On Neutrosophic Vague Graphs

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Abstract: In this work, the new concept of neutrosophic vague graphs are introduced form the ideas of neutrosophic vague sets. Moreover, some remarkable properties of strong neutrosophic vague graphs, complete neutrosophic vague graphs and self-complementary neutrosophic vague graphs are investigated and the proposed concepts are described with suitable examples.

Keywords: Neutrosophic vague graphs, Complete neutrosophic vague graph, Strong neutrosophic vague graph.

1. Introduction

Initially, vague set theory was first investigated by Gau and Buehrer [30] which is an extension of fuzzy set theory. Vague sets are regarded as a special case of context-dependent fuzzy sets. In order to handle the indeterminate and inconsistent information, the neutrosophic set is introduced by the author Smarandache and studied extensively about neutrosophic set [14] - [37] and it receives applications in many fields. In neutrosophic set, the indeterminacy value is quantified explicitly and truth-membership, indeterminacy membership, and false-membership are defined completely independent, if the sum of these values lies between 0 and 3. The new developments of neutrosophic theory are extensively studied in [1] - [6]. Molodtsoy [28] firstly introduced the soft set theory as a general mathematical tool to with uncertainty and vagueness. Akram [9] established the certain notions including strong neutrosophic soft graphs and complete neutrosophic soft graphs. The authors [7] first introduce the concept of neutrosophic vague soft expert set which is a combination of neutrosophic vague set and soft expert set to improve the reasonability of decision making in reality. Neutrosophic vague set theory are introduced in [8]. The operations on single valued neutrosophic graphs are studied in [11]. Further, intuitionistic neutrosophic soft set and graphs are developed in [13]. Now, the domination in vague graphs and its is application are discussed in [16]. Intuitionistic neutrosophic soft set are studied in [18]. Interval valued neutrosophic graphs are developed by the author Broumi [22,23,25]. Interval neutrosophic vague sets are initiated in [31]. Motivation of the aforementioned works, we introduced the concept of neutrosophic vague graphs and strong neutrosophic vague graphs. This is a new developed theory which is the combination of neutrosophic graphs and vague graphs. Here the sum of Truth, Intermediate and False membership value lies between 0 and 2 since the truth and false membership are dependent variables. Here the complement of neutrosophic vague graphs is again neutrosophic.
vague graphs. This development theory will be applied in Operation Research, Social network problems. Particularly, fake profile is one of the big problems of social networks. Now, it has become easier to create a fake profile. People often use fake profile to insult, harass someone, involve in unsocial activities, etc. This model can be reformulated in the abstract form to be applied in neutrosophic vague graphs. The major contribution of this work as follows:

- Newly introduced neutrosophic vague graphs, neutrosophic vague subgraphs, constant neutrosophic vague graphs with examples.
- Further we presented some remarkable properties of strong neutrosophic vague graphs with suitable examples.

2 Preliminaries

**Definition 2.1** [10] A vague set $A$ on a non empty set $X$ is a pair $(T_A, F_A)$, where $T_A: X \to [0,1]$ and $F_A: X \to [0,1]$ are true membership and false membership functions, respectively, such that
\[
0 \leq T_A(r) + F_A(r) \leq 1 \text{ for any } r \in X.
\]
Let $X$ and $Y$ be two non-empty sets. A vague relation $R$ of $X$ to $Y$ is a vague set $R$ on $X \times Y$ that is $R = (T_R, F_R)$, where $T_R: X \times Y \to [0,1], F_R: X \times Y \to [0,1]$ which satisfies the condition:
\[
0 \leq T_R(r,s) + F_R(r,s) \leq 1 \text{ for any } r \in X.
\]
Let $G = (R, S)$ be a graph. A pair $G = (J, K)$ is named as a vague graph on $G'$ or a vague graph where $J = (T_J, F_J)$ is a vague set on $R$ and $K = (T_K, F_K)$ is a vague set on $S \subseteq R \times R$ such that for each $rs \in S$,
\[
T_K(rs) \leq (T_J(r) \land T_J(s)) \& (T_F(r) \lor F_J(s)).
\]

**Definition 2.2** [9] A Neutrosophic set $A \in B$, (i.e) $A \subseteq C$ if $\forall r \in X, T_A(r) \leq T_B(r), I_A(r) \geq I_B(r)$ and $F_A(r) \geq F_B(r)$.

