Neutrosophic Sets and Systems, Vol. 48, 2022
University of New Mexico

Single-Valued Pentapartitioned Neutrosophic Dice Similarity Measure and Its Application in the Selection of Suitable Metal Oxide Nano-Additive for Biodiesel Blend on Environmental Aspect

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Abstract: In this paper, an attempt has been made to introduce a new similarity measure namely single-valued pentapartitioned neutrosophic dice similarity measure (SVPNDSM) under the single-valued pentapartitioned neutrosophic set (SVPNS) environment, and to formulate several interesting results on SVPNDSM and SVPNWDSM. In this present work, the SVPNDSM under the SVPNS framework is combined with a multi-attribute decision making (MADM) strategy. This proposed method is used to select suitable metal oxide nano-additive for biodiesel blends on the basis of environmental aspects. The effects of nano-additives on engine emissions have been reported here from six different literatures. The SVPNDSM applied under the SVPNS environment enables the selection of the best nano-additive among relevant literatures. Alternative L_4 comes out as the best from the proposed method. The proposed MADM method is shown to be well suited to this problem after it has been compared with two existing methods.

Keywords: *Neutrosophic Set*; *Indeterminacy*; *SVPNS*; *Dice Similarity*.

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1. Introduction: The notion of fuzzy set (FS) theory was grounded by Zadeh [49] to deal with the events having uncertainty, where every element has membership value between 0 and 1. Later on, Atanassov [2] introduced the concept of intuitionistic fuzzy set (IFS) by generalizing the notion of FS in the year 1986, where every element has membership and non-membership values. Till now many researchers around the globe have applied the concept of FS and IFS in the area of theoretical and practical research.

We come across many situations involving indeterminacy, incompleteness which cannot be easily determined by the degrees of membership and non-membership. Keeping in mind, Smarandache [40] introduced the notion of neutrosophic set (NS) on generalizing the idea of FS and IFS to deal with the events having indeterminacy. In an NS, every element has three independent memberships values namely truth, indeterminacy and false membership values respectively, those lie between 0 and 1 each. In the recent past, many researcher around the globe used the concept of NS and their extensions for theoretical research [4-6, 8-11, 14, 24, 42, etc.]. Degree of indeterminacy membership of a mathematical expression plays an important role in every MADM problem of this real world. Afterwards, Wang et al. [45] introduced the notion of single-valued neutrosophic set (SVNS) in 2010, which is a subclass of NS. The notion of SVNS is more useful to deal with the situation involving incomplete and indeterminate information. Till now, many researcher have applied SVNS and their extensions in different branches of real-world such as fault diagnosis [46, 47], medical diagnosis [35, 36], decision-making problems [3, 7, 12-13, 15, 17-23, 25-28, 32-34, 43, 48], etc.

Recently, Mallick and Pramanik [24] investigated the notion of single-valued pentapartitioned neutrosophic set (SVPNS) by splitting the degree of indeterminacy membership into three independent components namely contradiction membership, ignorance membership and unknown membership. Das et al. [6] presented the notion of pentapartitioned neutrosophic Q-ideals of Q-algebra in 2021. Das et al. [13] proposed a MADM strategy based on the tangent similarity measure of SVPNS. Later on, Das et al. [12] established a MADM strategy based on grey relational analysis under the SVPNS environment. A MADM strategy based on cosine similarity measure of SVPNSs was established by Majumder et al. [23] to identify the most significant risk factor of COVID-19 in economy.

In this article, a new similarity measure called SVPNDSM is proposed used to select suitable metal oxide nano-additive for biodiesel blends on the basis of environmental aspects under the SVPNS environment and generate several interesting results. In addition, a MADM technique is established based on SVPNDSM within the SVPNS environment.

Research Gap: In the literature review, no study is found relating to SVPNDSM based MADM strategy in SVPNS.

Motivation: To explore the unexplored MADM strategy in SVPNS environment, a new MADM strategy under SVPNS environment based on SVPNDSM between SVPNSs is presented in this present work.

The rest of this paper has been split into the following sections:

Section-2 presents several basic definitions and operations on SVPNSs those are very useful for developing the main results of this paper. In section-3, Single-Valued Pentapartitioned Neutrosophic Dice Similarity Measure and Single-Valued Pentapartitioned Neutrosophic Weighted Dice Similarity Measure under the SVPNS environment is proposed. Further, we formulate some interesting results on SVPNDSM and SVPNWDSM. A MADM strategy using SVPNWDSM under the SVPNS environment is discussed in section-4. In section-5 the proposed MADM strategy is applied to a real world problem. Finally, in section 6, a comparative study has been conducted to validate the results obtained from the proposed method. In section-7, wrap up the work presented in this article.

