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Parameter Reduction of Neutrosophic Soft Sets and Their Applications

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Abstract: Parameter reduction can be treated as an effective tool in many fields, including pattern recognition. Many reduction techniques have been reported so far for soft sets, fuzzy soft sets and bipolar fuzzy soft sets to solve decision-making problems. However, there is almost no attention to the parameter reduction of neutrosophic soft sets. In this present paper we focus our discussion on the parameter reduction of neutrosophic soft sets as an extension of parameter reduction of soft sets and fuzzy soft sets. To do that, using the concept of indiscernibility relation, we first define the terms 'dispensable set' and 'indispensible set'. We utilize these definitions to define the terms 'decision partition', 'parameter reduction' and 'degree of importance of a parameter' with a suitable example. Next we present an algorithm based on the concept of degree of importance and parameter reduction of a neutrosophic soft set. An illustrative example is employed to show the feasibility and validity of our proposed algorithm based on parameter reduction of neutrosophic soft sets in real life decision making problem.

Keywords: Neutrosophic set, neutrosophic soft set, parameter reduction, decision making.

1. Introduction

Molodstov **[31]** initiated the concept of soft set theory as a fundamental mathematical tool for modelling uncertainty, vague concepts and not clearly defined objects. Although various traditional tools, including but not limited to rough set theory **[33]**, fuzzy set theory **[41]**, intuitionistic fuzzy set theory **[10]** etc. have been used by many researchers to extract useful information hidden in the uncertain data, but there are inherent complications connected with each of these theories.

Additionally, all these approaches lack in parameterizations of the tools and hence they couldn't be applied effectively in real life problems, especially in areas like environmental, economic and social problems. Soft set theory is standing uniquely in the sense that it is free from the above mentioned impediments and obliges approximate illustration of an object from the beginning, which makes this theory a natural mathematical formalism for approximate reasoning.

The Theory of soft set has excellent potential for application in various directions some of which are reported by Molodtsov **[31]** in his pioneer work. Later on Maji et al. **[27]** introduced some new annotations on soft sets such as subset, complement, union and intersection of soft sets and discussed in detail its applications in decision making problems. Ali et al. **[7]** defined some new operations on soft sets and shown that De Morgan's laws holds in soft set theory with respect to these newly defined operations. Atkas and Cagman **[6]** compared soft sets with fuzzy sets and rough sets to show that every fuzzy set and every rough set may be considered as a soft set. Jun **[24]** connected soft sets to the theory of BCK/BCI-algebra and introduced the concept of soft BCK/BCI-algebras. Feng et al.**[21]** characterized soft semi rings and a few related notions to establish a relation between soft sets and semi rings.

Chen et al. **[15]** introduced the concept of parameter reduction of soft sets in 2005. In 2008, Z. Kong et al **[25]** introduced the definition of normal parameter reduction in soft sets and presented a heuristic algorithm of normal parameter reduction. The soft sets mentioned above are based on complete information. However, incomplete information widely exists in various real life problems. H. Qin et al **[34]** studied the data filling approach of incomplete soft sets. Y. Zou et al **[42]** investigated data analysis approaches of soft sets under incomplete information. In 2001, Maji et al. **[28]** defined the concept of fuzzy soft set by combining of fuzzy sets **[41]** and soft sets **[31]**. Roy and Maji **[35]** proposed a fuzzy soft set based decision making method.

Xiao et al. **[39]** presented a combined forecasting method based on fuzzy soft set. Feng et al. **[22]** discussed the validity of the Roy-Maji method **[35]** and presented an adjustable decision-making method based on fuzzy soft set. Yang et al. **[40]** initiated the idea of interval valued fuzzy soft set (IVFS-set) and analyzed a decision making method using the IVFS-sets. The notion of intuitionistic fuzzy set (IFS) was initiated by Atanassov **[10]** as a significant generalization of fuzzy set **[41]**. Intuitionistic fuzzy sets are very useful in situations when description of a problem by a linguistic variable, given in terms of a membership function only, seems too complicated. Recently intuitionistic fuzzy sets have been applied to many fields such as logic programming, medical diagnosis, decision making problems etc.