**Definition 2.3** [12, 26, 30] Let $X$ be a space of points (objects), with a generic elements in $X$ known by $r$. A single valued neutrosophic set (SVNS) $A$ in $X$ is characterized by truth-membership function $T_A(r)$, indeterminacy-membership function $I_A(r)$ and falsity-membership function $F_A(r)$.

For each point $r$ in $X$, $T_A(r), F_A(r), I_A(r) \in [0,1]$.

**Definition 2.4** [19, 20] A neutrosophic graph is represented as a pair $G' = (V, E)$ where

(i) $V = \{r_1, r_2, \ldots, r_n\}$ such that $T_1 = R \rightarrow [0,1], I_1 = R \rightarrow [0,1]$ and $F_1 = R \rightarrow [0,1]$ denote the degree of truth-membership function, indeterminacy function and falsity-membership function, respectively and
\[
0 \leq T_A(r) + I_A(r) + F_A(r) \leq 3
\]

(ii) $S \subseteq R \times R$ where $T_2 = S \rightarrow [0,1], I_2 = S \rightarrow [0,1]$ and $F_2 = S \rightarrow [0,1]$ are such that
\[
T_2(rs) \leq (T_2(r) \land T_2(s)),
\]
\[
I_2(rs) \geq (I_2(r) \lor I_2(s))
\]
\[
F_2(rs) \geq (F_2(r) \lor F_2(s))
\]
and $0 \leq T_2(rs) + I_2(rs) + F_2(rs) \leq 3, \forall rs \in R$

**Definition 2.5** [8] A neutrosophic vague set $A_{NV}$ (NVS in short) on the universe of discourse $X$ written as
\[ A_{NV} = \{(r, T_{NV}(r), \tilde{T}_{NV}(r), \tilde{I}_{NV}(r), \tilde{F}_{NV}(r)), r \in X\} \]
whose truth-membership, indeterminacy membership and falsity-membership function is defined as
\[ T_{NV}(r) = [\tilde{T}^-(r), \tilde{T}^+(r)], \tilde{I}_{NV}(r) = [\tilde{I}^-(r), \tilde{I}^+(r)], [\tilde{F}^-(r), \tilde{F}^+(r)], \]
where \( T^+(r) = 1 - F^-(r), F^+(r) = 1 - T^-(r) \), and \( 0 \leq T^-(r) + I^-(r) + F^-(r) \leq 2 \).

**Definition 2.6** [8] The complement of NVS \( A_{NV} \) is denoted by \( A_{NV}^c \) and it is represented by
\[ T_{NV}^c(r) = [1 - T^+(r), 1 - T^-(r)], \]
\[ \tilde{T}_{NV}^c(r) = [1 - \tilde{T}^+(r), 1 - \tilde{T}^-(r)], \]
\[ \tilde{I}_{NV}^c(r) = [1 - \tilde{I}^+(r), 1 - \tilde{I}^-(r)], \]
\[ \tilde{F}_{NV}^c(r) = [1 - \tilde{F}^+(r), 1 - \tilde{F}^-(r)]. \]

**Definition 2.7** [8] Let \( A_{NV} \) and \( B_{NV} \) be two NVSs of the universe \( U \). If for all \( r_i \in U \),
\[ T_{NV}(r_i) = \tilde{T}_{NV}(r_i), \tilde{I}_{NV}(r_i) = \tilde{I}_{NV}(r_i), \tilde{F}_{NV}(r_i) = \tilde{F}_{NV}(r_i) \]
then the NVS \( A_{NV} \) are included by \( B_{NV} \), denoted by \( A_{NV} \subseteq B_{NV} \) where \( 1 \leq i \leq n \).

**Definition 2.8** [7] The union of two NVSs \( A_{NV} \) and \( B_{NV} \) is a NVSs, \( C_{NV} \), written as \( C_{NV} = A_{NV} \cup B_{NV} \),
whose truth membership function, indeterminacy-membership function and false-membership function are related to those of \( A_{NV} \) and \( B_{NV} \) by
\[ T_{CNV}(x) = \max(T_{ANV}(x), T_{BNV}(x)), \min(T_{ANV}(x), T_{BNV}(x)), \]
\[ \tilde{T}_{CNV}(x) = \min(\tilde{T}_{ANV}(x), \tilde{T}_{BNV}(x)), \max(\tilde{T}_{ANV}(x), \tilde{T}_{BNV}(x)), \]
\[ \tilde{I}_{CNV}(x) = \min(I_{ANV}(x), I_{BNV}(x)), \max(I_{ANV}(x), I_{BNV}(x)), \]
\[ \tilde{F}_{CNV}(x) = \min(F_{ANV}(x), F_{BNV}(x)), \max(F_{ANV}(x), F_{BNV}(x)), \]
\[ F_{CNV}(x) = \max(F_{ANV}(x), F_{BNV}(x)), \min(F_{ANV}(x), F_{BNV}(x)), \]
\[ F_{CNV}(x) = \max(F_{ANV}(x), F_{BNV}(x)), \min(F_{ANV}(x), F_{BNV}(x)), \]
\[ F_{CNV}(x) = \max(F_{ANV}(x), F_{BNV}(x)), \min(F_{ANV}(x), F_{BNV}(x)). \]

