Neutrosophic Sets and Systems, Vol. 59, 2023
University of New Mexico

On The Symbolic 2-Plithogenic Real Analysis by Using Algebraic Functions

Nabil Khuder Salman

College Of Pharmacy, AL-Farahidi University, Baghdad, Iraq

nsalman@uoalfarahidi.edu.iq

Abstract:

Symbolic 2-plithogenic sets as a generalization of classical concept of sets were applicable to algebraic structures. The symbolic 2-plithogenic rings and fields are good generalizations of classical corresponding systems.

In this paper, we study the symbolic 2-plithogenic real functions with one variable by using a special algebraic function called AH-isometry. In addition, we discuss the symbolic 2-plithogenic simple differential equations and conic sections by using this isometry. Also, many examples will be presented to explain the novelty of this work.

Keywords: symbolic 2-plithogenic set, symbolic 2-plithogenic real function, symbolic 2-plithogenic circle, symbolic 2-plithogenic ellipse.

Introduction and basic definitions

Symbolic n-plithogenic algebraic structures are considered as new generalizations of classical algebraic structures [1-3], such as symbolic 2-plithogenic integers, modules, and vector spaces [4-8].

zenodo.10031172/10.5281

Symbolic 2-plithogenic structures have a similar structure to the refined neutrosophic structures and many non classical algebraic structures defined by many authors in [9-17,20-23, 24-30].

In the literature, many mathematical approaches were carried out on neutrosophic and refined neutrosophic structures, were a special function called AH-isometry was used to study the analytical properties and conic sections [11-12, 18-19], and that occurs by taking the direct image of neutrosophic elements to the classical Cartesian product of the real field with itself.

In this work, we follow the previous efforts, and we define for the first time a special AH-isometry on the symbolic 2-plithogenic field of reals, and we use this isometry to obtain many formulas and properties about the symbolic 2-plithogenic analytical concepts such as differentiability, continuity, and integrability. Also, symbolic 2-plithogenic conic sections will find a place in our study.

Definition.

The symbolic 2-plithogenic ring of real numbers is defined as follows:

$$2-SP_R=\{t_0+t_1P_1+t_2P_2; t_i\in R, P_1\times P_2=P_2\times P_1=P_2, {P_1}^2={P_2}^2=P_2\}$$

The addition operation on $2-SP_R$ is defined as follows:

$$(t_0+t_1P_1+t_2P_2)+(t'_0+t'_1P_1+t'_2P_2)=(t_0+t'_0)+(t_1+t'_1)P_1+(t_2+t'_2)P_2$$

The multiplication on $2-SP_R$ is defined as follows:

$$(t_0+t_1P_1+t_2P_2)(t'_0+t'_1P_1+t'_2P_2)$$
$$=t_0t'_0+(t_0t'_1+t_1t'_0+t_1t'_1)P_1+(t_0t'_2+t_1t'_2+t_2t'_2+t_2t'_0+t_2t'_1)P_2$$

Remark.

If $T=t_0+t_1P_1+t_2P_2\in 2-SP_R$, then:

$T^{-1}=\frac{1}{T}=\frac{1}{t_0}+\left[\frac{1}{t_0+t_1}-\frac{1}{t_0}\right]P_1+\left[\frac{1}{t_0+t_1+t_2}-\frac{1}{t_0+t_1}\right]P_2$, with $t_0\neq 0, t_0+t_1\neq 0, t_0+t_1+t_2\neq 0$.

Main Results

Definition.

Let $2-SP_R=\{a+bP_1+cP_2; a,b,c\in R\}$ be the 2-plithogenic field of real numbers, a function $f=f(X): 2-SP_R\to 2-SP_R$ is called one variable symbolic 2-plithogenic real function, with

$X=x_0+x_1P_1+x_2P_2\in 2-SP_R$.

Definition.

Let $2-SP_R$ be the symbolic 2-plithogenic field of reals, we define its AH-isometry as follows:

$I: 2-SP_R\to R\times R\times R$ such that:

$I(x+yP_1+zP_2)=(x,x+y,x+y+z)$.

It is easy to see that I is a ring isomorphism with the inverse:

$I^{-1}: R\times R\times R\to 2-SP_R$ such that:

$I^{-1}(x,y,z)=x+(y-x)P_1+(z-y)P_2$

Definition.

Let $f: 2-SP_R\to 2-SP_R$ be a symbolic 2-plithogenic real function with one variable, we define the canonical formula as follows:

$$I^{-1}\circ I(f): 2-SP_R\to 2-SP_R$$

Example.

