On Some Properties of Neutrosophic Quadruple H_v-rings

Madeleine Al-Tahan[1] and Bijan Davvaz[2,*]
[1]Department of Mathematics, Lebanese International University, Bekaa, Lebanon; madeline.tahan@liu.edu.lb
[2]Department of Mathematics, Yazd University, Yazd, Iran; davvaz@yazd.ac.ir
*Correspondence: davvaz@yazd.ac.ir

Abstract. Hyperstructure theory, an 86 years old theory, has been of great interest for many algebraists where their researches were divided in to two categories: theory and applications. On the other hand, neutrosophic theory which is the study of neutralities, was introduced and developed by F. Smarandache in 1995 as an extension of dialectics. The purpose of this paper is to study some connections between the two theories: Neutrosophy and hyperstructures. In this regard, we define neutrosophic quadruple H_v-rings, neutrosophic quadruple H_v-subrings, and neutrosophic quadruple homomorphism and study their various properties.

Keywords: H_v-ring; neutrosophic quadruple number; neutrosophic quadruple H_v-ring; neutrosophic homomorphism.

---

1. Introduction

The concept of neutrosophic quadruple numbers was introduced by Smarandache [14] in 2015. Where he defined and presented some arithmetic operations of these numbers such as addition, subtraction, multiplication, and scalar multiplication. Later in 2017, Akinleye et al. [2] considered the set of neutrosophic quadruple numbers and defined some operations on it and discussed neutrosophic quadruple algebraic structures. A generalization of the latter work was done in 2016 where Agboola et al. [1] considered the set of neutrosophic quadruple numbers and defined some hyperoperations on it and discussed neutrosophic quadruple hyperstructures. For more details about neutrosophy and its applications, we refer to [3–7, 10, 13, 15, 16].

A generalization of hyperstructures, known as H_v-structures was introduced by T. Vougiouklis [19, 20]. We refer to [19, 20] for basic definitions and results on H_v-rings. Al Tahan and Davvaz in [3] discussed neutrosophic H_v-groups and studied their properties. In this work, we extend the results to H_v-rings and it is constructed as follows: after an Introduction, in

Section 2, we present some basic definitions about hyperstructures that are used throughout the paper. In Section 3, we define neutrosophic quadruple H_v-rings and provide some examples on it. In Section 4, we define neutrosophic quadruple H_v-subrings and neutrosophic quadruple homomorphism and study their properties.

2. Basic definitions about algebraic hyperstructures

In this section, we present some definitions and theorems related to hyperstructure theory that are used throughout the paper. (See [8, 9, 19].)

Let H be a non-empty set and $\mathcal{P}^*(H)$ the set of all non-empty subsets of H. Then, a mapping $\circ : H \times H \to \mathcal{P}^*(H)$ is called a *binary hyperoperation* on H. The couple $(H, \circ)$ is called a *hypergroupoid*. In this definition, if X and Y are two non-empty subsets of H and $h \in H$, then we define:

$$X \circ Y = \bigcup_{\substack{x \in X \\ y \in Y}} x \circ y,\ h \circ X = \{h\} \circ X \text{ and } X \circ h = X \circ \{h\}.$$

Definition 2.1. A hypergroupoid $(H, \circ)$ is called a:

(1) *semihypergroup* if for every $x, y, z \in H$, we have $x \circ (y \circ z) = (x \circ y) \circ z$;

(2) *quasi-hypergroup* if for every $x \in H$, $x \circ H = H = H \circ x$ (The latter condition is called the reproduction axiom);

(3) *hypergroup* if it is a semihypergroup and a quasi-hypergroup.

T. Vougiouklis [19, 20] introduced H_v-structures as a generalization of the well-known algebraic hyperstructures. The equality in some axioms of classical algebraic hyperstructures is replaced by non-empty intersection in H_v-structures. The majority of H_v-structures are applied in representation theory.

Definition 2.2. A hypergroupoid $(H, \circ)$ is called an H_v*-semigroup* if the weak associative axiom is satisfied. i.e., $(x \circ (y \circ z)) \cap ((x \circ y) \circ z) \neq \emptyset$ for all $x, y, z \in H$.

An element $0 \in H$ is called an *identity* if $x \in 0 \circ h \cap h \circ 0$ for all $h \in H$ and it is called a *scalar identity* if $h = 0 \circ h = h \circ 0$ for all $h \in H$. A scalar identity (if it exists) is unique. A hypergroupoid $(H, \circ)$ is called an H_v*-group* if it is a quasi-hypergroup and an H_v-semigroup. A non-empty subset M of an H_v-group $(H, \circ)$ is called H_v*-subgroup of* H if $(M, \circ)$ is an H_v-group.

Definition 2.3. Let R be a non-empty set and "+", "·" be hyperoperations. Then $(R, +, \cdot)$ is a *hyperring* if the following conditions hold. (1) $(R, +)$ is a hypergroup; (2) $(R, \cdot)$ is is a semihypergroup; (3) $\cdot$ is distributive with respect to $+$. And it is an H_v*-ring* if (1) $(R, +)$ is an H_v-group; (2) $(R, \cdot)$ is is an H_v-semigroup; (3) $\cdot$ is weak distributive with respect to $+$.

$(R,+,\cdot)$ is said to be commutative if $x+y=y+x$ and $x\cdot y=y\cdot x$ for all $x,y\in R$. An element $1\in R$ is called a *unit* if $x\in 1\cdot x\cap x\cdot 1$ for all $x\in R$ and it is called a *scalar unit* if $x=1\cdot x=x\cdot 1$ for all $x\in R$. If the scalar unit exists then it is unique. A subset M of an H_v-ring $(R,+,\cdot)$ is called an *H_v-subring* if $(M,+,\cdot)$ is an H_v-ring. To prove that $(M,+,\cdot)$ is an H_v-subring of $(M,+,\cdot)$, it suffices to show that $m+M=M+m=M$ and $M\cdot M\subseteq M$ for all $m\in M$.

Let $(R,+,\star)$ and $(R',+',\star')$ be two H_v-rings. Then $f:R\to R'$ is said to be *H_v-ring homomorphism* if $f(r+s)=f(r)+'f(s)$ and $f(r\star s)=f(r)\star' f(s)$ for all $r,s\in R$. $(R,+,\star)$ and $(S,+',\star')$ are called *isomorphic H_v-rings*, and written as $R\cong S$, if there exists a bijective homomorphism $f:R\to S$.

The concept of very thin hyperstructures was introduced and studied by Vougioklis [17,18].

