Pairwise Neutrosophic \( b \)-Continuous Function in Neutrosophic Bitopological Spaces

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Abstract: The main focus of this article is to procure the notions of pairwise neutrosophic continuous and pairwise neutrosophic \( b \)-continuous mappings in neutrosophic bitopological spaces. Then, we formulate some results on them via neutrosophic bitopological spaces.

Keywords: Neutrosophic Topology; Neutrosophic Bitopology; Pairwise Neutrosophic \( b \)-Interior; Pairwise Neutrosophic \( b \)-Closure; Pairwise Neutrosophic Continuous.

1. Introduction

Zadeh [31] presented the notions of fuzzy set (in short FS) in the year 1965. Afterwards, Chang [4] applied the idea of topology on fuzzy sets and introduced the fuzzy topological space. In the year 2017, Dutta and Tripathy [15] studied on fuzzy \( b \)-0 open sets via fuzzy topological space. Later on, Smarandache [23] grounded the idea of neutrosophic set (in short N-set) in the year 1998, as an extension of the concept of intuitionistic fuzzy set (in short IF-set) [3], where every element has three-independent memberships values namely truth, indeterminacy, and false membership values respectively. Afterwards, Salama and Alblowi [21] applied the notion of topology on N-sets and introduced neutrosophic topological space (in short NT-space) by extending the notions of fuzzy topological spaces. Salama and Alblowi [22] also defined generalized N-set and introduced the concept of generalized NT-space. Later on, Arockiarani et al. [2] introduced the ideas of neutrosophic point and studied some functions in neutrosophic topological spaces. The notions of neutrosophic pre-open (in short NP-O) and neutrosophic pre-closed (in short NP-C) sets via NT-spaces are studied by Rao and Srinivasa [20]. The idea of \( b \)-open sets via topological spaces was established by Andrijevic [1]. Afterwards, Ebenjar et al. [16] presents the concept of neutrosophic \( b \)-open set (in short N-\( b \)-O-set) via NT-spaces. In the year 2020, Das and Pramanik [8] presents the generalized neutrosophic \( b \)-open sets in NT-spaces. The notions of neutrosophic \( \Phi \)-open set and neutrosophic \( \Phi \)-continuous functions via NT-spaces was also presented by Das and Pramanik [9]. The concept of neutrosophic simply soft open set in neutrosophic soft topological space was studied by Das and Pramanik [10]. In the year 2021, Das and Tripathy [14] presented the notions of neutrosophic simply \( b \)-open set via NT-spaces. In the year 2020, Das and Tripathy [12] grounded the notions of neutrosophic multiset and applied topology on it. In the year 2021, Das et al. [5] studied the concept of quadripartitioned neutrosophic topological spaces. The notion of bitopological space was introduced by Kelly [17] in the year 1963. In the year 2011, Tripathy and Sarma [26] studied on \( b \)-locally open sets via bitopological spaces. The idea of pairwise \( b \)-locally...
open and \( b \)-locally closed functions in bitopological spaces was studied by Tripathy and Sarma [27]. Tripathy and Sarma [28] also studied on weakly \( b \)-continuous mapping via bitopological spaces in the year 2013. Later on, the concept of generalized \( b \)-closed sets in ideal bitopological spaces was studied by Tripathy and Sarma [29]. Afterwards, Tripathy and Debnath [25] presented the notions of fuzzy \( b \)-locally open sets in fuzzy bitopological space. Thereafter, Ozturk and Ozkan [19] introduced the idea of neutrosophic bitopological space (in short NBi-T-space) in the year 2019. Recently, Das and Tripathy [13] presented the idea of pairwise N-\( b \)-O-sets and studied their different properties.

The main focus of this article is to procure the notions of pairwise \( \tau_p \)-neutrosophic-\( b \)-interior (in short \( P \tau_p \)-N\( _b \)-int), pairwise \( \tau_q \)-neutrosophic-\( b \)-closure (in short \( P \tau_q \)-N\( _b \)-cl), \( \tau \)-neutrosophic continuous mapping (in short \( P \tau \)-N-C-mapping), pairwise neutrosophic \( b \)-continuous mapping (in short pairwise N-\( b \)-C-mapping) via NBi-T-spaces.

2. Preliminaries and Definitions:

The notion of N-set is defined as follows:

Let \( X \) be a fixed set. Then, an N-set [23] \( L \) over \( X \) is denoted as follows:

\[
L=(t, T, t; T_1, t_1; T_2, t_2): t \in X, \quad T, T_1, T_2 : X \rightarrow [0, 1]
\]

where \( T, T_1, T_2 \) are called the truth-membership, indeterminacy-membership and false-membership functions and \( 0 \leq T(t) + T_1(t) + T_2(t) \leq 3 \), for all \( t \in X \).

The neutrosophic null set (0\( _N \)) and neutrosophic whole set (1\( _N \)) over a fixed set \( X \) are defined as follows:

(i) \( 0_N = \{(t, 0, 0, 1): t \in X \} \);

(ii) \( 1_N = \{(t, 1, 0, 0): t \in X \} \).

The N-sets 0\( _N \) and 1\( _N \) also has three other representations. They are given below:

\[
0_N = \{(t, 0, 0, 0): t \in X \} \quad & \quad 1_N = \{(t, 1, 1, 1): t \in X \} \; \quad & \quad 0_N = \{(t, 0, 1, 0): t \in X \} \quad & \quad 1_N = \{(t, 1, 0, 1): t \in X \} \; \quad & \quad 0_N = \{(t, 0, 1, 1): t \in X \} \quad & \quad 1_N = \{(t, 1, 1, 0): t \in X \}.
\]

Let \( p, q, r \in [0, 1] \). An neutrosophic point (in short N-point) [2] \( x_{p,q,r} \) is an N-set over \( X \) given by

\[
x_{p,q,r}(y) = \begin{cases} (p, q, r), & \text{if } x = y, \\ (0, 0, 1), & \text{if } x \neq y, \end{cases}
\]

where \( p, q, r \) denotes the truth, indeterminacy and false membership value of \( x_{p,q,r} \).

The notion of NT-space is defined as follows:

A family \( \tau \) of N-sets over \( X \) is called an [21] neutrosophic topology (in short N-topology) on \( X \) if the following axioms hold:

(i) \( 0_N, 1_N \in \tau \);

(ii) \( L \subseteq \tau \Rightarrow \bigcup L \in \tau \) and \( \bigcap L \in \tau \);

(iii) \( \bigcup L \in \tau \), for every \( \{L: i \in \Delta\} \subseteq \tau \), where \( \Delta \) is the support set.