**3 NEUTROSOFT VAGUE GRAPH**

**Definition 3.1** Let \( G^* = (R, S) \) be a graph. A pair \( G = (J, K) \) is named as a neutrosophic vague graph (NVG) on \( G^* \) or a neutrosophic graph where \( J = (T_J, \tilde{T}_J, \tilde{I}_J, \tilde{F}_J) \) is a neutrosophic vague set on \( R \) and \( K = (T_K, \tilde{T}_K, \tilde{I}_K, \tilde{F}_K) \) is a neutrosophic vague set \( S \subseteq R \times R \) such that
\[ T_J : R \rightarrow [0, 1], I_J : R \rightarrow [0, 1], F_J : R \rightarrow [0, 1] \]
which satisfies the condition \( F_J^- = [1 - T_J^-] \)
\[ T_J^+ : R \rightarrow [0, 1], I_J^+ : R \rightarrow [0, 1], F_J^+ : R \rightarrow [0, 1] \]
which satisfies the condition \( F_J^+ = [1 - T_J^-] \) indicates the degree of truth membership function, indeterminacy membership and falsity membership of the element \( r_i \in R \), and
\[ 0 \leq T_J^-(r_i) + I_J^-(r_i) + F_J^-(r_i) \leq 2 \]
\[ 0 \leq T_J^+(r_i) + I_J^+(r_i) + F_J^+(r_i) \leq 2 \]
(2) \( S \subseteq R \times R \) where
\[ T_K : R \times R \rightarrow [0, 1], I_K : R \times R \rightarrow [0, 1], F_K : R \times R \rightarrow [0, 1] \]
\[ T_K^+ : R \times R \rightarrow [0, 1], I_K^+ : R \times R \rightarrow [0, 1], F_K^+ : R \times R \rightarrow [0, 1] \]
indicates the degree of truth membership function, indeterminacy membership and falsity membership of the element \( r_i, r_j \in S \) respectively and such that

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\[ 0 \leq T_K(r_i) + I_K(r_i) + F_K(r_i) \leq 2. \]

such that

\[ T_K(rs) \leq \{ T_J^-(r) \land T_J^-(s) \} \]
\[ I_K(rs) \leq \{ I_J^-(r) \land I_J^-(s) \} \]
\[ F_K(rs) \leq \{ F_J^-(r) \lor F_J^-(s) \}. \]

similarly

\[ T_K^+(rs) \leq \{ T_J^+(r) \land T_J^+(s) \} \]
\[ I_K^+(rs) \leq \{ I_J^+(r) \land I_J^+(s) \} \]
\[ F_K^+(rs) \leq \{ F_J^+(r) \lor F_J^+(s) \}. \]

**Example 3.2** A neutrosophic vague graph \( G = (J, K) \) such that \( J = \{a, b, c\} \) and \( K = \{ab, bc, ca\} \) indicated by

\[ a = T[0.5, 0.6], I[0.4, 0.3], F[0.4, 0.5], b = T[0.4, 0.6], I[0.7, 0.3], F[0.4, 0.6], \]
\[ c = T[0.4, 0.4], I[0.5, 0.3], F[0.6, 0.6] \]
\[ a^- = (0.5, 0.4, 0.4), b^- = (0.4, 0.7, 0.4), c^- = (0.4, 0.5, 0.6) \]
\[ a^+ = (0.6, 0.3, 0.5), b^+ = (0.6, 0.3, 0.6), c^+ = (0.4, 0.3, 0.6) \]

\[ \text{Figure 1} \]

**Definition 3.3** A neutrosophic vague graph \( H = (J'(r), K'(r)) \) is meant to be a neutrosophic vague subgraph of the NVG \( G = (J, K) \) if \( J'(r) \subseteq J(r) \) and \( K'(rs) \subseteq K(rs) \) in other words, if

\[ T_J^-(r) \leq T_J^-(r) \]
\[ I_J^-(r) \leq I_J^-(r) \]
\[ F_J^-(r) \leq F_J^-(r) \]
$F^+_J(r) \geq F^-_J(r) \forall r \in R$

and

$\frac{J^+}{J^+}(rs) \leq T^+_K(rs)$

$I^+_K(rs) \leq I^-_K(rs)$

$F^+_K(rs) \geq F^-_K(rs) \forall (rs) \in S$.