List of abbreviations are shown in below:

List of abbreviations	
Full Form	**Short Form**
Fuzzy Set	FS
Intuitionistic Fuzzy Set	IFS
Neutrosophic Set	NS
Single Valued Neutrosophic Set	SVNS
Single Valued Pentapartitioned Neutrosophic Set	SVPNS
Multi-Attribute Decision Making	MADM
Single Valued Pentapartitioned Neutrosophic Dice Similarity Measure	SVPNDSM
Single Valued Pentapartitioned Neutrosophic Weighted Dice Similarity Measure	SVPNWDSM
Positive Ideal Solution	PIS

2. Basic Preliminaries:

In this section some basic definitions and results are described.

Definition 2.1.[24] Suppose that X be a fixed set. Then R, an SVPNS over X is defined as follows:

$$R = \{(\delta, \Delta_R(\delta), \Gamma_R(\delta), \Pi_R(\delta), \Omega_R(\delta), \Phi_R(\delta)) : \delta \in X\},$$

where Δ_R, Γ_R, Π_R, Ω_R, Φ_R: $X \rightarrow [0, 1]$ represents the truth, contradiction, ignorance, unknown and falsity membership functions respectively such that $0 \leq \Delta_R(\delta) + \Gamma_R(\delta) + \Pi_R(\delta) + \Omega_R(\delta) + \Phi_R(\delta) \leq 5$, for all $\delta \in X$.

Remark 2.1.[24] Suppose that $R = \{(\delta, \Delta_R(\delta), \Gamma_R(\delta), \Pi_R(\delta), \Omega_R(\delta), \Phi_R(\delta)) : \delta \in X\}$ be an SVPNS over X. Then, for any $\delta \in X$, $(\Delta_R(\delta), \Gamma_R(\delta), \Pi_R(\delta), \Omega_R(\delta), \Phi_R(\delta))$ is called an single-valued pentapartitioned neutrosophic number (SVPNN) over X.

Definition 2.2.[24] Suppose that W={(δ, $\Delta_W(\delta)$, $\Gamma_W(\delta)$, $\Pi_W(\delta)$, $\Omega_W(\delta)$, $\Phi_W(\delta)$) : δ∈X} and M={(δ, $\Delta_M(\delta)$, $\Gamma_M(\delta)$, $\Pi_M(\delta)$, $\Omega_M(\delta)$, $\Phi_M(\delta)$) : δ∈X} be two SVPNSs over X. Then,

i. $W\subseteq M \Leftrightarrow \Delta_W(\delta)\leq\Delta_M(\delta)$, $\Gamma_W(\delta)\leq\Gamma_M(\delta)$, $\Pi_W(\delta)\geq\Pi_M(\delta)$, $\Omega_W(\delta)\geq\Omega_M(\delta)$, $\Phi_W(\delta)\geq\Phi_M(\delta)$, for all δ∈X;

ii. W^c={(δ, $\Phi_W(\delta)$, $\Omega_W(\delta)$, 1-$\Pi_W(\delta)$, $\Gamma_W(\delta)$, $\Delta_W(\delta)$) : δ∈X} and M^c={(δ, $\Phi_M(\delta)$, $\Omega_M(\delta)$, 1-$\Pi_M(\delta)$, $\Gamma_M(\delta)$, $\Delta_M(\delta)$) : δ∈X};

iii. W∪M={(δ, max {$\Delta_W(\delta)$, $\Delta_M(\delta)$}, max {$\Gamma_W(\delta)$, $\Gamma_M(\delta)$}, min {$\Pi_W(\delta)$, $\Pi_M(\delta)$}, min {$\Omega_W(\delta)$, $\Omega_M(\delta)$}, min {$\Phi_W(\delta)$, $\Phi_M(\delta)$}) : δ∈X};

iv. W∩M={(δ, min {$\Delta_W(\delta)$, $\Delta_M(\delta)$}, min {$\Gamma_W(\delta)$, $\Gamma_M(\delta)$}, max {$\Pi_W(\delta)$, $\Pi_M(\delta)$}, max {$\Omega_W(\delta)$, $\Omega_M(\delta)$}, max {$\Phi_W(\delta)$, $\Phi_M(\delta)$}) : δ∈X}.