Then, \( (X, \tau) \) is called an NT-space. Each element of \( \tau \) is an neutrosophic open set (in short NO-set). If \( L \) is an NO-set in \( (X, \tau) \), then \( L^c \) is called an neutrosophic closed set (in short NC-set).
The notion of NBi-T-space is defined as follows:

Let τ₁ and τ₂ be two different N-topologies on X. Then, (X,τ₁,τ₂) is [19] called an NBi-T-space. An N-set L is called a pairwise NO-set in (X,τ₁,τ₂), if there exist an NO-set L₁ in τ₁ and an NO-set L₂ in τ₂ such that L=L₁∪L₂. The complement of L i.e., L’ is called a pairwise neutrosophic closed set (in short pairwise NC-set) in (X,τ₁,τ₂).

Remark 2.1.[13] In an NBi-T-space(X,τ₁,τ₂), every τ₀-NO-set is a pairwise τ₀-NO-set.

Remark 2.2. Let G be an N-set over X and (X,τ₁,τ₂) be an NBi-T-space. Then, we shall use the following notations throughout the article:

(i) $N^i_{cl}(G)=\text{Neutrosophic closure of } G \text{ in } (X,\tau)$ (i=1, 2);

(ii) $N^b_{int}(G)=\text{Neutrosophic interior of } G \text{ in } (X,\tau)$ (i=1, 2).

Definition 2.1.[13] Let (X,τ₁,τ₂) be an NBi-T-space. Then, P is called a

(i) τ-neutrosophic semi-open set (in short τ-NSO-set) if and only if $P \subseteq N^{b}_{cl}N^{i}_{int}(P)$;

(ii) τ-neutrosophic pre-open set (in short τ-NPO-set) if and only if $P \subseteq N^{b}_{int}N^{i}_{cl}(P)$;

(iii) τ-neutrosophic b-open set (in short τ-NbO-set) if and only if $P \subseteq N^{b}_{cl}N^{i}_{int}(P) \cup N^{i}_{int}N^{b}_{cl}(P)$.

Remark 2.3.[13] Let (X,τ₁,τ₂) be an NBi-T-space. Then, an N-set P over X is called a τ-neutrosophic b-closed set (in short τ-NbC-set) if and only if $P^c$ is a τ-NbO-set.

Proposition 2.1.[13] In an NBi-T-space (X,τ₁,τ₂), if P is a τ-NSO-set (τ-NSO-set), then P is a τ-NbO-set.

Proposition 2.2.[13] Let (X,τ₁,τ₂) be an NBi-T-space. Then, the union of any two τ-NbO-sets is a τ-NbO-set.

Definition 2.2.[13] Let (X,τ₁,τ₂) be an NBi-T-space. Then, P is called a

(i) τ₀-neutrosophic semi-open set (in short τ₀-NSO-set) if and only if $P \subseteq N^{b}_{cl}N^{i}_{int}(P)$;

(ii) τ₀-neutrosophic pre-open set (in short τ₀-NPO-set) if and only if $P \subseteq N^{b}_{int}N^{i}_{cl}(P)$;

(iii) τ₀-neutrosophic b-open set (in short τ₀-NbO-set) if and only if $P \subseteq N^{b}_{cl}N^{i}_{int}(P) \cup N^{i}_{int}N^{b}_{cl}(P)$.

Remark 2.4.[13] An N-set L over X is called a τ₀-neutrosophic b-closed set (in short τ₀-NbC-set) if and only if $L^c$ is a τ₀-NbO-set in (X,τ₁,τ₂).

Theorem 2.1.[13] Let (X,τ₁,τ₂) be an NBi-T-space. Then, every τ₀-NSO-set (τ₀-NPO-set) is a τ₀-NbO-set.

Definition 2.3.[13] An N-set L is called a pairwise τ₀-NPO-set (pairwise τ₀-NSO-set) in an NBi-T-space(X,τ₁,τ₂) if $L=K\cup M$, where K is a τ₀-NPO-set (τ₀-NSO-set) and M is a τ₀-NPO-set (τ₀-NSO-set) in (X,τ₁,τ₂).

Definition 2.4.[13] An N-set L is called a pairwise τ₀-NbO-set in a NBi-T-space(X,τ₁,τ₂) if $L=K\cup M$, where K is a τ₀-NbO-set and M is a τ₀-NbO-set in (X,τ₁,τ₂). If L is a pairwise τ₀-NbO-set in (X,τ₁,τ₂), then $L^c$ is called a pairwise τ₀-neutrosophic b-closed set (in short pairwise τ₀-NbC-set) in (X,τ₁,τ₂).

Lemma 2.1.[13] In an NBi-T-space(X,τ₁,τ₂), every pairwise τ₀-NPO-set (pairwise τ₀-NSO-set) is a pairwise τ₀-NbO-set.

Proposition 2.3.[13] Let(X,τ₁,τ₂) be an NBi-T-space. Then, the union of two pairwise τ₀-NbO-set in (X,τ₁,τ₂) is also a pairwise τ₀-NbO-set.

Theorem 2.2. Let (X,τ₁,τ₂) be an NBi-space. Then, the union of two pairwise τ₀-NbO-set in (X,τ₁,τ₂) is also a pairwise τ₀-NSO-set.
Proof. Let \( L \) and \( M \) be two pairwise \( \tau_b \)-NSO-sets in an NBI-T-space \((X,\tau_1,\tau_2)\). So, one can write 
\( L = L_1 \cup L_2 \) and \( M = M_1 \cup M_2 \) where \( L_1, M_1 \) are \( \tau_b \)-NSO-sets and \( L_2, M_2 \) are \( \tau_b \)-NSO-sets in \((X,\tau_1,\tau_2)\). Since, \( L_1 \) and \( M_1 \) are \( \tau_b \)-NSO-sets, so \( L_1 \subseteq N_{\tau_1}^j N_{\tau_1}^i (L_1) \) and \( M_1 \subseteq N_{\tau_1}^j N_{\tau_1}^i (M_1) \). Further, Since \( L_2 \) and \( M_2 \) are 
\( \tau_b \)-NSO-sets, so \( L_2 \subseteq N_{\tau_1}^j N_{\tau_1}^i (L_2) \), \( M_2 \subseteq N_{\tau_1}^j N_{\tau_1}^i (M_2) \). 
Now, \( L \cup M = (L_1 \cup L_2) \cup (M_1 \cup M_2) = (L_1 \cup M_1) \cup (L_2 \cup M_2) \). 
Therefore, \( L \cup M \subseteq N_{\tau_1}^j N_{\tau_1}^i (L_1) \cup N_{\tau_1}^j N_{\tau_1}^i (M_1) \). 
\( \equiv N_{\tau_1}^j N_{\tau_1}^i (L_1 \cup M_1) \). 
This implies, \( L \cup M \) is a \( \tau_b \)-NSO-set in \((X,\tau_1,\tau_2)\).