An H_v-structure is called a *very thin H_v-structure*, denoted as VT-H_v-structure, if all hyperoperations are operations except one which has all hyperproducts singletons except only one. For example an H_v-ring $(H,\star,\circ)$ is said to be a VT-H_v-ring if there exists only one $(x,y)\in H^2$ with the property $|x\star y|>1$ or $|x\circ y|>1$.

3. Construction of neutrosophic quadruple H_v-rings

Symbolic (or Literal) neutrosophic theory is referring to the use of abstract symbols (i.e. the letters T, I, F, representing the neutrosophic components: truth, indeterminacy, and falsehood) in neutrosophics.

In [1,2], Agboola et al. and Akinleye et al. respectively based their study of neutrosophic quadruple algebraic structures (hyperstructures) on quadruple numbers based on the set of real numbers. In this section, we consider neutrosophic quadruple numbers based on a set instead of real or complex numbers and we use them to define neutrosophic quadruple H_v-rings.

Definition 3.1. [11] Let X be a nonempty set. A neutrosophic quadruple X-number is an ordered quadruple (a,bT,cI,dF) where $a,b,c,d\in X$ and T,I,F have their usual neutrosophic logical meanings.

The set of all neutrosophic quadruple X-numbers is denoted by $NQ(X)$, that is,

$$NQ(X)=\{(a,bT,cI,dF):a,b,c,d\in X\}.$$

With respect to the preference law $T<I<F$, we define the Absorbance Law for the multiplications of T, I, and F, in the sense that the bigger one absorbs the smaller one (or the big fish eats the small fish); for example:

$FT=TF=F$ (because F is bigger), $TT=T$ (T absorbs itself), $TI=IT=I$ (because I is bigger), (because F is bigger), and $FI=IF=I$ (because F is bigger).

Let $(R,+,\cdot)$ be an H_v-ring with zero "0" and unit "1"and define "$\oplus$" and "$\odot$" on $NQ(R)$ as follows:

$$(x_1, x_2T, x_3I, x_4F) \oplus (y_1, y_2T, y_3I, y_4F)$$
$$= \{(a, bT, cI, dF) : \ a \in x_1 + y_1, b \in x_2 + y_2, c \in x_3 + y_3, d \in x_4 + y_4\}.$$

and

$$(x_1, x_2T, x_3I, x_4F) \odot (y_1, y_2T, y_3I, y_4F)$$
$$= \{(a, bT, cI, dF) : \ a \in x_1 \cdot y_1, b \in x_1 \cdot y_2 \cup x_2 \cdot y_1 \cup x_2 \cdot y_2,$$
$$c \in x_1 \cdot y_3 \cup x_2 \cdot y_3 \cup x_3 \cdot y_1 \cup x_3 \cdot y_2 \cup x_3 \cdot y_3,$$
$$d \in x_1 \cdot y_4 \cup x_2 \cdot y_4 \cup x_3 \cdot y_4 \cup x_4 \cdot y_1 \cup x_4 \cdot y_2 \cup x_4 \cdot y_3 \cup x_4 \cdot y_4\}.$$

Throughout this section, $T < I < F$ and $(R,+,\cdot)$ is an H_v-ring with identity "0", unit "1", $0 + 0 = 0$, $1 \cdot 1 = 1$ and $x \cdot 0 = 0 \cdot x = 0$ for all $x \in R$ (i.e, 0 is an absorbing element).

Theorem 3.2. *[3] Let R be a set with $0 \in R$. Then $(NQ(R), \oplus)$ is an H_v-group (called neutrosophic H_v-group) with identity $\overline{0} = (0, 0T, 0I, 0F)$ if and only if $(R,+)$ is an H_v-group with identity "0" and $0 + 0 = 0$.*

Theorem 3.3. *[3] Let R be a set with $0 \in R$. Then $(NQ(R), \oplus)$ is a hypergroup (called neutrosophic hypergroup) with identity $\overline{0} = (0, 0T, 0I, 0F)$ if and only if $(R,+)$ is a hypergroup with identity "0" and $0 + 0 = 0$.*

In [1], Agboola et al. gave an example on a hypergroup of order 3 (Example 2.4) and said that it is a neutrosophic hypergroup which is an impossible case. We illustrate it by the following remark.

Remark 3.4. A neutrosophic H_v-group (hypergroup) $NQ(R) = \{(a, bT, cI, dF) : a, b, c, d \in R\}$ is either infinite or of order $|R|^4$ where $|R|$ is the number of elements in R in case R is finite. This is clear by using Theorem 3.2 and Theorem 3.3 respectively.

Theorem 3.5. *[3] Let R be a set with $0 \in R$. Then $(NQ(R), \oplus)$ is a commutative H_v-group with identity $\overline{0} = (0, 0T, 0I, 0F)$ if and only if $(R,+)$ is a commutative H_v-group with identity "0" and $0 + 0 = 0$.*

Proposition 3.6. *Let R be a set containing "0" and "1" with a hyperoperation "$\cdot$". Then $(NQ(R), \odot)$ is a quadruple H_v-semigroup with unit 1 if and only if $(R,\cdot)$ is an H_v-semigroup with unit $\overline{1} = (1, 0T, 0I, 0F)$.*

Proof. Let $(NQ(R), \odot)$ be a quadruple H_v-semigroup and let $a, b, c \in R$. Having $x = (a, 0T, 0I, 0F) \in NQ(R)$, $y = (b, 0T, 0I, 0F) \in NQ(R)$, $z = (c, 0T, 0I, 0F) \in NQ(R)$ and $(x \odot (y \odot z)) \cap ((x \odot y) \odot z) \neq \emptyset$ implies that $(a \cdot (b \cdot c)) \cap ((a \cdot b) \cdot c) \neq \emptyset$.

Let $(R,\cdot)$ be an H_v-semigroup and let $x, y, z \in NQ(R)$. Then there exist $x_i, y_i, z_i \in R$ with $i = 1, 2, 3, 4$ such that $x = (x_1, x_2T, x_3I, x_4F)$, $y = (y_1, y_2T, y_3I, y_4F)$ and $z =$

(z_1, z_2T, z_3I, z_4F). We have $(x_i \cdot (y_i \cdot z_i)) \cap (x_i \cdot y_i) \cdot z_i) \neq \emptyset$ for $i = 1, 2, 3, 4$. Applying the latter with some computations on $x \odot (y \odot z)$ and on $(x \odot y) \odot z$, we get $(x \odot (y \odot z)) \cap ((x \odot y) \odot z) \neq \emptyset$. □

Proposition 3.7. *R be a set containing "0" and "1" with a hyperoperation "$\cdot$". Then $(NQ(R), \odot)$ is a quadruple semihypergroup with $\overline{1} = (1, 0T, 0I, 0F)$ as unit if and only if $(R, \cdot)$ is a semihypergroup with 1 as unit.*