**Example 3.4** A neutrosophic vague graph $G = (J, K)$ in Figure (1)

In this case, $r$ and $s$ are known to be neighbours and $(rs)$ is incident at $r$ and $s$ also.

**Definition 3.5** The two vertices are said to be adjacent in a neutrosophic vague graph $G = (J, K)$ if

$T^+_K(rs) = \{T^+_J(r) \land T^+_J(s)\}$

$I^+_K(rs) = \{I^+_J(r) \land I^+_J(s)\}$

$F^+_K(rs) = \{F^+_J(r) \lor F^+_J(s)\}$ 

and

$T^-_K(rs) = \{T^-_J(r) \land T^-_J(s)\}$

$I^-_K(rs) = \{I^-_J(r) \land I^-_J(s)\}$

$F^-_K(rs) = \{F^-_J(r) \lor F^-_J(s)\}$

In this case, $r$ and $s$ are known to be neighbours and $(rs)$ is incident at $r$ and $s$ also.

**Definition 3.6** A path $\rho$ in a NVG $G = (J, K)$ is a sequence of distinct vertices $r_0, r_1, \ldots, r_n$ such that

$T^+_K(r_{i-1}, r_i) > 0, \quad I^+_K(r_{i-1}, r_i) > 0, \quad F^+_K(r_{i-1}, r_i) > 0, \quad I^-_K(r_{i-1}, r_i) > 0, \quad I^-_K(r_{i-1}, r_i) > 0, \quad F^-_K(r_{i-1}, r_i) > 0,$

for $0 \leq i \leq 1$, here $n \leq 1$ is called the length of the path $\rho$. A single node or single vertex $r_i$ may all consider as a path.

**Definition 3.7** A neutrosophic vague graph $G = (J, K)$ is said to be connected if every pair of vertices has at least on neutrosophic vague path between them otherwise it is disconnected.

**Definition 3.8** A vertex $r_i \in R$ of neutrosophic vague graph $G = (J, K)$ called as a pendent vertex if there is no effective edge incident at $x_i$.
**Definition 3.9** A vertex in a neutrosophic vague graph \( G = (J,K) \) having exactly one neighbours is called a isolated vertex. otherwise, it is called non-isolated vertex. An edge in a neutrosophic vague graph incident with a isolated vertex is called a isolated edge other words it is called non-isolated edge. A vertex in a neutrosophic vague graph adjacent to the isolated vertex is called a support of the pendant edge.

**Example 3.10** A neutrosophic vague graph \( G = (J,K) \) in figure (3)

![Figure 3](image)

**Figure 3**

**NEUTROSOPHIC VAGUE GRAPH**

In figure (3), the neutrosophic vague vertex \( b \) is an pendent vertex.

**Definition 3.11** Let \( G = (J,K) \) be a neutrosophic vague graph. Then the degree of a vertex \( r \in G \) is a sum of degree truth membership, sum of indeterminacy membership and sum of falsity membership of all those edges which are incident on vertex \( r \) represented by

\[
\begin{align*}
\text{degree} & = d_T(r) + d_I(r) + d_F(r) \\
& = \sum_{s \neq r} (T_{rk}^-(rs), F_{rk}^-), \\
& = \sum_{s \neq r} (T_{rk}^+(rs), F_{rk}^+),
\end{align*}
\]

where

- \( d_T^-(r) = \sum_{s \neq r} T_{rk}^- \)
- \( d_I^-(r) = \sum_{s \neq r} I_{rk}^- \)
- \( d_F^-(r) = \sum_{s \neq r} F_{rk}^- \)
- \( d_T^+(r) = \sum_{s \neq r} T_{rk}^+ \)
- \( d_I^+(r) = \sum_{s \neq r} I_{rk}^+ \)
- \( d_F^+(r) = \sum_{s \neq r} F_{rk}^+ \)

