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On The Representation of Some n-Plithogenic Differential Operators by Matrices

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Abstract

The main goal of this paper is to study the representation of the symbolic n-plithogenic differential operator for many different values of n by classical algebraic matrices and plithogenic matrices. We present many examples about the representation of symbolic n-plithogenic differential operators by matrices. As well as, we compute the symbolic 2-plithogenic, 3-plithogenic, and 4-plithogenic Wronsckian, and anti-Wronsckian.

Keywords: Differential operator, Wronsckian, anti-Wronsckian, symbolic n-plithogenic matrix

Introduction

Symbolic n-plithogenic structures and sets are defined for the first time by Smarandache [4], as extensions of classical algebraic structures. Where they were used widely by many researchers to generalize famous algebraic structures. For example, we can see symbolic n-plithogenic rings, probability, spaces, and matrices [1-3, 5-8, 14-19].

The main results about symbolic n-plithogenic structures are the similarity between them and refined neutrosophic structures, see [9-13].

In this work, we concentrate on the analytical side of symbolic n-plithogenic algebraic structures, where we provide many examples about the applications of matrices in representing symbolic n-plithogenic differential operators. Also, we present the concept of symbolic n-plithogenic Wronsckian, and anti-Wronsckian,

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with many computable examples. For the definitions of symbolic n-plithogenic rings and structures, check [1,6,8,19].

Main Discussion

Definition:

Let $f: 2-SP_R \rightarrow 2-SP_R$ be a symbolic 2-plithogenic real function, we define the symbolic 2-plithogenic differential operator as: $D_2(f)=\acute{f}$.

Definition.

Let $f: 3-SP_R \rightarrow 3-SP_R$ be a symbolic 3-plithogenic real function, we define the symbolic 2-plithogenic differential operator as: $D_3(f)=\acute{f}$.

Definition.

Let $f: 4-SP_R \rightarrow 4-SP_R$ be a symbolic 4-plithogenic real function, we define the symbolic 2-plithogenic differential operator as: $D_4(f)=\acute{f}$.

Definition.

Let $f: 5-SP_R \rightarrow 5-SP_R$ be a symbolic 5-plithogenic real function, we define the symbolic 2-plithogenic differential operator as: $D_5(f)=\acute{f}$.

Example.

Consider $f: 2-SP_R \rightarrow 2-SP_R; f(X)=X^2+(P_1+P_2)X-P_1$,where $X=x_0+x_1P_1+x_2P_2 \in 2-SP_R$, then $D_2(f)=2X+(P_1+P_2)$.

Consider $g: 3-SP_R \rightarrow 3-SP_R; g(X)=X^2+P_3X+P_3+P_2$,where $X=x_0+x_1P_1+x_2P_2+x_3P_3 \in 3-SP_R$, then $D_3(g)=2X+P_3$.

Consider $h: 4-SP_R \rightarrow 4-SP_R; h(X)=X^3+(P_1+P_4)X-1$,where $X=x_0+x_1P_1+x_2P_2+x_3P_3+x_4P_4 \in 4-SP_R$, then $D_3(h)=3X^2+P_1+P_4$.

Example.

Consider D_2 the symbolic 2-plithogenic differential operator on the space of symbolic 2-plithogenic quadratic polynomials $\{aX^2+bX+c; a,b,c,X \in 2-SP_R\}$, then:

$$\begin{cases} D_2(X^2)=2X=2x_0+2x_1P_1+2x_2P_2=0X^2+2X+0.1 \\ D_2(X)=1=0X^2+0.X+1.1 \\ D_2(1)=0=0X^2+0.X+0.1 \end{cases}$$

Hence $[D_2] = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$.

Example.

Consider D_3, D_4, D_5 be the symbolic 3-plithogenic, 4-plithogenic, 5-plithogenic differential operators on the spaces of cubic symbolic 30plithogenic, 4-plithogenic, and 5-plithogenic spaces.