Proof. The proof is the same as that of Proposition 3.6 but instead of nonempty intersection, we have equality. □

Proposition 3.8. *Let $(NQ(R), \oplus, \odot)$ be an H_v-ring with zero "$\overline{0}$" and unit "$\overline{1}$". Then for all $a, b, c \in R$, we have:*

$$(a \cdot (b + c)) \cap ((a \cdot b) + (a \cdot c)) \neq \emptyset.$$

Proof. Let $a, b, c \in R$. Then $x = (a, 0T, 0I, 0F), y = (b, 0T, 0I, 0F), z = (c, 0T, 0I, 0F) \in NQ(R)$. Since $(x \odot (y \oplus z)) \cap ((x \odot y) \oplus (x \odot z)) \neq \emptyset$, it follows that $(a \cdot (b + c)) \cap ((a \cdot b) + (a \cdot c)) \neq \emptyset$. □

Proposition 3.9. *Let $(R, +, \cdot)$ be an H_v-ring with identity "0" and unit "1". Then for all $x, y, z \in NQ(R)$, we have:*

$$(x \odot (y \oplus z)) \cap ((x \odot y) \oplus (y \odot z)) \neq \emptyset.$$

Proof. Let $x = (x_1, x_2T, x_3I, x_4F), y = (y_1, y_2T, y_3I, y_4F), z = (z_1, z_2T, z_3I, z_4F) \in NQ(R)$. We have:

$x \odot (y \oplus z) = \{(t_1, t_2, t_3, t_4) : t_1 \in x_1 \cdot (y_1 + z_1), t_2 \in x_1 \cdot (y_2 + z_2) \cup x_2 \cdot (y_1 + z_1) \cup x_2 \cdot (y_2 + z_2),$
$t_3 \in x_1 \cdot (y_3 + z_3) \cup x_2 \cdot (y_3 + z_3) \cup x_3 \cdot (y_1 + z_1) \cup x_3 \cdot (y_2 + z_2) \cup x_3 \cdot (y_3 + z_3),$
$t_4 \in x_1 \cdot (y_4 + z_4) \cup x_2 \cdot (y_4 + z_4) \cup x_3 \cdot (y_4 + z_4) \cup x_4 \cdot (y_1 + z_1) \cup x_4 \cdot (y_2 + z_2) \cup x_4 \cdot (y_3 + z_3) \cup x_4 \cdot (y_4 + z_4)\}$.

On the other hand, we have:

$(x \odot y) \oplus (x \odot z) = \{s = (s_1, s_2T, s_3I, s_4F) : q = (q_1, q_2T, q_3I, q_4F) \in x \odot y, r = (r_1, r_2T, r_3I, r_4F) \in x \odot z, s_i \in q_i + r_i$ for $i = 1, 2, 3, 4\}$.

Having $q = (q_1, q_2T, q_3I, q_4F) \in x \cdot y$ and $r = (r_1, r_2T, r_3I, r_4F) \in x \cdot z$ implies that $q_1 \in x_1 \cdot y_1$, $q_2 \in x_1 \cdot y_2 \cup x_2 \cdot y_1 \cup x_2 \cdot y_2$, $q_3 \in x_1 \cdot y_3 \cup x_2 \cdot y_3 \cup x_3 \cdot y_1 \cup x_3 \cdot y_2 \cup x_3 \cdot y_3$, $q_4 \in x_1 \cdot y_4 \cup x_2 \cdot y_4 \cup x_3 \cdot y_4 \cup x_4 \cdot y_1 \cup x_4 \cdot y_2 \cup x_4 \cdot y_3 \cup x_4 \cdot y_4$, $r_1 \in x_1 \cdot z_1$, $r_2 \in x_1 \cdot z_2 \cup x_2 \cdot z_1 \cup x_2 \cdot z_2$, $r_3 \in x_1 \cdot z_3 \cup x_2 \cdot z_3 \cup x_3 \cdot z_1 \cup x_3 \cdot z_2 \cup x_3 \cdot z_3$ and $r_4 \in x_1 \cdot z_4 \cup x_2 \cdot z_4 \cup x_3 \cdot z_4 \cup x_4 \cdot z_1 \cup x_4 \cdot z_2 \cup x_4 \cdot z_3 \cup x_4 \cdot z_4$. Since $x_i \cdot (y_i + z_i) \cap (x_i \cdot y_i + x_i \cdot z_i) \neq \emptyset$ for $i = 1, 2, 3, 4$, it follows that $(x \odot (y \oplus z)) \cap ((x \odot y) \oplus (y \odot z)) \neq \emptyset$. □

Proposition 3.10. *Let $(NQ(R), \oplus, \odot)$ be an hyperring with zero "$\overline{0}$" and unit "$\overline{1}$". Then for all $a, b, c \in R$, we have:*

$$(a \cdot (b + c)) = ((a \cdot b) + (a \cdot c)).$$

Proof. The proof is the same as that of Proposition 3.8 but instead of nonempty intersection, we have equality. □

Proposition 3.11. *Let $(R,+,\cdot)$ be a hyperring with identity "0" and unit "1". Then for all $x,y,z \in NQ(R)$, we have:*

$$x \odot (y \oplus z) \subseteq (x \odot y) \oplus (y \odot z).$$

Proof. The proof is straightforward. □

Remark 3.12. The equality in Proposition 3.11 may not hold. We illustrate it by the following example.

Example 3.13. Let $R = \mathbb{Z}_2$ be the ring of integers under standard addition and multiplication modulo 2 and let $x = (1, 1T, 0I, 0F)$, $y = (0, 1T, 0I, 0F)$ and $z = (1, 0T, 0I, 0F)$. Having $x \odot (y \oplus z) = (1, 1T, 0I, 0F)$ and $(x \odot y) \oplus (x \odot z) = \{(1, 0T, 0I, 0F), (1, 1T, 0I, 0F)\}$ implies that $x \odot (y \oplus z) \neq (x \odot y) \oplus (y \odot z)$.

In the proof of Theorem 2.11, [1], the proof of distributivity contains a gap. Our example, Example 3.13 can be used as an illustration.

Notation 1. *Let $(R,+,\cdot)$ be an H_v-ring with "0" and "1" as zero and unit respectively satisfying $0+0=0$, $1\cdot 1=1$ and $0\cdot x = x\cdot 0 = 0$ for all $x \in R$. Then $(NQ(R),\oplus,\odot)$ is called neutrosophic quadruple H_v-ring.*

Notation 2. *Let $(NQ(R),\oplus,\odot)$ be a hyperring. Then we call it a neutrosophic quadruple hyperring.*

Remark 3.14. Let $(R,+,\cdot)$ be a hyperring. Then $(NQ(R),\oplus,\odot)$ may fail to be a hyperring. One can easily see that $(NQ(R),\oplus,\odot)$ in Example 3.13 is not a hyperring (as the distributivity law does not hold.).