Consider$f(X)=X^2+2-P_1+P_2$, its canonical formula is:

$I\big(f(X)\big)=[I(X)]^2+I(2-P_1+P_2)=(x_0,x_0+x_1,x_0+x_1+x_2)^2+(2,1,2)$

$$=({x_0}^2+2,(x_0+x_1)^2+1,(x_0+x_1+x_2)^2+2)$$

$I^{-1}\circ I\big(f(X)\big)=$

${x_0}^2+2+P_1[(x_0+x_1)^2-{x_0}^2-1]+P_2[(x_0+x_1+x_2)^2-(x_0+x_1)^2+1]$

For example:

$f(1+P_1)=(1+P_1)^2+2-P_1+P_2=1+P_1+2P_1+2-P_1+P_2=3+2P_1+P_2$.

If we put values $x_0=1, x_1=1, x_2=0$,

in the canonical formula, then we get:

$I^{-1}\circ I\big(f(X)\big)=(1)^2+2+P_1[(2)^2-(1)^2-1]+P_2[(2)^2-(2)^2+1]=3+2P_1+P_2$.

The canonical formulas of famous functions:

1. The exponent function:

$e^X; X = x_0 + x_1P_1 + x_2P_2, I^{-1} \circ I(e^X) =$
$I^{-1}(e^{x_0}, e^{x_0+x_1}, e^{x_0+x_1+x_2}) =$
$e^{x_0} + P_1[e^{x_0+x_1} - e^{x_0}] + P_2[e^{x_0+x_1+x_2} - e^{x_0+x_1}]$

2. The logarithmic function:

$\ln(X)\,; X = x_0 + x_1P_1 + x_2P_2,$

$$I^{-1} \circ I(\ln(X)) = I^{-1}(\ln(x_0)\,, \ln(x_0 + x_1)\,, \ln(x_0 + x_1 + x_2)) =$$

$\ln(x_0) + P_1[\ln(x_0 + x_1) - \ln(x_0)] + P_2[\ln(x_0 + x_1 + x_2) - \ln(x_0 + x_1)].$

3. Famous trigonometric functions:

$\sin(X) = \sin(x_0) + P_1[\sin(x_0 + x_1) - \sin(x_0)] + P_2[\sin(x_0 + x_1 + x_2) - \sin(x_0 + x_1)]$

$$\cos(X) = \cos(x_0) + P_1[\cos(x_0 + x_1) - \cos(x_0)] + P_2[\cos(x_0 + x_1 + x_2) - \cos(x_0 + x_1)]$$

$$\tan(X) = \tan(x_0) + P_1[\text{taan}(x_0 + x_1) - \tan(x_0)] + P_2[\tan(x_0 + x_1 + x_2) - \tan(x_0 + x_1)]$$

And so no.

Definition.

A symbolic 2-plithogenic real number $T = t_0 + t_1P_1 + t_2P_2$ is called positive if and only if

$t_0 \geq 0, t_0 + t_1 \geq 0, t_0 + t_1 + t_2 \geq 0.$

For example $3 + 2P_1 - P_2 > 0$, that is because, $3 > 0$, $5 > 0, 4 > 0$.

Definition.

Let $f: 2 - SP_R \to 2 - SP_R$ be a symbolic 2-plithogenic real function with one variable $X = x_0 + x_1P_1 + x_2P_2$, then:

a. f is differentiable if and only if $I\big(f(X)\big)$ is differentiable.

b. f is continuous if and only if $I\big(f(X)\big)$ is continuous.

c. f is integrable if and only if $I\big(f(X)\big)$ is integrable.

Example.

Find the derivation of $f(X) = X^2 + X + P_1$ in two different ways.

Solution.

The regular way is $\acute{f}(X) = 2X + 1$.

The canonical way is:

$I^{-1} \circ I\big(f(X)\big) =$

$$I^{-1}({x_0}^2,(x_0+x_1)^2,(x_0+x_1+x_2)^2)+I^{-1}(x_0,x_0+x_1,x_0+x_1+x_2)+I^{-1}(0,1,1)$$
$$= {x_0}^2 + x_0 + P_1[(x_0+x_1)^2 - {x_0}^2 + (x_0+x_1) - x_0 + 1]$$
$$+ P_2[(x_0+x_1+x_2)^2 - (x_0+x_1)^2 + (x_0+x_1+x_2) - (x_0+x_1) + 0] =$$
$$= {x_0}^2 + x_0 + P_1[(x_0+x_1)^2 - {x_0}^2 + x_1 + 1]$$
$$+ P_2[(x_0+x_1+x_2)^2 - (x_0+x_1)^2 + x_1]$$

First, we have: $({x_0}^2 + x_0)'_{x_0} = 2x_0 + 1.$

$[(x_0+x_1)^2 - {x_0}^2 + x_1 + 1 + {x_0}^2 + x_0]'_{x_0+x_1} =$

$[(x_0+x_1)^2 + (x_0+x_1) + 1]'_{x_0+x_1} = 2(x_0+x_1)+1$

$[(x_0+x_1+x_2)^2 - (x_0+x_1)^2 + x_2 + (x_0+x_1)^2 - {x_0}^2 + x_1 + 1]'_{x_0+x_1+x_2}$

$$= [(x_0+x_1+x_2)^2 + (x_0+x_1+x_2) + 1]'_{x_0+x_1+x_2} = 2(x_0+x_1+x_2)+1$$

Thus, $\acute{f}(X) = 2x_0 + 1 + P_1[2(x_0+x_1)+1-2x_0-1] + P_2[2(x_0+x_1+x_2)+1-2(x_0+x_1)-1] = (2x_0+1) + 2x_1P_1 + 2x_2P_2 = 2X+1$

Example.