$L_1 = \{aX^3 + bX^2 + bX + d; a, b, c, d, X \in 3 - SP_R\}$

$L_2 = \{aX^3 + bX^2 + bX + d; a, b, c, d, X \in 4 - SP_R\}$

$L_3 = \{aX^3 + bX^2 + bX + d; a, b, c, d, X \in 5 - SP_R\}$

Then $D_n(X^3) = 3X^2 = 0.X^3 + 3X^2 + 0.X + 0.1$.

$D_n(X^2) = 2X = 0.X^3 + 0.X^2 + 2.X + 0.1$.

$D_n(X) = 1 = 0.X^3 + 0.X^2 + 0.X + 1.1$.

$D_n(1) = 0 = 0.X^3 + 0.X^2 + 0.X + 0.1$

For all $3 \le n \le 5$, hence:

$$[D_n] = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

Example.

For $sinX = sin(x_0 + x_1P_1 + x_2P_2), cosX = cos(x_0 + x_1P_1 + x_2P_2)$.

We have:

$$\begin{cases} D_2(sinX) = cosX = 0.sinX + 1.cosX + 0.1 \\ D_2(cosX) = -sinX = -1.sinX + 0..cosX + 0.1 \\ D_2(1) = 0 = 0.sinX + 0.cosX + 0.1 \end{cases}$$

Hence $[D_2] = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

For $sinX = sin(x_0 + x_1P_1 + x_2P_2 + x_3P_3), cosX = cos(x_0 + x_1P_1 + x_2P_2 + x_3P_3)$.

We have:

$$\begin{cases} D_3(sinX) = cosX \\ D_3(cosX) = -sinX \\ D_3(1) = 0 \end{cases}$$

Hence $[D_3] = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

For $sinX = sin(x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4), cosX = cos(x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4)$.

We have:

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Hence $[D_4] = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

Example.

For $\{1, X, X^2, X^3, X^4\}; X = x_0 + x_1P_1 + x_2P_2$,we have:

$$\begin{cases} D_2(X^4) = 4X^3 \\ D_2(X^3) = 3X^2 \\ D_2(X^2) = 2X \\ D_2(X) = 1 \\ D_2(1) = 0 \end{cases}$$

Hence $[D_2] = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$

For $X = x_0 + \sum_{i=1}^{5} x_iP_i$, we have:

$$\begin{cases} D_5(X^4) = 4X^3 \\ D_5(X^3) = 3X^2 \\ D_5(X^2) = 2X \\ D_5(X) = 1 \\ D_5(1) = 0 \end{cases}$$

Hence $[D_5] = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$

Example.

Consider $A = \{1, e^X, e^{2X}\}$, where $X = x_0 + x_1P_1 + x_2P_2$, $B = \{1, e^Y, e^{2Y}\}$, where $Y = y_0 + y_1P_1 + y_2P_2 + y_3P_3$, $C = \{1, e^Z, e^{2Z}\}$, where $Z = z_0 + z_1P_1 + z_2P_2 + z_3P_3 + z_4P_4$, $D = \{1, e^T, e^{2T}\}$, where $T = t_0 + + \sum_{i=1}^{5} t_iP_i$, then:

$$\begin{cases} D_2(1) = 0 \\ D_2(e^X) = e^X \\ D_2(e^{2X}) = 2e^{2X} \end{cases}$$

Hence $[D_3] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.

$$\begin{cases} D_3(1) = 0 \\ D_3(e^Y) = e^Y \\ D_3(e^{2Y}) = 2e^{2Y} \end{cases}$$

Hence $[D_3] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.

$$\begin{cases} D_4(1) = 0 \\ D_4(e^Z) = e^Z \\ D_4(e^{2Z}) = 2e^{2Z} \end{cases}$$

Hence $[D_4] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.

$$\begin{cases} D_5(1) = 0 \\ D_5(e^T) = e^T \\ D_5(e^{2T}) = 2e^{2T} \end{cases}$$

Hence $[D_5] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.

Another possible representation.

We have shown that symbolic n-plithogenic differential operators can be represented by classical real matrices, now we will try to explain how they can be represented by plithogenic matrices.

