



# Pairwise Neutrosophic $b$ -Continuous Function in Neutrosophic Bitopological Spaces

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**Abstract:** The main focus of this article is to procure the notions of pairwise neutrosophic continuous and pairwise neutrosophic  $b$ -continuous mappings in neutrosophic bitopological spaces. Then, we formulate some results on them via neutrosophic bitopological spaces.

**Keywords:** Neutrosophic Topology; Neutrosophic Bitopology; Pairwise Neutrosophic  $b$ -Interior; Pairwise Neutrosophic  $b$ -Closure; Pairwise Neutrosophic Continuous.

## 1. Introduction

Zadeh [31] presented the notions of fuzzy set (in short FS) in the year 1965. Afterwards, Chang [4] applied the idea of topology on fuzzy sets and introduced the fuzzy topological space. In the year 2017, Dutta and Tripathy [15] studied on fuzzy  $b$ - $\theta$  open sets via fuzzy topological space. Later on, Smarandache [23] grounded the idea of neutrosophic set (in short N-set) in the year 1998, as an extension of the concept of intuitionistic fuzzy set (in short IF-set) [3], where every element has three independent membership values namely truth, indeterminacy, and false membership values respectively. Afterwards, Salama and Alblowi [21] applied the notions of topology on N-sets and introduced neutrosophic topological space (in short NT-space) by extending the notions of fuzzy topological spaces. Salama and Alblowi [22] also defined generalized N-set and introduced the concept of generalized NT-space. Later on, Arokiarani et al. [2] introduced the ideas of neutrosophic point and studied some functions in neutrosophic topological spaces. The notions of neutrosophic pre-open (in short NP-O) and neutrosophic pre-closed (in short NP-C) sets via NT-spaces are studied by Rao and Srinivasa [20]. The idea of  $b$ -open sets via topological spaces was established by Andrijevic [1]. Afterwards, Ebenanjar et al. [16] presents the concept of neutrosophic  $b$ -open set (in short N- $b$ -O-set) via NT-spaces. In the year 2020, Das and Pramanik [8] presents the generalized neutrosophic  $b$ -open sets in NT-spaces. The notions of neutrosophic  $\Phi$ -open set and neutrosophic  $\Phi$ -continuous functions via NT-spaces was also presented by Das and Pramanik [9]. The concept of neutrosophic simply soft open set in neutrosophic soft topological space was studied by Das and Pramanik [10]. In the year 2021, Das and Tripathy [14] presented the notions of neutrosophic simply  $b$ -open set via NT-spaces. In the year 2020, Das and Tripathy [12] grounded the notions of neutrosophic multiset and applied topology on it. In the year 2021, Das et al. [5] studied the concept of quadripartitioned neutrosophic topological spaces. The notion of bitopological space was introduced by Kelly [17] in the year 1963. In the year 2011, Tripathy and Sarma [26] studied on  $b$ -locally open sets via bitopological spaces. The idea of pairwise  $b$ -locally

open and  $b$ -locally closed functions in bitopological spaces was studied by Tripathy and Sarma [27]. Tripathy and Sarma [28] also studied on weakly  $b$ -continuous mapping via bitopological spaces in the year 2013. Later on, the concept of generalized  $b$ -closed sets in ideal bitopological spaces was studied by Tripathy and Sarma [29]. Afterwards, Tripathy and Debnath [25] presented the notions of fuzzy  $b$ -locally open sets in fuzzy bitopological space. Thereafter, Ozturk and Ozkan [19] introduced the idea of neutrosophic bitopological space (in short NBi-T-space) in the year 2019. Recently, Das and Tripathy [13] presented the idea of pairwise N- $b$ -O-sets and studied their different properties.

The main focus of this article is to procure the notions of pairwise  $\tau_{ij}$ -neutrosophic- $b$ -interior (in short P- $\tau_{ij}$ -N $b$ -int), pairwise  $\tau_{ij}$ -neutrosophic- $b$ -closure (in short P- $\tau_{ij}$ -N $b$ -cl), pairwise neutrosophic continuous mapping (in short P-N-C-mapping), pairwise neutrosophic  $b$ -continuous mapping (in short pairwise N- $b$ C-mapping) via NBi-T-spaces.

## 2. Preliminaries and Definitions:

The notion of N-set is defined as follows:

Let  $X$  be a fixed set. Then, an N-set [23]  $L$  over  $X$  is denoted as follows:  
 $L = \{(t, T_L(t), I_L(t), F_L(t)) : t \in X\}$ , where  $T_L, I_L, F_L : X \rightarrow [0,1]$  are called the truth-membership, indeterminacy-membership and false-membership functions and  $0 \leq T_L(t) + I_L(t) + F_L(t) \leq 3$ , for all  $t \in X$ .

The neutrosophic null set ( $0_N$ ) and neutrosophic whole set ( $1_N$ ) over a fixed set  $X$  are defined as follows:

- (i)  $0_N = \{(t, 0, 0, 1) : t \in X\}$ ;
- (ii)  $1_N = \{(t, 1, 0, 0) : t \in X\}$ .

The N-sets  $0_N$  and  $1_N$  also has three other representations. They are given below:

$$0_N = \{(t, 0, 0, 0) : t \in X\} \ \& \ 1_N = \{(t, 1, 1, 1) : t \in X\};$$

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$$0_N = \{(t, 0, 1, 1) : t \in X\} \ \& \ 1_N = \{(t, 1, 1, 0) : t \in X\}.$$

Let  $p, q, r \in [0,1]$ . An neutrosophic point (in short N-point) [2]  $x_{p,q,r}$  is an N-set over  $X$  given by

$$x_{p,q,r}(y) = \begin{cases} (p, q, r), & \text{if } x = y, \\ (0, 0, 1), & \text{if } x \neq y, \end{cases}$$

where  $p, q, r$  denotes the truth, indeterminacy and false membership value of  $x_{p,q,r}$ .

The notion of NT-space is defined as follows:

A family  $\tau$  of N-sets over  $X$  is called an [21] neutrosophic topology (in short N-topology) on  $X$  if the following axioms hold:

- (i)  $0_N, 1_N \in \tau$ ;
- (ii)  $L_1, L_2 \in \tau \Rightarrow L_1 \cap L_2 \in \tau$ ;
- (iii)  $\cup L_i \in \tau$ , for every  $\{L_i : i \in \Delta\} \subseteq \tau$ , where  $\Delta$  is the support set.

Then,  $(X, \tau)$  is called an NT-space. Each element of  $\tau$  is an neutrosophic open set (in short NO-set). If  $L$  is an NO-set in  $(X, \tau)$ , then  $L^c$  is called an neutrosophic closed set (in short NC-set).

The notion of NBI-T-space is defined as follows:

Let  $\tau_1$  and  $\tau_2$  be two different N-topologies on  $X$ . Then,  $(X, \tau_1, \tau_2)$  is [19] called an NBI-T-space. An N-set  $L$  is called a pairwise NO-set in  $(X, \tau_1, \tau_2)$ , if there exist an NO-set  $L_1$  in  $\tau_1$  and an NO-set  $L_2$  in  $\tau_2$  such that  $L=L_1 \cup L_2$ . The complement of  $L$  i.e.,  $L^c$  is called a pairwise neutrosophic closed set (in short pairwise NC-set) in  $(X, \tau_1, \tau_2)$ .