Definition 2.3.[24] The absolute SVPNS (1_X) and null SVPNS (0_X) over a fixed set X are defined by:

i. 1_X= {(δ, 1, 1, 0, 0, 0) : δ∈X};

ii. 0_X= {(δ, 0, 0, 1, 1, 1) : δ∈X}.

Clearly, $0_X \subseteq R \subseteq 1_X$, for any SVPNS R over X.

3. Single-Valued Pentapartitioned Neutrosophic Dice Similarity Measure:

This section introduces the notion of Single-Valued Pentapartitioned Neutrosophic Dice Similarity Measure and Single-Valued Pentapartitioned Neutrosophic Weighted Dice Similarity Measure, and formulates several interesting results on them under the SVPNS environment.

Definition 3.1. Assume that W = {(θ, $\Delta_W(\theta)$, $\Gamma_W(\theta)$, $\Pi_W(\theta)$, $\Omega_W(\theta)$, $\Phi_W(\theta)$) : θ∈U} and M = {(θ, $\Delta_M(\theta)$, $\Gamma_M(\theta)$, $\Pi_M(\theta)$, $\Omega_M(\theta)$, $\Phi_M(\theta)$) : θ∈U} be two SVPNSs over a fixed set U. Then, the SVPNDSM between W and M is defined by:

$D_{SVPNDSM}$(W, M)

$$=\frac{1}{n}\sum_{\theta\in U}\frac{2[\Delta_W(\theta).\Delta_M(\theta)+\Gamma_W(\theta).\Gamma_M(\theta)+\Pi_W(\theta).\Pi_M(\theta)+\Omega_W(\theta).\Omega_M(\theta)+\Phi_W(\theta).\Phi_M(\theta)]}{[(\Delta_W(\theta))^2+(\Gamma_W(\theta))^2+(\Pi_W(\theta))^2+(\Omega_W(\theta))^2+(\Phi_W(\theta))^2+(\Delta_M(\theta))^2+(\Gamma_M(\theta))^2+(\Pi_M(\theta))^2+(\Omega_M(\theta))^2+(\Phi_M(\theta))^2]} \quad (1)$$

Example 3.1. Let W={(a,0.6,0.3,0.1,0.2,0.1), (b,0.9,0.1,0.0,0.1,0.2)} and M={(a,1.0,0.0,0.1,0.1, 0.2), (b,0.8,0.0,0.0,0.1,0.0)}be two SVPNSs over U={a, b}. Then, SVPNDSM (W, M) = 0.8942758967.

Theorem 3.1. Suppose that $D_{SVPNDSM}$ (W, M) be the SVPNDSM between the SVPNSs W and M. Then,

(i) 0 ≤ $D_{SVPNDSM}$ (W, M) ≤ 1;

(ii) $D_{SVPNDSM}$ (W, M) = $D_{SVPNDSM}$ (M, W);

(iii) W = M ⇒$D_{SVPNDSM}$ (W, M) = 1.

Proof. (i) Let $D_{SVPNDSM}$ (W, M) be the SVPNDSM between W and M.

Therefore, $D_{SVPNDSM}$ (W, M)

$$= \frac{1}{n}\sum_{\theta\in U}\frac{2[\Delta_W(\theta).\Delta_M(\theta)+\Gamma_W(\theta).\Gamma_M(\theta)+\Pi_W(\theta).\Pi_M(\theta)+\Omega_W(\theta).\Omega_M(\theta)+\Phi_W(\theta).\Phi_M(\theta)]}{[(\Delta_W(\theta))^2+(\Gamma_W(\theta))^2+(\Pi_W(\theta))^2+(\Omega_W(\theta))^2+(\Phi_W(\theta))^2+(\Delta_M(\theta))^2+(\Gamma_M(\theta))^2+(\Pi_M(\theta))^2+(\Omega_M(\theta))^2+(\Phi_M(\theta))^2]}.$$

It is known that, 0 ≤$\Delta_W(\theta)$ ≤ 1, 0 ≤$\Delta_M(\theta)$ ≤ 1, 0 ≤$\Gamma_W(\theta)$ ≤ 1, 0 ≤$\Gamma_M(\theta)$ ≤ 1, 0 ≤$\Pi_W(\theta)$ ≤ 1, 0 ≤$\Pi_M(\theta)$ ≤ 1, 0 ≤$\Omega_W(\theta)$ ≤ 1, 0 ≤$\Omega_M(\theta)$ ≤ 1, 0 ≤$\Phi_W(\theta)$ ≤ 1 and 0 ≤$\Phi_M(\theta)$ ≤ 1, for each θ∈U.