Similarly, it can be established that \( L \cup M \) is a \( \tau_b \)-NSO-set in \((X,\tau_1,\tau_2)\). Therefore, \( L \cup M \) is a pairwise \( \tau_b \)-NSO-set in \((X,\tau_1,\tau_2)\). Hence, the union of two pairwise \( \tau_b \)-NSO-set in \((X,\tau_1,\tau_2)\) is again a pairwise \( \tau_b \)-NSO-set in \((X,\tau_1,\tau_2)\).

Theorem 2.4. Let \((X,\tau_1,\tau_2)\) be an NBI-T-space. Then, the union of two pairwise \( \tau_b \)-NPO-set in \((X,\tau_1,\tau_2)\) is a pairwise \( \tau_b \)-NPO-set.

Proof. Let \( L \) and \( M \) be two pairwise \( \tau_b \)-NPO-sets in an NBI-T-space \((X,\tau_1,\tau_2)\). So, one can write 
\( L = L_1 \cup L_2 \) and \( M = M_1 \cup M_2 \) where \( L_1, M_1 \) are \( \tau_b \)-NPO-sets and \( L_2, M_2 \) are \( \tau_b \)-NPO-sets in \((X,\tau_1,\tau_2)\). Since, \( L_1 \) and \( M_1 \) are \( \tau_b \)-NPO-sets, so \( L_1 \subseteq N_{\tau_1}^j N_{\tau_1}^i (L_1) \) and \( M_1 \subseteq N_{\tau_1}^j N_{\tau_1}^i (M_1) \). Further, since \( L_2 \) and \( M_2 \) are 
\( \tau_b \)-NPO-sets, so \( L_2 \subseteq N_{\tau_1}^j N_{\tau_1}^i (L_2) \) and \( M_2 \subseteq N_{\tau_1}^j N_{\tau_1}^i (M_2) \). 
Now, \( L \cup M = (L_1 \cup L_2) \cup (M_1 \cup M_2) = (L_1 \cup M_1) \cup (L_2 \cup M_2) \). 
Therefore, \( L \cup M \subseteq N_{\tau_1}^j N_{\tau_1}^i (L_1) \cup N_{\tau_1}^j N_{\tau_1}^i (M_1) \). 
\( \equiv N_{\tau_1}^j N_{\tau_1}^i (L_1 \cup M_1) \). 
This implies, \( L \cup M \) is a \( \tau_b \)-NPO-set in \((X,\tau_1,\tau_2)\). Similarly, it can be established that \( L \cup M \) is a \( \tau_b \)-NPO-set in \((X,\tau_1,\tau_2)\). Therefore, \( L \cup M \) is a pairwise \( \tau_b \)-NPO-set in \((X,\tau_1,\tau_2)\). Hence, the union of two pairwise \( \tau_b \)-NPO-sets in \((X,\tau_1,\tau_2)\) is again a pairwise \( \tau_b \)-NPO-set.

3. Pairwise \( b \)-Continuous Function:

In this section, we procure the notions of pairwise \( b \)-continuous functions via neutrosophic bitopological space and formulate some results on it.

Definition 3.1. Let \((X,\tau_1,\tau_2)\) be an NBI-T-space. Then, the pairwise \( \tau_b \)-neutrosophic-\( b \)-interior (in short \( P_{\tau_b L} N_{\tau_b X} \)) of an \( N \)-set \( L \) is the union of all pairwise \( \tau_b \)-N-bO-sets contained in \( L \), i.e.
\( P_{\tau_b L} N_{\tau_b X} = \bigcup K \) \( K \) is a pairwise \( \tau_b \)-N-bO-set in \( X \) and \( K \subseteq L \).

Clearly, \( P_{\tau_b L} N_{\tau_b X} = \bigcup K \) is the largest pairwise \( \tau_b \)-N-bO-set which contained in \( L \).

Definition 3.2. Let \((X,\tau_1,\tau_2)\) be an NBI-T-space. Then, the pairwise \( \tau_b \)-neutrosophic-\( b \)-closure (in short \( P_{\tau_b L} N_{\tau_b X} \)) of an \( N \)-set \( L \) is the intersection of all pairwise \( \tau_b \)-N-bC-sets containing \( L \), i.e.
\( P_{\tau_b L} N_{\tau_b X} = \bigcap K \) \( K \) is a pairwise \( \tau_b \)-N-bC-set in \( X \) and \( L \subseteq K \).

Clearly, \( P_{\tau_b L} N_{\tau_b X} = \bigcap K \) is the smallest pairwise \( \tau_b \)-N-bC-set which containing \( L \).

Theorem 3.1. Let \( L \) and \( K \) be two neutrosophic subsets of an NBI-T-space \((X,\tau_1,\tau_2)\). Then,
(i) \( P_{\tau_b L} N_{\tau_b X} (0) = 0 \); \( P_{\tau_b L} N_{\tau_b X} (1) = 1 \);
(ii) \( P_{\tau_b L} N_{\tau_b X} (L) = L \);
(iii) \( L \subseteq M \Rightarrow P_{\tau_b L} N_{\tau_b X} (L) \subseteq P_{\tau_b L} N_{\tau_b X} (M) \).
(iv) \( P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(L)=L \) if \( L \) is a pairwise \( \tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}} \)-set.

**Proof.** (i) Straightforward.

(ii) By Definition 3.1, we have \( P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(L)=\bigcup \{ K: K \text{ is a pairwise } \tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}} \text{-set in } X \text{ and } K \subseteq L \} \). Since, each \( K \subseteq L \), so \( \bigcup \{ K: K \text{ is a pairwise } \tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}} \text{-set in } X \text{ and } K \subseteq L \} \), i.e. \( P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(L) \subseteq L \). Therefore, \( P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(L) \subseteq L \).

(iii) Let \( L \) and \( M \) be two neutrosophic subset of an NBi-T-space \((X, \tau_{\mathfrak{p}}, \tau_{\mathfrak{b}})\) such that \( L \subseteq M \).

Now, \( P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(L)=\bigcup \{ K: K \text{ is a pairwise } \tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}} \text{-set in } X \text{ and } K \subseteq L \} \)
\[ \subseteq \bigcup \{ K: K \text{ is a pairwise } \tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}} \text{-set in } X \text{ and } K \subseteq M \} \] [since \( L \subseteq M \)]
\[ =P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(M). \]

Therefore, \( L \subseteq M \Rightarrow P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(L) \subseteq P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(M) \).

(iv) Let \( L \) be a pairwise \( \tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}} \)-set in an NBi-T-space \((X, \tau_{\mathfrak{p}}, \tau_{\mathfrak{b}})\).