**Remark 2.1.**[13] In an NBI-T-space  $(X, \tau_1, \tau_2)$ , every  $\tau_i$ -NO-set is a pairwise  $\tau_{ij}$ -NO-set.

**Remark 2.2.** Let  $G$  be an N-set over  $X$  and  $(X, \tau_1, \tau_2)$  be an NBI-T-space. Then, we shall use the following notations throughout the article:

(i)  $N_{cl}^i(G)$ = Neutrosophic closure of  $G$  in  $(X, \tau_i)$  ( $i=1, 2$ );

(ii)  $N_{int}^i(G)$ = Neutrosophic interior of  $G$  in  $(X, \tau_i)$  ( $i=1, 2$ ).

**Definition 2.1.**[13] Let  $(X, \tau_1, \tau_2)$  be an NBI-T-space. Then,  $P$  is called a

(i)  $\tau_i$ -neutrosophic semi-open set (in short  $\tau_i$ -NSO-set) if and only if  $P \subseteq N_{cl}^i N_{int}^i(P)$ ;

(ii)  $\tau_i$ -neutrosophic pre-open set (in short  $\tau_i$ -NPO-set) if and only if  $P \subseteq N_{int}^i N_{cl}^i(P)$ ;

(iii)  $\tau_i$ -neutrosophic  $b$ -open set (in short  $\tau_i$ -N-bO-set) if and only if  $P \subseteq N_{cl}^i N_{int}^i(P) \cup N_{int}^i N_{cl}^i(P)$ .

**Remark 2.3.**[13] Let  $(X, \tau_1, \tau_2)$  be an NBI-T-space. Then, an N-set  $P$  over  $X$  is called a  $\tau_i$ -neutrosophic  $b$ -closed set (in short  $\tau_i$ -N-bC-set) if and only if  $P^c$  is a  $\tau_i$ -N-bO-set.

**Proposition 2.1.**[13] In an NBI-T-space  $(X, \tau_1, \tau_2)$ , if  $P$  is  $\tau_i$ -NSO-set ( $\tau_i$ -NPO-set), then  $P$  is a  $\tau_i$ -N-bO-set.

**Proposition 2.2.**[13] Let  $(X, \tau_1, \tau_2)$  be an NBI-T-space. Then, the union of any two  $\tau_i$ -N-bO-sets is a  $\tau_i$ -N-bO-set.

**Definition 2.2.**[13] Let  $(X, \tau_1, \tau_2)$  be an NBI-T-space. Then,  $P$  is called a

(i)  $\tau_{ij}$ -neutrosophic semi-open set (in short  $\tau_{ij}$ -NSO-set) if and only if  $P \subseteq N_{cl}^i N_{int}^j(P)$ ;

(ii)  $\tau_{ij}$ -neutrosophic pre-open set (in short  $\tau_{ij}$ -NPO-set) if and only if  $P \subseteq N_{int}^j N_{cl}^i(P)$ ;

(iii)  $\tau_{ij}$ -neutrosophic  $b$ -open set (in short  $\tau_{ij}$ -N-bO-set) if and only if  $P \subseteq N_{cl}^i N_{int}^j(P) \cup N_{int}^j N_{cl}^i(P)$ .

**Remark 2.4.**[13] An N-set  $L$  over  $X$  is called a  $\tau_{ij}$ -neutrosophic  $b$ -closed set (in short  $\tau_{ij}$ -N-bC-set) if and only if  $L^c$  is a  $\tau_{ij}$ -N-bO-set in  $(X, \tau_1, \tau_2)$ .

**Theorem 2.1.**[13] Let  $(X, \tau_1, \tau_2)$  be an NBI-T-space. Then, every  $\tau_{ij}$ -NSO-set ( $\tau_{ij}$ -NPO-set) is a  $\tau_{ij}$ -N-bO-set.

**Definition 2.3.**[13] An N-set  $L$  is called a pairwise  $\tau_{ij}$ -NPO-set (pairwise  $\tau_{ij}$ -NSO-set) in an NBI-T-space  $(X, \tau_1, \tau_2)$  if  $L=K \cup M$ , where  $K$  is a  $\tau_{ij}$ -NPO-set ( $\tau_{ij}$ -NSO-set) and  $M$  is a  $\tau_{ji}$ -NPO-set ( $\tau_{ji}$ -NSO-set) in  $(X, \tau_1, \tau_2)$ .

**Definition 2.4.**[13] An N-set  $L$  is called a pairwise  $\tau_{ij}$ -N-bO-set in a NBI-T-space  $(X, \tau_1, \tau_2)$  if  $L=K \cup M$ , where  $K$  is a  $\tau_{ij}$ -N-bO-set and  $M$  is a  $\tau_{ji}$ -N-bO-set in  $(X, \tau_1, \tau_2)$ . If  $L$  is a pairwise  $\tau_{ij}$ -N-bO-set in  $(X, \tau_1, \tau_2)$ , then  $L^c$  is called a pairwise  $\tau_{ij}$ -neutrosophic- $b$ -closed set (in short pairwise  $\tau_{ij}$ -N-bC-set) in  $(X, \tau_1, \tau_2)$ .

**Lemma 2.1.**[13] In an NBI-T-space  $(X, \tau_1, \tau_2)$ , every pairwise  $\tau_{ij}$ -NPO-set (pairwise  $\tau_{ij}$ -NSO-set) is a pairwise  $\tau_{ij}$ -N-bO-set.

**Proposition 2.3.**[13] Let  $(X, \tau_1, \tau_2)$  be an NBI-T-space. Then, the union of two pairwise  $\tau_{ij}$ -N-bO-set in  $(X, \tau_1, \tau_2)$  is also a pairwise  $\tau_{ij}$ -N-bO-set.

**Theorem 2.2.** Let  $(X, \tau_1, \tau_2)$  be an NBI-T-space. Then, the union of two pairwise  $\tau_{ij}$ -NSO-set in  $(X, \tau_1, \tau_2)$  is also a pairwise  $\tau_{ij}$ -NSO-set.

**Proof.** Let  $L$  and  $M$  be two pairwise  $\tau_{ij}$ -NSO-sets in an NBI-T-space  $(X, \tau_1, \tau_2)$ . So, one can write  $L=L_1 \cup L_2$  and  $M=M_1 \cup M_2$ , where  $L_1, M_1$  are  $\tau_{ij}$ -NSO-sets and  $L_2, M_2$  are  $\tau_{ij}$ -NSO-sets in  $(X, \tau_1, \tau_2)$ . Since,  $L_1$  and  $M_1$  are  $\tau_{ij}$ -NSO-sets, so  $L_1 \subseteq N_{cl}^i N_{int}^j(L_1)$  and  $M_1 \subseteq N_{cl}^i N_{int}^j(M_1)$ . Further, Since  $L_2$  and  $M_2$  are  $\tau_{ij}$ -NSO-sets, so  $L_2 \subseteq N_{cl}^j N_{int}^i(L_2)$ ,  $M_2 \subseteq N_{cl}^j N_{int}^i(M_2)$ .