Find the value of $\int_0^{1+P_1+P_2} e^X dX$ in two different ways.

Solution;

The regular way: $\int_0^{1+P_1+P_2} e^X dX = [e^X]_0^{1+P_1+P_2} = e^{1+P_1+P_2} - e^0 = e^{1+P_1+P_2} - 1.$

The canonical formula way:

$I^{-1} \circ I\big(f(X)\big) = e^{x_0} + P_1[e^{x_0+x_1} - e^{x_0}] + P_2[e^{x_0+x_1+x_2} - e^{x_0+x_1}]$

We have:

$$\int_0^1 e^{x_0} dx_0 = e - 1, \int_0^2 e^{e^{x_0+x_1}} d(x_0+x_1) = e^2 - 1, \int_0^3 e^{x_0+x_1+x_2} d(x_0+x_1+x_2) = e^3 - 1$$

Thus, $\int_0^{1+P_1+P_2} e^X dX = e - 1 + P_1[e^2 - 1 - e + 1] + P_2[e^3 - 1 - e^2 + 1] = e - 1 + P_1[e^2 - e] + P_2[e^3 - e^2] = e^{1+P_1+P_2} - 1$

Applications to differential equations.

Example.

Solve the equation $\acute{Y} = C; Y = y_0 + y_1P_1 + y_2P_2$ is a function, and $C = c_0 + c_1P_1 + c_2P_2$ is a constant.

We have $y_0 = f_0, y_1 = f_1, y_2 = f_2: R \to R.$

$$\acute{Y} = (y_0)'_{x_0} + P_1\left[(y_0+y_1)'_{x_0+x_1} - (y_0)'_{x_0}\right] +$$

$P_2\left[(y_0+y_1+y_2)'_{x_0+x_1+x_2} - (y_0+y_1)'_{x_0+x_1}\right] = c_0 + c_1P_1 + c_2P_2,$

so that:

$$\begin{cases} (y_0)'_{x_0} = c_0 \\ (y_0+y_1)'_{x_0+x_1} = c_1 \\ (y_0+y_1+y_2)'_{x_0+x_1+x_2} = c_2 \end{cases} \Longrightarrow \begin{cases} y_0 = c_0x_0 + m_0 \\ y_0+y_1 = c_1(x_0+x_1) + m_1 \\ y_0+y_1+y_2 = c_2(x_0+x_1+x_2) + m_2 \end{cases}$$

This implies that:

$$Y = (c_0x_0 + m_0) + P_1[c_1(x_0+x_1) + m_1 - (c_0x_0+m_0)] +$$

$P_2[c_2(x_0+x_1+x_2) + m_2 - c_1(x_0+x_1) - m_1]$; x_i are real variables, m_i are real constants.

Example.

Solve the differential equation $\acute{Y} = CY$, where $C = c_0 + c_1P_1 + c_2P_2, Y = y_0 + y_1P_1 + y_2P_2$.

Solution.

$\acute{Y} = CY$ equivalents:

$$\begin{cases} (y_0)'_{x_0} = c_0y_0 \\ (y_0+y_1)'_{x_0+x_1} = (c_0+c_1)(y_0+y_1) \\ (y_0+y_1+y_2)'_{x_0+x_1+x_2} = (c_0+c_1+c_2)(y_0+y_1+y_2) \end{cases}$$

So that:

$$\begin{cases} y_0 = k_0e^{c_0x_0} \\ y_0+y_1 = k_1e^{(c_0+c_1)(x_0+x_1)} \\ y_0+y_1+y_2 = k_2e^{(c_0+c_1+c_2)(x_0+x_1+x_2)} \end{cases}$$

Thus: $Y = k_0e^{c_0x_0} + P_1\left[k_1e^{(c_0+c_1)(x_0+x_1)} - k_0e^{c_0x_0}\right] + P_2\left[k_2e^{(c_0+c_1+c_2)(x_0+x_1+x_2)} - k_1e^{(c_0+c_1)(x_0+x_1)}\right]$.

Example.

Solve the differential equation $Y'' = C$, where $C = c_0 + c_1P_1 + c_2P_2, Y = y_0 + y_1P_1 + y_2P_2$.

Solution.