Now, \( P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(L) \subseteq \bigcup \{ K: K \text{ is a pairwise } \tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}} \text{-set in } X \text{ and } K \subseteq L \} \). Since, \( L \) is a pairwise \( \tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}} \)-set in \((X, \tau_{\mathfrak{p}}, \tau_{\mathfrak{b}})\), so \( L \) is the largest pairwise \( \tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}} \)-set in \((X, \tau_{\mathfrak{p}}, \tau_{\mathfrak{b}})\), which is contained in \( L \). Therefore, \( \bigcup \{ K: K \text{ is a pairwise } \tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}} \text{-set in } X \text{ and } K \subseteq L \} = L \). This implies, \( P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(L) = L \).

**Theorem 3.2.** Let \( L \) and \( K \) be two neutrosophic subsets of an NBi-T-space \((X, \tau_{\mathfrak{p}}, \tau_{\mathfrak{b}})\). Then,

(i) \( P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(b_0)=b_0 \) & \( P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(1_N)=1_N \);

(ii) \( L \subseteq P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(L) \);

(iii) \( L \subseteq M \Rightarrow P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(L) \subseteq P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(M) \);

(iv) \( P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(L) = L \) if \( L \) is a pairwise \( \tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{c}} \)-set.

**Proof.** (i) Straightforward.

(ii) It is clear that \( P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(L) = \bigcap \{ K: K \text{ is a pairwise } \tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{c}} \text{-set in } X \text{ and } L \subseteq K \} \).

Since, each \( L \subseteq K \), so \( L \subseteq \bigcap \{ K: K \text{ is a pairwise } \tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{c}} \text{-set in } X \text{ and } L \subseteq K \} \), i.e. \( L \subseteq P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(L) \).

(iii) Let \( L \) and \( M \) be two neutrosophic subset of an NBi-T-space \((X, \tau_{\mathfrak{p}}, \tau_{\mathfrak{b}})\) such that \( L \subseteq M \).

Now, \( P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(L) = \bigcap \{ K: K \text{ is a pairwise } \tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{c}} \text{-set in } X \text{ and } L \subseteq K \} \).
\[ \bigcap \{ K: K \text{ is a pairwise } \tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{c}} \text{-set in } X \text{ and } M \subseteq K \} \] [since \( L \subseteq M \)]
\[ =P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(M). \]

Therefore, \( L \subseteq M \Rightarrow P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(L) \subseteq P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(M) \).

(iv) Let \( L \) be a pairwise \( \tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{c}} \)-set in an NBi-T-space \((X, \tau_{\mathfrak{p}}, \tau_{\mathfrak{b}})\). Now, \( P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(L) = \bigcap \{ K: K \text{ is a pairwise } \tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{c}} \text{-set in } X \text{ and } L \subseteq K \} \). Since, \( L \) is a pairwise \( \tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{c}} \)-set in \((X, \tau_{\mathfrak{p}}, \tau_{\mathfrak{b}})\), so \( L \) is the smallest pairwise \( \tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{c}} \)-set, which contains \( L \). This implies, \( \bigcap \{ K: K \text{ is a pairwise } \tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{c}} \text{-set in } X \text{ and } L \subseteq K \} = L \). Therefore, \( P_{\tau_{\mathfrak{p}} N_{\mathfrak{b} \mathfrak{o}}}(L) = L \).
This implies, \(| P \cap T_{\text{N-bC-set}}(L) | = \{(w, \forall T_{\text{N-bC-set}}(w), \forall I_{\text{N-bC-set}}(w), \forall F_{\text{N-bC-set}}(w)) : w \in X \}.

Here \( T_{\text{N-bC-set}}(w) = T_{\text{N-bC-set}}(w), I_{\text{N-bC-set}}(w) = I_{\text{N-bC-set}}(w), F_{\text{N-bC-set}}(w) = F_{\text{N-bC-set}}(w) \), for each \( w \in X \).

Therefore, \( P \cap T_{\text{N-bC-set}}(L) = \{(w, \forall T_{\text{N-bC-set}}(w), \forall I_{\text{N-bC-set}}(w), \forall F_{\text{N-bC-set}}(w)) : w \in X \} \)

\( \cap \{L \mid p \in \Delta \text{ and } L_p \text{ is a pairwise } \tau \text{-N-bC-set in } X \text{ such that } L \subseteq L_p \}. \)

Hence, \( P \cap T_{\text{N-bC-set}}(L) = P \cap T_{\text{N-bC-set}}(L) \).

(ii) Let \( (X, \tau, \tau) \) be an NBi-T-space and \( L = \{(w, T_{\text{N-bC-set}}(w), I_{\text{N-bC-set}}(w), F_{\text{N-bC-set}}(w)) : w \in X \} \) be a N-set over \( X \). Then,

\[ P \cap T_{\text{N-bC-set}}(L) = \{(w, \forall T_{\text{N-bC-set}}(w), \forall I_{\text{N-bC-set}}(w), \forall F_{\text{N-bC-set}}(w)) : w \in X \} \]

where \( L_p \) is a pairwise \( \tau \text{-N-bC-set in } X \text{ such that } L \subseteq L_p \), for each \( p \in \Delta \).

This implies, \( P \cap T_{\text{N-bC-set}}(L) = \{(w, \forall T_{\text{N-bC-set}}(w), \forall I_{\text{N-bC-set}}(w), \forall F_{\text{N-bC-set}}(w)) : w \in X \} \).

Here, \( \forall T_{\text{N-bC-set}}(w) \supseteq T_{\text{N-bC-set}}(w), \forall I_{\text{N-bC-set}}(w) \subseteq I_{\text{N-bC-set}}(w), \forall F_{\text{N-bC-set}}(w) \subseteq F_{\text{N-bC-set}}(w) \), for each \( w \in X \).

Therefore, \( P \cap T_{\text{N-bC-set}}(L) = \{(w, \forall T_{\text{N-bC-set}}(w), \forall I_{\text{N-bC-set}}(w), \forall F_{\text{N-bC-set}}(w)) : w \in X \} \)

\( \cap \{L \mid p \in \Delta \text{ and } L_p \text{ is a pairwise } \tau \text{-N-bO-set in } X \text{ such that } L \subseteq L_p \}. \)

Hence, \( P \cap T_{\text{N-bC-set}}(L) = P \cap T_{\text{N-bC-set}}(L) \).

**Theorem 3.1.** Let \( (X, \tau, \tau) \) be an NBi-T-space. Then, the neutrosophic null set \((0_\tau)\) and the neutrosophic whole set \((1_\tau)\) are both \( \tau \)-N-bO-set and \( \tau \)-N-bC-set.

**Proof.** Let \( (X, \tau, \tau) \) be an NBi-T-space. Now, \( N^j_{\text{cl}} N^i_{\text{int}} (0_\tau) \cup N^j_{\text{cl}} N^i_{\text{int}} (0_\tau) = N^j_{\text{cl}} N^i_{\text{int}} (0_\tau) = 0_\tau \cup 0_\tau = 0_\tau \).