Now,  $L \cup M = (L_1 \cup L_2) \cup (M_1 \cup M_2) = (L_1 \cup M_1) \cup (L_2 \cup M_2)$ .

$$\begin{aligned} \text{Therefore, } L_1 \cup M_1 &\subseteq N_{cl}^i N_{int}^j(L_1) \cup N_{cl}^i N_{int}^j(M_1) \\ &= N_{cl}^i(N_{int}^j(L_1) \cup N_{int}^j(M_1)) \\ &\subseteq N_{cl}^i N_{int}^j(L_1 \cup M_1). \end{aligned}$$

This implies,  $L_1 \cup M_1$  is a  $\tau_{ij}$ -NSO-set in  $(X, \tau_1, \tau_2)$ .

Similarly, it can be established that  $L_2 \cup M_2$  is a  $\tau_{ij}$ -NSO-set in  $(X, \tau_1, \tau_2)$ . Therefore,  $L \cup M$  is a pairwise  $\tau_{ij}$ -NSO-set in  $(X, \tau_1, \tau_2)$ . Hence, the union of two pairwise  $\tau_{ij}$ -NSO-set in  $(X, \tau_1, \tau_2)$  is again a pairwise  $\tau_{ij}$ -NSO-set in  $(X, \tau_1, \tau_2)$ .

**Theorem 2.4.** Let  $(X, \tau_1, \tau_2)$  be an NBI-T-space. Then, the union of two pairwise  $\tau_{ij}$ -NPO-set in  $(X, \tau_1, \tau_2)$  is a pairwise  $\tau_{ij}$ -NPO-set.

**Proof.** Let  $L$  and  $M$  be two pairwise  $\tau_{ij}$ -NPO-sets in an NBI-T-space  $(X, \tau_1, \tau_2)$ . So, one can write  $L=L_1 \cup L_2$  and  $M=M_1 \cup M_2$ , where  $L_1, M_1$  are  $\tau_{ij}$ -NPO-sets and  $L_2, M_2$  are  $\tau_{ij}$ -NPO-sets in  $(X, \tau_1, \tau_2)$ . Since,  $L_1$  and  $M_1$  are  $\tau_{ij}$ -NPO-sets, so  $L_1 \subseteq N_{int}^j N_{cl}^i(L_1)$  and  $M_1 \subseteq N_{int}^j N_{cl}^i(M_1)$ . Further, since  $L_2$  and  $M_2$  are  $\tau_{ij}$ -NPO-sets, so  $L_2 \subseteq N_{int}^i N_{cl}^j(L_2)$  and  $M_2 \subseteq N_{int}^i N_{cl}^j(M_2)$ .

Now,  $L \cup M = (L_1 \cup L_2) \cup (M_1 \cup M_2) = (L_1 \cup M_1) \cup (L_2 \cup M_2)$ .

$$\begin{aligned} \text{Therefore, } L_1 \cup M_1 &\subseteq N_{int}^j N_{cl}^i(L_1) \cup N_{int}^j N_{cl}^i(M_1) \\ &= N_{int}^j(N_{cl}^i(L_1) \cup N_{cl}^i(M_1)) \\ &\subseteq N_{int}^j N_{cl}^i(L_1 \cup M_1). \end{aligned}$$

This implies,  $L_1 \cup M_1$  is a  $\tau_{ij}$ -NPO-set in  $(X, \tau_1, \tau_2)$ . Similarly, it can be established that  $L_2 \cup M_2$  is a  $\tau_{ij}$ -NPO-set in  $(X, \tau_1, \tau_2)$ . Therefore,  $L \cup M$  is a pairwise  $\tau_{ij}$ -NPO-set in  $(X, \tau_1, \tau_2)$ . Hence, the union of two pairwise  $\tau_{ij}$ -NPO-sets in  $(X, \tau_1, \tau_2)$  is again a pairwise  $\tau_{ij}$ -NPO-set.

### 3. Pairwise $b$ -Continuous Function:

In this section, we procure the notions of pairwise  $b$ -continuous functions via neutrosophic bitopological space and formulate some results on it.

**Definition 3.1.** Let  $(X, \tau_1, \tau_2)$  be an NBI-T-space. Then, the pairwise  $\tau_{ij}$ -neutrosophic- $b$ -interior (in short  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}$ ) of an N-set  $L$  is the union of all pairwise  $\tau_{ij}$ -N- $b$ O-sets contained in  $L$ , i.e.  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L) = \cup\{K: K \text{ is a pairwise } \tau_{ij}\text{-N-}b\text{O-set in } X \text{ and } K \subseteq L\}$ .

Clearly,  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L)$  is the largest pairwise  $\tau_{ij}$ -N- $b$ O-set which contained in  $L$ .

**Definition 3.2.** Let  $(X, \tau_1, \tau_2)$  be an NBI-T-space. Then, the pairwise  $\tau_{ij}$ -neutrosophic- $b$ -closure (in short  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}$ ) of an N-set  $L$  is the intersection of all pairwise  $\tau_{ij}$ -N- $b$ C-sets containing  $L$ , i.e.  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L) = \cap\{K: K \text{ is a pairwise } \tau_{ij}\text{-N-}b\text{C-set in } X \text{ and } L \subseteq K\}$ .

Clearly,  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)$  is the smallest pairwise  $\tau_{ij}$ -N- $b$ C-set which containing  $L$ .

**Theorem 3.1.** Let  $L$  and  $K$  be two neutrosophic subsets of an NBI-T-space  $(X, \tau_1, \tau_2)$ . Then,

- (i)  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(0_N) = 0_N$ ,  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(1_N) = 1_N$ ;
- (ii)  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L) \subseteq L$ ;
- (iii)  $L \subseteq M \Rightarrow P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L) \subseteq P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(M)$ ;

(iv)  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L)=L$  if  $L$  is a pairwise  $\tau_{ij}\text{-}N\text{-}bO$ -set.

**Proof.** (i) Straight forward.

(ii) By Definition 3.1, we have  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L)=\cup\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bO\text{-set in } X \text{ and } K\subseteq L\}$ . Since, each  $K\subseteq L$ , so  $\cup\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bO\text{-set in } X \text{ and } K\subseteq L\}\subseteq L$ , i.e.  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L)\subseteq L$ . Therefore,  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L)\subseteq L$ .

(iii) Let  $L$  and  $M$  be two neutrosophic subset of an NBI-T-space  $(X, \tau_1, \tau_2)$  such that  $L\subseteq M$ .

$$\begin{aligned} \text{Now, } P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L) &= \cup\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bO\text{-set in } X \text{ and } K\subseteq L\} \\ &\subseteq \cup\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bO\text{-set in } X \text{ and } K\subseteq M\} \quad [\text{since } L\subseteq M] \\ &= P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(M) \end{aligned}$$

$$\Rightarrow P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L)\subseteq P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(M).$$

Therefore,  $L\subseteq M \Rightarrow P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L)\subseteq P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(M)$ .

