1-6, 9, 21,23, 24-26]. Furthermore, Smarandache and Ali deliberated neutrosophic triplet theory particularly NTG's in [28, 12, 30]. Later on, netroshophic triplet theory has been studied with fixed point theory in [19, 20].

Moreover, a new direction in the theory of fixed point was recently given by Jachymski [11] and gave some generalization of the Banach contraction principle for mapping on a metric spaces endowed with a graph in 2007. Jachymski [11] generalized and unified the results existing in the literature using the languages of graph theory and opened an avenue for further development of fixed point theory in this direction. His work is considered as a reference in this domain. The fixed point theory with graph is a very curious way in the field of research and have wide number of applications in other fields. Motivated by the remarkable work of Jachymski [11], a lot of work in fixed point theory with graph have been done by several authors in various spaces with various contractive conditions, see in [7,8,16,22] and etc.

Sahin et al. [18-20] proposed the NT-b-MS, NTMS, NTPMS respectively by combing the fixed point theory with neutrosophy which is a new interesting approach in this direction. But in their paper [19] (pp. 699), according to their Definition 4.1 of NTMS, Example 4.3 doesn't support the definition 4.1. For this we give a counter Example 2.1 in this paper in Section 2. Also, in 2018 Sahin et al.[20] introduced the concept of NTPMS but we get the disparity of their Definition 4. with Example 1. in [20] (pp. 3). For this, we demonstrate a counter Example 2.2 in Section 2. Also, in their paper [20] (pp. 5) in Theorem 4, we can't write inequality (8)

i.e, $p_n(\varkappa_n, \varkappa_k) \leq p_n(\varkappa_n, \varkappa_k * \text{neut}(\varkappa_{n-1}))$

for any arbitrary element $\varkappa_{n-1}$, since in Definition 8 [20] (pp. 5) they assumed that there exist any element c (any one) in A such that

$$p_n(a, b) \leq p_n(a, b*\text{neut}(c)) \text{ for all a, b in A.}$$

If they assumed condition (i) of Definition 8 in [20](pp.5) for all 'c' elements then all these properties of Definition 4 in [20] (pp.3) becomes properties of the partial metric space. Therefore, Theorem 4 in [20] (pp. 5) becomes the existing result in partial metric space. But Sahin et al.[18] again redefined their Definition 4.1[19] of NTMS and Definition 4. of NTPMS which is in corrected form. Here, we also discussed what is the difference between taking "any element" or "atleast one element" in triangular inequality of NTMS and NTPMS.

Recently, Ur Rahaman[17] in 2015, introduced the topological properties of dislocated-quasi-b-metric space and proved some fixed point theorems. Motivated by Smarandache and Ali[28], Sahin et al.[18-20] and Ur Rahaman[17], we define a new topological structure NT-qdb-MS which is different from classical quasi-dislocated-b-metric space. A great benefit of NT-qdb-MS is that it gives a new space structure to those structures which are not quasi-dislocated-b- metric space with respect to some functions that not satisfy triangular inequality for all $\varkappa$, $\gamma, \mathfrak{z}$ since we don't need to verify the triangular inequality for all $\varkappa$, $\gamma, \mathfrak{z}$ in NT-qdb-MS as we can see in Definition 2.3. defined in Section 2. Triangular inequality in NT-qdb-MS is much weaker assumption as compare to the triangular inequality in quasi-dislocated-b- metric space. We also studied some interesting properties of this newly born structure. At the end, we obtained some fixed point results such as famous Banach fixed theorem(generalized version) and Kannan fixed point theorem inspired by [7,10,14] in this topological structure and provided an example to explain the results.

Now, we call some basic definitions from neutrosophic triplet theory following as:

Definition 1.1 [28]. Let S be a non–empty set with a binary operation ◊ then it is called a **NTS** if for any s∈S, there exist a neutral of s in S denoted by neut(s) different from classical algebraic unitary element and also there exist antineutral of s in S named as anti (s) such that

$$s \lozenge neut(s) = neut(s) \lozenge s = s \quad \text{and} \quad s \lozenge anti(s) = anti(s) \lozenge s = neut(s)$$

The triplet (s, neut(s), anti(s)) represents neutrosophic triplet. For the same element, there may be more neutrals to it neut(s)'s and more opposite of it anti(s)'s.

Definition 1.2 [28]. A non-empty set S with binary operation ◊ is called a **NTG** if it satisfies following properties:

(i) $s_1 \lozenge s_2 \in S$ for all $s_1, s_2 \in S$;

(ii) $(s_1 \lozenge s_2) \lozenge s_3 = s_1 \lozenge (s_2 \lozenge s_3)$ for all $s_1, s_2, s_3 \in S$;

(iii) for each $s_1 \in S$, there exist a neutral of s_1 in S denoted by neut(s_1) different from classical algebraic unitary element such that

$$s_1 \lozenge neut(s_1) = neut(s_1) \lozenge s_1 = s_1$$

(iv) and for each $s_1 \in S$, there exist anti-neutral of s_1 in S named as anti (s_1) such that

$$s_1 \lozenge anti(s_1) = anti(s_1) \lozenge s_1 = neut(s_1).$$

Definition 1.3 [7]: A mapping $\varphi: R^+ \to R^+$is said to be comparison function if it satisfies:

1. φ is monotonic increasing;
2. The sequence $\{\varphi^n (t)\}$ converges to zero for all $t \in R^+$.

2. Neutrosophic triplet quasi–dislocated-b-metric space and Revised definition of NTMS and NTPMS

In this section, first we define the revised definition of NTMS and NTPMS then we define NT-qdb-MS and its properties.

Definition 2.1 [19]. Let (M, ◊) be a NTS with $\varkappa$ ◊ y ∈ M for all $\varkappa$, y in M. A mapping
d: M×M→ [0, ∞) is called a NTM on M if satisfying the following properties for all $\varkappa$, z ∈ M,

(i) d($\varkappa$, z) ≥ 0;

(ii) If $\varkappa$ = z then d($\varkappa$, z) = 0;

(iii) d($\varkappa$, z) = d(z, $\varkappa$);

(iv) If there exist at least one element y in M different from $\varkappa$ and z in M such that

d($\varkappa$, z) ≤ d($\varkappa$, z ◊ neut(y)) then d($\varkappa$, z ◊ neut(y)) ≤ d($\varkappa$, y) + d(y, z).

The space ((M, ◊), d) is known as NTMS.

Remark 2.1. In metric space, we have to verify the triangular inequality for all $\varkappa$, y, z ∈ M and it is much stronger assumption as compare to the triangular inequality in NTMS since we don't need to verify the triangular inequality for all $\varkappa$, y, z ∈ M in NTMS. In fact, we have to verify it for at least one element y in M different from $\varkappa$ and z in M such that

d($\varkappa$, z) ≤ d($\varkappa$, z ◊ neut(y)) then d($\varkappa$, z ◊ neut(y)) ≤ d($\varkappa$, y) + d(y, z) and it implies

d($\varkappa$, z) ≤ d($\varkappa$, y) + d(y, z).

A big advantage of neutrosophic triplet metric space is that it gives a new space structure to those structures which are not metric spaces with respect to some functions that not satisfy triangular inequality for all $\varkappa$, y, z ∈ M.