Smarandache **[38]** introduced the concept of neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. Maji **[30]** introduced the concept of neutrosophic soft set and established some operations on these sets. Mukherjee et al **[32]** introduced the concept of interval valued neutrosophic soft sets and studied their basic properties. In 2013, Broumi and Smarandache **[12, 13]** combined the intuitionistic neutrosophic and soft set which lead to a new mathematical model called" intuitionistic neutrosophic soft set". They studied the notions of intuitionistic neutrosophic soft set union, intuitionistic neutrosophic soft set intersection, complement of intuitionistic neutrosophic soft set and several other properties of intuitionistic neutrosophic soft set along with examples and proofs of certain results.

Also, in **[11]** S. Broumi presented the concept of "generalized neutrosophic soft set" by combining the generalized neutrosophic sets **[11]** and soft set models, studied some properties on it, and presented an application of generalized neutrosophic soft set **[11]** in decision making problem. Recently, Deli **[17]** introduced the concept of interval valued neutrosophic soft set as a combination of interval neutrosophic set and soft set. In 2014, S. Broumi et al. **[14]** initiated the concept of relations on interval valued neutrosophic soft sets.I. Deli **[18]** proposed a new notation called expansion and reduction of the neutrosophic classical soft sets that are based on the linguistic modifiers. Saha et al. **[36]** proposed the concept of data filling of neutrosophic soft sets having incomplete/missing data. Few more works on neutrosophic soft sets can be found in **[9, 19, 23, 37]**.

Parameter reduction can be treated an effective tool in many fields, including pattern recognition. Many reduction techniques **[8, 15, 16, 20, 25, 26]** have been reported so far for soft sets, fuzzy soft sets and bipolar fuzzy soft sets to solve decision-making problems. However, there is almost no attention to the parameter reduction of neutrosophic soft sets. In this present paper we focus our discussion on the parameter reduction of neutrosophic soft sets as an extension of parameter reduction of soft sets and fuzzy soft sets.

This present paper is organized as follows:

Section-2 presents some basic definitions related to fuzzy set theory with their generalizations and soft set theory with their generalizations. In section-3, we first present the concept of indiscernibility relations and then based on it, we define the terms 'dispensable set', 'indispensible set', 'decision partition', 'parameter reduction', 'degree of importance of a parameter' with a suitable example in neutrosophic soft environment. In the next section (section-4), we have presented an algorithm based on the concept of degree of importance and parameter reduction supported by an illustrative example to show the feasibility and validity of our algorithm.

2. Preliminaries:

2.1 Definition: [41] Let U be a non empty set. Then afuzzy set τ on U is a set having the form $\tau = \{(x, \mu_\tau(x)) : x \in U\}$ where the function $\mu_\tau : U \to [0, 1]$ is called the membership function and $\mu_\tau(x)$ represents the degree ofmembership of each element $x \in U$.

2.2 Definition: [10] Let U be a non empty set. Then an intuitionistic fuzzy set (IFS for short) τ is an object having the form $\tau = \{\langle x, \mu_\tau(x), \gamma_\tau(x)\rangle : x \in U\}$ where the functions $\mu_\tau : U \to [0, 1]$ and $\gamma_\tau : U \to [0, 1]$ are called membership function and non-membership function respectively.

$\mu_\tau(x)$ and $\gamma_\tau(x)$ represent the degree ofmembership and the degree of non-membership respectively of each element $x \in U$ and $0 \le \mu_\tau(x) + \gamma_\tau(x) \le 1$ for each $x \in U$. We denote the class of all intuitionistic fuzzy sets on U by IFS^U.

2.3 Definition: [31] Let U be a universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subseteq E$. Then the pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.

In other words, the soft set is not a kind of set, but a parameterized family of subsets of U. For $e \in A$, $F(e) \subseteq U$ may be considered as the set of e-approximate elements of the soft set (F, A).

2.4 Definition: [28] Let U be a universe set, E be a set of parameters and $A \subseteq E$. Then the pair (F, A) is called a fuzzy soft setover U, where F is a mapping given by $F : A \to FS^U$.

2.5 Definition: [29] Let U be a universe set, E be a set of parameters and $A \subseteq E$. Then the pair (F, A) is called an intuitionistic fuzzy soft set over U, where F is a mapping given by $F : A \to IFS^U$. For $e \in A$, $F(e)$ is an intuitionistic fuzzy subset of U and is called the intuitionistic fuzzy value set of the parameter 'e'.

Let us denote $\mu_{F(e)}(x)$ by the membership degree that object 'x' holds parameter 'e' and $\gamma_{F(e)}(x)$ by the membership degree that object 'x' doesn't hold parameter 'e' , where $e \in A$ and $x \in U$. Then $F(e)$ can be written as an intuitionistic fuzzy set such that $F(e) = \{(x, \mu_{F(e)}(x), \gamma_{F(e)}(x)) : x \in U\}$.