(iv) Let  $L$  be a pairwise  $\tau_{ij}\text{-}N\text{-}bO$ -set in an NBI-T-space  $(X, \tau_1, \tau_2)$ .

Now,  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L)=\cup\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bO\text{-set in } X \text{ and } K\subseteq L\}$ . Since,  $L$  is a pairwise  $\tau_{ij}\text{-}N\text{-}bO$ -set in  $(X, \tau_1, \tau_2)$ , so  $L$  is the largest pairwise  $\tau_{ij}\text{-}N\text{-}bO$ -set in  $(X, \tau_1, \tau_2)$ , which is contained in  $L$ . Therefore,  $\cup\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bO\text{-set in } X \text{ and } K\subseteq L\}=L$ . This implies,  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L)=L$ .

**Theorem 3.2.** Let  $L$  and  $K$  be two neutrosophic subsets of an NBI-T-space  $(X, \tau_1, \tau_2)$ . Then,

(i)  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(0_N)=0_N$  &  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(1_N)=1_N$ ;

(ii)  $L\subseteq P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)$ ;

(iii)  $L\subseteq M \Rightarrow P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)\subseteq P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(M)$ ;

(iv)  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)=L$  if  $L$  is a pairwise  $\tau_{ij}\text{-}N\text{-}bC$ -set.

**Proof.** (i) Straightforward.

(ii) It is clear that  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)=\cap\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bC\text{-set in } X \text{ and } L\subseteq K\}$ .

Since, each  $L\subseteq K$ , so  $L\subseteq \cap\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bC\text{-set in } X \text{ and } L\subseteq K\}$ , i.e.  $L\subseteq P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)$ .

(iii) Let  $L$  and  $M$  be two neutrosophic subset of an NBI-T-space  $(X, \tau_1, \tau_2)$  such that  $L\subseteq M$ .

$$\begin{aligned} \text{Now, } P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L) &= \cap\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bC\text{-set in } X \text{ and } L\subseteq K\} \\ &\subseteq \cap\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bC\text{-set in } X \text{ and } M\subseteq K\} \quad [\text{since } L\subseteq M] \\ &= P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(M) \end{aligned}$$

$$\Rightarrow P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)\subseteq P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(M).$$

Therefore,  $L\subseteq M \Rightarrow P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)\subseteq P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(M)$ .

(iv) Let  $L$  be a pairwise  $\tau_{ij}\text{-}N\text{-}bC$ -set in an NBI-T-space  $(X, \tau_1, \tau_2)$ . Now,  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)=\cap\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bC\text{-set in } X \text{ and } L\subseteq K\}$ . Since,  $L$  is a pairwise  $\tau_{ij}\text{-}N\text{-}bC$ -set in a  $(X, \tau_1, \tau_2)$ , so  $L$  is the smallest pairwise  $\tau_{ij}\text{-}N\text{-}bC$ -set, which contains  $L$ . This implies,  $\cap\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bC\text{-set in } X \text{ and } L\subseteq K\}=L$ . Therefore,  $P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)=L$ .

**Proposition 3.3.** Let  $L$  be a neutrosophic subset of an NBI-T-space  $(X, \tau_1, \tau_2)$ . Then,

(i)  $[P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L)]^c = P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L^c)$ ;

(ii)  $[P\text{-}\tau_{ij}\text{-}N_{b\text{-}cl}(L)]^c = P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L^c)$ .

**Proof.** (i) Let  $(X, \tau_1, \tau_2)$  be an NBI-T-space. Let  $L=\{(w, T_L(w), I_L(w), F_L(w)): w\in X\}$  be an neutrosophic subset of  $(X, \tau_1, \tau_2)$ .

$$\begin{aligned} \text{Now, } P\text{-}\tau_{ij}\text{-}N_{b\text{-}int}(L) &= \cup\{K:K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}bO\text{-set in } X \text{ and } K\subseteq L\} \\ &= \{(w, \vee T_{L_p}(w), \wedge I_{L_p}(w), \wedge F_{L_p}(w)): w\in X\}, \end{aligned}$$

where  $L_p$  is a pairwise  $\tau_{ij}\text{-}N\text{-}bO$ -set in  $X$  such that  $L_p\subseteq L$ , for each  $p\in\Delta$ .

This implies,  $[P-\tau_{ij}\text{-}N_{b\text{-}int}(L)]^c = \{(w, \wedge T_{L_p}(w), \vee I_{L_p}(w), \vee F_{L_p}(w)) : w \in X\}$ .

Here  $\wedge T_{L_p}(w) \leq T_L(w), I_{L_p}(w) \geq I_L(w), F_{L_p}(w) \geq F_L(w)$ , for each  $w \in X$ .

Therefore,  $P-\tau_{ij}\text{-}N_{b\text{-}int}(L^c) = \{(w, \wedge T_{L_p}(w), \vee I_{L_p}(w), \vee F_{L_p}(w)) : w \in X\}$

$$= \cap \{L_p : p \in \Delta \text{ and } L_p \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}b\text{C}\text{-set in } X \text{ such that } L^c \subseteq L_p\}$$

Hence,  $[P-\tau_{ij}\text{-}N_{b\text{-}int}(L)]^c = P-\tau_{ij}\text{-}N_{b\text{-}cl}(L^c)$ .

(ii) Let  $(X, \tau_1, \tau_2)$  be an NBi-T-space and  $L = \{(w, T_L(w), I_L(w), F_L(w)) : w \in X\}$  be a N-set over  $X$ . Then,

$P-\tau_{ij}\text{-}N_{b\text{-}cl}(L) = \cap \{K : K \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}b\text{C}\text{-set in } X \text{ and } L \subseteq K\}$

$$= \{(w, \wedge T_{L_p}(w), \vee I_{L_p}(w), \vee F_{L_p}(w)) : w \in X\},$$

where  $L_p$  is a pairwise  $\tau_{ij}\text{-}N\text{-}b\text{C}\text{-set}$  in  $X$  such that  $L \subseteq L_p$ , for each  $p \in \Delta$ .

This implies,  $[P-\tau_{ij}\text{-}N_{b\text{-}cl}(L)]^c = \{(w, \vee T_{L_p}(w), \wedge I_{L_p}(w), \wedge F_{L_p}(w)) : w \in X\}$ .

Here,  $\vee T_{L_p}(w) \geq T_L(w), \wedge I_{L_p}(w) \leq I_L(w), \wedge F_{L_p}(w) \leq F_L(w)$ , for each  $w \in X$ .

Therefore,  $P-\tau_{ij}\text{-}N_{b\text{-}int}(L^c) = \{(w, \vee T_{L_p}(w), \wedge I_{L_p}(w), \wedge F_{L_p}(w)) : w \in X\}$

$$= \cup \{L_p : p \in \Delta \text{ and } L_p \text{ is a pairwise } \tau_{ij}\text{-}N\text{-}b\text{O}\text{-set in } X \text{ such that } L_p \subseteq L^c\}.$$

Hence,  $[P-\tau_{ij}\text{-}N_{b\text{-}cl}(L)]^c = P-\tau_{ij}\text{-}N_{b\text{-}int}(L^c)$ .