Example 2.1. Let M = {1, 2} and the power set of M, Ƥ(M) = {∅, {1}, {2}, {1,2}} together with binary operation ◊ = ∪ form a NTS where neut(J) = J and anti(J) = J, for all J ∈ Ƥ(M). Define a function d: Ƥ(M)× Ƥ(M)→[0, ∞] such that

d(J, Ƙ) = | n(J)- n(Ƙ) | where n(J) denotes the cardinality of J.

Clearly (i), (ii) and (iii) of Definition 4.1 in [19] are satisfied.

Now, we will see condition (iv).

Take, J = {1}, Ł = {2} and Ƙ = {1}

0 = d(J, Ł) ≤ d(J, Ł ◊ neut(Ƙ))

= d(J, Ł∪ Ƙ)

= | n(J)- n(Ł∪ Ƙ) | =1

But 1= d(J, Ł ◊ neut(Ƙ)) ≰ d(J, Ƙ) + d(Ƙ, Ł) = 0.

This shows that there exist an element y ∈ Ƥ(M) such that

d($\varkappa$, z) ≤ d($\varkappa$, z ◊ neut (y))

but d($\varkappa$, z ◊ neut (y))≤ d($\varkappa$, y) +d(y, z) doesn't hold which contradict Example 4.3. provided in [19] (pp.699),

i.e, Example 4.3[19] doesn't satisfy the triangular inequality of Definition 4.1 in [19].

Remark 2.2. According to above Definition 2.1, Example 4.3 in [19] is accurate.

Definition 2.2 [20]. Let (M, ◊) be a NTS with $\varkappa$ ◊ y ∈ M for all $\varkappa$, y in M. A NTPM is a mapping P_N : M×M→ [0, ∞) such that for all $\varkappa$, z ∈ M,

(i) $P_N(\varkappa, \varkappa) \leq P_N(\varkappa, ʒ)$;

(ii) If $P_N(\varkappa, ʒ) = P_N(\varkappa, \varkappa) = P_N(ʒ, ʒ)$ then $\varkappa = ʒ$;

(iii) $P_N(\varkappa, ʒ) = P_N(ʒ, \varkappa)$;

(iv) If there exist atleast one element y in M different from $\varkappa$ and $ʒ$ in M such that

$P_N(\varkappa, ʒ) \leq P_N(\varkappa, ʒ \lozenge neut(y))$ then $P_N(\varkappa, ʒ \lozenge neut(y)) \leq P_N(\varkappa, y) + P_N(y, ʒ) - P_N(y, y)$.

The space $((M, \lozenge), P_N)$ is known as NTPMS.

Remark 2.3. Concept of NTPMS(NTMS) is different from partial metric space(metric space respectively), neither of them is generalization of each other. First we see that, Is PMS implies NTPMS?. For this, we have to identify triangular inequality i.e, (iv) condition of NTPMS since all other conditions of NTPMS are satisfied by PMS.

If $P_N(\varkappa, ʒ) \leq P_N(\varkappa, y) + P_N(y, ʒ) - P_N(y, y)$ and $P_N(\varkappa, ʒ) \leq P_N(\varkappa, ʒ \lozenge neut(y))$
then $P_N(\varkappa, ʒ \lozenge neut(y)) \leq P_N(\varkappa, y) + P_N(y, ʒ) - P_N(y, y)$ for atleast one element y, which is not possible always, if this is possible then (iv) is meaningless.

Clearly, NTPMS doesn't implies PMS.

If we take assumption (iv) in Definition 2.2 for any element y in M as defined by Sahin et al. [20] (in Definition 4.[20] (pp. 3)) then examples which we have constructed not form NTPMS and it is difficult to find the examples for NTPMS which satisfy the properties of NTPMS defined by Sahin et al. in [20] since it is much stronger assumption as compare to assumption (iv) in Definition 2.2 and triangular inequality of partial metric space.

Moreover, we take here element y in M is different from $\varkappa$ and $ʒ$ since there exist always an element $y = ʒ$ for all $\varkappa, ʒ \in M$ such that

$P_N(\varkappa, ʒ) \leq P_N(\varkappa, ʒ \lozenge neut(ʒ)) = P_N(\varkappa, ʒ)$ then $P_N(\varkappa, ʒ \lozenge neut(ʒ)) \leq P_N(\varkappa, ʒ) + P_N(ʒ, ʒ) - P_N(ʒ, ʒ)$
and the property (iv) becomes meaningless.

Counter example 2.2. Let M be any set, Ƥ(M) be the power set of M with binary operation $\lozenge = \cup$ then (Ƥ(M), ∪) is a NTS where neut(Ɉ) = Ɉ and anti(Ɉ) = Ɉ, for all Ɉ∈ Ƥ(M).

Define a map P_N : Ƥ(M)× Ƥ(M)→ $[0,\infty)$ such that

P_N(Ɉ, Ꝁ) = max {n(Ɉ), n(Ꝁ)} where n(Ɉ) denotes the cardinality of Ɉ.

Condition (i), (ii) and (iii) are easy to verified of Definition 4. [20] (pp.3). Here we see condition (iv).

If we take the sets, Ɉ, Ꝁ, Ł in Ƥ(M) such that

n(Ɉ)= 25, n(Ꝁ)= 22, n(Ł) =10 and n(Ꝁ∩ Ł)= 4.

Now,

P_N(Ɉ, Ł) ≤ P_N (Ɉ, Ł ◊ neut(Ꝁ)) = P_N (Ɉ, Ł∪ Ꝁ)

i.e, max {n(Ɉ), n(Ł)} ≤ max{ n(Ɉ), n(Ł)+ n(Ꝁ)-n(Ł∪ Ꝁ) }

i.e., $25 \leq 28$

which is true.

But P_N (Ɉ, Ł ◊ neut(Ꝁ)) ≰ P_N (Ɉ, Ꝁ) + P_N (Ꝁ, Ł) - P_N(Ꝁ, Ꝁ)

since P_N (Ɉ, Ł∪ Ꝁ) = 28 and P_N (Ɉ, Ꝁ) + P_N (Ꝁ, Ł) - P_N(Ꝁ, Ꝁ) = 25+22-22=25.

Thus the condition (iv) of Definition 4. in [20] doesn't satisfied. Hence (Ƥ(M),∪) is not a NTPMS according to Definition 4. [20] but it becomes a NTPMS according to Definition 2.2 as defined above. Since for any elements $\varkappa$ = Ɉ $\neq \emptyset$, ƴ = Ꝁ $\neq \emptyset$, there exist ʒ = $\emptyset$ such that

$$P_N(Ɉ, Ꝁ) \leq P_N(Ɉ, Ꝁ \lozenge neut(\emptyset)) = P_N(Ɉ, Ꝁ)$$

then $P_N(Ɉ, Ꝁ \lozenge neut(\emptyset)) \leq P_N(Ɉ, \emptyset) + P_N(\emptyset, Ꝁ) - P_N(\emptyset,\emptyset)$

and for $\varkappa$ = Ɉ $\neq \emptyset$, ƴ = Ꝁ = $\emptyset$, there exist ʒ = Ł in Ƥ(M) different from $\emptyset$, Ɉ such that

$$P_N(Ɉ, Ꝁ) \leq P_N(Ɉ, Ꝁ \lozenge neut(Ł)) = P_N(Ɉ, Ꝁ \cup Ł) = P_N(Ɉ, Ł)$$

then $P_N(Ɉ, Ꝁ \lozenge neut(Ł)) \leq P_N(Ɉ, Ł) + P_N(Ł, Ꝁ) - P_N(Ł,Ł)$.