**Example 3.12** A neutrosophic vague graph \( G = (J,K) \) in figure (1), we have the degree of each vertex as follows

\[
\begin{align*}
d_T^-(a) &= (0.6,0.7,0.9), d_T^+(b) = (0.7,0.8,1.3), d_T^-(c) = (0.7,0.7,1.0), \\
d_F^-(a) &= (0.9,0.6,1.0), d_F^+(b) = (0.9,0.6,1.0), d_F^-(c) = (0.8,0.6,1.0)
\end{align*}
\]

**Definition 3.13** A neutrosophic vague graph \( G = (J,K) \) is called constant if degree of each vertex is \( A = (A_1, A_2, A_3) \) that is \( d(x) = (A_1, A_2, A_3) \) for all \( x \in V \).

**Example 3.14** Consider a neutrosophic vague graph \( G = (J,K) \) in figure (4) defined by

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\[ \tilde{a} = T[0.5,0.6], I[0.6,0.4], F[0.4,0.5], \tilde{b} = T[0.4,0.4], I[0.4,0.6], F[0.6,0.6], \]
\[ \tilde{c} = T[0.4,0.6], I[0.7,0.3], F[0.4,0.6], \tilde{d} = T[0.6,0.4], I[0.3,0.7], F[0.6,0.4] \]
\[ a^- = (0.5,0.6,0.4), b^- = (0.4,0.4,0.6), c^- = (0.4,0.7,0.4), d^- = (0.6,0.3,0.6) \]
\[ a^+ = (0.6,0.4,0.5), b^+ = (0.4,0.6,0.6), c^+ = (0.6,0.3,0.6), d^+ = (0.4,0.7,0.4) \]

**Figure 4**

**CONSTANT NEUTROSOFTIC VAGUE GRAPH**

Clearly as it is seen in figure(4) \(G\) is constant neutrosophic vague graph since the degree of \( (\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}) \)
and \( \tilde{d} = (0.6,0.6,1,2) \).

**Definition 3.15** The complement of neutrosophic vague graph \( G = (J, K) \) on \( G^c \) is a neutrosophic vague graph \( G^c \) on \( G^c \) where

- \( J^c(r) = J(r) \)
- \( T_{j}^\pm(r) = T_{j}^\pm(r), I_{j}^\pm(r) = I_{j}^\pm(r), F_{j}^\pm(r) = F_{j}^\pm(r) \) for all \( r \in R \).
- \( T_{k}^\pm(rs) = (T_{j}^\pm(r) \wedge T_{j}^\pm(s)) - T_{k}^\pm(rs) \)
  \[ I_{k}^\pm(rs) = (I_{j}^\pm(r) \wedge I_{j}^\pm(s)) - I_{k}^\pm(rs) \]
  \[ F_{k}^\pm(rs) = (F_{j}^\pm(r) \vee F_{j}^\pm(s)) - F_{k}^\pm(rs) \] for all \( (rs) \in S \)
- \( T_{k}^\pm(rs) = (T_{j}^\pm(r) \wedge T_{j}^\pm(s)) - T_{k}^\pm(rs) \)
  \[ I_{k}^\pm(rs) = (I_{j}^\pm(r) \wedge I_{j}^\pm(s)) - I_{k}^\pm(rs) \]
  \[ F_{k}^\pm(rs) = (F_{j}^\pm(r) \vee F_{j}^\pm(s)) - F_{k}^\pm(rs) \] for all \( (rs) \in S \)

**4 Strong Neutrosophic Vague Graphs**

**Definition 4.1** A neutrosophic vague graph \( G = (J, K) \) of \( G^c = (R, S) \) is named as a strong neutrosophic vague graph if

\[ T_{k}^\pm(rs) = (T_{j}^\pm(r) \wedge T_{j}^\pm(s)) \]
\[ I_{k}^\pm(rs) = (I_{j}^\pm(r) \wedge I_{j}^\pm(s)) \]
\[ F_{k}^\pm(rs) = (F_{j}^\pm(r) \vee F_{j}^\pm(s)) \] and
\[ T_{k}^\pm(rs) = (T_{j}^\pm(r) \wedge T_{j}^\pm(s)) \]
\[ I_{k}^\pm(rs) = (I_{j}^\pm(r) \wedge I_{j}^\pm(s)) \]
\[ F_{k}^\pm(rs) = (F_{j}^\pm(r) \vee F_{j}^\pm(s)) \] for all \( (rs) \in S \)
Example 4.2 A neutrosophic vague graph $G = (J,K)$ such that $J = \{a,b,c\}$ and $K = \{ab,bc,ca\}$ defined by $\hat{a} = T[0.3,0.4], I[0.4,0.6], F[0.6,0.7], \hat{b} = T[0.6,0.4], I[0.6,0.7], F[0.6,0.4], \hat{c} = T[0.7,0.7], I[0.5,0.6], F[0.3,0.3]$