$Y'' = C$ equivalents:

$$\begin{cases} (Y_{x_0})'' = c_0 \\ (y_0 + y_1)_{x_0+x_1}'' = c_1 \\ (y_0 + y_1 + y_2)_{x_0+x_1+x_2}'' = c_2 \end{cases}$$

So that:

$$\begin{cases} y_0 = \frac{c_0}{2} {x_0}^2 + m_0 x_0 + n_0; m_0, n_0 \in R \\ y_0 + y_1 = \frac{c_1}{2}(x_0 + x_1)^2 + m_1(x_0 + x_1) + n_1; m_1, n_1 \in R \\ y_0 + y_1 + y_2 = \frac{c_2}{2}(x_0 + x_1 + x_2)^2 + m_2(x_0 + x_1 + x_2) + n_2; m_2, n_2 \in R \end{cases}$$

And

$$Y = \left(\frac{c_0}{2}{x_0}^2 + m_0 x_0 + n_0\right) +$$

$$P_1\left[\frac{c_1}{2}(x_0 + x_1)^2 + m_1(x_0 + x_1) + n_1 - \frac{c_0}{2}{x_0}^2 - m_0 x_0 - n_0\right] +$$

$$P_2\left[\frac{c_2}{2}(x_0 + x_1 + x_2)^2 + m_2(x_0 + x_1 + x_2) + n_2 - \frac{c_1}{2}(x_0 + x_1)^2 - m_1(x_0 + x_1) - n_1\right].$$

Applications to geometric shapes:

Definition.

1). We define the symbolic 2-plithogenic circle as follows:

$(X - A)^2 + (Y - B)^2 = R^2; A = a_0 + a_1 P_1 + a_2 P_2, B = b_0 + b_1 P_1 + b_2 P_2, R = r_0 + r_1 P_1 + r_2 P_2, Y = y_0 + y_1 P_1 + y_2 P_2, x = x_0 + x_1 P_1 + x_2 P_2$, with $a_i, b_i, r_i, x_i, y_i \in R$

2). We define the symbolic 2-plithogenic sphere as follows:

$(X - A)^2 + (Y - B)^2 + (Z - C)^2 = R^2; X, A, B, C, R, Y, Z \in 2 - SP_R$

3). We define the symbolic 2-plithogenic ellipse as follows:

$\frac{(X-A)^2}{T^2} + \frac{(Y-B)^2}{S^2} = 1; X, A, B, T, S, Y \in 2 - SP_R$ and T, S invertible.

4). We define the symbolic 2-plithogenic hyperbola as follows:

$\frac{(X-A)^2}{T^2} - \frac{(Y-B)^2}{S^2} = 1; X, A, B, T, S, Y \in 2 - SP_R$ and T, S invertible.

Example.

1). $(X - 1 - P_1 + P_2)^2 + (Y - 3 + 2P_1 - P_2)^2 = (1 + P_2)^2$ is a 2-plithogenic circle.

2). $(X - 10 + P_1)^2 + (Y + P_2)^2 + (Z - P_1 + P_2)^2 = (1 + P_1 + 13P_2)^2$ is a 2-plithogenic sphere.

3). $\frac{(X-P_1)^2}{(1+P_1+P_2)^2} + \frac{(Y+P_1-P_2)^2}{(2-P_1+5P_2)^2} = 1$ is a 2-plithogenic ellipse.

4). $\frac{(X-2P_2)^2}{(1+P_1+3P_2)^2}-\frac{(Y+1+4P_2)^2}{(13-2P_1+P_2)^2}=1$ is a 2-plithogenic hyperbola.

Theorem.

1. Any symbolic 2-plithogenic circle is equivalent to three classical circles.
2. Any symbolic 2-plithogenic sphare is equivalent to three classical spheres.
3. Any symbolic 2-plithogenic ellipse is equivalent to three classical ellipses.
4. Any symbolic 2-plithogenic hyperbola is equivalent to three classical hyperbolas.

Proof.

1. Consider the symbolic 2-plithogenic circle:

$(X-A)^2+(Y-B)^2=R^2$, then by using the isomprphism defined before, we get:

$$I[(X-A)^2]=[I(X)-I(A)]^2$$
$$=\left(x_0-a_0,(x_0+x_1)-(a_0+a_1),(x_0+x_1+x_2)-(a_0+a_1+a_2)\right)^2$$
$$=\left((x_0-a_0)^2,\left((x_0+x_1)-(a_0+a_1)\right)^2,\left((x_0+x_1+x_2)-(a_0+a_1+a_2)\right)^2\right)$$

$$I[(Y-B)^2]=[I(Y)-I(B)]^2$$
$$=\left((y_0-b_0)^2,\left((y_0+y_1)-(b_0+b_1)\right)^2,\left((y_0+y_1+y_2)-(b_0+b_1+b_2)\right)^2\right)$$

$$I(R^2)=[I(R)]^2=({r_0}^2,(r_0+r_1)^2,(r_0+r_1+r_2)^2)$$

Thus, it is equivalent to:

$$\begin{cases} (x_0-a_0)^2+(y_0-b_0)^2={r_0}^2 \\ \left((x_0+x_1)-(a_0+a_1)\right)^2+\left((y_0+y_1)-(b_0+b_1)\right)^2=(r_0+r_1)^2 \\ \left((x_0+x_1+x_2)-(a_0+a_1+a_2)\right)^2+\left((y_0+y_1+y_2)-(b_0+b_1+b_2)\right)^2=(r_0+r_1+r_2)^2 \end{cases}$$