Example 2.3. Let M = {0,4,8,9} be a NTG together binary operation ◊ = multiplication modulo 12 in (Z_{12}, ×). Neutrosophic triplet are:

(0,0,0), (4,4,4), (8,4,8), (9,9,9) where ($\varkappa$, ƴ, ʒ) denote here, $\varkappa \in M$ be any element, ƴ = neut($\varkappa$) and ʒ = anti ($\varkappa$).

Now, we define a map P_N: M× M→ [0,∞) such that

$$P_N(\varkappa, ƴ) = \max\{\varkappa, ƴ\} \text{ for all } \varkappa, ƴ \in M$$

Clearly, conditions (i), (ii) and (iii) are satisfied. Now, we identify condition (iv) for all $\varkappa$, ƴ, ʒ ∈M;

For $\varkappa$ = 4, ƴ = 0 ∃ ʒ = 8 such that

$4=P_N(\varkappa, ƴ) \leq P_N(\varkappa, ƴ \lozenge neut(ʒ)) = P_N(\varkappa,0) = 4$ then

$4=P_N(\varkappa, ƴ \lozenge neut(ʒ)) \leq P_N(\varkappa, ʒ) + P_N(ʒ, ƴ) - P_N(ʒ, ʒ)= 8+8-8=8$.

For $\varkappa$ = 8, ƴ = 0 ∃ ʒ = 4 such that

$8=P_N(\varkappa, ƴ) \leq P_N(\varkappa, ƴ \lozenge neut(ʒ)) = P_N(\varkappa,0)=8$

then $8=P_N(\varkappa, ƴ \lozenge neut(ʒ)) \leq P_N(\varkappa, ʒ) + P_N(ʒ, ƴ) - P_N(ʒ, ʒ) = 8+4-4=8$.

Similarly, for $\varkappa$ = 9, ƴ = 0 ∃ ʒ = 4, for $\varkappa$ = 8, ƴ = 4 ∃ ʒ = 9, for $\varkappa$ = 9, ƴ = 4 ∃ ʒ = 8, for $\varkappa$ = 9, ƴ = 8 ∃ ʒ = 4, for $\varkappa$ = 0, ƴ = 4 ∃ ʒ = 8 and for $\varkappa$ = 0, ƴ =8 ∃ ʒ = 4 such that

$P_N(\varkappa, ƴ) \leq P_N(\varkappa, ƴ \lozenge neut(ʒ))$ then $P_N(\varkappa, ƴ \lozenge neut(ʒ)) \leq P_N(\varkappa, ʒ) + P_N(ʒ, ƴ) - P_N(ʒ, ʒ)$.

But for $\varkappa$ = 0, ƴ = 9, for $\varkappa$ = 4, ƴ = 8, for $\varkappa$ = 4, ƴ = 9 and for $\varkappa$ = 8, ƴ = 9 there does not exist a different element ʒ such that $P_N(\varkappa, ƴ) \leq P_N(\varkappa, ƴ \lozenge neut(ʒ))$ hold so, we will not see

$P_N(\varkappa, ƴ \lozenge neut(ʒ)) \leq P_N(\varkappa, ʒ) + P_N(ʒ, ƴ) - P_N(ʒ, ʒ)$.

Hence ((M, ◊), P_N) is a NTPMS.

Definition 2.3. Let (M, ◊) be a NTS with $\varkappa$ ◊ ƴ ∈ M for all $\varkappa$, ƴ in M. Then a NT-qdb-M is a mapping N_{qdb} : M×M→ [0, ∞) such that for all $\varkappa$, ƴ ∈ M,

(Nqdb1.) $N_{qdb}(\varkappa, ƴ) = N_{qdb}(ƴ, \varkappa) = 0$ implies $\varkappa$ = ƴ

(Nqdb2.) If there exist atleast one element ʒ in M/{ $\varkappa$, ƴ } such that

$$N_{qdb}(\varkappa, ƴ) \leq N_{qdb}(\varkappa, ƴ \lozenge neut(ʒ)) \text{ then } N_{qdb}(\varkappa, ƴ \lozenge neut(ʒ)) \leq s[N_{qdb}(\varkappa, ʒ) + N_{qdb}(ʒ, ƴ)]$$

where s ≥ 1 be a real number.

The space ((M, ◊), N_{qdb}) is known as NT-qdb-MS.

Remark 2.4. Concept of NT-qdb-MS is different from dislocated-quasi-b-metric space. Here, N_{qdb} (ϰ, ƴ) $\neq N_{qdb}$ (ƴ, ϰ) and N_{qdb} (ϰ, ϰ) = 0 may not be possible. For s=1, the space NT-qdb-MS ((M, ◊), N_{qdb}) becomes neutrosophic triplet quasi dislocated metric space.

Example 2.4. Let M= {0,2,3,4} be a NTG together binary operation ◊=$\otimes_6$ in (Z_6, ×). Neutrosophic triplet are:

(0,0,0), (2,4,2), (3,3,3) (4,4,4) where (ϰ, ƴ, ʒ) denote here, ϰ ∈ M be any element, ƴ = neut(ϰ) and ʒ = anti (ϰ).

Now, we define a map N_{qdb} : M×M→ [0,∞) such that

$N_{qdb}(0,0) = 0$, $N_{qdb}(0,2) = 4$, $N_{qdb}(0,3) = 9$, $N_{qdb}(0,4) = 16$,

$N_{qdb}(2,2) = 4$, $N_{qdb}(3,3) = 9$, $N_{qdb}(4,4) = 16$, $N_{qdb}(2,0) = 8$,

$N_{qdb}(3,0) = 18$, $N_{qdb}(4,0) = 32$, $N_{qdb}(2,3) = 5$, $N_{qdb}(3,2) = 10$,

$N_{qdb}(2,4) = 8$, $N_{qdb}(4,2) = 20$, $N_{qdb}(3,4) = 10$, $N_{qdb}(4,3) = 17$

Next, we identify the conditions (Nqdb1.) and (Nqdb2.) of NT-qdb-MS.

(Nqdb1.) N_{qdb} (ϰ, ƴ) = N_{qdb} (ƴ, ϰ) = 0 for only ϰ = ƴ = 0 implies ϰ = ƴ.

(Nqdb2.) Take s = $\frac{3}{2}$.

For ϰ = 2, ƴ = 0 ∃ ʒ = 3 in M such that

$N_{qdb}(2,0) \leq N_{qdb}(2, 0 \otimes_6 \text{neut}(3)) = N_{qdb}(2,0) = 8$ then

$8 = N_{qdb}(2, 0 \otimes_6 \text{neut}(3)) \leq s[N_{qdb}(2,3) + N_{qdb}(3,0)] = s[5+18] = 23s$.