![Figure 5]

**STRONG NEUTROSOPHIC VAGUE GRAPH**

Remark 4.3 If $G = (J,K)$ is a neutrosophic vague graph on $G^*$ then from above definition, it follow that $G^c$ is given by the neutrosophic vague graph $G^c = J^c, K^c$ on $G^*$ where

- $(J^c)^c(r) = I(r)$
- $(T_J^c)^c(r) = T_J^c(r), (I_J^c)^c(r) = I_J^c(r), (F_J^c)^c(r) = F_J^c(r)$ for all $r \in R$
- $(T_K^c)^c(rs) = (T_K^c(r) \land T_K^c(s)) - T_K^c(rs)$
  - $(I_K^c)^c(rs) = (I_K^c(r) \land I_K^c(s)) - I_K^c(rs)$
  - $(F_K^c)^c(rs) = (F_K^c(r) \lor F_K^c(s)) - F_K^c(rs)$ for all $(rs) \in S$
- $(T_K^c)^c(rs) = (T_K^c(r) \land T_K^c(s)) - T_K^c(rs)$
  - $(I_K^c)^c(rs) = (I_K^c(r) \land I_K^c(s)) - I_K^c(rs)$
  - $(F_K^c)^c(rs) = (F_K^c(r) \lor F_K^c(s)) - F_K^c(rs)$ for all $(rs) \in S$

for any neutrosophic vague graph $G, G^c$ is strong neutrosophic graph and $G \equiv G^c$

Definition 4.4 A strong neutrosophic graph $G$ is called self-complementery if $G \equiv G^c$ where $G^c$ is the complement of neutrosophic vague graph $G$

Example 4.5 A neutrosophic vague graph $G = (J,K)$ such that $J = \{a,b,c,d\}$ and $K = \{ab,bc,cd,da\}$ defined as follows: consider a neutrosophic vague graph $G$ as in figure(6)
Clearly, as it is seen in figure (6) $G \cong G^c$. Hence $G$ is self complementary.

**Proposition 4.6** Let $G = (J, K)$ be a strong neutrosophic vague graph if

$$T_K\left( rs \right) = \left\{ T_J^-(r) \land T_J^-(s) \right\}$$

$$I_K\left( rs \right) = \left\{ I_J^-(r) \land I_J^-(s) \right\}$$

$$F_K\left( rs \right) = \left\{ F_J^+(r) \lor F_J^+(s) \right\}$$

Then $G$ is self complementary.

Proof. Let $G = (J, K)$ be a strong neutrosophic vague graph such that

$$\overline{T}_K\left( rs \right) = \frac{1}{2} \min\left[ \overline{T}_J(r), \overline{T}_J(s) \right]$$

$$\overline{I}_K\left( rs \right) = \frac{1}{2} \min\left[ \overline{I}_J(r), \overline{I}_J(s) \right]$$

$$\overline{F}_K\left( rs \right) = \frac{1}{2} \max\left[ \overline{F}_J(r), \overline{F}_J(s) \right]$$
for all $rs \in J$ then $G \approx G^{rs}$ under the identity map $1: J \to J$. Hence $G$ is self complementary

**Proposition 4.7** Let $G$ be a self complementary neutrosophic vague graph then

$$
\sum_{rs} \tilde{T}_K(rs) = \frac{1}{2} \sum_{rs} \min\{\tilde{T}_I(r), \tilde{T}_I(s)\}
$$

$$
\sum_{rs} I_K(rs) = \frac{1}{2} \sum_{rs} \min\{I_I(r), I_I(s)\}
$$

$$
\sum_{rs} \bar{F}_K(rs) = \frac{1}{2} \sum_{rs} \max\{\bar{F}_I(r), \bar{F}_I(s)\}
$$