2. Consider the sphere $(X-A)^2+(Y-B)^2+(Z-C)^2=R^2$, we use the isomorphism I, to get:

$$I[(X-A)^2]=\left((x_0-a_0)^2,\left((x_0+x_1)-(a_0+a_1)\right)^2,\left((x_0+x_1+x_2)-(a_0+a_1+a_2)\right)^2\right)$$

$$I[(Y-B)^2]=\left((y_0-b_0)^2,\left((y_0+y_1)-(b_0+b_1)\right)^2,\left((y_0+y_1+y_2)-(b_0+b_1+b_2)\right)^2\right)$$

$$I[(Z-C)^2]=\left((z_0-c_0)^2,\left((z_0+z_1)-(c_0+c_1)\right)^2,\left((z_0+z_1+z_2)-(c_0+c_1+c_2)\right)^2\right)$$

$I(R^2)=[I(R)]^2=({r_0}^2,(r_0+r_1)^2,(r_0+r_1+r_2)^2)$, hence we get:

$$\begin{cases}(x_0-a_0)^2+(y_0-b_0)^2+(z_0-c_0)^2={r_0}^2\\ \left((x_0+x_1)-(a_0+a_1)\right)^2+\left((y_0+y_1)-(b_0+b_1)\right)^2+\left((z_0+z_1)-(c_0+c_1)\right)^2=(\\ \left((x_0+x_1+x_2)-(a_0+a_1+a_2)\right)^2+\left((y_0+y_1+y_2)-(b_0+b_1+b_2)\right)^2+\left((z_0+z_1+z_2)-(c_0+c\end{cases}$$

3. Consider the ellipse $\frac{(X-A)^2}{T^2}+\frac{(Y-B)^2}{S^2}=1$, we use the isomorphism I to get:

$$I\left[\frac{(X-A)^2}{T^2}\right]=\left(\frac{(x_0-a_0)^2}{{t_0}^2},\frac{\left((x_0+x_1)-(a_0+a_1)\right)^2}{(t_0+t_1)^2},\frac{\left((x_0+x_1+x_2)-(a_0+a_1+a_2)\right)^2}{(t_0+t_1+t_2)^2}\right)$$

$$I\left[\frac{(Y-B)^2}{S^2}\right]=\left(\frac{(y_0-b_0)^2}{{s_0}^2},\frac{\left((y_0+y_1)-(b_0+b_1)\right)^2}{(s_0+s_1)^2},\frac{\left((y_0+y_1+y_2)-(b_0+b_1+b_2)\right)^2}{(s_0+s_1+s_2)^2}\right)$$

$I(1)=(1,1,1)$, thus:

$$\begin{cases}\frac{(x_0-a_0)^2}{{t_0}^2}+\frac{(y_0-b_0)^2}{{s_0}^2}=1\\ \frac{\left((x_0+x_1)-(a_0+a_1)\right)^2}{(t_0+t_1)^2}+\frac{\left((y_0+y_1)-(b_0+b_1)\right)^2}{(s_0+s_1)^2}=1\\ \frac{\left((x_0+x_1+x_2)-(a_0+a_1+a_2)\right)^2}{(t_0+t_1+t_2)^2}+\frac{\left((y_0+y_1+y_2)-(b_0+b_1+b_2)\right)^2}{(s_0+s_1+s_2)^2}=1\end{cases}$$

4. Consider the hyperbola $\frac{(X-A)^2}{T^2}-\frac{(Y-B)^2}{S^2}=1$, by a similar discussion, we get

$$\begin{cases}\frac{(x_0-a_0)^2}{{t_0}^2}-\frac{(y_0-b_0)^2}{{s_0}^2}=1\\ \frac{\left((x_0+x_1)-(a_0+a_1)\right)^2}{(t_0+t_1)^2}-\frac{\left((y_0+y_1)-(b_0+b_1)\right)^2}{(s_0+s_1)^2}=1\\ \frac{\left((x_0+x_1+x_2)-(a_0+a_1+a_2)\right)^2}{(t_0+t_1+t_2)^2}-\frac{\left((y_0+y_1+y_2)-(b_0+b_1+b_2)\right)^2}{(s_0+s_1+s_2)^2}=1\end{cases}$$

Example.