Therefore, \( 0_\tau \subseteq 0_\tau \cup N^j_{\text{cl}} N^i_{\text{int}} (0_\tau) \). Hence, the neutrosophic null set \((0_\tau)\) is a \( \tau \)-N-bO-set.

Similarly, it can be established that the neutrosophic whole set \((1_\tau)\) is a \( \tau \)-N-bC-set.

Further, one can show that the neutrosophic whole set \((1_\tau)\) are both \( \tau \)-N-bO-set and \( \tau \)-N-bC-set.

**Theorem 3.2.** In an NBi-T-space \((X, \tau, \tau)\), every \( \tau \)-NO-set is a \( \tau \)-N-bO-set.

**Proof.** Let \( (X, \tau, \tau) \) be an NBi-T-space.\( (X, \tau, \tau) \). Therefore, \( N^i_{\text{int}} (L) = N^i_{\text{int}} (L) \).

This implies, \( L \subseteq N^i_{\text{int}} (L) \). Hence, \( L \) is a \( \tau \)-N-bO-set in \( (X, \tau, \tau) \).

**Theorem 3.3.** In an NBi-T-space \((X, \tau, \tau)\),

(i) every \( \tau \)-N-bO-set is a pairwise \( \tau \)-N-bO-set;

(ii) every \( \tau \)-N-bC-set is a pairwise \( \tau \)-N-bC-set;

(iii) every \( \tau \)-N-bC-set is a pairwise \( \tau \)-N-bC-set.

**Proof.** (i) Let \( (X, \tau, \tau) \) be a \( \tau \)-N-bO-set in an NBi-T-space \((X, \tau, \tau)\). Then, \( L \) can be expressed as \( L = L \cap 0_\tau \), where \( L \) is a \( \tau \)-N-bO-set and \( 0_\tau \) is a \( \tau \)-N-bO-set in \((X, \tau, \tau)\). This implies, \( L \) is a pairwise \( \tau \)-N-bO-set in \((X, \tau, \tau)\).

(ii) Straightforward.

(iii) Let \( (X, \tau, \tau) \) be a \( \tau \)-N-bC-set in an NBi-T-space \((X, \tau, \tau)\). Then, \( L \) can be expressed as \( L = L \cap 1_\tau \), where \( L \) is a \( \tau \)-NC-set and \( 1_\tau \) is a \( \tau \)-NC-set in \((X, \tau, \tau)\). This implies, \( L \) is a pairwise \( \tau \)-N-bC-set in \((X, \tau, \tau)\).

(iv) Straightforward.

**Theorem 3.4.** In an NBi-T-space \((X, \tau, \tau)\), every \( \tau \)-NO-set is a pairwise \( \tau \)-N-bO-set.

**Proof.** Let \( (X, \tau, \tau) \) be an NBi-T-space \((X, \tau, \tau)\). By Theorem 3.2, it is clear that \( L \) is a \( \tau \)-N-bO-set. Further, by Theorem 3.3, it is clear that \( L \) is a pairwise \( \tau \)-N-bO-set.

**Theorem 3.5.** Let \((X, \tau, \tau)\) be an NBi-T-space. Then, \( 0_\tau \) and \( 1_\tau \) are both pairwise \( \tau \)-N-bO-set and pairwise \( \tau \)-N-bO-set.

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Proof. Let \((X, \tau, \mu)\) be an NBi-T-space. One can write \(0_N = A \cup B\), where \(A = 0_N\) is a \(\tau_N\)-N-bO-set and \(B = 0_N\) is a \(\tau_N\)-N-bO-set in \((X, \tau, \mu)\). This implies, \(0_N\) is a pairwise \(\tau_N\)-N-bO-set in \((X, \tau, \mu)\).

Similarly, it can be established that \(0_N\) is a pairwise \(\tau_N\)-N-bO-set in \((X, \tau, \mu)\).

Again, one can write \(1_N = L \cup M\), where \(L = 1_N\) is a \(\tau_N\)-N-bO-set and \(M = 1_N\) is a \(\tau_N\)-N-bO-set in \((X, \tau, \mu)\). This implies, \(1_N\) is a pairwise \(\tau_N\)-N-bO-set in \((X, \tau, \mu)\).

Similarly, it can be also established that \(1_N\) is a pairwise \(\tau_N\)-N-bO-set in \((X, \tau, \mu)\).

**Theorem 3.6.** Let \((X, \tau, \mu)\) be an NBi-T-space. Then, both \(0_N\) and \(1_N\) are pairwise \(\tau_N\)-N-bC-set and pairwise \(\tau_N\)-N-bC-set.

**Proof.** By Theorem 3.5, it is clear that \(0_N\) is both pairwise \(\tau_N\)-N-bO-set and pairwise \(\tau_N\)-N-bO-set. Hence, its complement \(1_N\) is both pairwise \(\tau_N\)-N-bC-set and pairwise \(\tau_N\)-N-bC-set.

Similarly, from Theorem 3.5, it is clear that \(1_N\) is both pairwise \(\tau_N\)-N-bO-set and pairwise \(\tau_N\)-N-bO-set. Hence, its complement \(0_N\) is both pairwise \(\tau_N\)-N-bC-set and pairwise \(\tau_N\)-N-bC-set.

**Remark 3.1.** Throughout the article, we denote \(\tau^i_{ji}\) as a collection of all pairwise \(\tau_N\)-N-bO-sets and \(\tau^i_{ji}\) as a collection of all pairwise \(\tau_N\)-N-bC-sets in \((X, \tau, \mu)\). The collection \(\tau^i_{ji}\) forms an neutrosophic supra topology on \(X\).

**Definition 3.3.** Let \((X, \tau, \mu)\) and \((Y, \delta, \beta)\) be two NBi-T-spaces. Then, an one to one and onto mapping \(\xi: (X, \tau, \mu) \rightarrow (Y, \delta, \beta)\) is called a

(i) pairwise neutrosophic semi continuous mapping (in short P-NS-C-mapping) if and only if \(\xi^{-1}(L)\) is a \(\tau_N\)-NSO-set in \(X\), whenever \(L\) is a pairwise \(\delta_N\)-NO-set in \(Y\).

(ii) pairwise neutrosophic pre continuous mapping (in short P-NP-C-mapping) if and only if \(\xi^{-1}(L)\) is a \(\tau_N\)-NPO-set in \(X\), whenever \(L\) is a pairwise \(\delta_N\)-NO-set in \(Y\).