$\Rightarrow$ $0 \le \Delta_W(\theta).\Delta_M(\theta) \le 1$, $0 \le \Gamma_W(\theta).\Gamma_M(\theta) \le 1$, $0 \le \Pi_W(\theta).\Pi_M(\theta) \le 1$, $0 \le \Omega_W(\theta).\Omega_M(\theta) \le 1$, $0 \le \Phi_W(\theta).\Phi_M(\theta) \le 1$, $0 \le (\Delta_W(\theta))^2 \le 1$, $0 \le (\Gamma_W(\theta))^2 \le 1$, $0 \le (\Pi_W(\theta))^2 \le 1$, $0 \le (\Omega_W(\theta))^2 \le 1$, $0 \le (\Phi_W(\theta))^2 \le 1$, for each $\theta \in U$.

$\Rightarrow$ $0 \le (\Delta_W(\theta))^2 + (\Gamma_W(\theta))^2 + (\Pi_W(\theta))^2 + (\Omega_W(\theta))^2 + (\Phi_W(\theta))^2 + (\Delta_M(\theta))^2 + (\Gamma_M(\theta))^2 + (\Pi_M(\theta))^2 + (\Omega_M(\theta))^2 + (\Phi_M(\theta))^2 \le 10$, $0 \le \Delta_W(\theta).\Delta_M(\theta) + \Gamma_W(\theta).\Gamma_M(\theta) + \Pi_W(\theta).\Pi_M(\theta) + \Omega_W(\theta).\Omega_M(\theta) + \Phi_W(\theta).\Phi_M(\theta) \le 5$, for each $\theta \in U$.

$$\Rightarrow 0 \le \frac{2[\Delta_W(\theta).\Delta_M(\theta)+\Gamma_W(\theta).\Gamma_M(\theta)+\Pi_W(\theta).\Pi_M(\theta)+\Omega_W(\theta).\Omega_M(\theta)+\Phi_W(\theta).\Phi_M(\theta)]}{[(\Delta_W(\theta))^2+(\Gamma_W(\theta))^2+(\Pi_W(\theta))^2+(\Omega_W(\theta))^2+(\Phi_W(\theta))^2+(\Delta_M(\theta))^2+(\Gamma_M(\theta))^2+(\Pi_M(\theta))^2+(\Omega_M(\theta))^2+(\Phi_M(\theta))^2]} \le 1,\ \forall \theta \in U.$$

$$\Rightarrow 0 \le \sum_{\theta \in U} \frac{2[\Delta_W(\theta).\Delta_M(\theta)+\Gamma_W(\theta).\Gamma_M(\theta)+\Pi_W(\theta).\Pi_M(\theta)+\Omega_W(\theta).\Omega_M(\theta)+\Phi_W(\theta).\Phi_M(\theta)]}{[(\Delta_W(\theta))^2+(\Gamma_W(\theta))^2+(\Pi_W(\theta))^2+(\Omega_W(\theta))^2+(\Phi_W(\theta))^2+(\Delta_M(\theta))^2+(\Gamma_M(\theta))^2+(\Pi_M(\theta))^2+(\Omega_M(\theta))^2+(\Phi_M(\theta))^2]} \le n.$$

$$\Rightarrow 0 \le \frac{1}{n}\sum_{\theta \in U} \frac{2[\Delta_W(\theta).\Delta_M(\theta)+\Gamma_W(\theta).\Gamma_M(\theta)+\Pi_W(\theta).\Pi_M(\theta)+\Omega_W(\theta).\Omega_M(\theta)+\Phi_W(\theta).\Phi_M(\theta)]}{[(\Delta_W(\theta))^2+(\Gamma_W(\theta))^2+(\Pi_W(\theta))^2+(\Omega_W(\theta))^2+(\Phi_W(\theta))^2+(\Delta_M(\theta))^2+(\Gamma_M(\theta))^2+(\Pi_M(\theta))^2+(\Omega_M(\theta))^2+(\Phi_M(\theta))^2]} \le 1$$

$\Rightarrow 0 \le D_{SVPNDSM}(W, M) \le 1$.

Therefore, $0 \le D_{SVPNDSM}(W, M) \le 1$.