Proof. If $G$ be an self complementary neutrosophic vague graph then there exist an isomorphism $f: I_1 \to I_2$ satisfy

$$
\tilde{T}_{K_1}(f(r)) = \tilde{T}_{I_1}(f(r)) = \tilde{T}_{I_1}(r)
$$

$$
I_{K_1}(f(r)) = I_{I_1}(f(r)) = I_{I_1}(r)
$$

$$
\bar{F}_{K_1}(f(r)) = \bar{F}_{I_1}(f(r)) = \bar{F}_{I_1}(r)
$$

and

$$
\tilde{T}_{K_1}(f(r), f(s)) = \tilde{T}_{K_1}(f(r), f(s)) = \tilde{T}_{K_1}(rs)
$$

$$
I_{K_1}(f(r), f(s)) = I_{K_1}(f(r), f(s)) = I_{K_1}(rs)
$$

$$
\bar{F}_{K_1}(f(r), f(s)) = \bar{F}_{K_1}(f(r), f(s)) = \bar{F}_{K_1}(rs)
$$

we have $\tilde{T}_{K_1}(f(r), f(s)) = \min(\tilde{T}_{I_1}(r), \tilde{T}_{I_1}(s)) - \tilde{T}_{K_1}(f(r), f(s))$. $\tilde{T}_{K_1}(rs) = \min(\tilde{T}_{I_1}(r), \tilde{T}_{I_1}(s)) - \tilde{T}_{K_1}(f(r), f(s))$. $\tilde{T}_{K_1}(rs) = \sum_{rs} \tilde{T}_{K_1}(rs) + \sum_{rs} \bar{T}_{K_1}(rs) = \sum_{rs} \min(\tilde{T}_{I_1}(r), \tilde{T}_{I_1}(s))$.

Similarly, $\tilde{T}_{K_1}(rs) = \sum_{rs} \tilde{T}_{K_1}(rs)$

$$
\sum_{rs} \tilde{T}_{K_1}(rs) + \sum_{rs} \bar{T}_{K_1}(rs) = \sum_{rs} \min(\tilde{T}_{I_1}(r), \tilde{T}_{I_1}(s))
$$

$$
2 \sum_{rs} \tilde{T}_{K_1}(rs) = \sum_{rs} \min(\tilde{T}_{I_1}(r), \tilde{T}_{I_1}(s))
$$

$$
2 \sum_{rs} I_{K_1}(rs) = \sum_{rs} \min(I_{I_1}(r), I_{I_1}(s))
$$

$$
2 \sum_{rs} \bar{F}_{K_1}(rs) = \sum_{rs} \max(\bar{F}_{I_1}(r), \bar{F}_{I_1}(s))
$$

from the equation of the proposition (4.8) holds.

**Proposition 4.8** Let $G_1$ and $G_2$ be strong neutrosophic vague graph $G_1 \approx G_2$(isomorphism)

Proof. Assume that $G_1$ and $G_2$ are isomorphic there exist a bijective map $f: I_1 \to I_2$ satisfying,

$$
\tilde{T}_{I_1}(r) = \tilde{T}_{I_2}(f(r)), I_{I_1}(r) = I_{I_2}(f(r)), \bar{F}_{I_1}(r) = \bar{F}_{I_2}(f(r)), \forall r \in I_1
$$

and

$$
\tilde{T}_{K_1}(rs) = \tilde{T}_{K_2}(f(r), f(s))
$$

$$
I_{K_1}(rs) = I_{K_2}(f(r), f(s))
$$

$$
\bar{F}_{K_1}(rs) = \bar{F}_{K_2}(f(r), f(s)) \forall rs \in K_1
$$

by definition (4.3) we have

---

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\[ T_{K_1}(rs) = \min(T_{I_1}(r), T_{I_1}(s)) - T_{K_1}(rs) \]
\[ = \min(T_{I_2}(f(r), T_{I_2}(f(s))) - T_{K_2}(f(r)f(s)) \]
\[ = T_{K_2}(f(r)f(s)) \]
\[ I_{K_1}(rs) = \min(I_{I_1}(r), I_{I_1}(s)) - I_{K_1}(rs) \]
\[ = \min(I_{I_2}(f(r), I_{I_2}(f(s))) - I_{K_2}(f(r)f(s)) \]
\[ = I_{K_2}(f(r)f(s)) \]
\[ F_{K_1}(rs) = \max(F_{I_1}(r), F_{I_1}(s)) - F_{K_1}(rs) \]
\[ = \max(F_{I_2}(f(r), F_{I_2}(f(s))) - F_{K_2}(f(r)f(s)) \]
\[ = F_{K_2}(f(r)f(s)) \]

Hence \( G_1^c \approx G_2^c \) for all \((rs) \in K_1\)