Example.

Consider $\{1, X, X^2\}; X = x_0 + x_1P_1 + x_2P_2$, with D_2 the symbolic 2-plithogenic differential operator, then:

$$\begin{cases} D_2(1) = 0 = 0.x_0 + 0.x_1P_1 + 0.x_2P_2 \\ D_2(X) = 1 \\ D_2(X^2) = 2X \end{cases}$$

The basis $\{1, X, X^2\}$ can be represented as follows:

$$B_1 = \{1, x_0, {x_0}^2\}, B_2 = \{1, (x_0 + x_1), (x_0 + x_1)^2\}, B_3 = \{1, (x_0 + x_1 + x_2), (x_0 + x_1 + x_2)^2\}$$

Any quadratic polynomial $P(X) = aX^2 + bX + c; a, b, c, X \in 2 - SP_R$, with:

$$\begin{cases} a = a_0 + a_1P_1 + a_2P_2 \\ b = b_0 + b_1P_1 + b_2P_2 \\ c = c_0 + c_1P_1 + c_2P_2 \\ X = x_0 + x_1P_1 + x_2P_2 \end{cases}$$

$$\begin{aligned} P(X) = aX^2 + bX + c \\ &= (a_0{x_0}^2 + b_0x_0 + c_0) \\ &+ P_1[(a_0 + a_1)(x_0 + x_1)^2 - a_0{x_0}^2 + (b_0 + b_1)(x_0 + x_1) - b_0x_0 + c_1] \\ &+ P_2[(a_0 + a_1 + a_2)(x_0 + x_1 + x_2)^2 - (a_0 + a_1)(x_0 + x_1)^2 \\ &+ (b_0 + b_1 + b_2)(x_0 + x_1 + x_2) - (b_0 + b_1)(x_0 + x_1) + c_2] \\ &= q_1(x_0) + P_1[q_2(x_0 + x_1) - q_1(x_0)] \\ &+ P_2[q_3(x_0 + x_1 + x_2) - q_2(x_0 + x_1)] \end{aligned}$$

Hence, $D_2\big(P(X)\big) = D_2(q_1) + P_1[D_2(q_2) - D_2(q_1)] + P_2[D_2(q_3) - D_2(q_2)]$, hence:

$$[D_2] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix} + P_1\left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix}\right] + P_2\left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix}\right]$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 + 2P_1 + 2P_2 \\ 0 & 1 + P_1 + P_2 & 0 \end{pmatrix}$$

The symbolic plithogenic Wronsckian.

Consider the following functions independent set:

$E = \{f_1, \dots, f_n\}$, their wronsckian is defined as follows:

$$W(E) = \begin{vmatrix} f_1 & \cdots & f_n \\ \acute{f}_1 & \cdots & \acute{f}_n \\ \vdots & \vdots & \vdots \\ {f_1}^{(n-1)} & \cdots & {f_n}^{(n-1)} \end{vmatrix}$$

We show some examples for finding the symbolic n-plithogenic Wronsckian.

Example.

Consider $E_1 = \{e^X, e^{2X}; X = x_0 + x_1P_1 + x_2P_2\}, E_2 = \{1, sinX, cosX; X = x_0 + x_1P_1 + x_2P_2 + x_3P_3\}, E_3 = \{1, tanX; X = x_0 + x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4\}, E_4 = \left\{1, X, X^2, X^3; X = x_0 + \sum_{i=1}^{5} x_iP_i\right\}$, we have:

$$W(E_1) = \begin{vmatrix} {D_2}^{(0)}(e^X) & {D_2}^{(0)}(e^{2X}) \\ D_2(e^X) & D_2(e^{2X}) \end{vmatrix} = \begin{vmatrix} e^{x_0+x_1P_1+x_2P_2} & e^{2x_0+2x_1P_1+2x_2P_2} \\ e^{x_0+x_1P_1+x_2P_2} & 2e^{2x_0+2x_1P_1+2x_2P_2} \end{vmatrix}$$