(ii) We have, $D_{SVPNDSM}(W, M)$

$$= \frac{1}{n}\sum_{\theta \in U} \frac{2[\Delta_W(\theta).\Delta_M(\theta)+\Gamma_W(\theta).\Gamma_M(\theta)+\Pi_W(\theta).\Pi_M(\theta)+\Omega_W(\theta).\Omega_M(\theta)+\Phi_W(\theta).\Phi_M(\theta)]}{[(\Delta_W(\theta))^2+(\Gamma_W(\theta))^2+(\Pi_W(\theta))^2+(\Omega_W(\theta))^2+(\Phi_W(\theta))^2+(\Delta_M(\theta))^2+(\Gamma_M(\theta))^2+(\Pi_M(\theta))^2+(\Omega_M(\theta))^2+(\Phi_M(\theta))^2]}$$

$$= \frac{1}{n}\sum_{\theta \in U} \frac{2[\Delta_M(\theta).\Delta_W(\theta)+\Gamma_M(\theta).\Gamma_W(\theta)+\Pi_M(\theta).\Pi_W(\theta)+\Omega_M(\theta).\Omega_W(\theta)+\Phi_M(\theta).\Phi_W(\theta)]}{[(\Phi_W(\theta))^2+(\Delta_M(\theta))^2+(\Gamma_M(\theta))^2+(\Pi_M(\theta))^2+(\Omega_M(\theta))^2+(\Phi_M(\theta))^2+(\Delta_W(\theta))^2+(\Gamma_W(\theta))^2+(\Pi_W(\theta))^2+(\Omega_W(\theta))^2]}$$

$= D_{SVPNDSM}(M, W)$

Therefore, $D_{SVPNDSM}(W, M) = D_{SVPNDSM}(M, W)$.

(iii) Suppose that W and M be two SVPNSs over a fixed set U such that W = M. Let $D_{SVPNDSM}(W, M)$ be the SVPNDSM between the SVPNSs W and M.

Now, W = M

$\Rightarrow \Delta_W(\theta)=\Delta_M(\theta)$, $\Gamma_W(\theta)=\Gamma_M(\theta)$, $\Pi_W(\theta)=\Pi_M(\theta)$, $\Omega_W(\theta)=\Omega_M(\theta)$ and $\Phi_W(\theta)=\Phi_M(\theta)$, for each $\theta \in U$.

$\Rightarrow$ $\Delta_W(\theta).\Delta_M(\theta) = (\Delta_W(\theta))^2$, $\Gamma_W(\theta).\Gamma_M(\theta) = (\Gamma_W(\theta))^2$, $\Pi_W(\theta).\Pi_M(\theta) = (\Pi_W(\theta))^2$, $\Omega_W(\theta).\Omega_M(\theta)=(\Omega_W(\theta))^2$ and $\Phi_W(\theta).\Phi_M(\theta)=(\Phi_W(\theta))^2$, for each $\theta \in U$.

$\Rightarrow$ $2[\Delta_W(\theta).\Delta_M(\theta) + \Gamma_W(\theta).\Gamma_M(\theta) + \Pi_W(\theta).\Pi_M(\theta) + \Omega_W(\theta).\Omega_M(\theta) + \Phi_W(\theta).\Phi_M(\theta)] = [(\Delta_W(\theta))^2 + (\Gamma_W(\theta))^2 + (\Pi_W(\theta))^2 + (\Omega_W(\theta))^2 + (\Phi_W(\theta))^2 + (\Delta_M(\theta))^2 + (\Gamma_M(\theta))^2 + (\Pi_M(\theta))^2 + (\Omega_M(\theta))^2 + (\Phi_M(\theta))^2]$, for each $\theta \in U$.

$$\Rightarrow \frac{2[\Delta_M(\theta).\Delta_W(\theta)+\Gamma_M(\theta).\Gamma_W(\theta)+\Pi_M(\theta).\Pi_W(\theta)+\Omega_M(\theta).\Omega_W(\theta)+\Phi_M(\theta).\Phi_W(\theta)]}{[(\Phi_W(\theta))^2+(\Delta_M(\theta))^2+(\Gamma_M(\theta))^2+(\Pi_M(\theta))^2+(\Omega_M(\theta))^2+(\Phi_M(\theta))^2+(\Delta_W(\theta))^2+(\Gamma_W(\theta))^2+(\Pi_W(\theta))^2+(\Omega_W(\theta))^2]} = 1,\ \forall \theta \in U.$$

$$\Rightarrow \sum_{\theta \in U} \frac{2[\Delta_M(\theta).\Delta_W(\theta)+\Gamma_M(\theta).\Gamma_W(\theta)+\Pi_M(\theta).\Pi_W(\theta)+\Omega_M(\theta).\Omega_W(\theta)+\Phi_M(\theta).\Phi_W(\theta)]}{[(\Phi_W(\theta))^2+(\Delta_M(\theta))^2+(\Gamma_M(\theta))^2+(\Pi_M(\theta))^2+(\Omega_M(\theta))^2+(\Phi_M(\theta))^2+(\Delta_W(\theta))^2+(\Gamma_W(\theta))^2+(\Pi_W(\theta))^2+(\Omega_W(\theta))^2]} = n.$$