(iii) pairwise neutrosophic continuous mapping (in short P-N-C-mapping) if and only if \(\xi^{-1}(L)\) is a \(\tau_N\)-NO-set in \(X\), whenever \(L\) is a pairwise \(\delta_N\)-NO-set in \(Y\).

(iv) pairwise neutrosophic b-continuous mapping (in short P-N-b-C-mapping) if and only if \(\xi^{-1}(L)\) is a \(\tau_N\)-bO-set in \(X\), whenever \(L\) is a pairwise \(\delta_N\)-NO-set in \(Y\).

**Theorem 3.7.** Let \((X, \tau, \mu)\) and \((Y, \delta, \beta)\) be two NBi-T-spaces. Then, every P-N-C-mapping from \((X, \tau, \mu)\) to \((Y, \delta, \beta)\) is a P-NS-C-mapping (P-NS-C-mapping).

**Proof.** Let \(L\) be a pairwise \(\delta_N\)-NO-set in \((Y, \delta, \beta)\). Since, \(\xi(X, \tau, \mu) \rightarrow (Y, \delta, \beta)\) is a P-N-C-mapping from \((X, \tau, \mu)\) to \((Y, \delta, \beta)\), so \(\xi^{-1}(L)\) is a \(\tau_N\)-NO-set in \((X, \tau, \mu)\). It is known that every \(\tau_N\)-NO-set is a \(\tau_N\)-NPO-set (\(\tau_N\)-NSO-set). Therefore, \(\xi^{-1}(L)\) is a \(\tau_N\)-NPO-set (\(\tau_N\)-NSO-set) in \((X, \tau, \mu)\). Hence, \(\xi: (X, \tau, \mu) \rightarrow (Y, \delta, \beta)\) is a P-NP-C-mapping (P-NS-C-mapping).

**Theorem 3.8.** Let \((X, \tau, \mu)\) and \((Y, \delta, \beta)\) be two NBi-T-spaces. Then, every P-N-C-mapping (P-NP-C-mapping) from \((X, \tau, \mu)\) to \((Y, \delta, \beta)\) is a P-N-b-C-mapping.

**Proof.** Let \(L\) be a pairwise \(\delta_N\)-NO-set in \((Y, \delta, \beta)\). Since, \(\xi(X, \tau, \mu) \rightarrow (Y, \delta, \beta)\) is a P-N-C-mapping (P-NP-C-mapping) from \((X, \tau, \mu)\) to \((Y, \delta, \beta)\), so \(\xi^{-1}(L)\) is a \(\tau_N\)-NO-set (\(\tau_N\)-NPO-set) in \((X, \tau, \mu)\). It is known that, every \(\tau_N\)-NO-set (\(\tau_N\)-NPO-set) is a \(\tau\)-N-bO-set. Therefore, \(\xi^{-1}(L)\) is a \(\tau\)-N-bO-set in \((X, \tau, \mu)\). Hence, \(\xi: (X, \tau, \mu) \rightarrow (Y, \delta, \beta)\) is a P-N-b-C-mapping.

**Theorem 3.9.** Let \((X, \tau, \mu)\) and \((Y, \delta, \beta)\) be two NBi-T-spaces. Then, every P-N-C-mapping from \((X, \tau, \mu)\) to \((Y, \delta, \beta)\) is a P-N-b-C-mapping.

**Proof.** Let \(L\) be a pairwise \(\delta_N\)-NO-set in \((Y, \delta, \beta)\). Since, \(\xi(X, \tau, \mu) \rightarrow (Y, \delta, \beta)\) is a P-N-C-mapping from \((X, \tau, \mu)\) to \((Y, \delta, \beta)\), so \(\xi^{-1}(L)\) is a \(\tau\)-NO-set in \((X, \tau, \mu)\). It is known that, every \(\tau\)-NO-set is a \(\tau\)-N-bO-set. Therefore, \(\xi^{-1}(L)\) is a \(\tau\)-N-bO-set in \((X, \tau, \mu)\). It is known that, every \(\tau\)-NO-set is a
\(\tau\)-N-b-O-set. Therefore, \(\xi_i^j(L)\) is a \(\tau\)-N-b-O-set in \((X,\tau_1,\tau_2)\). Hence, \(\xi_i(X,\tau_1,\tau_2)\rightarrow(Y,\delta_1,\delta_2)\) is a \(\rho\)-N-b-C-mapping.

**Theorem 3.10.** If \(\xi_i(X,\tau_1,\tau_2)\rightarrow(Y,\delta_1,\delta_2)\) and \(\chi_j(Y,\delta_1,\delta_2)\rightarrow(\tau,\theta_1,\theta_2)\) be two \(\rho\)-C-mapping, then the composition mapping \(\chi_j^i\xi_i(X,\tau_1,\tau_2)\rightarrow(\tau,\theta_1,\theta_2)\) is also a \(\rho\)-N-C-mapping.

**Proof.** Let \(\xi_i(X,\tau_1,\tau_2)\rightarrow(Y,\delta_1,\delta_2)\) and \(\chi_j(Y,\delta_1,\delta_2)\rightarrow(\tau,\theta_1,\theta_2)\) be two \(\rho\)-C-mappings. Let \(L\) be a pairwise \(\theta_1\)-NO-set in \((\tau,\theta_1,\theta_2)\). Since, \(\chi_j(Y,\delta_1,\delta_2)\rightarrow(\tau,\theta_1,\theta_2)\) is a \(\rho\)-N-C-mapping, so \(\chi_j^i(L)\) is a \(\delta_1\)-NO-set in \(Y\). Since, \(\xi_i(X,\tau_1,\tau_2)\rightarrow(Y,\delta_1,\delta_2)\) is a \(\rho\)-N-C-mapping, so \(\xi_i^j(\chi_j^i(L))=\chi_j^i(\xi_i(L))\) is a \(\tau\)-NO-set in \(X\).

**Theorem 3.11.** If \(\xi_i(X,\tau_1,\tau_2)\rightarrow(Y,\delta_1,\delta_2)\) be an one to one and onto mapping between two NBi-T-spaces, then the following two are equivalent:

(i) \(\xi_i\) is a \(\rho\)-N-b-C-mapping,

(ii) \(\xi_i(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}A(\xi_i))\subseteq(\tau\text{-}N\text{-}b\text{-}u\text{-}A(\xi_i)),\) for every neutrosophic subset \(A\) of \(Y\).