Theorem 3.15. *Let R be any set with two hyperoperations "+" and "·". Then $(NQ(R),\oplus,\odot)$ is a neutrosophic H_v-ring with zero and unit $\overline{0} = (0, 0T, 0I, 0F)$ and $\overline{1} = (1, 0T, 0I, 0F)$ respectively if and only if $(R,+,\cdot)$ is an H_v-ring with zero and unit "0" and "1" respectively.*

Proof. The proof follows from Theorem 3.2, Proposition 3.6, Proposition 3.8 and Proposition 3.9. □

Corollary 3.16. *Let $(R,+,\cdot)$ be an H_v-ring containing an identity and absorbing element 0 and a unit 1 with the property that $0+0=0$, $1\cdot 1=1$. Then we can construct infinite number of neutrosophic quadruple H_v-rings.*

Proof. Theorem 3.15 asserts that $(NQ(R), \oplus, \odot)$ is an H_v-ring with zero and unit $\overline{0} = (0, 0T, 0I, 0F)$ and $\overline{1} = (1, 0T, 0I, 0F)$ respectively. Applying Theorem 3.15 on $(NQ(R), \oplus, \odot)$, we get $NQ(NQ(R))$ is a quadruple H_v-ring. Continuing on this pattern, we can construct infinite number of quadruple H_v-rings. Particularly, we have $NQ(NQ(\dots NQ(\dots(R))\dots)$ is a quadruple H_v-ring. □

Proposition 3.17. *Let $(R, +, \cdot)$ be any ring with unit. Then $(NQ(R), \oplus, \odot)$ is a neutrosophic H_v-ring. Moreover, $(NQ(R), \oplus, \odot)$ is not a ring.*

Proof. We can consider the ring $(R, +, \cdot)$ as an H_v-ring with zero and unit. Theorem 3.15 asserts that $(NQ(R), \oplus, \odot)$ is a neutrosophic H_v-ring.
Having $x = (1, 0T, 0I, 0F), y = (1, T, 0I, 0F) \in NQ(R)$ implies that $x \odot y \subseteq NQ(R)$. It is clear that $(1, 0T, 0I, 0F), (1, T, 0I, 0F) \in x \odot y$. Thus, $|x \odot y| > 1$. □

Example 3.18. Let $R_1 = \{0, 1\}$ and define $(R_1, +_1, \cdot_1)$ as follows:

$+_1$	0	1
0	0	1
1	1	R_1

$\cdot_1$	0	1
0	0	0
1	0	1

Then $(NQ(R_1), \oplus, \odot)$ is a quadruple H_v-ring with 16 elements.

By setting

$$\begin{array}{lll} \overline{1} = (1, 0T, 0I, 0F), & a_6 = (0, 0T, I, F), & a_{11} = (1, 0T, 0I, F), \\ a_2 = (0, T, 0I, 0F), & a_7 = (0, T, I, 0F), & a_{12} = (1, T, 0I, F), \\ a_3 = (0, 0T, I, 0F), & a_8 = (0, T, 0I, F), & a_{13} = (1, 0T, I, F), \\ a_4 = (0, 0T, 0I, F), & a_9 = (1, T, 0I, 0F), & a_{14} = (1, T, I, 0F), \\ a_5 = (0, T, I, F), & a_{10} = (1, 0T, I, 0F), & a_{15} = (1, T, I, F), \end{array}$$

we present some of the results for $a_i \oplus a_j = a_j \oplus a_i$, $i, j = 1, 2, \ldots, 15$ in the following table.

$\overline{0} \oplus x = \{x\}$ for all $x \in NQ(R_1)$	$\overline{1} \oplus \overline{1} = \{\overline{1}\}$
$\overline{1} \oplus a_2 = \{a_9\}$	$a_2 \oplus a_5 = \{a_5, a_6\}$
$a_3 \oplus a_4 = \{a_6\}$	$\overline{1} \oplus a_3 = \{a_{10}\}$
$a_5 \oplus a_5 = \{\overline{0}, \overline{1}, a_2, a_4, a_5, a_6, a_7, a_8, a_{10}\}$	$a_5 \oplus a_6 = \{a_5, a_7, a_8\}$
$\overline{1} \oplus a_4 = \{a_{11}\}$	$a_5 \oplus a_7 = \{a_4, a_5, a_6, a_8\}$
$a_5 \oplus a_8 = \{a_5, a_6, a_7, a_{10}\}$	$\overline{1} \oplus a_5 = \{a_{15}\}$
$a_5 \oplus a_9 = \{a_{13}, a_{14}, a_{15}\}$	$a_5 \oplus a_{10} = \{a_5, a_8\}$
$\overline{1} \oplus a_6 = \{a_{13}\}$	$a_5 \oplus a_{11} = \{a_{14}, a_{15}\}$
$a_5 \oplus a_{12} = \{a_{13}, a_{14}, a_{15}\}$	$\overline{1} \oplus a_7 = \{a_{14}\}$
$a_5 \oplus a_{13} = \{a_9, a_{12}, a_{14}, a_{15}\}$	$a_5 \oplus a_{14} = \{a_{11}, a_{13}, a_{15}\}$
$\overline{1} \oplus a_8 = \{a_{12}\}$	$a_4 \oplus a_{14} = \{a_{15}\}$
$a_4 \oplus a_{15} = \{a_{14}, a_{15}\}$	$\overline{1} \oplus a_9 = \{a_2, a_9\}$
$a_{14} \oplus a_{14} = \{\overline{1}, a_2, a_3, a_7, a_9, a_{10}, a_{14}\}$	$a_{14} \oplus a_{15} = \{a_4, a_5, a_6, a_8, a_{11}, a_{13}, a_{15}\}$
$\overline{1} \oplus a_{10} = \{a_3, a_{10}\}$	$a_{15} + a_{15} = NQ(R_1)$
$a_{15} \oplus a_3 = \{a_{12}, a_{15}\}$	$\overline{1} \oplus a_{11} = \{a_4, a_{10}\}$

and we present some of the results for $a_i \odot a_j = a_j \odot a_i$, $i, j = 1, 2, \ldots, 15$ in the following table.