2.6 Definition: [38] A neutrosophicset A on the universe of discourse U is defined as $A = \{\langle x, \mu_A(x), \gamma_A(x), \delta_A(x)\rangle : x \in U\}$, where $\mu_A, \gamma_A, \delta_A : U \to]^-0, 1^+[$ are functions such that the condition: $\forall x \in U, {}^-0 \le \mu_A(x) + \gamma_A(x) + \delta_A(x) \le 3^+$ is satisfied.

Here $\mu_A(x), \gamma_A(x), \delta_A(x)$ represent the truth-membership, indeterminacy-membership and falsity-membership (hesitancy membership) respectively of the element $x \in U$.

Smarandache **[25]** applied neutrosophic sets in many directions after giving examples of neutrosophic sets. Then he introduced the neutrosophic set operations namely-complement, union, intersection, difference, Cartesian product etc.

2.7 Definition: **[30]** Let U be an initial universe, E be a set of parameters and $A \subseteq E$. Let $NP(U)$ denotes the set of all neutrosophic sets of U. Then the pair (f, A) is termed to be the neutrosophicsoftset over U, where f is a mapping given by $f : A \rightarrow NP(U)$.

2.8 Example: Let us consider a neutrosophic soft set (f, A) which describes the "attractiveness of the house". Suppose $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be the set of six houses under consideration and $E = \{e_1 \text{(beautiful)}, e_2 \text{(expensive)}, e_3 \text{(cheap)}, e_4 \text{(good location)}, e_5 \text{(wooden)}\}$ be the set of parameters. Then a neutrosophic soft set (f, A) over U can be given by:

U	e_1	e_2	e_3	e_4	e_5
u_1	(0.8,0.5,0.2)	(0.3,0.4,0.6)	(0.1,0.6,0.4)	(0.7,0.3,0.6)	(0.3,0.4,0.6)
u_2	(0.4,0.1,0.7)	(0.8,0.2,0.4)	(0.4,0.1,0.7)	(0.2,0.4,0.4)	(0.1,0.1,0.3)
u_3	(0.2,0.6,0.4)	(0.5,0.5,0.5)	(0.8,0.1,0.7)	(0.5,0.3,0.5)	(0.5,0.5,0.5)
u_4	(0.3,0.4,0.4)	(0.1,0.3,0.3)	(0.3,0.4,0.4)	(0.6,0.6,0.6)	(0.1,0.1,0.5)
u_5	(0.1,0.1,0.7)	(0.2,0.6,0.7)	(0.4,0.2,0.1)	(0.8,0.6,0.1)	(0.6,0.7,0.7)
u_6	(0.5,0.3,0.9)	(0.3,0.6,0.6)	(0.1,0.5,0.5)	(0.3,0.6,0.5)	(0.4,0.4,0.4)

3. Parameter reduction of neutrosophic soft sets:

Suppose $U = \{x_1, x_2, x_3, \ldots\ldots, x_n\}$ be the universe set of objects and $E = \{e_1, e_2, e_3, \ldots\ldots, e_m\}$ be the set of parameters. Consider a neutrosophic soft set (f, E) given by $f(e) = \{\langle x, m_{f(e)}(x), g_{f(e)}(x), d_{f(e)}(x)\rangle : x \in U\}$ for $e \in E$. Let us define a function $\tilde{f}_E$ given by: $\tilde{f}_E(x_i) = \sum_j \left(m_{f(e_j)}(x_i) + g_{f(e_j)}(x_i) + d_{f(e_j)}(x_i)\right)$, $x_i \in U$.

We use $\tilde{f}_{e_j}(x_i)$ to denote $m_{f(e_j)}(x_i) + g_{f(e_j)}(x_i) + d_{f(e_j)}(x_i)$.

3.1 Definition: For any subset of parameters $B \subseteq E$, an indiscernibility relation IND_B is defined as: $IND_B = \{(x_i, x_j) \in U \times U : \tilde{f}_B(x_i) = \tilde{f}_B(x_j)\}$.