For ϰ = 3, ƴ = 0 ∃ ʒ = 2 in M such that

$N_{qdb}(3,0) \leq N_{qdb}(3, 0 \otimes_6 \text{neut}(2)) = N_{qdb}(3,0) = 18$ then

$18 = N_{qdb}(3, 0 \otimes_6 \text{neut}(2)) \leq s[N_{qdb}(3,2) + N_{qdb}(2,0)] = s[10+8] = 18s$.

For ϰ = 4, ƴ = 0 ∃ ʒ = 2 in M such that

$N_{qdb}(4,0) \leq N_{qdb}(4, 0 \otimes_6 \text{neut}(2)) = N_{qdb}(4,0) = 32$ then

$32 = N_{qdb}(4, 0 \otimes_6 \text{neut}(2)) \leq s[N_{qdb}(4,2) + N_{qdb}(2,0)] = s[20+8] = 28s$.

For ϰ = 2, ƴ = 3 ∃ ʒ = 0 in M such that

$5 = N_{qdb}(2,3) \leq N_{qdb}(2, 3 \otimes_6 \text{neut}(0)) = N_{qdb}(2,0) = 8$ then

$8 = N_{qdb}(2, 3 \otimes_6 \text{neut}(0)) \leq s[N_{qdb}(2,0) + N_{qdb}(0,3)] = s[8+9] = 17s$.

For ϰ = 2, ƴ = 4 ∃ ʒ = 0 in M such that

$8 = N_{qdb}(2,4) \leq N_{qdb}(2, 4 \otimes_6 \text{neut}(0)) = N_{qdb}(2,0) = 8$ then

$8 = N_{qdb}(2, 4 \otimes_6 \text{neut}(0)) \leq s[N_{qdb}(2,0) + N_{qdb}(0,4)] = s[8+16] = 24s$.

For ϰ = 3, ƴ = 4 ∃ ʒ = 0 in M such that

$10 = N_{qdb}(3,4) \le N_{qdb}(3, 4 \otimes_6 \text{ neut}(0)) = N_{qdb}(3,0) = 18$ then

$18 = N_{qdb}(3, 4 \otimes_6 \text{ neut}(0)) \le s[N_{qdb}(3,0) + N_{qdb}(0,4)] = s[18+16] = 34s$.

Similarly, for $\varkappa = 0$, $y = 2 \ \exists \ z = 4$, for $\varkappa = 0$, $y = 4 \ \exists \ z = 2$, for $\varkappa = 3$, $y = 2 \ \exists \ z = 0$, for $\varkappa = 4$, $y = 2 \ \exists \ z = 0$ and for $\varkappa = 4$, $y = 3 \ \exists \ z = 0$ such that

$N_{qdb}(\varkappa, y) \le N_{qdb}(\varkappa, y \lozenge \text{neut}(z))$ then $N_{qdb}(\varkappa, y \lozenge \text{neut}(z)) \le s[N_{qdb}(\varkappa, z) + N_{qdb}(z, y)]$.

See for $\varkappa = 0$, $y = 3$, $N_{qdb}(0,3) \nleq N_{qdb}(0, 3 \otimes_6 \text{neut}(z))$ for any element $z \in M/\{\varkappa, y\}$

so, we will not see $N_{qdb}(\varkappa, y \lozenge \text{neut}(z)) \le s[N_{qdb}(\varkappa, z) + N_{qdb}(z, y)]$.

Hence $((M, \lozenge), N_{qdb})$ is a NT-qdb-MS.

Example 2.5. Let M be any infinite set, $\mathcal{P}(M)$ be the power set of M with binary operation $\lozenge = \cup$ then $(\mathcal{P}(M), \cup)$ is a NTS where neut(J) = J and anti(J) = J, for all $J \in \mathcal{P}(M)$.

Define a map $N_{qdb}: \mathcal{P}(M) \times \mathcal{P}(M) \to [0, \infty)$ such that

$N_{qdb}(J, K) = |n(J)-n(K)|^2 + |n(J)|^2$ where n(J) denotes the cardinality of J.

(Nqdb1.) $N_{qdb}(J, K) = N_{qdb}(K, J) = 0$

implies that $|n(J)-n(K)|^2 + |n(J)|^2 = |n(J)-n(K)|^2 + |n(K)|^2 = 0$

i.e, $|n(J)|^2 = |n(K)|^2 = 0$

or $n(J) = n(K) = 0$ implies $J = K = \emptyset$.

(Nqdb2.) For any sets $J \neq \emptyset$, $K \neq \emptyset$ in $\mathcal{P}(M)$ there exist set $\emptyset$ in $\mathcal{P}(M)$ such that

$$N_{qdb}(J, K) = N_{qdb}(J, K \lozenge \text{neut}(\emptyset)) = N_{qdb}(J, K)$$

then $|n(J)-n(K)|^2 + |n(J)|^2 = N_{qdb}(J, K \lozenge \text{neut}(\emptyset)) \le s[N_{qdb}(J, \emptyset) + N_{qdb}(\emptyset, K)]$

$= s[\,|n(J)-n(\emptyset)|^2 + |n(J)|^2 + |n(\emptyset)-n(K)|^2 + |n(\emptyset)|^2]$

$= s[\,2|n(J)|^2 + |n(K)|^2]$

and for $\varkappa = J \neq \emptyset$, $y = K = \emptyset$, we have

$2|n(J)|^2 = N_{qdb}(J, \emptyset) \le N_{qdb}(J, \emptyset \cup Ł) = N_{qdb}(J, Ł)$

$= |n(J)-n(Ł)|^2 + |n(J)|^2$ for any $Ł \in \mathcal{P}(M) \setminus \{J, K\}$ with $n(Ł) \ge 2n(J)$

implies

$N_{qdb}(J, Ł) = N_{qdb}(J, \emptyset \cup Ł) \le N_{qdb}(J, Ł) + N_{qdb}(Ł, \emptyset)$

Also, $|n(J)|^2 = N_{qdb}(\emptyset, J) \le N_{qdb}(\emptyset, J \cup Ł) = |n(J \cup Ł)|^2$ for any $Ł \in \mathcal{P}(M) \setminus \{J, \emptyset\}$ with $n(Ł) \ge n(J)$.

implies $|n(J \cup Ł)|^2 = N_{qdb}(\emptyset, J \cup Ł)$

$\le s[N_{qdb}(\emptyset, Ł) + N_{qdb}(Ł, J)]$

$= s[\,|n(Ł)|^2 + |n(Ł)-n(J)|^2 + |n(Ł)|^2\,]$ for s = 2.

Hence, $(\mathcal{P}(M), \cup)$ is a NT-qdb-MS.

Remark 2.5. If M is finite set then it is also NT-qdb-MS.

Since for J = M itself, $|n(M)|^2 = N_{qdb}(\emptyset, M) \le N_{qdb}(\emptyset, M \cup Ł) = |n(M \cup Ł)|^2$

for any $Ł \in \mathcal{P}(M) \setminus \{M, \emptyset\}$ with $n(Ł) = n(M)-1$.

implies $| n(M \cup Ł) |^2 = N_{qdb} (\emptyset, M \cup Ł)$

$\leq s[N_{qdb} (\emptyset, Ł)+ N_{qdb} (Ł, M)]$

$= s[| n(Ł) |^2 + | n(Ł)-n(M) |^2 + | n(Ł) |^2]$ for $s = 2$.