$$= e^{3x_0+3x_1P_1+3x_2P_2}$$

$$W(E_2)=\begin{vmatrix} 1 & sin\left(x_0+\sum_{i=1}^{3}x_iP_i\right) & cos\left(x_0+\sum_{i=1}^{3}x_iP_i\right) \\ 0 & cos\left(x_0+\sum_{i=1}^{3}x_iP_i\right) & -sin\left(x_0+\sum_{i=1}^{3}x_iP_i\right) \\ 0 & -sin\left(x_0+\sum_{i=1}^{3}x_iP_i\right) & -cos\left(x_0+\sum_{i=1}^{3}x_iP_i\right) \end{vmatrix}=-cos^2\left(x_0+\sum_{i=1}^{3}x_iP_i\right)$$

$$W(E_3)=\begin{vmatrix} 1 & tan\left(x_0+\sum_{i=1}^{4}x_iP_i\right) \\ 0 & 1+tan^2\left(x_0+\sum_{i=1}^{4}x_iP_i\right) \end{vmatrix}=1+tan^2\left(x_0+\sum_{i=1}^{4}x_iP_i\right)$$

$$W(E_4)=\begin{vmatrix} 1 & x_0+\sum_{i=1}^{5}x_iP_i & \left(x_0+\sum_{i=1}^{5}x_iP_i\right)^2 & \left(x_0+\sum_{i=1}^{5}x_iP_i\right)^3 \\ 0 & 1 & 2\left(x_0+\sum_{i=1}^{5}x_iP_i\right) & 3\left(x_0+\sum_{i=1}^{5}x_iP_i\right) \\ 0 & 0 & 2 & 6\left(x_0+\sum_{i=1}^{5}x_iP_i\right) \\ 0 & 0 & 0 & 6 \end{vmatrix}=12$$

Example.

Consider $E_1=\{lnX,e^X;X=x_0+\sum_{i=1}^{5}x_iP_i\}, E_2=\{lnX,\sqrt{X};X=x_0+\sum_{i=1}^{4}x_iP_i\}, E_3=\{e^X,sinX;X=x_0+\sum_{i=1}^{3}x_iP_i\}$, then:

$$W(E_1)=\begin{vmatrix} lnX & e^X \\ \frac{1}{X} & e^X \end{vmatrix}=e^{x_0+\sum_{i=1}^{3}x_iP_i}\left[ln\left(x_0+\sum_{i=1}^{3}x_iP_i\right)-\frac{1}{x_0+\sum_{i=1}^{3}x_iP_i}\right]$$

$$W(E_2)=\begin{vmatrix} lnX & \sqrt{X} \\ \frac{1}{X} & \frac{1}{2\sqrt{X}} \end{vmatrix}=\frac{lnX}{2\sqrt{X}}-\frac{1}{\sqrt{X}}=\frac{lnX-2}{2\sqrt{X}}$$

$$=\frac{1}{\sqrt{x_0+\sum_{i=1}^{4}x_iP_i}}\left[ln\left(x_0+\sum_{i=1}^{4}x_iP_i\right)-2\right]$$

$$W(E_3)=\begin{vmatrix} e^X & sinX \\ e^X & cosX \end{vmatrix}=e^{x_0+\sum_{i=1}^{5}x_iP_i}\left[cos\left(x_0+\sum_{i=1}^{5}x_iP_i\right)-sin\left(x_0+\sum_{i=1}^{5}x_iP_i\right)\right]$$

Symbolic n-plithogenic anti-Wronsckian.

Let $E=\{f_1,\dots,f_n\}$ be a set of n functions, then:

$$AW(E) = \begin{vmatrix} f_1 & f_2 & \cdots & f_n \\ \int f_1 & \int f_2 & \cdots & \vdots \\ \int\left(\int f_1\right) & \vdots & \cdots & \vdots \\ \vdots & & & \vdots \\ \underbrace{\int f_1}_{n-1\ times} & \underbrace{\int f_2}_{n-1\ times} & \vdots \cdots & \underbrace{\int f_n}_{n-1\ times} \end{vmatrix}$$

Now, we will clarify how (AW) can be computed.