**Proof.** (i) \(\Rightarrow\)(ii)

Let \(\xi_i(X,\tau_1,\tau_2)\rightarrow(Y,\delta_1,\delta_2)\) be a \(\rho\)-N-b-C-mapping. Let \(A\) be a neutrosophic subset of \(Y\). Here, \(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}A(\xi_i)\) is a pairwise \(\delta_1\)-NO-set in \(Y\) and \(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}A(\xi_i)\subseteq A\). This implies, \(\xi_i(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}A(\xi_i))\subseteq\xi_i(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}A(\xi_i))\). By the hypothesis, \(\xi_i(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}A(\xi_i))\) is a \(\tau\)-N-b-O-set in \(X\). Therefore, \(\xi_i(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}A(\xi_i))\) is a \(\tau\)-N-b-O-set in \(X\) such that \(\xi_i(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}A(\xi_i))\subseteq\xi_i(\tau\text{-}N\text{-}b\text{-}u\text{-}A(\xi_i))\). It is known that \(\tau\text{-}N\text{-}b\text{-}u\text{-}A(\xi_i)\) is the largest \(\tau\)-N-b-O-set in \(X\), which is contained in \(\xi_i(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}A(\xi_i))\). Hence, \(\xi_i(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}A(\xi_i))\subseteq\tau\text{-}N\text{-}b\text{-}u\text{-}A(\xi_i)\).

(ii) \(\Rightarrow\)(i)

Let \(A\) be a pairwise \(\delta_1\)-NO-set in \((Y,\delta_1,\delta_2)\). Therefore, \(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}A(\xi_i)=A\). By hypothesis, \(\xi_i(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}A(\xi_i))\subseteq\tau\text{-}N\text{-}b\text{-}u\text{-}A(\xi_i)\). This implies, \(\xi_i(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}A(\xi_i))\subseteq\xi_i(\tau\text{-}N\text{-}b\text{-}u\text{-}A(\xi_i))\). It is known that \(\tau\text{-}N\text{-}b\text{-}u\text{-}A(\xi_i)\subseteq\xi_i(\tau\text{-}N\text{-}b\text{-}u\text{-}A(\xi_i))\). Therefore, \(\tau\text{-}N\text{-}b\text{-}u\text{-}A(\xi_i)=\xi_i(\tau\text{-}N\text{-}b\text{-}u\text{-}A(\xi_i))\). Hence, \(\xi_i(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}A(\xi_i))\) is a \(\tau\)-N-b-O-set in \((X,\tau_1,\tau_2)\). Therefore, \(\xi_i\) is a \(\rho\)-N-b-C-mapping from an NBi-T-space \((X,\tau_1,\tau_2)\) to another NBi-T-space \((Y,\delta_1,\delta_2)\).

**Theorem 3.12.** An one to one and onto mapping \(\xi_i(X,\tau_1,\tau_2)\rightarrow(Y,\delta_1,\delta_2)\) is a \(\rho\)-N-b-C-mapping if and only if \(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}A(\xi_i)\subseteq\xi_i(\tau\text{-}N\text{-}b\text{-}u\text{-}A(\xi_i))\), for every \(N\)-set \(A\) over \(X\) and \(i=1,2\), and \(i\in\mathbb{R}\).

**Proof.** Let \(\xi_i(X,\tau_1,\tau_2)\rightarrow(Y,\delta_1,\delta_2)\) be a \(\rho\)-N-b-C-mapping. Let \(A\) be an \(N\)-set over \(X\). Then, \(\xi_i(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}A(\xi_i))\) is also an \(N\)-set over \(Y\). By Theorem 3.11, we have \(\xi_i(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}A(\xi_i))\subseteq\tau\text{-}N\text{-}b\text{-}u\text{-}A(\xi_i(\xi_i(A)))\). This implies, \(\xi_i(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}A(\xi_i))\subseteq\tau\text{-}N\text{-}b\text{-}u\text{-}A(\xi_i(A))\). Hence, \(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}A(\xi_i(\xi_i(A)))\subseteq\tau\text{-}N\text{-}b\text{-}u\text{-}A(\xi_i(A))\). Therefore, \(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}A(\xi_i(\xi_i(A)))\subseteq\tau\text{-}N\text{-}b\text{-}u\text{-}A(\xi_i(A))\), for every \(N\)-set \(A\) of \(Y\). Hence, by Theorem 3.11., the mapping \(\xi_i(X,\tau_1,\tau_2)\rightarrow(Y,\delta_1,\delta_2)\) is a \(\rho\)-N-b-C-mapping.

**Corollary 3.1.** If \(\xi_i(X,\tau_1,\tau_2)\rightarrow(Y,\delta_1,\delta_2)\) is an one to one and onto mapping from an NBi-T-space \((X,\tau_1,\tau_2)\) to another NBi-T-space \((Y,\delta_1,\delta_2)\), then the following two are equivalent:

(i) \(\xi_i\) is a \(\rho\)-N-C-mapping,

(ii) \(\xi_i(\rho\text{-}N\text{-}b\text{-}N\text{-}u\text{-}Q)\subseteq\tau\text{-}N\text{-}u\text{-}A(\xi_i(Q))\), for every \(N\)-set \(Q\) over \(Y\).

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Definition 3.4. Let \((X,\tau_1,\tau_2)\) be an NBi-T-space. Let \(x_{ab}\) be an N-point in \(X\). Then, an N-set \(Q\) over \(X\) is called a pairwise \(\tau_\theta\)-neutrosophic \(b\)-neighbourhood (in short \(P-\tau_\theta\)-\(N\)-\(b\)-nbhd) of \(x_{ab}\), if there exist a pairwise \(\tau_\theta\)-\(N\)-\(b\)-O-set \(U\) such that \(x_{ab}\in U\subseteq Q\).

Theorem 3.13. Let \((X,\tau_1,\tau_2)\) be an NBi-T-space. An N-set \(Q\) over \(X\) is a pairwise \(\tau_\theta\)-\(N\)-\(b\)-O-set if and only if \(Q\) is a \(P-\tau_\theta\)-\(N\)-\(b\)-nbhd of all of its N-points.

Proof. Let \(Q\) be a pairwise \(\tau_\theta\)-\(N\)-\(b\)-O-set in an NBi-T-space \((X,\tau_1,\tau_2)\). Let \(x_{ab}\) be an N-point in \(X\) such that \(x_{ab}\in Q\). Therefore, \(x_{ab}\in Q\subseteq Q\). This implies, \(Q\) is a \(P-\tau_\theta\)-\(N\)-\(b\)-nbhd of \(x_{ab}\). Hence, \(Q\) is the \(P-\tau_\theta\)-\(N\)-\(b\)-nbhd of all of its N-points.

Conversely, let \(Q\) be a \(P-\tau_\theta\)-\(N\)-\(b\)-nbhd of all of its N-points. Assume that \(x_{ab}\) be an N-point in \(X\), such that \(x_{ab}\in Q\). Therefore, there exist a pairwise \(\tau_\theta\)-\(N\)-\(b\)-O-set \(G\) such that \(x_{ab}\in G\subseteq Q\).