$\overline{0} \odot x = \{\overline{0}\}$ for all $x \in NQ(R_1)$	$\overline{1} \odot \overline{1} = \{\overline{1}\}$
$\overline{1} \odot a_2 = \{\overline{0}, a_2\}$	$\overline{1} \odot a_3 = \{\overline{0}, a_3\}$
$\overline{1} \odot a_4 = \{\overline{0}, a_4\}$	$\overline{1} \odot a_5 = \{\overline{0}, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$
$\overline{1} \odot a_6 = \{\overline{0}, a_3, a_4, a_6\}$	$\overline{1} \odot a_7 = \{\overline{0}, a_2, a_3, a_7\}$
$\overline{1} \odot a_8 = \{\overline{0}, a_2, a_4, a_8\}$	$\overline{1} \odot a_9 = \{\overline{1}, a_9\}$
$\overline{1} \odot a_{10} = \{\overline{1}, a_{10}\}$	$\overline{1} \odot a_{11} = \{\overline{1}, a_{11}\}$
$\overline{1} \odot a_{12} = \{\overline{1}, a_9, a_{11}, a_{12}\}$	$\overline{1} \odot a_{13} = \{\overline{1}, a_{10}, a_{11}, a_{13}\}$
$\overline{1} \odot a_{14} = \{\overline{1}, a_9, a_{10}, a_{14}\}$	$\overline{1} \odot a_{15} = \{\overline{1}, a_9, a_{10}, a_1, a_{12}, a_{13}, a_{14}, a_{15}\}$
$a_2 \odot a_2 = \{\overline{0}, a_2\}$	$a_3 \odot a_3 = \{\overline{0}, a_3\}$

It is clear that $(NQ(R_1), \oplus, \odot)$ is a commutative quadruple H_v-ring.

Proposition 3.19. *Let $(R, +, \cdot)$ be an H_v-ring. Then "1" is the scalar unit of $(R, +, \cdot)$ if and only if $\overline{1} = (1, 0T, 0I, 0F)$ is the scalar unit of $(NQ(R), \oplus, \odot)$.*

Proof. The proof is straightforward by applying the uniqueness of the scalar unit. □

Proposition 3.20. *Let $(R, +, \cdot)$ be an H_v-ring. Then $(R, +, \cdot)$ is a commutative H_v-ring if and only if $(NQ(R), \oplus, \odot)$ is a commutative H_v-ring.*

Proof. Theorem 3.5 asserts that $(NQ(R), \oplus)$ is a commutative H_v-group if and only if $(R, +)$ is a commutative H_v-group. We need to show that $(NQ(R), \odot)$ is a commutative H_v-semigroup if and only if $(R, \cdot)$ is a commutative H_v-semigroup. Suppose that $(R, \cdot)$ is a commutative H_v-semigroup and let $x = (x_1, x_2T, x_3I, x_4F), y = (y_1, y_2T, y_3I, y_4F) \in NQ(R)$. Easy computations show that $x \odot y = y \odot x$. Thus, $(NQ(R), \odot)$ is a commutative H_v-semigroup.

Conversely, let $(NQ(R), \odot)$ be a commutative H_v-group and $a, b \in R$. Having $x = (a, 0T, 0I, 0F), y = (b, T, 0I, 0F) \in NQ(R)$ implies that $x \odot y = (a \cdot b, 0T, 0I, 0F) = y \odot x = (b \cdot a, 0T, 0I, 0F)$. Thus, $a \cdot b = b \cdot a$. Therefore, $(R, \cdot)$ is a commutative H_v-semigroup. □

Proposition 3.21. *If $(R, +, \cdot)$ is a VT-H_v-ring then $(NQ(R), \oplus, \odot)$ is not a VT-H_v-ring.*

Proof. Suppose that $(R, +)$ is a VT-H_v-ring. Then there exist $a, b \in R$ with either $|a + b| > 1$ or $|a \cdot b| > 1$.

- Case $|a + b| > 1$. Having $0 + 0 = 0$ implies that either $a \neq 0$ or $b \neq 0$ (or both are not equal to zero). Without loss of generality, we take $b \neq 0$. Let $x = (a, aI, 0T, 0F), y = (b, 0I, 0T, 0F), z = (0, bT, bI, bF) \in NQ(R)$. It is clear that $y \neq z$, $|x \oplus y| > 1$ and that $|x \oplus z| > 1$.
- Case $|a \cdot b| > 1$. Having $1 \cdot 1 = 1$ implies that either $a \neq 1$ or $b \neq 1$ (or both are not equal to 1). Without loss of generality, we take $b \neq 1$. Let $x = (a, 0I, 0T, 0F), y = (b, 0I, 0T, 0F), z = (0, bT, 0I, 0F) \in NQ(R)$. It is clear that $y \neq z$, $|x \odot y| > 1$ and that $|x \odot z| > 1$.

Therefore, $(NQ(R), \oplus, \odot)$ is not a VT-H_v-ring. □

4. **Neutrosophic quadruple H_v-subrings and neutrosophic homomorphisms**

In this section, we define neutrosophic quadruple H_v-subrings and neutrosophic homomorphisms and investigate some of their properties.

Definition 4.1. Let $(NQ(R), \oplus, \odot)$ be a neutrosophic quadruple H_v-ring and T be a non-empty subset of $NQ(R)$. Then $(T, \oplus, \odot)$ is called a *neutrosophic quadruple H_v-subring of $NQ(R)$* if $(T, \oplus, \odot)$ is a neutrosophic quadruple H_v-ring.

Remark 4.2. Neutrosophic H_v-rings have no proper neutrosophic H_v-ideals. This is clear as if $NQ(J)$ is a neutrosophiv H_v-ideal of $NQ(R)$ then $(1, 0T, 0I, 0F) \in NQ(J)$. The latter implies that $(a, bT, cI, dF) = (a, bT, cI, dF) \odot (1, 0T, 0I, 0F) \in NQ(J)$ for all $(a, bT, cI, dF) \in NQ(R)$.

Theorem 4.3. *[3] Let $(R, +)$ be an H_v-group with identity "0", $S \subseteq R$ and $0 \in S$. Then $(NQ(S), \oplus)$ is an H_v-subgroup of $(NQ(R), \oplus)$ if and only if $(S, +)$ is an H_v-subgroup of $(R, +)$.*

Theorem 4.4. *Let $(R, +, \cdot)$ be an H_v-ring with identity "0" and unit 1, $S \subseteq R$ and $0, 1 \in S$. Then $(NQ(S), \oplus, \odot)$ is an H_v-subring of $(NQ(R), \oplus, \odot)$ if and only if $(S, +, \cdot)$ is an H_v-subring of $(R, +, \cdot)$.*

Proof. Theorem 4.3 asserts that $(NQ(S), \oplus)$ is an H_v-subgroup of $(NQ(R), \oplus)$ if and only if $(S, +)$ is an H_v-subgroup of $(R, +)$. We need to show that $(NQ(S), \odot)$ is an H_v-subsemigroup of $(NQ(R), \odot)$ if and only if $(S, \cdot)$ is an H_v-subsemigroup of $(R, \cdot)$. Suppose that $(S, \cdot)$ is an H_v-subsemigroup of $(R, \cdot)$. We need to show that $x \odot NQ(S) \cup NQ(S) \odot x \subseteq NQ(S)$ for all $x = (x_1, x_2T, x_3I, x_4F) \in NQ(S)$ which is clear.