Let ((M, ◊), N_{qdb}) be NT-qdb-MS and *G* = (V, E) is a reflexive digraph where the vertex set V(G) = M and the set E(G) of its edges contains no parallel edges. By G^{-1} means that the graph obtained from *G* by reversing the direction of edges. By disregarding the direction of edges of G, we acquire $\tilde{G}$, the undirected graph from *G*. So, $E(\tilde{G}) = E(G) \cup E(G^{-1})$. Literally, it will be better appropriate for us to approach $\tilde{G}$ as a digraph for which the set of its edges is symmetric.

Definition 2.4. Let ((M, ◊), N_{qdb}) be NT-qdb-MS with graph G then

(1.) A sequence $\{\varkappa_n\}$ in ((M, ◊), N_{qdb}) is said to be **0-Convergent** to $\varkappa \in M$ if for each $\varepsilon > 0$, there exist a positive integer $n_0 > 0$ such that $N_{qdb}(\varkappa_n, \varkappa) < \varepsilon$ and $N_{qdb}(\varkappa, \varkappa_n) < \varepsilon$ for all $n \geq n_0$.

(2.) The sequence $\{\varkappa_n\}$ in ((M, ◊), N_{qdb}) is said to be **0-Cauchy** if for each $\varepsilon>0$, there exist a positive integer $n_0 > 0$ such that $N_{qdb}(\varkappa_n, \varkappa_m) < \varepsilon$ and $N_{qdb}(\varkappa_m, \varkappa_n) < \varepsilon$ for all $n, m \geq n_0$.

(3.) ((M, ◊), N_{qdb}) is said to be **complete** if every **0-Cauchy sequence** $\{\varkappa_n\}$ in M converges to a point y in M.

(4.) Mapping f : M→ M is said to be **continuous** at $\varkappa \in M$, if for each $\varepsilon>0$, there exist a $\delta > 0$ such that whenever $N_{qdb}(\varkappa, ʒ) < \delta$ and $N_{qdb}(ʒ, \varkappa) < \delta$ implies $N_{qdb}(f\varkappa, fʒ) < \varepsilon$ and $N_{qdb}(fʒ, f\varkappa) < \varepsilon$ for all $ʒ \in M$.

Sequentially continuous if whenever $\varkappa_n \to \varkappa$ then $f\varkappa_n \to f\varkappa$ as $n \to \infty$

i.e, if $\lim_{n\to\infty} N_{qdb}(\varkappa_n, \varkappa) = 0$ and $\lim_{n\to\infty} N_{qdb}(\varkappa, \varkappa_n) = 0$

then $\lim_{n\to\infty} N_{qdb}(f\varkappa_n, f\varkappa) = 0$ and $\lim_{n\to\infty} N_{qdb}(f\varkappa, f\varkappa_n) = 0$

(5.) A mapping f: M → M is called **G-continuous** if given $\varkappa \in M$ and a sequence $(\varkappa_n)_{n\in N}$, $\varkappa_n \to \varkappa$ $as\ n \to \infty$ and $(\varkappa_n, \varkappa_{n+1}) \in E(G)$ for n∈N imply $f\varkappa_n \to f\varkappa\ as\ n \to \infty$.

Remark 2.6. Here, a convergent sequence in NT-qdb-MS may not be Cauchy sequence and need not necessary limit of the sequence is unique. Also, a constant sequence need not be convergent. For instance, we can see in Example 2.4, the sequence {2,2,2,2………} is not a convergent sequence. In fact, it is not a Cauchy sequence.

3. Main results

In this component, we shall obtain some fixed point results in context of complete NT-qdb-MS by proving Lemma 3.1, Lemma 3.2. and present an example in the support of obtained results.

Lemma 3.1: Let $\{\varkappa_n\}$ be a 0-convergent sequence converges to ʒ in ((M, ◊), N_{qdb}) be NT-qdb-MS and $N_{qdb}(\varkappa_n, \varkappa_m) \leq N_{qdb}(\varkappa_n, \varkappa_m \lozenge neut(ʒ))$ for all n, m then $\{\varkappa_n\}$ is a 0- Cauchy sequence in NT-qdb-MS.

Proof. Since $\{\varkappa_n\}$ be a 0-convergent sequence and it converges to ʒ in M. Therefore, for given $\varepsilon > 0$ there exist a positive integer k > 0 such that

$$N_{qdb}(\varkappa_n, ʒ) < \varepsilon/2s \text{ and } N_{qdb}(ʒ, \varkappa_n) < \varepsilon/2s \quad \text{for all } n \geq k$$

By given assumption and triangle inequality (NTqdb2.), for all n, m≥ k, we have

$$N_{qdb}(\varkappa_n, \varkappa_m) \leq N_{qdb}(\varkappa_n, \varkappa_m \lozenge neut(z)) \leq s\,[N_{qdb}(\varkappa_n, ʒ)+ N_{qdb}(ʒ, \varkappa_m)] < s[\varepsilon/2s + \varepsilon/2s] = \varepsilon$$

i.e, $N_{qdb}(\varkappa_n, \varkappa_m) < \varepsilon$

Similarly, $N_{qdb}(\varkappa_m, \varkappa_n) < \varepsilon$

Thus, $\{\varkappa_n\}$ is a 0-Cauchy sequence in NT-qdb-MS.

Lemma 3.2. Let $\{\varkappa_n\}$ be a 0-convergent sequence in NT-qdb-MS say $((M, \Diamond), N_{qdb})$, let it converges to $\varkappa$ and y in M. Assume that $N_{qdb}(\varkappa, y) \leq N_{qdb}(\varkappa, y \Diamond neut(\varkappa_k))$ and $N_{qdb}(y, \varkappa) \leq N_{qdb}(y, \varkappa \Diamond neut(\varkappa_k))$ for any k∈N then limit of the sequence $\{\varkappa_n\}$ is unique.

Proof: By the assumption,

$$N_{qdb}(\varkappa, y) \leq N_{qdb}(\varkappa, y \Diamond neut(\varkappa_m))$$

and by (NTqdb2)

$$N_{qdb}(\varkappa, y \Diamond neut(\varkappa_m)) \leq s\,[N_{qdb}(\varkappa, \varkappa_m) + N_{qdb}(\varkappa_m, y)]$$

Since $\{\varkappa_n\}$ is a 0-convergent sequence and converges to $\varkappa$ and y, so right hand side of above equation tends to zero as $m \to \infty$ that is, we have

$$N_{qdb}(\varkappa, y) = 0.$$

Similarly, we can show that

$$N_{qdb}(y, \varkappa) = 0$$

Hence, by (NTqdb1), $\varkappa = y$ which completes the proof.