Consider the symbolic 2-plithogenic circle:

$(X-1+P_1)^2+(Y-3+2P_1+2P_2)^2=(1+4P_1+4P_2)^2$, it is equivalent to:

$$\begin{cases} (x_0-1)^2+(y_0-3)^2=1 \\ \left((x_0+x_1)\right)^2+\left((y_0+y_1)-1\right)^2=25 \\ \left((x_0+x_1+x_2)\right)^2+\left((y_0+y_1+y_2)\right)^2=81 \end{cases}$$

The first circle has $(1,3)$ as entre and radius 1.

The second circle has $(0,1)$ as entre and radius 5.

The third circle has $(0,-1)$ as entre and radius 9.

Example:

Consider the symbolic 2-plithogenic sphere:

$(X-1+P_1+3P_2)^2+(Y-P_2)^2+(Z-4+P_2)^2=(3-P_1+P_2)^2$, it is equivalent to:

$$\begin{cases} (x_0-2)^2+(y_0)^2+(z_0-4)^2=9 \\ \left((x_0+x_1)-1\right)^2+\left((y_0+y_1)\right)^2+\left((z_0+z_1)-1\right)^2=4 \\ \left((x_0+x_1+x_2)+4\right)^2+\left((y_0+y_1+y_2)-1\right)^2+\left((z_0+z_1+z_2)-3\right)^2=9 \end{cases}$$

The first sphere has $(2,0,4)$ as entre and radius 3.

The second sphere has $(1,0,4)$ as entre and radius 2.

The third sphere has $(-4,1,3)$ as entre and radius 3.

Example:

Consider the symbolic 2-plithogenic ellipse:

$\frac{(X-1+P_1+3P_2)^2}{(3+P_1+P_2)^2}+\frac{(Y-1-P_2)^2}{(4+2P_1+3P_2)^2}=1$, it is equivalent to:

$$\begin{cases} \frac{(x_0)^2}{3^2}+\frac{(y_0-1)^2}{4^2}=1 \\ \frac{\left((x_0+x_1)-4\right)^2}{4^2}+\frac{\left((y_0+y_1)-1\right)^2}{6^2}=1 \\ \frac{\left((x_0+x_1+x_2)-3\right)^2}{5^2}+\frac{\left((y_0+y_1+y_2)-2\right)^2}{9^2}=1 \end{cases}$$

Example:

Consider the symbolic 2-plithogenic hyperbola:

$\frac{(X-1)^2}{(5+2P_1)^2}-\frac{(Y-P_1)^2}{(1+P_1+P_2)^2}=1$, it is equivalent to:

$$\begin{cases} \dfrac{(x_0-1)^2}{5^2}-\dfrac{(y_0)^2}{1^2}=1 \\ \dfrac{\left((x_0+x_1)-1\right)^2}{7^2}-\dfrac{\left((y_0+y_1)-1\right)^2}{2^2}=1 \\ \dfrac{\left((x_0+x_1+x_2)-1\right)^2}{7^2}-\dfrac{\left((y_0+y_1+y_2)-1\right)^2}{3^2}=1 \end{cases}$$

Example.

Let us the parametric representation of the symbolic 2-plithogenic ellipse:

$$\frac{(X-1-P_1-P_2)^2}{(2+P_1)^2}+\frac{(Y-2-3P_1)^2}{(1+2P_1-P_2)^2}=1$$

The previous ellipse is equivalent to:

$$\begin{cases} \dfrac{(x_0-1)^2}{2^2}+\dfrac{(y_0-2)^2}{1^2}=1 \ldots (1) \\ \dfrac{\left((x_0+x_1)-2\right)^2}{3^2}+\dfrac{\left((y_0+y_1)-5\right)^2}{3^2}=1 \ \ldots (2) \\ \dfrac{\left((x_0+x_1+x_2)-3\right)^2}{3^2}+\dfrac{\left((y_0+y_1+y_2)-5\right)^2}{2^2}=1 \ \ldots (3) \end{cases}$$

Equation (1) implies $\frac{x_0-1}{2}=cos\theta_0$, $\frac{y_0-2}{1}=sin\theta_0$, hence $x_0=2cos\theta_0+1, y_0=sin\theta_0+2$.

Equation (2) implies $\frac{(x_0+x_1)-2}{3}=cos\theta_1, \frac{(y_0+y_1)-5}{2}=sin\theta_1$, hence $x_1=3cos\theta_1+2-x_0=3cos\theta_1-2cos\theta_0+1, y_1=2sin\theta_1+5-y_0=2sin\theta_1-sin\theta_0+3$

Equation (3) implies $\frac{(x_0+x_1+x_2)-3}{3}=cos\theta_2$, $\frac{(y_0+y_1+y_2)-5}{2}=sin\theta_2$, hence $x_2=3cos\theta_2-(x_0+x_1)+3=3cos\theta_2-3cos\theta_1+1, y_2=2sin\theta_2-(y_0+y_1)+5=2sin\theta_2-2sin\theta_1$

This means that:

$X=(2cos\theta_0+1)+P_1[3cos\theta_1-2cos\theta_0+1]+P_2[3cos\theta_2-3cos\theta_1+1]$

$Y=(sin\theta_0+2)+P_1[2sin\theta_1-sin\theta_0+3]+P_2[2sin\theta_2-2sin\theta_1]$.