For the neutrosophic soft set (f, E), we denote $C_E^U = \{\{x_1, x_2, x_3, \ldots\ldots, x_i\}_{x_1}, \{x_{i+1}, x_{i+2}, \ldots\ldots, x_j\}_{x_2}, \ldots\ldots\ldots$ $\{x_k, x_{k+1}, \ldots\ldots, x_n\}_{x_s}\}$ as a partition of objects in U which partitions and ranks the objects according to the value of $\tilde{f}_E(x_i)$ based on the indiscernibility relation IND_E. C_E^U is called the decision partition, where the sub classes are: $\{x_1, x_2, x_3, \ldots\ldots, x_i\}, \{x_{i+1}, x_{i+2}, \ldots\ldots, x_j\}, \ldots\ldots\ldots, \{x_k, x_{k+1}, \ldots\ldots, x_n\}$ where s is the number of sub-classes, and $x_1 \geq x_2 \geq x_3 \geq \ldots\ldots \geq x_s$.

For any sub-class $\{x_z, x_{z+1}, \ldots.., x_{z+h}\}_{x_q}, \left[\tilde{f}_E(x_z)\right] = \left[\tilde{f}_E(x_{z+1})\right] = \ldots\ldots.. = \left[\tilde{f}_E(x_{z+h})\right] = x_q$, where $[.]$ denotes the greatest integer function. Thus objects from U with the same value of $\tilde{f}_E(.)$ are included into a same class.

3.2 Example: Let $U = \{x_1, x_2, x_3, \ldots.., x_6\}$ be the set of six houses and $E = \{e_1, e_2, e_3, \ldots.., e_6\}$ be the set of parameters where the parameters $e_1, e_2, e_3, e_4, e_5, e_6$ represents 'beautiful', 'in the main town', 'expensive', 'concrete', 'in green surroundings', 'wooden' respectively. Consider the neutrosophic soft set (f, E) which describes the attractiveness and physical trait of the houses given by the following table (table-1).

Table-1

U	e_1	e_2	e_3	e_4	e_5	e_6	$\tilde{f}_E(.)$
x_1	(0.3,0.7,0.4)	(0.4,0.5,0.1)	(0.2,0.2,0.4)	(0.6,0.3,0.4)	(0.1,0.1,0.3)	(0.2,0.4,0.6)	**6.2**
x_2	(0.4,0.5,0.5)	(0.2,0.2,0.6)	(0.5,0.5,0.1)	(0.2,0.8,0.3)	(0.4,0.3,0.2)	(0.6,0.3,0.4)	**7.0**
x_3	(0.2,0.5,0.7)	(0.3,0.2,0.5)	(0.8,0.2,0.4)	(0.5,0.5,0.3)	(0.2,0.4,0.2)	(0.9,0.6,0.6)	**8.0**
x_4	(0.5,0.3,0.6)	(0.6,0.3,0.1)	(0.2,0.5,0.6)	(0.4,0.4,0.5)	(0.7,0.3,0.2)	(0.5,0.5,0.8)	**8.0**
x_5	(0.3,0.5,0.6)	(0.4,0.4,0.2)	(0.3,0.3,0.5)	(0.6,0.1,0.6)	(0.7,0.8,0.1)	(0.4,0.6,0.6)	**8.0**
x_6	(0.7,0.3,0.4)	(0.3,0.5,0.2)	(0.4,0.8,0.5)	(0.5,0.3,0.5)	(0.1,0.2,0.3)	(0.4,0.4,0.2)	**7.0**

In this case, $C_E^U = \left\{\{x_3, x_4, x_5\}_{x_1}, \{x_2, x_6\}_{x_2}, \{x_1\}_{x_3}\right\}$ as $\tilde{f}_E(x_1) = 6.2, \tilde{f}_E(x_2) = 7.0, \tilde{f}_E(x_3) = 8.0, \tilde{f}_E(x_4) = 8.0,$ $\tilde{f}_E(x_5) = 8.0, \tilde{f}_E(x_6) = 7.0;$ where $x_1 = 8, x_2 = 7, x_3 = 6$.