Theorem 3.1. Let $((M, \Diamond), N_{qdb})$ be a complete NT-qdb-MS with graph G and coefficient $s \geq 1$. Let $T: M \to M$ be a G-continuous mapping satisfying

$$N_{qdb}(T\varkappa, Ty) \leq \varphi\,(N_{qdb}(\varkappa, y)) \quad \text{for all } \varkappa, y \in E(\tilde{G}) \tag{1}$$

where φ is a comparison function, with the following properties:

(a) for a set $O(\varkappa) = \{\varkappa, T\varkappa, T^2\varkappa, T^3\varkappa\ldots\}$, assume that $(T^n\varkappa, T^m\varkappa) \in E(\tilde{G})$ for all n, m and

$$N_{qdb}(T^n\varkappa, T^m\varkappa) \leq N_{qdb}(T^n\varkappa, T^m\varkappa \Diamond neut(v)) \text{ for any } v \in M \text{ and for all n, m.}$$

(b) If sequence $\{\varkappa_n\}$ be converges to $\varkappa$ and y in M then $N_{qdb}(\varkappa, y) \leq N_{qdb}(\varkappa, y \Diamond neut(\varkappa_k))$ and $N_{qdb}(y, \varkappa) \leq N_{qdb}(y, \varkappa \Diamond neut(\varkappa_k))$ for any $k \in N$.

In addition, if $\varkappa$ and $\varkappa^*$ are two fixed points with $(\varkappa, \varkappa^*) \in E(\tilde{G})$ then T has a unique fixed point.

Proof. Take $\varkappa_0 \in M$ be any arbitrary point but it's fixed. Define a iterative sequence in M as follows:

$$\varkappa_n = T\varkappa_{n-1} \quad \text{where } n = 1,2,3,4\ldots\ldots\ldots$$

$$\text{i.e,} \quad \varkappa_1 = T\varkappa_0,\ \varkappa_2 = T\varkappa_1,\ \varkappa_3 = T\varkappa_2 \ \ldots..$$

If we assume that $\varkappa_{n+1} = \varkappa_n$ for some $n \in Z^+$, then it follows that $\varkappa_n = \varkappa_{n+1} = T\varkappa_n$. So, $\varkappa_n$ is fixed point and proof is finished. Therefore, we assume that

$$\varkappa_n \neq \varkappa_{n+1} \quad \text{for each } n \in Z^+.$$

We claim that $\{\varkappa_n\}$ is a 0-Cauchy sequence in M. Now, for $\varkappa = \varkappa_n$, $y = \varkappa_{n+1}$ with assumption (a), contractive condition (1) becomes,

$$N_{qdb}(\varkappa_n, \varkappa_{n+1}) = N_{qdb}(T\varkappa_{n-1}, T\varkappa_n) \leq \varphi\,(N_{qdb}(\varkappa_{n-1}, \varkappa_n))$$

and $$N_{qdb}(\varkappa_{n-1}, \varkappa_n) = N_{qdb}(T\varkappa_{n-2}, T\varkappa_{n-1}) \leq \varphi\,(N_{qdb}(\varkappa_{n-2}, \varkappa_{n-1}))$$

By assumption of φ,

$$\varphi\,(N_{qdb}(\varkappa_{n-1}, \varkappa_n)) \leq \varphi^2\,(N_{qdb}(\varkappa_{n-2}, \varkappa_{n-1}))$$

Computing repeatedly in this way, we obtain

$$\begin{aligned} N_{qdb}(\varkappa_n, \varkappa_{n+1}) &\leq \varphi\,(N_{qdb}(\varkappa_{n-1}, \varkappa_n) \\ &\leq \varphi^2\,(N_{qdb}(\varkappa_{n-2}, \varkappa_{n-1})) \\ &\leq \varphi^3\,(N_{qdb}(\varkappa_{n-3}, \varkappa_{n-2})) \end{aligned}$$

……………….

$\leq \varphi^n(N_{qdb}(\varkappa_0\,,\ \varkappa_1))$ (2)

Proceeding in similar way, we can obtain

$N_{qdb}(\varkappa_{n+1}\,,\ \varkappa_n)\ \leq\ \varphi^n(N_{qdb}(\varkappa_1\,,\ \varkappa_0))$ (3)

If $N_{qdb}(\varkappa_0\,,\ \varkappa_1) = 0$ and $N_{qdb}(\varkappa_1\,,\ \varkappa_0) = 0$ then $\varkappa_0 = T\varkappa_0$ i.e., $\varkappa_0$ is a fixed point. Therefore, assume that $N_{qdb}(\varkappa_0\,,\ \varkappa_1) > 0$ and $N_{qdb}(\varkappa_1\,,\ \varkappa_0) > 0$

To prove $\{\varkappa_n\}$ is a 0-Cauchy sequence, consider m $>$ n and using (a), (Nqdb2.)

$N_{qdb}(\varkappa_n\,,\ \varkappa_m)\ \leq\ s\,N_{qdb}(\varkappa_n\,,\ \varkappa_{n+1}) +\ s^2 N_{qdb}(\varkappa_{n+1}\,,\ \varkappa_{n+2})\ +\ s^3 N_{qdb}(\varkappa_{n+2}\,,\ \varkappa_{n+3}) +\ldots$ (4)

Using (2), (4) becomes

$N_{qdb}(\varkappa_n\,,\ \varkappa_m)\ \leq\ s\ \varphi^n(N_{qdb}(\varkappa_0\,,\ \varkappa_1)) +\ s^2\varphi^{n+1}(N_{qdb}(\varkappa_0\,,\ \varkappa_1)) + s^3\ \varphi^{n+2}(N_{qdb}(\varkappa_0\,,\ \varkappa_1)) +\ldots$

By the definition of function φ and letting n, m $\to \infty$, we have

$$\lim_{n,m\to\infty} N_{qdb}(\varkappa_n\,,\ \varkappa_m) = 0$$

Similarly, we can show that,

$$\lim_{n,m\to\infty} N_{qdb}(\varkappa_m\,,\ \varkappa_n) = 0$$

which shows that $\{\varkappa_n\}$ is a 0-Cauchy sequence in M. Since M is complete NT- qdb- MS, there exist p$\in$ M such that $\varkappa_n \to$ p as n$\to\infty$. Now, here we will show that p is a fixed point in M.

As $\varkappa_n \to$ p as n$\to\infty$ and using G- continuity of T, it follows that

$$\lim_{n\to\infty} T\varkappa_n = Tp$$

and we can write above equation as

$$\lim_{n\to\infty} \varkappa_{n+1} = Tp$$

Thus, p is a fixed point in M by using assumption (b) and Lemma 3.2.

Now, we want to show that p is a unique fixed point. For this, suppose p^* be another fixed point.

Consider, $N_{qdb}(p,\ p^*) = N_{qdb}(Tp,\ Tp^*)\ \leq\ \varphi\ (N_{qdb}(p,\ p^*))$ by using (1).

By assumption of φ, above inequality implies that $N_{qdb}(p,\ p^*) =0$, also $N_{qdb}(\ p^*, p) =0$ by following same process as we have done above. Hence by (NTqdb1.) p = p^*.

Corollary 3.1: Let ((M, ◊), N_{qdb}) be a complete Neutrosophic quasi-dislocated-b-metric space with coefficient s$\geq$ 1. Let T: M$\to$ M be a G-continuous mapping satisfying

$$N_{qdb}(T\varkappa, T\,y) \leq\ \alpha\ N_{qdb}(\varkappa, y) \text{ for all } \varkappa,\ y\ \in\ E(\tilde{G})$$

where $\alpha \in (0,1)$; with the following properties:

(a) for a set O($\varkappa$) = {$\varkappa$, T$\varkappa$, $T^2\varkappa$, $T^3\varkappa$…}, assume that ($T^n\varkappa$, $T^m\varkappa$) $\in$ E($\tilde{G}$) for all n, m and

$N_{qdb}(T^n\varkappa,\ T^m\varkappa) \leq\ N_{qdb}(T^n\varkappa,\ T^m\varkappa \Diamond \text{neut}(v))$ for any v $\in$ M and for all n, m.