**Theorem 3.1.** Let  $(X, \tau_1, \tau_2)$  be an NBi-T-space. Then, the neutrosophic null set  $(0_N)$  and the neutrosophic whole set  $(1_N)$  are both  $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$  and  $\tau_{ij}\text{-}N\text{-}b\text{C}\text{-set}$ .

**Proof.** Let  $(X, \tau_1, \tau_2)$  be an NBi-T-space. Now,  $N_{cl}^i N_{int}^j(0_N) \cup N_{int}^j N_{cl}^i(0_N) = N_{cl}^i(0_N) \cup N_{int}^j(0_N) = 0_N \cup 0_N = 0_N$ . Therefore,  $0_N \subseteq 0_N = N_{cl}^i N_{int}^j(0_N) \cup N_{int}^j N_{cl}^i(0_N)$ . Hence, the neutrosophic null set  $(0_N)$  is a  $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$ .

Similarly, it can be established that the neutrosophic null set  $(0_N)$  is a  $\tau_{ji}\text{-}N\text{-}b\text{O}\text{-set}$ .

Further, one can show that the neutrosophic whole set  $(1_N)$  are both  $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$  and  $\tau_{ji}\text{-}N\text{-}b\text{O}\text{-set}$ .

**Theorem 3.2.** In an NBi-T-space  $(X, \tau_1, \tau_2)$ , every  $\tau_i\text{-}N\text{O}\text{-set}$  is a  $\tau_{ji}\text{-}N\text{-}b\text{O}\text{-set}$ .

**Proof.** Let  $L$  be a  $\tau_i\text{-}N\text{O}\text{-set}$  in an NBi-T-space  $(X, \tau_1, \tau_2)$ . Therefore,  $N_{int}^i(L) = L$ . Now,  $L \subseteq N_{cl}^j(L) = N_{cl}^j N_{int}^i(L)$ . This implies,  $L \subseteq N_{cl}^j N_{int}^i(L) \cup N_{int}^i N_{cl}^j(L)$ . Hence,  $L$  is a  $\tau_{ji}\text{-}N\text{-}b\text{O}\text{-set}$  in  $(X, \tau_1, \tau_2)$ .

**Theorem 3.3.** In an NBi-T-space  $(X, \tau_1, \tau_2)$ ,

- (i) every  $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$  is a pairwise  $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$ ;
- (ii) every  $\tau_{ji}\text{-}N\text{-}b\text{O}\text{-set}$  is a pairwise  $\tau_{ji}\text{-}N\text{-}b\text{O}\text{-set}$ ;
- (iii) every  $\tau_{ij}\text{-}N\text{-}b\text{C}\text{-set}$  is a pairwise  $\tau_{ij}\text{-}N\text{-}b\text{C}\text{-set}$ ;
- (iv) every  $\tau_{ji}\text{-}N\text{-}b\text{C}\text{-set}$  is a pairwise  $\tau_{ji}\text{-}N\text{-}b\text{C}\text{-set}$ .

**Proof.** (i) Let  $L$  be a  $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$  in an NBi-T-space  $(X, \tau_1, \tau_2)$ . Then,  $L$  can be expressed as  $L = L \cup 0_N$ , where  $L$  is a  $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$  and  $0_N$  is a  $\tau_{ji}\text{-}N\text{-}b\text{O}\text{-set}$  in  $(X, \tau_1, \tau_2)$ . This implies,  $L$  is a pairwise  $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$  in  $(X, \tau_1, \tau_2)$ .

(ii) Straightforward.

(iii) Let  $L$  be a  $\tau_{ij}\text{-}N\text{C}\text{-set}$  in an NBi-T-space  $(X, \tau_1, \tau_2)$ . Then,  $L$  can be expressed as  $L = L \cap 1_N$ , where  $L$  is a  $\tau_{ij}\text{-}N\text{C}\text{-set}$  and  $1_N$  is a  $\tau_{ji}\text{-}N\text{C}\text{-set}$  in  $(X, \tau_1, \tau_2)$ . This implies,  $L$  is a pairwise  $\tau_{ij}\text{-}N\text{-}b\text{C}\text{-set}$  in  $(X, \tau_1, \tau_2)$ .

(iv) Straightforward.

**Theorem 3.4.** In an NBi-T-space  $(X, \tau_1, \tau_2)$ , every  $\tau_i\text{-}N\text{O}\text{-set}$  is a pairwise  $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$ .

**Proof.** Let  $L$  be a  $\tau_i\text{-}N\text{O}\text{-set}$  in an NBi-T-space  $(X, \tau_1, \tau_2)$ . By Theorem 3.2., it is clear that  $L$  is a  $\tau_{ji}\text{-}N\text{-}b\text{O}\text{-set}$ . Further, by Theorem 3.3., it is clear that  $L$  is a pairwise  $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$ .

**Theorem 3.5.** Let  $(X, \tau_1, \tau_2)$  be an NBi-T-space. Then,  $0_N$  and  $1_N$  are both pairwise  $\tau_{ij}\text{-}N\text{-}b\text{O}\text{-set}$  and pairwise  $\tau_{ji}\text{-}N\text{-}b\text{O}\text{-set}$ .

**Proof.** Let  $(X, \tau_1, \tau_2)$  be an NBI-T-space. One can write  $0_N = A \cup B$ , where  $A = 0_N$  is a  $\tau_{ij}$ -N-*b*O-set and  $B = 0_N$  is a  $\tau_{ji}$ -N-*b*O-set in  $(X, \tau_1, \tau_2)$ . This implies,  $0_N$  is a pairwise  $\tau_{ij}$ -N-*b*O-set in  $(X, \tau_1, \tau_2)$ .

Similarly, it can be established that  $0_N$  is a pairwise  $\tau_{ji}$ -N-*b*O-set in  $(X, \tau_1, \tau_2)$ .

Again, one can write  $1_N = L \cup M$ , where  $L = 1_N$  is a  $\tau_{ij}$ -N-*b*O-set and  $M = 1_N$  is a  $\tau_{ji}$ -N-*b*O-set in  $(X, \tau_1, \tau_2)$ . This implies,  $1_N$  is a pairwise  $\tau_{ij}$ -N-*b*O-set in  $(X, \tau_1, \tau_2)$ .

Similarly, it can be also established that  $1_N$  is a pairwise  $\tau_{ji}$ -N-*b*O-set in  $(X, \tau_1, \tau_2)$ .

**Theorem 3.6.** Let  $(X, \tau_1, \tau_2)$  be an NBI-T-space. Then, both  $0_N$  and  $1_N$  are pairwise  $\tau_{ij}$ -N-*b*C-set and pairwise  $\tau_{ji}$ -N-*b*C-set.