Example.

Consider the symbolic 2-plithogenic ellipse:

$$\frac{(X-2-4P_1+3P_2)^2}{(1+5P_1+7P_2)^2}+\frac{(Y+1+P_2)^2}{(3-P_1-P_2)^2}=1$$

it is equivalent to:

$$\begin{cases} \dfrac{(x_0-2)^2}{1}+\dfrac{(y_0+1)^2}{3^2}=1 \\ \dfrac{[(x_0+x_1)-6]^2}{6^2}+\dfrac{[(y_0+y_1)+1]^2}{2^2}=1 \\ \dfrac{[(x_0+x_1+x_2)-3]^2}{13^2}+\dfrac{[(y_0+y_1+y_2)+2]^2}{1}=1 \end{cases}$$

Example.

Consider the symbolic 2-plithogenic circle:

$(X-2-4P_1+3P_2)^2+(Y+1+P_2)^2=1$

It is equivalent to:

$$\begin{cases} (x_0-2)^2+(y_0+1)^2=1 \\ [(x_0+x_1)-6]^2+[(y_0+y_1)+1]^2=1 \\ [(x_0+x_1+x_2)-3]^2+[(y_0+y_1+y_2)+2]^2=1 \end{cases}$$

Example.

Consider the symbolic 2-plithogenic hyperbola:

$$\frac{(X-2-4P_1+3P_2)^2}{(1+5P_1+7P_2)^2}-\frac{(Y+1+P_2)^2}{(3-P_1-P_2)^2}=1$$

it is equivalent to:

$$\begin{cases} \dfrac{(x_0-2)^2}{1}-\dfrac{(y_0+1)^2}{3^2}=1 \\ \dfrac{[(x_0+x_1)-6]^2}{6^2}-\dfrac{[(y_0+y_1)+1]^2}{2^2}=1 \\ \dfrac{[(x_0+x_1+x_2)-3]^2}{13^2}-\dfrac{[(y_0+y_1+y_2)+2]^2}{1}=1 \end{cases}$$

Conclusion

In this paper, we defined for the first time a special AH-isometry on the symbolic 2-plithogenic fields of reals, and we used this isometry to obtain many formulas and properties about the symbolic 2-plithogenic analytical concepts such as differentiability, continuity, and integrability. Also, symbolic 2-plithogenic conic sections were handled by using the mentioned isometry. In addition, many related examples were presented to clarify the novelty of our work.

References

1. Nader Mahmoud Taffach , Ahmed Hatip., "A Review on Symbolic 2-Plithogenic Algebraic Structures " Galoitica Journal Of Mathematical Structures and Applications, Vol.5, 2023.
2. Nader Mahmoud Taffach , Ahmed Hatip.," A Brief Review on The Symbolic 2-Plithogenic Number Theory and Algebraic Equations ", Galoitica Journal Of Mathematical Structures and Applications, Vol.5, 2023.
3. Merkepci, H., and Abobala, M., " On The Symbolic 2-Plithogenic Rings", International Journal of Neutrosophic Science, 2023.
4. Smarandache, F., " Introduction to the Symbolic Plithogenic Algebraic Structures (revisited)", Neutrosophic Sets and Systems, vol. 53, 2023.
5. Taffach, N., " An Introduction to Symbolic 2-Plithogenic Vector Spaces Generated from The Fusion of Symbolic Plithogenic Sets and Vector Spaces", Neutrosophic Sets and Systems, Vol 54, 2023.
6. Taffach, N., and Ben Othman, K., " An Introduction to Symbolic 2-Plithogenic Modules Over Symbolic 2-Plithogenic Rings", Neutrosophic Sets and Systems, Vol 54, 2023.
7. Merkepci, H., and Rawashdeh, A., " On The Symbolic 2-Plithogenic Number Theory and Integers ", Neutrosophic Sets and Systems, Vol 54, 2023.
8. Albasheer, O., Hajjari., A., and Dalla., R., " On The Symbolic 3-Plithogenic Rings and TheirAlgebraic Properties", Neutrosophic Sets and Systems, Vol 54, 2023.
9. Abobala, M., On Refined Neutrosophic Matrices and Their Applications In Refined Neutrosophic Algebraic Equations, Journal Of Mathematics, Hindawi, 2021
10. Olgun, N., Hatip, A., Bal, M., and Abobala, M., " A Novel Approach To Necessary and Sufficient Conditions For The Diagonalization of Refined

Neutrosophic Matrices", International Journal of neutrosophic Science, Vol. 16, pp. 72-79, 2021.