3.3 Definition: For a neutrosophic soft set (f, E) with $E = \{e_1, e_2, e_3, \ldots.., e_m\}$, if there exists a subset $A = \{e'_1, e'_2, e'_3, \ldots.., e'_p\} \subset E$ satisfying $\tilde{f}_A(x_1) = \tilde{f}_A(x_2) = \tilde{f}_A(x_3) = \ldots\ldots\ldots = \tilde{f}_A(x_n)$, then we say that A is dispensable, otherwise A is indispensable. Roughly speaking, $A \subset E$ is dispensable means that the difference between among all objects according to the parameters in A doesn't influence the final decision. $A \subset E$ is called a parameter reduction of E if A is indispensible and $\tilde{f}_{E-A}(x_1) = \tilde{f}_{E-A}(x_2)$ $= \tilde{f}_{E-A}(x_3) = \ldots\ldots\ldots = \tilde{f}_{E-A}(x_n)$ i.e; *E-A* is the maximal subset of E that keeps the value $\tilde{f}_{E-A}(.)$ constant.

Clearly after the parameter reduction of *E*, we have fewer parameters although the partition of objects have not been changed. In the above definition, $\tilde{f}_A(x_1) = \tilde{f}_A(x_2) = \tilde{f}_A(x_3) = \ldots\ldots\ldots = \tilde{f}_A(x_n)$ implies $C_E^U = C_{E-A}^U$.

3.4 Example: Using table-1, we have, $\tilde{f}_{\{e_1,e_2,e_4\}}(x_1) = \tilde{f}_{\{e_1,e_2,e_4\}}(x_2) = \tilde{f}_{\{e_1,e_2,e_4\}}(x_3) = \tilde{f}_{\{e_1,e_2,e_4\}}(x_4) = \tilde{f}_{\{e_1,e_2,e_4\}}(x_5) = \tilde{f}_{\{e_1,e_2,e_4\}}(x_6) = 3.7$. Hence the neutrosophic soft set (f,E) given by Table-1 has a parameter reduction $\{e_3,e_5,e_6\}$ and the corresponding neutrosophic soft set (f,A) is displayed in table-2 given below:

Table-2

U	e_3	e_5	e_6	$\tilde{f}_A(.)$
x_1	(0.2,0.2,0.4)	(0.1,0.1,0.3)	(0.2,0.4,0.6)	**2.5**
x_2	(0.5,0.5,0.1)	(0.4,0.3,0.2)	(0.6,0.3,0.4)	**3.3**
x_3	(0.8,0.2,0.4)	(0.2,0.4,0.2)	(0.9,0.6,0.6)	**4.3**
x_4	(0.2,0.5,0.6)	(0.7,0.3,0.2)	(0.5,0.5,0.8)	**4.3**
x_5	(0.3,0.3,0.5)	(0.7,0.8,0.1)	(0.4,0.6,0.6)	**4.3**
x_6	(0.4,0.8,0.5)	(0.1,0.2,0.3)	(0.4,0.4,0.2)	**3.3**

Table-1 shows that $\tilde{f}_E(x_1) = 6.2$, $\tilde{f}_E(x_2) = \tilde{f}_E(x_6) = 7$, $\tilde{f}_E(x_3) = \tilde{f}_E(x_4) = \tilde{f}_E(x_5) = 8$ and so x_3 or x_4 or x_5 is the optimal choice, x_2 or x_6 is the sub optional choice and x_1 is the inferior choice. Again according to Table-2, $\tilde{f}_A(x_1) = 2.5$, $\tilde{f}_A(x_2) = \tilde{f}_A(x_6) = 3.3$, $\tilde{f}_A(x_3) = \tilde{f}_A(x_4) = \tilde{f}_A(x_5) = 4.3$ and so in this case also x_3 or x_4 or x_5 is the optimal choice, x_2 or x_6 is the sub optional choice and x_1 is the inferior choice. Thus parameter reduction gives the same result as the original one.

We also have $C^U_{E-\{e_1,e_2,e_4\}} = \{\{x_3,x_4,x_5\}_4, \{x_2,x_6\}_3, \{x_1\}_2\}$.

For the neutrosophic soft set (f,E), $E = \{e_1,e_2,e_3,.....,e_m\}$ is the parameter set and $U = \{x_1,x_2,x_3,.....,x_n\}$ is the set of objects, $C^U_E = \{\{x_1,x_2,x_3,.....,x_i\}_{x_1}, \{x_{i+1},x_{i+2},.....,x_j\}_{x_2},........$ $\{x_k,x_{k+1},.....,x_n\}_{x_s}\}$is a decision partition of objects in U. Now deleting the parameter e_i from E, we get a new decision partition deleted e_i denoted by $C^U_{E-\{e_i\}}$, which is given by:

. $C^U_{E-\{e_i\}} = \{\{x'_1,x'_2,x'_3,.....,x'_i\}_{x'_1}, \{x'_{i+1},x'_{i+2},.....,x'_j\}_{x'_2},........, \{x'_k,x'_{k+1},.....,x'_n\}_{x'_s}\}$.