(b) If above sequence $\{\varkappa_n\}$ be converges to $\varkappa$ and y in M and suppose

$N_{qdb}(\varkappa, y) \leq N_{qdb}(\varkappa, y \Diamond \text{neut}(\varkappa_k))$ and $N_{qdb}(y, \varkappa) \leq\ N_{qdb}(y, \varkappa \Diamond \text{neut}(\varkappa_k))$ for any k $\in$ N.

In addition, if $\varkappa$ and $\varkappa^*$ are two fixed points with ($\varkappa$, $\varkappa^*$) $\in$ E($\tilde{G}$) then T has a unique fixed point.

Theorem 3.2: Let ((M, ◊), N_{qdb}) be a complete NT-qdb-MS with graph G (need not be reflexive) and coefficient s$\geq$ 1. Let T: M$\to$ M be a G-continuous mapping satisfying

$$N_{qdb}(T\varkappa, Ty) \leq \mu[N_{qdb}(\varkappa, T\varkappa)) + N_{qdb}(y, Ty)] \quad \text{for all } \varkappa, y \in E(\tilde{G}) \qquad (5)$$

where $0 < \mu < \frac{1}{s+1}$, with the following properties:

(a) for a sequence $\varkappa_{n+1} = T\varkappa_n$, n$\in N$ and $\varkappa_1 = \varkappa \in$M, assume that $(\varkappa_n, \varkappa_m) \in E(\tilde{G})$ for all n, m$\in$N and $N_{qdb}(\varkappa_n, \varkappa_m) \leq N_{qdb}(\varkappa_n, \varkappa_m \lozenge neut(\varkappa_k))$ for all n, m, k except some first finite few terms.

(b) If the above sequence $\{\varkappa_n\}$ converges to $\varkappa$ and y in M then $N_{qdb}(\varkappa, y) \leq N_{qdb}(\varkappa, y \lozenge neut(\varkappa_k))$ and

$N_{qdb}(y, \varkappa) \leq N_{qdb}(y, \varkappa \lozenge neut(\varkappa_k))$ for any k $\in N$.

Furthermore, if $\varkappa$ and $\varkappa^*$ are two fixed points with $(\varkappa, \varkappa^*) \in E(\tilde{G})$ then T has a unique fixed point.

Proof. Let $\varkappa \in$M be any arbitrary point but it's fixed. Construct an iteration sequence in M as follows:

$\varkappa$, $T\varkappa$, $T^2\varkappa$, $T^3\varkappa$ .....

i.e, $\varkappa = \varkappa_1$ and $\varkappa_{n+1} = T^n\varkappa = T\varkappa_n$ where n$\in N$.

If $\varkappa_{n+1} = \varkappa_n$ for some n $\in Z^+$, then it follows that $\varkappa_n = \varkappa_{n+1} = T\varkappa_n$ then $\varkappa_n$ is fixed point which completes the proof. Therefore, we suppose that $\varkappa_n \neq \varkappa_{n+1}$ for all n.

For $\varkappa = \varkappa$, $y = T\varkappa$ and using (a), (5) becomes

$N_{qdb}(T\varkappa, T^2\varkappa) \leq \mu[N_{qdb}(\varkappa, T\varkappa) + N_{qdb}(T\varkappa, T^2\varkappa)]$

i.e, $(1-\mu)\ N_{qdb}(T\varkappa, T^2\varkappa) \leq \mu\ N_{qdb}(\varkappa, T\varkappa)$

or $N_{qdb}(T\varkappa, T^2\varkappa) \leq \frac{\mu}{1-\mu} N_{qdb}(\varkappa, T\varkappa)$.

Again, for $\varkappa = T\varkappa$, $y = T^2\varkappa$, (5) becomes

$N_{qdb}(T^2\varkappa, T^3\varkappa) \leq \mu[N_{qdb}(T\varkappa, T^2\varkappa) + N_{qdb}(T^2\varkappa, T^3\varkappa)]$

$N_{qdb}(T^2\varkappa, T^3\varkappa) \leq \frac{\mu}{1-\mu} N_{qdb}(T\varkappa, T^2\varkappa)\ ) \leq (\frac{\mu}{1-\mu})^2\ N_{qdb}(\varkappa, T\varkappa) = t^2 N_{qdb}(\varkappa, T\varkappa)$ where $t = \frac{\mu}{1-\mu}$.

Continuing in this way, we obtain

$$N_{qdb}(T^n\varkappa, T^{n+1}\varkappa) \leq t^n N_{qdb}(\varkappa, T\varkappa) \qquad (6)$$
$$\to 0 \text{ as } n \to \infty, \text{ since } t \in [0,1).$$

That is,

$$\lim_{n\to\infty} N_{qdb}(T^n\varkappa, T^{n+1}\varkappa) = 0 \quad \text{i.e,} \quad \lim_{n\to\infty} N_{qdb}(\varkappa_{n+1}, \varkappa_{n+2}) = 0.$$

Next, it is desirable to show that $\{\varkappa_n\}$ is a 0-Cauchy sequence in M. For this, we take m, n are positive integers such that m $>$ n then by using definition of NT-qdb-MS and condition (a), (6), we have

$$\begin{aligned} N_{qdb}(\varkappa_n, \varkappa_m) &\leq s\,[N_{qdb}(\varkappa_n, \varkappa_{n+1}) + N_{qdb}(\varkappa_{n+1}, \varkappa_m)] \\ &\leq s\,N_{qdb}(\varkappa_n, \varkappa_{+1}) + s^2 N_{qdb}(\varkappa_{n+1}, \varkappa_{n+2}) + s^3 N_{qdb}(\varkappa_{n+2}, \varkappa_{n+3}) + \ldots + s^{m-n} N_{qdb}(\varkappa_{m-1}, \varkappa_m) \\ &\leq s\,t^{n-1} N_{qdb}(\varkappa, T\varkappa) + s^2 t^n N_{qdb}(\varkappa, T\varkappa) + s^3 t^{n+1} N_{qdb}(\varkappa, T\varkappa) + \ldots + s^{m-n} t^{m-2} N_{qdb}(\varkappa, T\varkappa) \\ &\leq \frac{st^{n-1}}{1-st} N_{qdb}(\varkappa, T\varkappa). \end{aligned}$$

Taking limit as n, m$\to \infty$, we have

$$\lim_{n,m\to\infty} N_{qdb}(\varkappa_n, \varkappa_m) = 0.$$

In similar way, we can obtain

$$\operatorname{Lim}_{n,m\to\infty} N_{qdb}(\varkappa_m, \varkappa_n) = 0.$$

Thus, $\{\varkappa_n\}$ is a 0-cauchy sequence in complete NT-qdb-MS. By definition of NT-qdb-MS, there exist a $\mathfrak{z}$ ∈M such that $\varkappa_n \to \mathfrak{z}$ as n → ∞ that is

$$\lim_{n\to\infty} N_{qdb}(\varkappa_n, \mathfrak{z}) = 0 = \lim_{n\to\infty} N_{qdb}(\mathfrak{z}, \varkappa_n) .$$

By using G- continuity of T and (a), it follow that

$$T(\lim_{n\to\infty} \varkappa_{n+1}) = T\mathfrak{z} = \lim_{n\to\infty} T^{n+1}\varkappa.$$

Now,

$$N_{qdb}(T\mathfrak{z}, \mathfrak{z}) \le s[N_{qdb}(T \mathfrak{z}, T^{n+1}\varkappa) + N_{qdb}(T^{n+1}\varkappa, \mathfrak{z})]$$

$$\to 0 \text{ as } n \to \infty$$

implies that $T\mathfrak{z} = \mathfrak{z}$ and also $N_{qdb}(\mathfrak{z}, \mathfrak{z}) = 0$.