Let $(NQ(S), \odot)$ be an H_v-subsemigroup of $(NQ(R), \odot)$ and let $x_1 \in S$. We need to show that $x_1 \cdot S \cup S \cdot x_1 \subseteq S$. For all $y_1 \in S$, we have $x = (x_1, 0T, 0I, 0F), y = (y_1, 0T, 0I, 0F) \in NQ(S)$. Since $x \odot y \subseteq NQ(S)$, it follows that $x_1 \cdot y_1 \subseteq S$. □

Example 4.5. Since $(R_1, +_1, \cdot_1)$ in Example 3.18 has only one H_v-subring (R_1) containing 0 and 1, it follows by applying Theorem 4.4 that $(NQ(R_1), \oplus, \odot)$ has only one neutrosophic H_v-subring: $(NQ(R_1), \oplus, \odot)$.

Example 4.6. Let $R_2 = \{0, 1, 2\}$ and define $(R_2, +_2, \cdot_2)$ as follows:

$+_2$	0	1	2
0	0	$\{0,1\}$	$\{0,2\}$
1	$\{0,1\}$	1	$\{1,2\}$
2	$\{0,2\}$	$\{1,2\}$	2

$\cdot_2$	0	1	2
0	0	0	0
1	0	1	$\{1,2\}$
2	0	$\{1,2\}$	2

It is clear that $(R_2, +_2, \cdot_2)$ is a commutative H_v-ring that has exactly two non-isomorphic H_v-subrings containing 0 and 1: $\{0, 1\}$ and R_2. We can deduce that $(NQ(R_2), \oplus, \odot)$ is a commutative neutrosophic quadruple H_v-ring and has two non-isomorphic neutrosophic quadruple H_v-subrings: $NQ(\{0, 1\}) = \{\overline{0}, \overline{1}\}$ and $NQ(R_2)$.

Proposition 4.7. *Let $n \geq 2$ be a natural number and $(\mathbb{Z}_n, +, \cdot)$ be the ring of integers under standard addition and multiplication modulo n. Then $(NQ(\mathbb{Z}_n), \oplus, \odot)$ has no proper neutrosophic H_v-subrings.*

Proof. Proposition 3.17 asserts that $(NQ(\mathbb{Z}_n), \oplus, \odot)$ is a neutrosophic H_v-ring. Let S be a subring of $\mathbb{Z}_n$. Then there exist $d \mid n$ with $1 \leq d \leq n$ such that $S = d\mathbb{Z}_n$. Since $1 \in S$ if and only if $d = 1$ and $(1, 0T, 0I, 0F) \in NQ(S)$, it follows that $NQ(S) = NQ(\mathbb{Z}_n)$. □

Proposition 4.8. *Let $(S, +, \cdot)$ be an H_v-subring of $(R, +, \cdot)$. Then $NQ(S) \oplus NQ(S) = NQ(S)$ and $NQ(S) \odot NQ(S) \subseteq NQ(S)$.*

Proof. The proof is straightforward. □

Definition 4.9. Let $(NQ(R), \oplus_1, \odot_1)$ and $(NQ(J), \oplus_2, \odot_2)$ be neutrosophic quadruple H_v-rings. A function $\phi : NQ(R) \to NQ(J)$ is called *neutosophic homomorphism* if

(1) $\phi(0_R, 0_R T, 0_R I, 0_R F) = (0_J, 0_J T, 0_J I, 0_J F)$;
(2) $\phi(1_R, 0_R T, 0_R I, 0_R F) = (1_J, 0_J T, 0_J I, 0_J F)$;
(3) $\phi(0_R, 1_R T, 0_R I, 0_R F) = (0_J, 1_J T, 0_J I, 0_J F)$;
(4) $\phi(0_R, 0_R T, 1_R I, 0_R F) = (0_J, 0_J T, 1_J I, 0_J F)$;
(5) $\phi(0_R, 0_R T, 0_R I, 1_R F) = (0_J, 0_J T, 0_J I, 1_J F)$;
(6) $\phi(x \oplus_1 y) = \phi(x) \oplus_2 \phi(y)$ for all $x, y \in NQ(R)$;
(7) $\phi(x \odot_1 y) = \phi(x) \odot_2 \phi(y)$ for all $x, y \in NQ(R)$.

If ϕ is a neutrosophic homomorphism and bijective then it is called *neutrosophic isomorphism* and we write $NQ(R) \cong NQ(J)$.

Example 4.10. Let $(R, +, \cdot)$ be an H_v-ring. Then $f : NQ(R) \to NQ(R)$ is an isomorphism, where $f(x) = x$ for all $x \in NQ(R)$.

Proposition 4.11. *Let $(R, +_1, \cdot_1)$ and $(J, +_2, \cdot_2)$ be H_v-rings. If there exist a homomorphism $f : R \to J$ with $f(0_R) = 0_J$ and $f(1_R) = 1_J$ then there exist a homomorphism from $(NQ(R), \oplus_1, \odot_1)$ to $(NQ(J), \oplus_2, \odot_2)$.*

Proof. Suppose that $f : R \to J$ is a homomorphism. We define $\phi : NQ(R) \to NQ(J)$ as follows: For $x = (x_1, x_2T, x_3I, x_4F) \in NQ(R)$

$$\phi((x_1, x_2T, x_3I, x_4F)) = (f(x_1), f(x_2)T, f(x_3)I, f(x_4)F).$$

It is clear that ϕ is well defined and that conditions 1. to 5. of Definition 4.9 are satisfied. Let $x = (x_1, x_2T, x_3I, x_4F), y = (y_1, y_2T, y_3I, y_4F) \in NQ(R)$. Since $f(x_i +_1 y_i) = f(x_i) +_2 f(y_i)$ for $i = 1, 2, 3, 4$, it follows that $\phi(x \oplus_1 y) = \phi(x) \oplus_2 \phi(y)$. Moreover, having $f(x_i \cdot_1 y_i) = f(x_i) \cdot_2 f(y_i)$ for $i = 1, 2, 3, 4$ implies that $\phi(x \odot_1 y) = \phi(x) \odot_2 \phi(y)$. □