**Proof.** By Theorem 3.5, it is clear that  $0_N$  is both pairwise  $\tau_{ij}$ -N-*b*O-set and pairwise  $\tau_{ji}$ -N-*b*O-set. Hence, its complement  $1_N$  is both pairwise  $\tau_{ij}$ -N-*b*C-set and pairwise  $\tau_{ji}$ -N-*b*C-set.

Similarly, from Theorem 3.5, it is clear that  $1_N$  is both pairwise  $\tau_{ij}$ -N-*b*O-set and pairwise  $\tau_{ji}$ -N-*b*O-set. Hence, its complement  $0_N$  is both pairwise  $\tau_{ij}$ -N-*b*C-set and pairwise  $\tau_{ji}$ -N-*b*C-set.

**Remark 3.1.** Throughout the article, we denote  $\tau_{ij}^b$  as a collection of all pairwise  $\tau_{ij}$ -N-*b*O-sets and  $\tau_{ij}^c$  as a collection of all pairwise  $\tau_{ij}$ -N-*b*C-sets in  $(X, \tau_1, \tau_2)$ . The collection  $\tau_{ij}^b$  forms a neutrosophic supra topology on  $X$ .

**Definition 3.3.** Let  $(X, \tau_1, \tau_2)$  and  $(Y, \delta_1, \delta_2)$  be two NBI-T-spaces. Then, an one to one and onto mapping  $\xi : (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  is called a

(i) pairwise neutrosophic semi continuous mapping (in short P-NS-C-mapping) if and only if  $\xi^{-1}(L)$  is a  $\tau_i$ -NSO-set in  $X$ , whenever  $L$  is a pairwise  $\delta_{ij}$ -NO-set in  $Y$ .

(ii) pairwise neutrosophic pre continuous mapping (in short P-NP-C-mapping) if and only if  $\xi^{-1}(L)$  is a  $\tau_i$ -NPO-set in  $X$ , whenever  $L$  is a pairwise  $\delta_{ij}$ -NO-set in  $Y$ .

(iii) pairwise neutrosophic continuous mapping (in short P-N-C-mapping) if and only if  $\xi^{-1}(L)$  is a  $\tau_i$ -NO-set in  $X$ , whenever  $L$  is a pairwise  $\delta_{ij}$ -NO-set in  $Y$ .

(iv) pairwise neutrosophic *b*-continuous mapping (in short P-N-*b*-C-mapping) if and only if  $\xi^{-1}(L)$  is a  $\tau_i$ -N-*b*O-set in  $X$ , whenever  $L$  is a pairwise  $\delta_{ij}$ -NO-set in  $Y$ .

**Theorem 3.7.** Let  $(X, \tau_1, \tau_2)$  and  $(Y, \delta_1, \delta_2)$  be two NBI-T-spaces. Then, every P-N-C-mapping from  $(X, \tau_1, \tau_2)$  to  $(Y, \delta_1, \delta_2)$  is a P-NP-C-mapping (P-NS-C-mapping).

**Proof.** Let  $L$  be a pairwise  $\delta_{ij}$ -NO-set in  $(Y, \delta_1, \delta_2)$ . Since,  $\xi : (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  is a P-N-C-mapping from  $(X, \tau_1, \tau_2)$  to  $(Y, \delta_1, \delta_2)$ , so  $\xi^{-1}(L)$  is a  $\tau_i$ -NO-set in  $(X, \tau_1, \tau_2)$ . It is known that every  $\tau_i$ -NO-set is a  $\tau_i$ -NPO-set ( $\tau_i$ -NSO-set). Therefore,  $\xi^{-1}(L)$  is a  $\tau_i$ -NPO-set ( $\tau_i$ -NSO-set) in  $(X, \tau_1, \tau_2)$ . Hence,  $\xi : (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  is a P-NP-C-mapping (P-NS-C-mapping).

**Theorem 3.8.** Let  $(X, \tau_1, \tau_2)$  and  $(Y, \delta_1, \delta_2)$  be two NBI-T-spaces. Then, every P-NS-C-mapping (P-NP-C-mapping) from  $(X, \tau_1, \tau_2)$  to  $(Y, \delta_1, \delta_2)$  is a P-N-*b*-C-mapping.

**Proof.** Let  $L$  be a pairwise  $\delta_{ij}$ -NO-set in  $(Y, \delta_1, \delta_2)$ . Since,  $\xi : (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  is a P-NS-C-mapping (P-NP-C-mapping) from  $(X, \tau_1, \tau_2)$  to  $(Y, \delta_1, \delta_2)$ , so  $\xi^{-1}(L)$  is a  $\tau_i$ -NSO-set ( $\tau_i$ -NPO-set) in  $(X, \tau_1, \tau_2)$ . It is known that, every  $\tau_i$ -NSO-set ( $\tau_i$ -NPO-set) is a  $\tau_i$ -N-*b*O-set. Therefore,  $\xi^{-1}(L)$  is a  $\tau_i$ -N-*b*O-set in  $(X, \tau_1, \tau_2)$ . Hence,  $\xi : (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  is a P-N-*b*-C-mapping.

**Theorem 3.9.** Let  $(X, \tau_1, \tau_2)$  and  $(Y, \delta_1, \delta_2)$  be two NBI-T-spaces. Then, every P-N-C-mapping from  $(X, \tau_1, \tau_2)$  to  $(Y, \delta_1, \delta_2)$  is a P-N-*b*-C-mapping.

**Proof.** Let  $L$  be a pairwise  $\delta_{ij}$ -NO-set in  $(Y, \delta_1, \delta_2)$ . Since,  $\xi : (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  is a P-N-C-mapping from  $(X, \tau_1, \tau_2)$  to  $(Y, \delta_1, \delta_2)$ , so  $\xi^{-1}(L)$  is a  $\tau_i$ -NO-set in  $(X, \tau_1, \tau_2)$ . It is known that, every  $\tau_i$ -NO-set is a

$\tau_i$ -N- $b$ -O-set. Therefore,  $\xi^{-1}(L)$  is a  $\tau_i$ -N- $b$ -O-set in  $(X, \tau_1, \tau_2)$ . Hence,  $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  is a p-N- $b$ -C-mapping.

**Theorem 3.10.** If  $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  and  $\chi: (Y, \delta_1, \delta_2) \rightarrow (Z, \theta_1, \theta_2)$  be two P-N-C-mapping, then the composition mapping  $\chi \circ \xi: (X, \tau_1, \tau_2) \rightarrow (Z, \theta_1, \theta_2)$  is also a P-N-C-mapping.

**Proof.** Let  $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  and  $\chi: (Y, \delta_1, \delta_2) \rightarrow (Z, \theta_1, \theta_2)$  be two P-N-C-mappings. Let  $L$  be a pairwise  $\theta_{ij}$ -NO-set in  $(Z, \theta_1, \theta_2)$ . Since,  $\chi: (Y, \delta_1, \delta_2) \rightarrow (Z, \theta_1, \theta_2)$  is a P-N-C-mapping, so  $\chi^{-1}(L)$  is a  $\delta_i$ -NO-set in  $Y$ . Since,  $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  is a P-N-C-mapping, so  $\xi^{-1}(\chi^{-1}(L)) = (\chi \circ \xi)^{-1}(L)$  is a  $\tau_i$ -NO-set in  $X$ .