$$\Rightarrow \frac{1}{n}\sum_{\theta \in U} \frac{2[\Delta_M(\theta).\Delta_W(\theta)+\Gamma_M(\theta).\Gamma_W(\theta)+\Pi_M(\theta).\Pi_W(\theta)+\Omega_M(\theta).\Omega_W(\theta)+\Phi_M(\theta).\Phi_W(\theta)]}{[(\Phi_W(\theta))^2+(\Delta_M(\theta))^2+(\Gamma_M(\theta))^2+(\Pi_M(\theta))^2+(\Omega_M(\theta))^2+(\Phi_M(\theta))^2+(\Delta_W(\theta))^2+(\Gamma_W(\theta))^2+(\Pi_W(\theta))^2+(\Omega_W(\theta))^2]} = 1.$$

$\Rightarrow D_{SVPNDSM}(M, W) = 1$.

Definition 3.2. Assume that W = {(θ, $\Delta_W(\theta)$, $\Gamma_W(\theta)$, $\Pi_W(\theta)$, $\Omega_W(\theta)$, $\Phi_W(\theta)$) : θ∈U} and M = {(θ, $\Delta_M(\theta)$, $\Gamma_M(\theta)$, $\Pi_M(\theta)$, $\Omega_M(\theta)$, $\Phi_M(\theta)$) : θ∈U} be two SVPNSs over a fixed set U. Then, the SVPNWDSM between two SVPNSs W and M is defined by:

$D_{SVPNWDSM}$ (W, M)

$$= \sum_{\theta\in U} w_\theta \cdot \frac{2[\Delta_W(\theta).\Delta_M(\theta)+\Gamma_W(\theta).\Gamma_M(\theta)+\Pi_W(\theta).\Pi_M(\theta)+\Omega_W(\theta).\Omega_M(\theta)+\Phi_W(\theta).\Phi_M(\theta)]}{[(\Delta_W(\theta))^2+(\Gamma_W(\theta))^2+(\Pi_W(\theta))^2+(\Omega_W(\theta))^2+(\Phi_W(\theta))^2+(\Delta_M(\theta))^2+(\Gamma_M(\theta))^2+(\Pi_M(\theta))^2+(\Omega_M(\theta))^2+(\Phi_M(\theta))^2]} \quad (2)$$

Example 3.2. Consider the SVPNSs W and M on U given in Example 3.1. Assume that w_1=0.6, w_2= 0.4 be the corresponding weights of the SVPNSs W and M respectively. Then, SVPNWDSM (W, M) = 0.8810258129.

Theorem 3.2. Suppose that $D_{SVPNWDSM}$ (W, M) be the SVPNWDSM between the SVPNSs W and M. Then, the following holds:

(i) 0 ≤ $D_{SVPNWDSM}$ (W, M) ≤ 1;

(ii) $D_{SVPNWDSM}$ (W, M) = $D_{SVPNWDSM}$ (M, W);

(iii) W = M ⇒ $D_{SVPNWDSM}$ (W, M) = 1.

Proof. (i) Suppose that $D_{SVPNWDSM}$ (W, M) be the SVPNWDSM between the SVPNSs W and M, where $D_{SVPNWDSM}$ (W, M)

$$= \sum_{\theta\in U} w_\theta . \frac{2[\Delta_W(\theta).\Delta_M(\theta)+\Gamma_W(\theta).\Gamma_M(\theta)+\Pi_W(\theta).\Pi_M(\theta)+\Omega_W(\theta).\Omega_M(\theta)+\Phi_W(\theta).\Phi_M(\theta)]}{[(\Delta_W(\theta))^2+(\Gamma_W(\theta))^2+(\Pi_W(\theta))^2+(\Omega_W(\theta))^2+(\Phi_W(\theta))^2+(\Delta_M(\theta))^2+(\Gamma_M(\theta))^2+(\Pi_M(\theta))^2+(\Omega_M(\theta))^2+(\Phi_M(\theta))^2]}.$$

It is known that,

$0\le \Delta_W(\theta)\le 1$, $0 \le\Delta_M(\theta)\le 1$, $0\le \Gamma_W(\theta)\le 1$, $0\le \Gamma_M(\theta)\le 1$, $0\le \Pi_W(\theta)\le 1$, $0\le \Pi_M(\theta)\le 1$, $0\le \Omega_W(\theta)\le 1$, $0 \le \Omega_M(\theta)\le 1$, $0\le \Phi_W(\theta)\le 1$ and $0\le \Phi_M(\theta)\le 1$, for each θ∈U.