For sake of convenience we denote:

$C^U_E = \{E_{x_1}, E_{x_2},........ E_{x_s}\}$and $C^U_{E-\{e_i\}} = \{\widetilde{E}-\{e_i\}_{x'_1}, \widetilde{E}-\{e_i\}_{x'_2},........, \widetilde{E}-\{e_i\}_{x'_s}\}$ where

$E_{x_1} = \{x_1, x_2, x_3, \ldots.., x_i\}_{x_1}$,
$E_{x_2} = \{x_{i+1}, x_{i+2}, \ldots.., x_j\}_{x_2}$,
....,
$E_{x_s} = \{x_k, x_{k+1}, \ldots.., x_n\}_{x_s}$,
$\overrightarrow{E} - \{e_i\}_{x_{1'}} = \{x_{1'}, x_{2'}, x_{3'}, \ldots.., x_{i'}\}_{x_{1'}}$,
$\overrightarrow{E} - \{e_i\}_{x_{2'}} = \{x_{i+1'}, x_{i+2'}, \ldots.., x_{j'}\}_{x_{2'}}$,
....,
$\overrightarrow{E} - \{e_i\}_{x_{s'}} = \{x_{k'}, x_{k+1'}, \ldots.., x_{n'}\}_{x_{s'}}$.

3.5 Definition: The degree of importance of e_r for the decision partition is denoted by $\mathrm{Im}(e_r)$ and is defined by $\mathrm{Im}(e_r) = \frac{1}{|U|}\sum_{q=1}^{s} \Omega_{q,e_r}$ where

$$\Omega_{q,e_r} = \begin{cases} \left| E_{x_q} - \overrightarrow{E} - \{e_r\}_{x_{y'}} \right|, & \text{if } \exists\, y' \text{ such that } x_q = x_{y'}, 1 \le y' \le s', 1 \le q \le s \\ \left| E_{x_q} \right|, & \text{otherwise} \end{cases}$$

3.6 Definition: For $A = \{e'_1, e'_2, e'_3, \ldots.., e'_p\} \subset E$, the decision partition deleted A is denoted by C^U_{E-A} and is given by $C^U_{E-A} = \left\{\overrightarrow{E} - A_{x_{1'}}, \overrightarrow{E} - A_{x_{2'}}, \ldots\ldots.., \overrightarrow{E} - A_{x_{s'}}\right\}$.
The degree of importance of A for the decision partition is defined by:
$\mathrm{Im}(A) = \frac{1}{|U|}\sum_{q=1}^{s} \Omega_{q,A}$ where

$$\Omega_{q,A} = \begin{cases} \left| E_{x_q} - \overrightarrow{E} - A_{x_{y'}} \right|, & \text{if } \exists\, y' \text{ such that } x_q = x_{y'}, 1 \le y' \le s', 1 \le q \le s \\ \left| E_{x_q} \right|, & \text{otherwise} \end{cases}$$

3.7 Example: Consider the neutrosophic soft set given in example 3.2. Then we have:
$C^U_E = \{\{x_3, x_4, x_5\}_8, \{x_2, x_6\}_7, \{x_1\}_6\}$, s=3 and $C^U_{E-\{e_1\}} = \{\{x_3, x_4, x_5\}_6, \{x_2, x_6\}_5, \{x_1\}_4\}$.
$\therefore\ \Omega_{1,e_1} = \left|\{x_1\} - \{x_1\}\right| = 0, \Omega_{2,e_1} = \left|\{x_2, x_6\}\right| = 2, \Omega_{3,e_1} = \left|\{x_3, x_4, x_5\}\right| = 3.$ So $\mathrm{Im}(e_1) = \frac{1}{6}(0+2+3) = 0.833.$

3.8 Proposition: For the neutrosophic soft set (f, E) where $E = \{e_1, e_2, \ldots.., e_m\}$, $0 \le \mathrm{Im}(e_r) \le 1, r = 1, 2, \ldots, m$.