**Theorem 3.11.** If  $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  be an one to one and onto mapping between two NBi-T-spaces, then the following two are equivalent:

(i)  $\xi$  is a P-N- $b$ -C-mapping.

(ii)  $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(A)) \subseteq_{\tau_i\text{-}N_{b\text{-}int}}(\xi^{-1}(A))$ , for every neutrosophic subset  $A$  of  $Y$ .

**Proof.** (i) $\Rightarrow$ (ii)

Let  $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  be a P-N- $b$ -C-mapping. Let  $A$  be an neutrosophic subset of  $Y$ . Here,  $P\text{-}\delta_{ij}\text{-}N_{int}(A)$  is a pairwise  $\delta_{ij}$ -NO-set in  $Y$  and  $P\text{-}\delta_{ij}\text{-}N_{int}(A) \subseteq A$ . This implies,  $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(A)) \subseteq \xi^{-1}(A)$ . By the hypothesis,  $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(A))$  is a  $\tau_i$ -N- $b$ -O-set in  $X$ . Therefore,  $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(A))$  is a  $\tau_i$ -N- $b$ -O-set in  $X$  such that  $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(A)) \subseteq \xi^{-1}(A)$ . It is known that  $\tau_i\text{-}N_{b\text{-}int}(\xi^{-1}(A))$  is the largest  $\tau_i$ -N- $b$ -O-set in  $X$ , which is contained in  $\xi^{-1}(A)$ . Hence,  $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(A)) \subseteq_{\tau_i\text{-}N_{b\text{-}int}}(\xi^{-1}(A))$ .

(ii) $\Rightarrow$ (i)

Let  $A$  be a pairwise  $\delta_{ij}$ -NO-set in  $(Y, \delta_1, \delta_2)$ . Therefore,  $P\text{-}\delta_{ij}\text{-}N_{int}(A) = A$ . By hypothesis,  $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(A)) \subseteq_{\tau_i\text{-}N_{b\text{-}int}}(\xi^{-1}(A))$ . This implies,  $\xi^{-1}(A) \subseteq_{\tau_i\text{-}N_{b\text{-}int}}(\xi^{-1}(A))$ . It is known that  $\tau_i\text{-}N_{b\text{-}int}(\xi^{-1}(A)) \subseteq \xi^{-1}(A)$ . Therefore,  $\tau_i\text{-}N_{b\text{-}int}(\xi^{-1}(A)) = \xi^{-1}(A)$ . Hence,  $\xi^{-1}(A)$  is a  $\tau_i$ -N- $b$ -O-set in  $(X, \tau_1, \tau_2)$ . Therefore,  $\xi$  is a P-N- $b$ -C-mapping from an NBi-T-space  $(X, \tau_1, \tau_2)$  to another NBi-T-space  $(Y, \delta_1, \delta_2)$ .

**Theorem 3.12.** An one to one and onto mapping  $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  is a P-N- $b$ -C-mapping if and only if  $P\text{-}\delta_{ij}\text{-}N_{int}(\xi(A)) \subseteq_{\xi}(\tau_i\text{-}N_{b\text{-}int}(A))$ , for every N-set  $A$  over  $X$  and  $i, j = 1, 2$ , and  $i \neq j$ .

**Proof.** Let  $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  be a P-N- $b$ -C-mapping. Let  $A$  be an N-set over  $X$ . Then,  $\xi(A)$  is also an N-set over  $Y$ . By Theorem 3.11, we have  $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(\xi(A))) \subseteq_{\tau_i\text{-}N_{b\text{-}int}}(\xi^{-1}(\xi(A)))$ . This implies,  $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(\xi(A))) \subseteq_{\tau_i\text{-}N_{b\text{-}int}}(A)$ . Hence,  $P\text{-}\delta_{ij}\text{-}N_{int}(\xi(A)) \subseteq_{\xi}(\tau_i\text{-}N_{b\text{-}int}(A))$ . Therefore,  $P\text{-}\delta_{ij}\text{-}N_{int}(\xi(A)) \subseteq_{\xi}(\tau_i\text{-}N_{b\text{-}int}(A))$ , for every N-set  $A$  over  $X$  and  $i, j = 1, 2$ ; and  $i \neq j$ .

Conversely, let  $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  be a mapping between two NBi-T-spaces such that

$$P\text{-}\delta_{ij}\text{-}N_{int}(\xi(A)) \subseteq_{\xi}(\tau_i\text{-}N_{b\text{-}int}(A)) \tag{1}$$

for every N-set  $A$  over  $X$  and  $i, j = 1, 2$ ; and  $i \neq j$ .

Let  $A$  be an N-set over  $Y$ . Then,  $\xi^{-1}(A)$  is an N-set over  $X$ . By putting  $A = \xi^{-1}(A)$  in eq. (1), we have,

$$P\text{-}\delta_{ij}\text{-}N_{int}(\xi(\xi^{-1}(A))) \subseteq_{\xi}(\tau_i\text{-}N_{b\text{-}int}(\xi^{-1}(A)))$$

$$\Rightarrow P\text{-}\delta_{ij}\text{-}N_{int}(A) \subseteq_{\xi}(\tau_i\text{-}N_{b\text{-}int}(\xi^{-1}(A)))$$

$$\Rightarrow \xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(A)) \subseteq_{\tau_i\text{-}N_{b\text{-}int}}(\xi^{-1}(A)).$$

Therefore,  $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(A)) \subseteq_{\tau_i\text{-}N_{b\text{-}int}}(\xi^{-1}(A))$ , for every N-set  $A$  of  $Y$ . Hence, by Theorem 3.11., the mapping  $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  is a P-N- $b$ -C-mapping.

**Corollary 3.1.** If  $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  is an one to one and onto mapping from an NBi-T-space  $(X, \tau_1, \tau_2)$  to another NBi-T-space  $(Y, \delta_1, \delta_2)$ , then the following two are equivalent:

(i)  $\xi$  is a P-N-C-mapping.

(ii)  $\xi^{-1}(P\text{-}\delta_{ij}\text{-}N_{int}(Q)) \subseteq_{\tau_i\text{-}N_{int}}(\xi^{-1}(Q))$ , for every N-set  $Q$  over  $Y$ .

**Definition 3.4.** Let  $(X, \tau_1, \tau_2)$  be an NBi-T-space. Let  $x_{a,b,c}$  be an N-point in  $X$ . Then, an N-set  $Q$  over  $X$  is called a pairwise  $\tau_{ij}$ -neutrosophic  $b$ -neighbourhood (in short P- $\tau_{ij}$ -N- $b$ -nbd) of  $x_{a,b,c}$ , if there exist a pairwise  $\tau_{ij}$ -N- $b$ O-set  $U$  such that  $x_{a,b,c} \in U \subseteq Q$ .

**Theorem 3.13.** Let  $(X, \tau_1, \tau_2)$  be an NBi-T-space. An N-set  $Q$  over  $X$  is a pairwise  $\tau_{ij}$ -N- $b$ O-set if and only if  $Q$  is a P- $\tau_{ij}$ -N- $b$ -nbd of all of its N-points.