Now, \(Q=U_{x_{a,b}\in G} \subseteq U_{x_{a,b}\in G} G\subseteq U_{x_{a,b}\in G} Q=Q\). This implies, \(Q=U_{x_{a,b}\in G} G\), which is a pairwise \(\tau_\theta\)-\(N\)-\(b\)-set. Therefore, \(Q\) is a pairwise \(\tau_\theta\)-\(N\)-\(b\)-O-set in \((X,\tau_1,\tau_2)\).

Theorem 3.14. An one to one and onto mapping \(\xi(X,\tau_1,\tau_2)\rightarrow(Y,\delta_1,\delta_2)\) is a \(P\)-\(N\)-\(b\)-C-mapping if and only if for every N-point \(x_{ab}\in Y\) and for any \(P-\delta_\phi\)-\(N\)-\(b\)-nbhd \(V\) of \(x_{ab}\) in \(Y\), there exist a \(\tau\)-neutrosophic-\(b\)-neighbourhood (in short \(\tau\)-\(N\)-\(b\)-nbhd) \(U\) of \(\xi(x_{ab})\) in \(X\) such that \(U\subseteq \xi^{-1}(V)\).

Proof. Let \(\xi(X,\tau_1,\tau_2)\rightarrow(Y,\delta_1,\delta_2)\) be a \(P\)-\(N\)-\(b\)-C-mapping. Let \(x_{ab}\) be an N-point in \(Y\) and \(V\) be a \(P-\delta_\phi\)-\(N\)-\(b\)-nbhd of \(x_{ab}\). Then, there exist a pairwise \(\delta_\phi\)-\(N\)-\(b\)-O-set \(G\) in \(Y\) such that \(x_{ab}\in G\subseteq V\). This implies, \(\xi^{-1}(x_{ab})\in \xi^{-1}(G)\subseteq \xi^{-1}(V)\). Since, \(\xi(X,\tau_1,\tau_2)\rightarrow(Y,\delta_1,\delta_2)\) is a \(P\)-\(N\)-\(b\)-C-mapping, so \(\xi^{-1}(G)\) is a \(\tau\)-\(N\)-\(b\)-O-set in \(X\). By taking \(U=\xi^{-1}(G)\), we see that \(U\) is a \(\tau\)-\(N\)-\(b\)-O-set in \(X\) such that \(\xi(x_{ab})\in U\subseteq \xi^{-1}(V)\). Hence, \(U=\xi^{-1}(G)\) is a \(\tau\)-\(N\)-\(b\)-nbhd of \(\xi^{-1}(x_{ab})\) and \(U\subseteq \xi^{-1}(V)\).

Conversely, let for every N-point \(x_{ab}\in Y\) and for any \(P-\delta_\phi\)-\(N\)-\(b\)-nbhd \(V\) of \(x_{ab}\) in \(Y\), there exist a \(\tau\)-\(N\)-\(b\)-nbhd \(U\) of \(\xi(x_{ab})\) in \(X\) such that \(U\subseteq \xi^{-1}(V)\). Let \(G\) be a pairwise \(\delta_\phi\)-\(N\)-\(b\)-O-set in \(Y\) and \(x_{ab}\in G\). By Theorem 3.13., \(G\) is a \(P-\delta_\phi\)-\(N\)-\(b\)-nbhd of \(x_{ab}\). By hypothesis, there exists a \(\tau\)-\(N\)-\(b\)-nbhd \(H\) of \(\xi^{-1}(x_{ab})\in X\) such that \(\xi^{-1}(x_{ab})\in H\subseteq \xi^{-1}(G)\). This implies, \(\xi^{-1}(G)\) is the \(\tau\)-\(N\)-\(b\)-nbhd of each of its N-points. Therefore, \(\xi^{-1}(G)\) is a \(\tau\)-\(N\)-\(b\)-O-set in \(X\). Hence, \(\xi(X,\tau_1,\tau_2)\rightarrow(Y,\delta_1,\delta_2)\) is a \(P\)-\(N\)-\(b\)-C-mapping.

Theorem 3.15. If \(\xi(X,\tau_1,\tau_2)\rightarrow(Y,\delta_1,\delta_2)\) be a \(P\)-\(N\)-\(b\)-C-mapping and \(\chi(Y,\delta_1,\delta_2)\rightarrow(Z,\theta_1,\theta_2)\) be a \(P\)-\(N\)-\(C\)-mapping, then the composition mapping \(\chi\circ\xi(X,\tau_1,\tau_2)\rightarrow(Z,\theta_1,\theta_2)\) be a \(P\)-\(N\)-\(b\)-C-mapping.

Proof. Let \(\xi(X,\tau_1,\tau_2)\rightarrow(Y,\delta_1,\delta_2)\) be a \(P\)-\(N\)-\(b\)-C-mapping and \(\chi(Y,\delta_1,\delta_2)\rightarrow(Z,\theta_1,\theta_2)\) be a \(P\)-\(N\)-\(C\)-mapping. Let \(L\) be a pairwise \(\theta_1\)-\(N\)-\(O\)-set in \((Z,\theta_1,\theta_2)\). Since, \(\chi(Y,\delta_1,\delta_2)\rightarrow(Z,\theta_1,\theta_2)\) be a \(P\)-\(N\)-\(C\)-mapping, so \(\chi^{-1}(L)\) be a \(\delta_\phi\)-\(N\)-\(O\)-set in \(Y\). Now, by Lemma 2.1., it is clear that \(\chi^{-1}(L)\) is a pairwise \(\delta_\phi\)-\(N\)-\(O\)-set in \((Y,\delta_1,\delta_2)\). Since, \(\xi(X,\tau_1,\tau_2)\rightarrow(Y,\delta_1,\delta_2)\) be a \(P\)-\(N\)-\(b\)-C-mapping, so \(\xi^{-1}(\chi^{-1}(L))=(\chi^{-1}(L))\) is a \(\tau\)-\(N\)-\(O\)-set in \(X\). Since, every \(\tau\)-\(N\)-\(O\)-set is a \(\tau\)-\(N\)-\(b\)-O-set, so \((\chi^{-1}(L))\) is a \(\tau\)-\(N\)-\(b\)-O-set in \(X\). Hence, \(\chi\circ\xi(X,\tau_1,\tau_2)\rightarrow(Z,\theta_1,\theta_2)\) be a \(P\)-\(N\)-\(b\)-C-mapping.

4. Conclusion
In this article, we introduce the notion of pairwise neutrosophic-\(b\)-interior, pairwise neutrosophic-\(b\)-closure, pairwise neutrosophic-\(b\)-continuous mapping, we prove some propositions and theorems on NBi-T-spaces. In the future, we hope that based on these notions in NBi-T-spaces, many new investigations can be carried out.

Conflict of Interest: The authors declare that they have no conflict of interest.

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