11. Merkepci, H., and Abobala, M., " The Application of AH-isometry In The Study Of Neutrosophic Conic Sections", Galoitica Journal Of Mathematical Structures And Applications, Vol.2, 2022.
12. Abobala, M., and Zeina, M.B., " A Study Of Neutrosophic Real Analysis By Using One Dimensional Geometric AH-Isometry", Galoitica Journal Of Mathematical Structures And Applications, Vol.3, 2023.
13. Khaldi, A., " A Study On Split-Complex Vector Spaces", Neoma Journal Of Mathematics and Computer Science, 2023.
14. Ahmad, K., " On Some Split-Complex Diophantine Equations", Neoma Journal Of Mathematics and Computer Science, 2023.
15. Ali, R., " On The Weak Fuzzy Complex Inner Products On Weak Fuzzy Complex Vector Spaces", Neoma Journal Of Mathematics and Computer Science, 2023.
16. Von Shtawzen, O., " Conjectures For Invertible Diophantine Equations Of 3-Cyclic and 4-Cyclic Refined Integers", Journal Of Neutrosophic And Fuzzy Systems, Vol.3, 2022.
17. Ben Othman, K., Albasheer, O., Nadweh, R., Von Shtawzen, O., and Ali, R., "On The Symbolic 6-Plithogenic and 7-Plithogenic Rings", Galoitica Journal of Mathematical Structures and Applications, Vol.8, 2023.
18. Bisher Ziena, M., and Abobala, M., " On The Refined Neutrosophic Real Analysis Based on Refined Neutrosophic Algebraic AH-Isometry", Neutrosophic Sets and Systems, vol. 54, 2023.
19. M. B. Zeina and M. Abobala, "A Novel Approach of Neutrosophic Continuous Probability Distributions using AH-Isometry with Applications

in Medicine," in *Cognitive Intelligence with Neutrosophic Statistics in Bioinformatics*, Elsevier, 2023.

20. M. B. Zeina, N. Altounji, M. Abobala, and Y. Karmouta, "Introduction to Symbolic 2-Plithogenic Probability Theory," *Galoitica: Journal of Mathematical Structures and Applications*, vol. 7, no. 1, 2023.
21. Ali, R., and Hasan, Z., "An Introduction To The Symbolic 3-Plithogenic Vector Spaces", Galoitica Journal Of Mathematical Structures and Applications, vol. 6, 2023.
22. Rawashdeh, A., "An Introduction To The Symbolic 3-plithogenic Number Theory", Neoma Journal Of Mathematics and Computer Science, 2023.
23. Ben Othman, K., "On Some Algorithms For Solving Symbolic 3-Plithogenic Equations", Neoma Journal Of Mathematics and Computer Science, 2023.
24. Florentin Smarandache, Plithogenic Algebraic Structures. Chapter in "Nidus idearum Scilogs, V: joining the dots" (third version), Pons Publishing Brussels, pp. 123-125, 2019.
25. Florentin Smarandache: Plithogenic Set, an Extension of Crisp, Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Sets – Revisited, *Neutrosophic Sets and Systems*, vol. 21, 2018, pp. 153-166.
26. Sarkis, M., " On The Solutions Of Fermat's Diophantine Equation In 3-refined Neutrosophic Ring of Integers", Neoma Journal of Mathematics and Computer Science, 2023.
27. Abobala, M., "On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations", Mathematical Problems in Engineering, Hindawi, 2021
28. Abobala, M., "A Study Of Nil Ideals and Kothe's Conjecture In Neutrosophic Rings", International Journal of Mathematics and Mathematical Sciences, hindawi, 2021

29. Abobala, M., "On The Characterization of Maximal and Minimal Ideals In Several Neutrosophic Rings", Neutrosophic sets and systems, Vol. 45, 2021.
30. Abobala, M., Bal, M., Aswad, M., "A Short Note On Some Novel Applications of Semi Module Homomorphisms", International journal of neutrosophic science, 2022.
31. Ben Othman, K., Von Shtawzen, O., Khaldi, A., and Ali, R., "On The Concept Of Symbolic 7-Plithogenic Real Matrices", Pure Mathematics For Theoretical Computer Science, Vol.1, 2023.
32. Ben Othman, K., Von Shtawzen, O., Khaldi, A., and Ali, R., "On The Symbolic 8-Plithogenic Matrices", Pure Mathematics For Theoretical Computer Science, Vol.1, 2023.
33. Abualkishik, A., Almajed, R., Thompson, W., "Improving The Performance of Fog-assisted Internet of Things Networks Using Bipolar Trapezoidal Neutrosophic Sets", Journal of Wireless and Ad Hoc Communication, 2023.

Received 10/4/2023, Accepted 7/10/2023