**Proof.** Let  $Q$  be a pairwise  $\tau_{ij}$ -N- $b$ O-set in an NBi-T-space  $(X, \tau_1, \tau_2)$ . Let  $x_{a,b,c}$  be an N-point in  $X$  such that  $x_{a,b,c} \in Q$ . Therefore,  $x_{a,b,c} \in Q \subseteq Q$ . This implies,  $Q$  is a P- $\tau_{ij}$ -N- $b$ -nbd of  $x_{a,b,c}$ . Hence,  $Q$  is the P- $\tau_{ij}$ -N- $b$ -nbd of all of its N-points.

Conversely, let  $Q$  be a P- $\tau_{ij}$ -N- $b$ -nbd of all of its N-points. Assume that  $x_{a,b,c}$  be an N-point in  $X$ , such that  $x_{a,b,c} \in Q$ . Therefore, there exist a pairwise  $\tau_{ij}$ -N- $b$ O-set  $G$  such that  $x_{a,b,c} \in G \subseteq Q$ .

Now,  $Q = \cup_{x_{a,b,c} \in Q} x_{a,b,c} \subseteq \cup_{x_{a,b,c} \in Q} G \subseteq \cup_{x_{a,b,c} \in Q} Q = Q$ . This implies,  $Q = \cup_{x_{a,b,c} \in Q} G$ , which is a pairwise  $\tau_{ij}$ -N- $b$ O-set. Therefore,  $Q$  is a pairwise  $\tau_{ij}$ -N- $b$ O-set in  $(X, \tau_1, \tau_2)$ .

**Theorem 3.14.** An one to one and onto mapping  $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  is a P-N- $b$ -C-mapping if and only if for every N-point  $x_{a,b,c} \in Y$  and for any P- $\delta_{ij}$ -N- $b$ -nbd  $V$  of  $x_{a,b,c}$  in  $Y$ , there exist a  $\tau_i$ -neutrosophic- $b$ -neighbourhood (in short  $\tau_i$ -N- $b$ -nbd)  $U$  of  $\xi^{-1}(x_{a,b,c})$  in  $X$  such that  $U \subseteq \xi^{-1}(V)$ .

**Proof.** Let  $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  be a P-N- $b$ -C-mapping. Let  $x_{a,b,c}$  be an N-point in  $Y$  and  $V$  be a P- $\delta_{ij}$ -N- $b$ -nbd of  $x_{a,b,c}$ . Then, there exist a pairwise  $\delta_{ij}$ -NO-set  $G$  in  $Y$  such that  $x_{a,b,c} \in G \subseteq V$ . This implies,  $\xi^{-1}(x_{a,b,c}) \in \xi^{-1}(G) \subseteq \xi^{-1}(V)$ . Since,  $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  is a P-N- $b$ -C-mapping, so  $\xi^{-1}(G)$  is a  $\tau_i$ -N- $b$ O-set in  $X$ . By taking  $U = \xi^{-1}(G)$ , we see that  $U$  is a  $\tau_i$ -N- $b$ O-set in  $X$  such that  $\xi^{-1}(x_{a,b,c}) \in U \subseteq \xi^{-1}(V)$ . Hence,  $U = \xi^{-1}(G)$  is a  $\tau_i$ -N- $b$ -nbd of  $\xi^{-1}(x_{a,b,c})$  and  $U \subseteq \xi^{-1}(V)$ .

Conversely, let for every N-point  $x_{a,b,c} \in Y$  and for any P- $\delta_{ij}$ -N- $b$ -nbd  $V$  of  $x_{a,b,c}$  in  $Y$ , there exist a  $\tau_i$ -N- $b$ -nbd  $U$  of  $\xi^{-1}(x_{a,b,c})$  in  $X$  such that  $U \subseteq \xi^{-1}(V)$ . Let  $G$  be a pairwise  $\delta_{ij}$ -NO-set in  $Y$  and  $x_{a,b,c} \in G$ . By Theorem 3.13.,  $G$  is a P- $\delta_{ij}$ -N- $b$ -nbd of  $x_{a,b,c}$ . By hypothesis, there exists a  $\tau_i$ -N- $b$ -nbd  $H$  of  $\xi^{-1}(x_{a,b,c}) \in X$  such that  $\xi^{-1}(x_{a,b,c}) \in H \subseteq \xi^{-1}(G)$ . This implies,  $\xi^{-1}(G)$  is the  $\tau_i$ -N- $b$ -nbd of each of its N-points. Therefore,  $\xi^{-1}(G)$  is a  $\tau_i$ -N- $b$ O-set in  $X$ . Hence,  $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  is a P-N- $b$ -C-mapping.

**Theorem 3.15.** If  $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  be a P-N- $b$ -C-mapping and  $\chi: (Y, \delta_1, \delta_2) \rightarrow (Z, \theta_1, \theta_2)$  be a P-N-C-mapping, then the composition mapping  $\chi \circ \xi: (X, \tau_1, \tau_2) \rightarrow (Z, \theta_1, \theta_2)$  is a P-N- $b$ -C-mapping.

**Proof.** Let  $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  be a P-N- $b$ -C-mapping and  $\chi: (Y, \delta_1, \delta_2) \rightarrow (Z, \theta_1, \theta_2)$  be a P-N-C-mapping. Let  $L$  be a pairwise  $\theta_{ij}$ -NO-set in  $(Z, \theta_1, \theta_2)$ . Since,  $\chi: (Y, \delta_1, \delta_2) \rightarrow (Z, \theta_1, \theta_2)$  is a P-N-C-mapping, so  $\chi^{-1}(L)$  is a  $\delta_i$ -NO-set in  $Y$ . Now, by Lemma 2.1., it is clear that  $\chi^{-1}(L)$  is a pairwise  $\delta_{ij}$ -NO-set in  $(Y, \delta_1, \delta_2)$ . Since,  $\xi: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$  is a P-N- $b$ -C-mapping, so  $\xi^{-1}(\chi^{-1}(L)) = (\chi \circ \xi)^{-1}(L)$  is a  $\tau_i$ -NO-set in  $X$ . Since, every  $\tau_i$ -NO-set is a  $\tau_i$ -N- $b$ O-set, so  $(\chi \circ \xi)^{-1}(L)$  is a  $\tau_i$ -N- $b$ O-set in  $X$ . Hence,  $\chi \circ \xi: (X, \tau_1, \tau_2) \rightarrow (Z, \theta_1, \theta_2)$  is a P-N- $b$ -C-mapping.

#### 4. Conclusion

In this article, we introduce the notion of pairwise neutrosophic- $b$ -interior, pairwise neutrosophic- $b$ -closure, pairwise neutrosophic  $b$ -continuous mapping, we prove some propositions and theorems on NBi-T-spaces. In the future, we hope that based on these notions in NBi-T-spaces, many new investigations can be carried out.

**Conflict of Interest:** The authors declare that they have no conflict of interest.

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