Proposition 4.12. *Let $(R, +_1, \cdot_1)$ and $(J, +_2, \cdot_2)$ be isomorphic H_v-rings, $0_R, 1_R \in R$ with $0_R + 0_R = 0_R$, $1_R \cdot 1_R = 1_R$, $0_R \cdot x = 0_R$ for all $x \in R$ and $f : (R, +_1, \cdot_1) \rightarrow (J, +_2, \cdot_2)$ be an isomorphism. Then $f(0_R) = 0_J$ and $f(1_R) = 1_J$.*

Proof. let $f(0_R) = a, f(1_R) = b$. Since $a = f(0_R) = f(0_R +_1 0_R) = a +_2 a$ and $a +_2 y = f(0_R +_1 x) \ni f(x) = y$ for all $y \in J$, it follows that a is a zero of J satisfying $a +_2 a = a$. Moreover, having $b = f(1_R \cdot_1 1_R) = b \cdot_2 b$ and $b \cdot_2 y = f(1_R \cdot_1 x) \ni f(x) = y$ for all $y \in J$ implies that b is a unit of J satisfying $1_J \cdot_2 1_J = 1_J$. □

Corollary 4.13. *Let $(R, +_1, \cdot_1)$ and $(J, +_2, \cdot_2)$ be isomorphic H_v-rings. Then*

$$(NQ(R), \oplus_1, \odot_1) \cong (NQ(J), \oplus_2, \odot_2).$$

Proof. The proof is straightforward by using Proposition 4.11 and Proposition 4.12. □

Corollary 4.14. *Let $(R, +_1, \cdot_1)$ and $(J, +_2, \cdot_2)$ be H_v-rings and let $Hom(R, J) = \{f : R \rightarrow J : f$ is homomorphism, $f(0_R) = 0_J$ and $f(1_R) = 1_J\}$. If $|Hom(R, J)| < \infty$ then*

$$|Hom(R, J)| \leq |Hom(NQ(R), NQ(J)|.$$

Proof. The proof is straightforward using Proposition 4.11. □

Let $(R, +)$ be a commutative H_v-ring with identity "0" and unit "1" and $S \subseteq R$ be an H_v-subring of R. Then $(R/S, +', \cdot')$ is an H_v-ring with: S as a zero, "$1 + S$" as a unit and $S +' S = S$. Here "$+'$" and "$\cdot'$" are defined as follows: For all $x, y \in R$,

$$(x + S) +' (y + S) = (x + y) + S \text{ and } (x + S) \cdot' (y + S) = x \cdot y + S.$$

Proposition 4.15. *Let $(S, +, \cdot)$ be an H_v-subring of a commutative H_v-ring $(R, +, \cdot)$. Then $(NQ(R/S), \oplus, \odot)$ is an H_v-ring.*

Proof. Since $(R, +, \cdot)$ is commutative, it follows that "$+'$" and "$\cdot'$" are well defined. The proof follows from having $(R/S, +', \cdot')$ an H_v-ring with S as zero, $1 + S$ as unit, $S \cdot' (x + S) = (x + S) \cdot' S = S$ and from Theorem 3.15. □

Proposition 4.16. *Let $(S, +, \cdot)$ be an H_v-subring of a commutative H_v-ring $(R, +, \cdot)$. Then $(NQ(R/S), \oplus, \odot) \cong (NQ(R)/NQ(S), \oplus', \odot')$.*

Proof. Let $g : NQ(R)/NQ(S) \to NQ(R/S)$ be defined as follows:

$$g((x_1, x_2T, x_3I, x_4F) \oplus NQ(S)) = (x_1 + S, (x_2 + S)T, (x_3 + S)I, (x_4 + S)F).$$

We claim that g is a neutrosophic isomorphism, that is, g is well defined, one-to-one, onto and neutrosophic homomorphism.

(1) g is well defined. Let $x \oplus NQ(S) = y \oplus NQ(S) \in NQ(R)/NQ(S)$. Then there exist $x_i, y_i \in R$, $i = 1, 2, 3, 4$ such that $x = (x_1, x_2T, x_3I, x_4F), y = (y_1, y_2T, y_3I, y_4F)$. We need to show that $x_i + S = y_i + S$ for $i = 1, 2, 3, 4$, that is $x_i + S \subseteq y_i + S$ and $y_i + S \subseteq x_i + S$ for $i = 1, 2, 3, 4$. We show that $x_i + S \subseteq y_i + S$ and $y_i + S \subseteq x_i + S$ is done in a similar manner. Since $x \oplus NQ(S) = y \oplus NQ(S)$, it follows that $x \in x \oplus z \subseteq y \oplus NQ(S)$ for all $z = (z_1, z_2T, z_3I, z_4F) \in NQ(S)$. The latter implies that there exist $s = (s_1, s_2T, s_3I, s_4F) \in NQ(S)$ such that $x \oplus z \in y \oplus s$. We get $x_i + z_i \in y_i + s_i \subseteq y_i + S$ for $i = 1, 2, 3, 4$. The latter implies that $x_i + S \subseteq y_i + S$ for $i = 1, 2, 3, 4$.

(2) g is onto. The proof is straightforward.

(3) g is one-to-one. Let $x \oplus NQ(S) = (x_1, x_2T, x_3I, x_4F) \oplus NQ(S), y \oplus NQ(S) = (y_1, y_2T, y_3I, y_4F) \oplus NQ(S) \in NQ(R)/NQ(S)$ with $h(x \oplus NQ(S)) = h(y \oplus NQ(S))$. We need to show that $x \oplus NQ(S) = y \oplus NQ(S)$, that is, $x \oplus NQ(S) \subseteq y \oplus NQ(S)$ and $y \oplus NQ(S) \subseteq x \oplus NQ(S)$. We prove $x \oplus NQ(S) \subseteq y \oplus NQ(S)$ and $y \oplus NQ(S) \subseteq x \oplus NQ(S)$ is done in a similar manner.
Having $h(x \oplus NQ(S)) = h(y \oplus NQ(S))$ implies that $(x_1 + S, (x_2 + S)T, (x_3 + S)I, (x_4 + S)F) = (y_1 + S, (y_2 + S)T, (y_3 + S)I, (y_4 + S)F)$. The latter implies that $x_i + S = y_i + S$ for $i = 1, 2, 3, 4$. Let $z = (z_1, z_2T, z_3I, z_4F) \in NQ(S)$. Having $x_i + S = y_i + S$ for $i = 1, 2, 3, 4$ implies that there exist s_i, $i = 1, 2, 3, 4$, such that $x_i + z_i \subseteq y_i + s_i$ for $i = 1, 2, 3, 4$. The latter implies that $x \oplus NQ(S) \subseteq y \oplus s \subseteq y \oplus NQ(S)$.