**Definition 4.9** A neutrosophic vague graph \( G = (J, K) \) is complete if

\[
T_K^{-}(rs) = \{ T_J^{-}(r) \land T_J^{-}(s) \}
\]
\[
I_K^{-}(rs) = \{ I_J^{-}(r), I_J^{-}(s) \}
\]
\[
F_K^{-}(rs) = \{ F_J^{-}(r) \lor F_J^{-}(s) \}
\]

similarly,

\[
T_K^{+}(rs) = \{ T_J^{+}(r) \land T_J^{+}(s) \}
\]
\[
I_K^{+}(rs) = \{ I_J^{+}(r), I_J^{+}(s) \}
\]
\[
F_K^{+}(rs) = \{ F_J^{+}(r) \lor F_J^{+}(s) \}
\]

**Example 4.10** Consider a neutrosophic vague graph \( G = (J, K) \) such that \( J = \{a, b, c, d\} \) and \( K = \{ab, bc, cd, da\} \) defined by

Figure 7

COMPLETE NEUTROSOPHIC VAGUE GRAPH
Definition 4.11 The complement of neutrosophic vague graph \( G = (J, K) \) of \( G^* = (V, E) \) is a neutrosophic vague complete graph \( G = (J^c, K^c) \) on \( G^* = (R, S^c) \) where

1. \( J^c(r_i) = J(r_i) \)
2. \( T^c_j(r_i) = \overline{T_j(r_i)} \), \( I^c_j(r_i) = \overline{I_j(r_i)} \), \( F^c_j(r_i) = \overline{F_j(r_i)} \) for all \( r_i \in J \)
3. \( T^c_k(r_i, s_j) = (\overline{T_k(r_i) \land T_j(s_j)}) - \overline{T_k(r_i, s_j)} \)
   \[ \overline{I_k(r_i, s_j)} = (\overline{I_k(r_i) \land I_j(s_j)}) - \overline{I_k(r_i, s_j)} \]
   \[ \overline{F_k(r_i, s_j)} = (\overline{F_k(r_i) \lor F_j(s_j)}) - \overline{F_k(r_i, s_j)} \] for all \( (r_i, s_j) \in K \)

Proposition 4.12 The complement of complete neutrosophic vague graph with no edge. or if \( G \) is complete then \( G^c \) the edge is empty.

Proof. Let \( G = (J, K) \) be a complete neutrosophic vague graph so

\[ \overline{T_k(r_i, s_j)} = (\overline{T_k(r_i) \land T_j(s_j)}) \]
\[ \overline{I_k(r_i, s_j)} = (\overline{I_k(r_i) \land I_j(s_j)}) \]
\[ \overline{F_k(r_i, s_j)} = (\overline{F_k(r_i) \lor F_j(s_j)}) \]

Hence in \( G^c \). Now,

\[ \overline{T^c_k(r_i, s_j)} = (\overline{T^c_k(r_i) \land \overline{T^c_j(s_j)})} - \overline{T^c_k(r_i, s_j)} \]
\[ = (\overline{T_k(r_i) \land \overline{T_j(s_j)}) - (\overline{T_k(r_i) \land \overline{T_j(s_j)})} \lor i,j,\ldots,n \]
\[ = 0 \forall i,j,\ldots,n. \]

and

\[ \overline{I^c_k(r_i, s_j)} = (\overline{I_k(r_i) \land \overline{I_j(s_j)})} - \overline{I_k(r_i, s_j)} \]
\[ = (\overline{I_k(r_i) \land \overline{I_j(s_j)})} - (\overline{I_k(r_i) \land I_j(s_j)}) \lor i,j,\ldots,n \]
\[ = 0 \forall i,j,\ldots,n. \]

Similarly \( \overline{F^c_k(r_i, s_j)} = 0 \). Thus, \( (\overline{T^c_k(r_i, s_j)}, \overline{I^c_k(r_i, s_j)}, \overline{F^c_k(r_i, s_j)}) = (0,0,0) \)

Hence, the edge set of \( G^c \) is empty if \( G \) is a complete neutrosophic vague graph.

Conclusion and future directions:

This work dealt with the new concept of neutrosophic vague graphs. Moreover, some remarkable properties of strong neutrosophic vague graphs, complete neutrosophic vague graphs and self-complementary neutrosophic vague graphs have been investigated and the proposed concepts were described with suitable examples. Further we can extend to investigate the regular and isomorphic properties of the proposed graph. This can be applied to social network model and operations research.

References


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