$\Rightarrow 0 \le \Delta_W(\theta).\Delta_M(\theta) \le 1$, $0 \le \Gamma_W(\theta).\Gamma_M(\theta) \le 1$, $0 \le \Pi_W(\theta).\Pi_M(\theta) \le 1$, $0 \le \Omega_W(\theta).\Omega_M(\theta) \le 1$, $0 \le \Phi_W(\theta).\Phi_M(\theta) \le 1$, $0 \le (\Delta_W(\theta))^2 \le 1$, $0 \le (\Gamma_W(\theta))^2 \le 1$, $0 \le (\Pi_W(\theta))^2 \le 1$, $0 \le (\Omega_W(\theta))^2 \le 1$, $0 \le (\Phi_W(\theta))^2 \le 1$, for each θ∈U.

$\Rightarrow 0\le (\Delta_W(\theta))^2 + (\Gamma_W(\theta))^2 + (\Pi_W(\theta))^2 + (\Omega_W(\theta))^2 + (\Phi_W(\theta))^2 + (\Delta_M(\theta))^2 + (\Gamma_M(\theta))^2 + (\Pi_M(\theta))^2 + (\Omega_M(\theta))^2 + (\Phi_M(\theta))^2 \le 10$, and $0\le \Delta_W(\theta).\Delta_M(\theta) + \Gamma_W(\theta).\Gamma_M(\theta) + \Pi_W(\theta).\Pi_M(\theta) + \Omega_W(\theta).\Omega_M(\theta) + \Phi_W(\theta).\Phi_M(\theta)\le 5$, for each θ∈U.

$$\Rightarrow 0 \le \frac{2[\Delta_W(\theta).\Delta_M(\theta)+\Gamma_W(\theta).\Gamma_M(\theta)+\Pi_W(\theta).\Pi_M(\theta)+\Omega_W(\theta).\Omega_M(\theta)+\Phi_W(\theta).\Phi_M(\theta)]}{[(\Delta_W(\theta))^2+(\Gamma_W(\theta))^2+(\Pi_W(\theta))^2+(\Omega_W(\theta))^2+(\Phi_W(\theta))^2+(\Delta_M(\theta))^2+(\Gamma_M(\theta))^2+(\Pi_M(\theta))^2+(\Omega_M(\theta))^2+(\Phi_M(\theta))^2]} \le 1, \ \forall\theta\in U$$

$$\Rightarrow 0\le\sum_{\theta\in U} w_\theta . \frac{2[\Delta_W(\theta).\Delta_M(\theta)+\Gamma_W(\theta).\Gamma_M(\theta)+\Pi_W(\theta).\Pi_M(\theta)+\Omega_W(\theta).\Omega_M(\theta)+\Phi_W(\theta).\Phi_M(\theta)]}{[(\Delta_W(\theta))^2+(\Gamma_W(\theta))^2+(\Pi_W(\theta))^2+(\Omega_W(\theta))^2+(\Phi_W(\theta))^2+(\Delta_M(\theta))^2+(\Gamma_M(\theta))^2+(\Pi_M(\theta))^2+(\Omega_M(\theta))^2+(\Phi_M(\theta))^2]} \le 1$$

⇒ 0 ≤ $D_{SVPNWDSM}$ (W, M) ≤ 1

Therefore, we have 0 ≤ $D_{SVPNWDSM}$ (W, M) ≤ 1.

(ii) We have, $D_{SVPNWDSM}$ (W, M)

$$= \sum_{\theta\in U} w_\theta . \frac{2[\Delta_W(\theta).\Delta_M(\theta)+\Gamma_W(\theta).\Gamma_M(\theta)+\Pi_W(\theta).\Pi_M(\theta)+\Omega_W(\theta).\Omega_M(\theta)+\Phi_W(\theta).\Phi_M(\theta)]}{[(\Delta_W(\theta))^2+(\Gamma_W(\theta))^2+(\Pi_W(\theta))^2+(\Omega_W(\theta))^2+(\Phi_W(\theta))^2+(\Delta_M(\theta))^2+(\Gamma_M(\theta))^2+(\Pi_M(\theta))^2+(\Omega_M(\theta))^2+(\Phi_M(\theta))^2]}$$