Proof:

If $\exists\, y'$ such that $x_q = x_{y'}, 1 \le y' \le s, 1 \le q \le s$, then $\Omega_{q,e_r} = \left| E_{x_q} - \tilde{E} - \{e_r\}_{x_{y'}} \right| \le \left| E_{x_q} \right|$ and $\Omega_{q,e_r} = \left| E_{x_q} \right|$, otherwise.

$$\therefore \operatorname{Im}(e_r) = \frac{1}{|U|}\sum_{q=1}^{s}\Omega_{q,e_r} \le \frac{1}{|U|}\sum_{q=1}^{s}\left|E_{x_q}\right| = \frac{1}{|U|}\left\{\left|E_{x_1}\right| + \left|E_{x_2}\right| + ...+ \left|E_{x_s}\right|\right\} = \frac{1}{|U|} \times |U| = 1.$$

Again it is easy to verify that $\operatorname{Im}(e_r) \ge 0$. Thus we have $0 \le \operatorname{Im}(e_r) \le 1$.

4. Decision making problem solving based on parameter reductionof neutrosophic soft set:

In this section we first develop an algorithm using parameter reduction of neutrosophic soft set and then we illustrate this with a real life application.

- **Algorithm:**

Step-1: Input the neutrosophic soft set (f, E).

Step-2: Choose a parameter reduction A of E.

Step-3: Compute the choice value of the object $x_i \in U$ using the formula given below:

$$c_i = \sum_j \operatorname{Im}(e_j) \times \tilde{f}_{e_j}(x_i) \text{ where } e_j \in A.$$

Step-4: Find k for which $c_k = \max_i c_i$.

Then c_k is the optimal choice object. If k has more than one values, then any one of them can be chosen by the decision maker.

- **An Illustrative example:** Consider the neutrosophic soft set given in example 3.2. Now suppose that Mr. John is interested to buy a house on the basis of his choice parameters $e_1, e_2, e_3, ..., e_6$, which means that out of the available houses in *U*, he will select that house that qualifies with all or maximum number of parameters in *E*.

Step-1: The neutrosophic soft set (f, E) is given below:

U	e_1	e_2	e_3	e_4	e_5	e_6	$\tilde{f}_E(.)$
x_1	(0.3,0.7,0.4)	(0.4,0.5,0.1)	(0.2,0.2,0.4)	(0.6,0.3,0.4)	(0.1,0.1,0.3)	(0.2,0.4,0.6)	**6.2**
x_2	(0.4,0.5,0.5)	(0.2,0.2,0.6)	(0.5,0.5,0.1)	(0.2,0.8,0.3)	(0.4,0.3,0.2)	(0.6,0.3,0.4)	**7.0**
x_3	(0.2,0.5,0.7)	(0.3,0.2,0.5)	(0.8,0.2,0.4)	(0.5,0.5,0.3)	(0.2,0.4,0.2)	(0.9,0.6,0.6)	**8.0**
x_4	(0.5,0.3,0.6)	(0.6,0.3,0.1)	(0.2,0.5,0.6)	(0.4,0.4,0.5)	(0.7,0.3,0.2)	(0.5,0.5,0.8)	**8.0**
x_5	(0.3,0.5,0.6)	(0.4,0.4,0.2)	(0.3,0.3,0.5)	(0.6,0.1,0.6)	(0.7,0.8,0.1)	(0.4,0.6,0.6)	**8.0**
x_6	(0.7,0.3,0.4)	(0.3,0.5,0.2)	(0.4,0.8,0.5)	(0.5,0.3,0.5)	(0.1,0.2,0.3)	(0.4,0.4,0.2)	**7.0**

Step-2: A parameter reduction of *E* is $A = \{e_3, e_5, e_6\}$. The corresponding neutrosophic soft set is given below:

U	e_3	e_5	e_6	$\tilde{f}_A(.)$
x_1	(0.2,0.2,0.4)	(0.1,0.1,0.3)	(0.2,0.4,0.6)	**2.5**
x_2	(0.5,0.5,0.1)	(0.4,0.3,0.2)	(0.6,0.3,0.4)	**3.3**
x_3	(0.8,0.2,0.4)	(0.2,0.4,0.2)	(0.9,0.6,0.6)	**4.3**
x_4	(0.2,0.5,0.6)	(0.7,0.3,0.2)	(0.5,0.5,0.8)	**4.3**
x_5	(0.3,0.3,0.5)	(0.7,0.8,0.1)	(0.4,0.6,0.6)	**4.3**
x_6	(0.4,0.8,0.5)	(0.1,0.2,0.3)	(0.4,0.4,0.2)	**3.3**

Step-3: $C_A^U = \{\{x_3, x_4, x_5\}_4, \{x_2, x_6\}_3, \{x_1\}_2\}$ and $s = 3$.