(4) g is neutrosophic homomorphism.
- $g(0, 0T, 0I, 0F) = (S, ST, SI, SF)$,
- $g(1, 0T, 0I, 0F) = (1 + S, ST, SI, SF)$,
- $g(0, 1T, 0I, 0F) = (S, (1 + S)T, SI, SF)$,
- $g(0, 0T, 1I, 0F) = (S, ST, (1 + S)I, SF)$,
- $g(0, 0T, 0I, 1F) = (S, ST, SI, (1 + S)F)$,
- We have $g((x_1, x_2T, x_3I, x_4F) \oplus NQ(S) \oplus' (y_1, y_2T, y_3I, y_4F) \oplus NQ(S)) = g((x_1 + y_1, (x_2 + y_2)T, (x_3 + y_3)I, (x_4 + y_4)F) \oplus NQ(S)) = (x_1 + y_1 + S, (x_2 + y_2 + S)T, (x_3 + y_3 + S)I, (x_4 + y_4 + S)F)$. On the other hand, we have $g((x_1, x_2T, x_3I, x_4F) \oplus NQ(S)) \oplus g((y_1, y_2T, y_3I, y_4F) \oplus NQ(S)) = (x_1 + S, (x_2 + S)T, (x_3 + S)I, (x_4 + S)F) \oplus (y_1 + S, (y_2 + S)T, (y_3 + S)I, (y_4 + S)F)$.

- We have:$(x_1, x_2T, x_3I, x_4F) \oplus NQ(S) \odot' (y_1, y_2T, y_3I, y_4F) \oplus NQ(S) = (x_1, x_2T, x_3I, x_4F) \odot (y_1, y_2T, y_3I, y_4F) \oplus NQ(S)$ and $g((x_1, x_2T, x_3I, x_4F) \oplus NQ(S)) \odot g((y_1, y_2T, y_3I, y_4F) \oplus NQ(S)) = (x_1 + S, (x_2 + S)T, (x_3 + S)I, (x_4 + S)F) \odot (y_1 + S, (y_2 + S)T, (y_3 + S)I, (y_4 + S)F)$. Simple computations imply that $g((x_1, x_2T, x_3I, x_4F) \oplus NQ(S) \odot' (y_1, y_2T, y_3I, y_4F) \oplus NQ(S)) = g((x_1, x_2T, x_3I, x_4F) \oplus NQ(S)) \odot g((y_1, y_2T, y_3I, y_4F) \oplus NQ(S))$.

Therefore, $(NQ(R/S), \oplus, \odot) \cong (NQ(R)/NQ(S), \oplus', \odot')$. □

Example 4.17. Let $R_2 = \{0, 1, 2\}$ and $S = \{0, 1\}$ in Example 4.6. Then $NQ(R_2/S) \cong NQ(R_2)/NQ(S)$.

5. Conclusion

This paper contributed to the study of neutrosophic hyperstructures by introducing neutrosophic quadruple H_v-rings and studying their properties. For future work, it will be interesting to introduce and study other neutrosophic quadruple H_v-structures such as neutrosophic H_v-modules and neutrosophic H_v-vectorspaces.

References

1. A. A. A. Agboola, B. Davvaz and F. Smarandache, *Neutrosophic quadruple algebraic hyperstructures*, Ann. Fuzzy Math. Inform. 14(2017): 29-42.
2. S. A. Akinleye, F. Smarandache and A. A. A. Agboola, *On neutrosophic quadruple algebraic structures*, Neutrosophic Sets and Systems, 12(2016): 122-126.
3. M. Al-Tahan and B. Davvaz, *On neutrosophic quadruple H_v-groups and their properties*, Southeast Asian Bulletin of Mathematics, to appear.
4. M. Al-Tahan and B. Davvaz, *Some results on single valued neutrosophic (weak) polygroups*, International Journal of Neutrosophic Science (IJNS), 2(1)(2020): 38-46.
5. M. Al-Tahan and B. Davvaz,, *On single valued neutrosophic sets and neutrosophic -structures: Applications on algebraic structures (hyperstructures)*, International Journal of Neutrosophic Science (IJNS), 3(2)(2020): 108-117.
6. M. Al-Tahan and B. Davvaz, *Refined neutrosophic quadruple (po-)hypergroups and their fundamental group*, Neutrosophic Sets and Systems, 27(2019): 138-153.
7. T. Bera, N.K. Mahapatra, *On neutrosophic soft topological space*, Neutrosophic Sets and Systems, 19 (2018): 3-15.
8. P. Corsini and V. Leoreanu, *Applications of hyperstructures theory*, Advances in Mathematics, Kluwer Academic Publisher, 2003.
9. B. Davvaz, *Polygroup Theory and Related Systems*, World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2013. viii+200 pp.
10. S. Dai, (α, β, γ)-*Equalities of single valued neutrosophic sets*, Neutrosophic Sets and Systems, 19(2018): 127-131.
11. Y. B. Jun , S-Z. Song, F. Smarandache and H.Bordbar, *Neutrosophic Quadruple BCK/BCI-Algebras*, Axioms 7(41)(2018), doi:10.3390/axioms7020041.

12. F. Marty, *Sur une generalization de la notion de group*, In 8*th* Congress Math. Scandenaves, (1934), 45-49.
13. M. Mullai, R. Surya, *Neutrosophic EOQ model with price break*, Neutrosophic Sets and Systems, 19 (2018): 24-29.
14. F. Smarandache, *Neutrosophic quadruple numbers, refined neutrosophic quadruple numbers, absorbance law, and the multiplication of neutrosophic quadruple numbers*, Neutrosophic Sets and Systems, 10(2015): 96-98.
15. G. Shahzadi, M. Akram, A. B. Saeid, *An application of single-valued neutrosophic sets in medical diagnosis*, Neutrosophic Sets and Systems, 18(2017): 80-88.
16. S. Topal, F. Tas, *Bezier surface modeling for neutrosophic data problems*, Neutrosophic Sets and Systems, 19 (2018): 19-23.
17. T. Vougiouklis, *The very thin hypergroups and the S-construction*, Combinatorics'88, Vol. 2 (Ravello, 1988), 471-477, Res. Lecture Notes Math., Mediterranean, Rende, 1991.
18. T. Vougiouklis, *Fundamentally very thin H_v-structures*, Algebras Groups Geom. 28(4) (2011), 387-400.
19. T. Vougiouklis, *Hyperstructures and their representations*, Hadronic Press Monographs in Mathematics, Hadronic Press, Inc., Palm Harbor, FL, 1994.
20. T. Vougiouklis, *The fundamental relation in hyperrings. The general hyperfield*, Algebraic hyperstructures and applications (Xanthi, 1990), 203-211, World Sci. Publ., Teaneck, NJ, 1991.

Received: 3 October 2019 / Accepted: 6 August 2020