Example.

Consider $E_1 = \{1, X, X^2; X = x_0 + \sum_{i=1}^{2} x_i P_i\}, E_2 = \{e^X, e^{2X}; X = x_0 + \sum_{i=1}^{3} x_i P_i\}, E_3 = \{sinX, cosX; X = x_0 + \sum_{i=1}^{4} x_i P_i\}$, then:

$$AW(E_1) = \begin{vmatrix} 1 & X & X^2 \\ X & \frac{1}{2}X^2 & \frac{1}{3}X^3 \\ \frac{1}{2}X^2 & \frac{1}{6}X^3 & \frac{1}{12}X^4 \end{vmatrix}$$

$$= 1.\begin{vmatrix} \frac{1}{2}X^2 & \frac{1}{3}X^3 \\ \frac{1}{6}X^3 & \frac{1}{12}X^4 \end{vmatrix} - X\begin{vmatrix} X & \frac{1}{3}X^3 \\ \frac{1}{2}X^2 & \frac{1}{12}X^4 \end{vmatrix} + X^2\begin{vmatrix} X & \frac{1}{2}X^2 \\ \frac{1}{2}X^2 & \frac{1}{6}X^3 \end{vmatrix}$$

$$= \left(\frac{1}{24} - \frac{1}{18}\right)X^6 - X\left(\frac{1}{12} - \frac{1}{6}\right)X^5 + X^2\left(\frac{1}{6} - \frac{1}{4}\right)X^4$$

$$= X^6\left(\frac{1}{24} - \frac{1}{18} - \frac{1}{12} + \frac{1}{6} + \frac{1}{6} - \frac{1}{4}\right) = X^6\left(\frac{1}{24} + \frac{8}{24} - \frac{6}{24} - \frac{2}{24} - \frac{1}{18}\right)$$

$$= X^6\left(\frac{1}{24} - \frac{1}{18}\right) = -\frac{1}{72}\left(x_0 + \sum_{i=1}^{3} x_i P_i\right)$$

$$AW(E_2) = \begin{vmatrix} e^X & e^{2X} \\ e^X & \frac{1}{2}e^{2X} \end{vmatrix} = \frac{1}{2}e^X e^{2X} - e^X e^{2X} = -\frac{1}{2}e^{3X} = -\frac{1}{2}e^{\left(x_0 + \sum_{i=1}^{3} x_i P_i\right)}$$

$$AW(E_3) = \begin{vmatrix} sinX & cosX & 1 \\ -cosX & -sinX & X \\ -sinX & -cosX & \frac{1}{2}X^2 \end{vmatrix}$$

$$= sinX. \begin{vmatrix} sinX & X \\ -cosX & \frac{1}{2}X^2 \end{vmatrix} - cosX \begin{vmatrix} -cosX & X \\ -sinX & \frac{1}{2}X^2 \end{vmatrix} + 1. \begin{vmatrix} -cosX & sinX \\ -sinX & -cosX \end{vmatrix}$$

$$= sinX\left(\frac{1}{2}X^2 sinX + XcosX\right) - cosX\left(-\frac{1}{2}X^2 cosX + XsinX\right)$$

$$+ (cos^2X + sin^2X)$$

$$= \frac{1}{2}X^2 sin^2X + XcosXsinX + \frac{1}{2}X^2 cos^2X - XcosXsinX + 1$$

$$= \frac{1}{2}X^2(1) + 1 = \frac{1}{2}\left(x_0 + \sum_{i=1}^{4} x_i P_i\right) + 1$$

Conclusion

In this paper, we have studied the representation of the symbolic n-plithogenic differential operator for many different values of n by classical algebraic matrices and plithogenic matrices. We presented many examples about the representation of symbolic n-plithogenic differential operators by matrices. As well as, we computed the symbolic 2-plithogenic, 3-plithogenic, and 4-plithogenic Wronsckian, and anti-Wronsckian.

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