$C_{A-\{e_3\}}^U = \{\{x_5\}_3, \{x_4\}_3, \{x_3\}_2, \{x_2\}_2, \{x_1\}_1, \{x_6\}_1\}$,

$C_{A-\{e_5\}}^U = \{\{x_3\}_3, \{x_4\}_3, \{x_5, x_6\}_2, \{x_2\}_2, \{x_1\}_2\}$,

$C_{A-\{e_6\}}^U = \{\{x_5\}_2, \{x_4\}_2, \{x_6\}_2, \{x_3\}_2, \{x_2\}_2, \{x_1\}_1\}$.

$$\therefore\ \Omega_{1,e_3} = |\{x_3, x_4, x_5\}| = 3, \Omega_{2,e_3} = |\{x_2, x_6\}| = 2, \Omega_{3,e_3} = |\{x_1\}| = 1;$$
$$\Omega_{1,e_5} = |\{x_3, x_4, x_5\}| = 3, \Omega_{2,e_5} = |\{x_2, x_6\}| = 2, \Omega_{3,e_5} = |\{x_1\} - \{x_1\}| = 0;$$
$$\Omega_{1,e_6} = |\{x_3, x_4, x_5\}| = 3, \Omega_{2,e_6} = |\{x_2, x_6\}| = 2, \Omega_{3,e_6} = |\{x_1\}| = 1.$$

Hence $\mathrm{Im}(e_3) = \frac{1}{|U|}\sum_{q=1}^{3} \Omega_{q,e_3} = \frac{1}{6}(3+2+1) = 1, \mathrm{Im}(e_5) = \frac{1}{|U|}\sum_{q=1}^{3} \Omega_{q,e_5} = \frac{1}{6}(3+2+0) = 0.83,$

$$\mathrm{Im}(e_6) = \frac{1}{|U|}\sum_{q=1}^{3} \Omega_{q,e_6} = \frac{1}{6}(3+2+1) = 1.$$

The computation table for obtaining the choice values is given by:

U	e_3	e_5	e_6	c_i
x_1	(0.2,0.2,0.4)	(0.1,0.1,0.3)	(0.2,0.4,0.6)	c_1 =(0.2+0.2+0.4)×1+(0.1+0.1+0.3)×0.83 +(0.2+0.4+0.6)×1=**2.415**
x_2	(0.5,0.5,0.1)	(0.4,0.3,0.2)	(0.6,0.3,0.4)	c_2 =(0.5+0.5+0.5)×1+(0.4+0.3+0.2)×0.83 +(0.6+0.3+0.4)×1=**3.147**
x_3	(0.8,0.2,0.4)	(0.2,0.4,0.2)	(0.9,0.6,0.6)	c_3 =(0.8+0.2+0.4)×1+(0.2+0.4+0.2)×0.83 +(0.9+0.6+0.6)×1=**4.164**
x_4	(0.2,0.5,0.6)	(0.7,0.3,0.2)	(0.5,0.5,0.8)	c_4 =(0.2+0.5+0.6)×1+(0.7+0.3+0.2)×0.83 +(0.5+0.5+0.8)×1=**4.096**
x_5	(0.3,0.3,0.5)	(0.7,0.8,0.1)	(0.4,0.6,0.6)	c_5 =(0.3+0.3+0.5)×1+(0.7+0.8+0.1)×0.83 +(0.4+0.6+0.6)×1=**3.828**
x_6	(0.4,0.8,0.5)	(0.1,0.2,0.3)	(0.4,0.4,0.2)	c_6 =(0.4+0.8+0.5)×1+(0.1+0.2+0.3)×0.83 +(0.4+0.4+0.2)×1=**3.198**

Step-4: Since the choice value c_3 is maximum, so house x_3 is the best option for Mr. John.

Conclusion

In this paper we have proposed the concept of parameter reduction for neutrosophic soft sets and we have used it to solve a decision making problem by developing an algorithm based on degree of importance of parameters. The experimental results prove that our proposed parameter reduction techniques delete the irrelevant parameters while keeping definite decision-making choices unchanged. The parameter reduction presented in this paper may play an important role in some knowledge discovery problem. Using the concept presented in this paper, one can think of parameter reduction of interval valued neutrosophic soft sets, hesitant neutrosophic soft sets and hesitant interval valued neutrosophic soft sets.

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