Hence, $\mathfrak{z}$ is a fixed point of T. For uniqueness, we assume that $\mathfrak{z}*$ be another fixed point of T different from z. Clearly, also $N_{qdb}(z*, z*) = 0$ by the above observation.

By the contractive condition (5), we have

$$N_{qdb}(\mathfrak{z}, \mathfrak{z}*) = N_{qdb}(T \mathfrak{z}, T\mathfrak{z}*) \le \mu [N_{qdb}(\mathfrak{z},T \mathfrak{z}) +N_{qdb}(\mathfrak{z}*, T\mathfrak{z}*)]$$

$$\le \mu [N_{qdb} \mathfrak{z}, \mathfrak{z}) +N_{qdb}(\mathfrak{z}*, \mathfrak{z}*)]$$

implies that $N_{qdb}(\mathfrak{z}, \mathfrak{z}*) = 0$ since $N_{qdb}(\mathfrak{z}, \mathfrak{z}) = 0$ and $N_{qdb}(\mathfrak{z}*, \mathfrak{z}*) = 0$ and which gives that $\mathfrak{z} = \mathfrak{z}*$.

Thus, z is the unique fixed point of T.

Example 3.1: Let M = {0,2,3,4} be a NTG together binary operation ◊ = $\otimes_6$ in (Z_6, ×). Neutrosophic triplet are:

(0,0,0), (2,4,2), (3,3,3) (4,4,4) where ($\varkappa$, y, $\mathfrak{z}$) denote here, $\varkappa \in M$ be any element, y= neut($\varkappa$) and $\mathfrak{z}$ = anti ($\varkappa$). Now, we define a map N_{qdb} : M×M→ [0,∞) such that

$N_{qdb}(0,0) = 0$, $N_{qdb}(0,2) = 4$, $N_{qdb}(0,3) = 9$, $N_{qdb}(0,4) = 16$,

$N_{qdb}(2,2) = 4$, $N_{qdb}(3,3) = 9$, $N_{qdb}(4,4) = 16$, $N_{qdb}(2,0) = 8$,

$N_{qdb}(3,0) = 18$, $N_{qdb}(4,0) = 32$, $N_{qdb}(2,3) = 5$, $N_{qdb}(3,2) = 10$,

$N_{qdb}(2,4) = 8$, $N_{qdb}(4,2) = 20$, $N_{qdb}(3,4) = 10$, $N_{qdb}(4,3) = 17$.

Hence, ((M, ◊), N_{qdb}) is a NT-qdb-MS with coefficient s≥ 1, as we have proved in Example 2.4. and also it is complete since {0,0,0,0…..} is only Cauchy sequence which converges in M.

A mapping T: M→M defined as T0 =0, T2=3, T3=0 and T4=0 and a graph G = (V, E) defined as V(G) = M and E(G)={(0,0), (3,3), (4,4), (0, 3), (0, 4), (2,4), (3,4), (3,0), (4,0), (3,2), (4,2), (4,3)}.

Now, we have the following cases to identify the contractive condition for all $\varkappa$, $y \in E(\tilde{G})$ as:

Case I: for $\varkappa$ = 0 and y =3

$0= N_{qdb}(T0, T3) \le \mu[N_{qdb}(0, T0)) + N_{qdb}(3, T3)] = 18 \mu.$

Case II: for $\varkappa$= 0 and y = 4

$0= N_{qdb}(T0, T4) \le \mu[N_{qdb}(0, T0)) + N_{qdb}(4, T4)] = 32 \mu.$

Case III: for $\varkappa$ =2 and y = 4

18= $N_{qdb}(T2, T4) \leq \mu[N_{qdb}(2, T2)) + N_{qdb}(4, T4)] = 37\ \mu$.

Case IV: for $\varkappa$ =3 and y = 4

0= $N_{qdb}(T3, T4) \leq \mu[N_{qdb}(3, T3)) + N_{qdb}(4, T4)] = 50\ \mu$.

Case V: for $\varkappa$ =3 and y = 0

0= $N_{qdb}(T3, T0) \leq \mu[N_{qdb}(3, T3)) + N_{qdb}(0, T0)] = 18\mu$.

Case VI: for $\varkappa$ = 4 and y = 0

0= $N_{qdb}(T4, T0) \leq \mu[N_{qdb}(4, T4)) + N_{qdb}(0, T0)] = 32\ \mu$.

Case VII: for $\varkappa$=3 and y = 2

9= $N_{qdb}(T3, T2) \leq \mu[N_{qdb}(3, T3)) + N_{qdb}(2, T2)] = 21\ \mu$

Case VIII: for $\varkappa$ = 4 and y = 2

9= $N_{qdb}(T4, T2) \leq \mu[N_{qdb}(4, T4)) + N_{qdb}(2, T2)] = 37\ \mu$.

Case IX: for $\varkappa$ = 4 and y = 3

0= $N_{qdb}(T4, T3) \leq \mu[N_{qdb}(4, T4)) + N_{qdb}(3, T3)] = 50\ \mu$.

Similarly, for $\varkappa$ = 0 and y = 0, for $\varkappa$=3 and y = 3 and for $\varkappa$ = 4 and y = 4

This shows that contractive condition of Theorem 3.2 is satisfied for $\mu = \frac{18}{37}$ for all $\varkappa$, $y \in$ E($\tilde{G}$). Thus all the conditions of Theorem 3.2 are satisfied. Therefore, 0 is a unique fixed point.

Conclusion: In this article, we reformulated the definition of NTMS and NTPMS and presented counter examples for dissimilarity of definitions with examples in [18,19]. Also, we established a new space NT-qdb-MS which is the generalization of the spaces established by Sahin et al. in [18,19]. Also, we studied some of their properties. Concept of NT-qdb-MS is absolutely different from classical quasi-dislocated b- metric space. The significance of NT-qdb-MS is that it provides a different space structure to those structures which are not quasi-dislocated-b- metric space with respect to some functions that not satisfy triangular inequality for all $\varkappa$, y, z. Finally, we proved generalize version of Banach fixed theorem and Kannan fixed point theorem in the framework of NT-qdb-MS with an example.

Open problems:

1. Can we also prove name theorems such as Chatterjee, Sehgal, Hardy and Rogers, Ciric, Meir-Keeler, F-contraction fixed point theorems in NTMS, NTPMS or NT-qdb-MS?
2. Can we extend the Theorem 3.1 and 3.2 for more than one mapping?

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