$$= \sum_{\theta\in U} w_\theta.\frac{2[\Delta_M(\theta).\Delta_W(\theta)+\Gamma_M(\theta).\Gamma_W(\theta)+\Pi_M(\theta).\Pi_W(\theta)+\Omega_M(\theta).\Omega_W(\theta)+\Phi_M(\theta).\Phi_W(\theta)]}{[(\Phi_W(\theta))^2+(\Delta_M(\theta))^2+(\Gamma_M(\theta))^2+(\Pi_M(\theta))^2+(\Omega_M(\theta))^2+(\Phi_M(\theta))^2+(\Delta_W(\theta))^2+(\Gamma_W(\theta))^2+(\Pi_W(\theta))^2+(\Omega_W(\theta))^2]}$$

= $D_{SVPNWDSM}$ (M, W)

Therefore, $D_{SVPNWDSM}$ (W, M) = $D_{SVPNWDSM}$ (M, W).

(iii) Suppose that W and M be two SVPNSs over a fixed set U such that W = M. Assume that $D_{SVPNWDSM}$ (W, M) be the SVPNWDSM between W and M.

Now, W = M

$\Rightarrow$ $\Delta_W(\theta)=\Delta_M(\theta)$, $\Gamma_W(\theta)=\Gamma_M(\theta)$, $\Pi_W(\theta)=\Pi_M(\theta)$, $\Omega_W(\theta)=\Omega_M(\theta)$ and $\Phi_W(\theta)=\Phi_M(\theta)$, for each $\theta\in U$.

$\Rightarrow$ $\Delta_W(\theta).\Delta_M(\theta) = (\Delta_W(\theta))^2$, $\Gamma_W(\theta).\Gamma_M(\theta)=(\Gamma_W(\theta))^2$, $\Pi_W(\theta).\Pi_M(\theta)=(\Pi_W(\theta))^2$, $\Omega_W(\theta).\Omega_M(\theta) = (\Omega_W(\theta))^2$, $\Phi_W(\theta).\Phi_M(\theta) = (\Phi_W(\theta))^2$, for each $\theta\in U$.

$\Rightarrow$ $2[\Delta_W(\theta).\Delta_M(\theta) + \Gamma_W(\theta).\Gamma_M(\theta) + \Pi_W(\theta).\Pi_M(\theta) + \Omega_W(\theta).\Omega_M(\theta) + \Phi_W(\theta).\Phi_M(\theta)] = [(\Delta_W(\theta))^2 + (\Gamma_W(\theta))^2 + (\Pi_W(\theta))^2 + (\Omega_W(\theta))^2 + (\Phi_W(\theta))^2 + (\Delta_M(\theta))^2 + (\Gamma_M(\theta))^2 + (\Pi_M(\theta))^2 + (\Omega_M(\theta))^2 + (\Phi_M(\theta))^2]$, for each $\theta\in U$.

$$\Rightarrow\frac{2[\Delta_M(\theta).\Delta_W(\theta)+\Gamma_M(\theta).\Gamma_W(\theta)+\Pi_M(\theta).\Pi_W(\theta)+\Omega_M(\theta).\Omega_W(\theta)+\Phi_M(\theta).\Phi_W(\theta)]}{[(\Phi_W(\theta))^2+(\Delta_M(\theta))^2+(\Gamma_M(\theta))^2+(\Pi_M(\theta))^2+(\Omega_M(\theta))^2+(\Phi_M(\theta))^2+(\Delta_W(\theta))^2+(\Gamma_W(\theta))^2+(\Pi_W(\theta))^2+(\Omega_W(\theta))^2]} = 1,\ \forall\theta\in U$$

$$\Rightarrow\sum_{\theta\in U} w_\theta.\frac{2[\Delta_M(\theta).\Delta_W(\theta)+\Gamma_M(\theta).\Gamma_W(\theta)+\Pi_M(\theta).\Pi_W(\theta)+\Omega_M(\theta).\Omega_W(\theta)+\Phi_M(\theta).\Phi_W(\theta)]}{[(\Phi_W(\theta))^2+(\Delta_M(\theta))^2+(\Gamma_M(\theta))^2+(\Pi_M(\theta))^2+(\Omega_M(\theta))^2+(\Phi_M(\theta))^2+(\Delta_W(\theta))^2+(\Gamma_W(\theta))^2+(\Pi_W(\theta))^2+(\Omega_W(\theta))^2]} = 1$$

$\Rightarrow$ $D_{SVPNWDSM}$ (M, W) = 1

Therefore, W = M $\Rightarrow$ $D_{SVPNWDSM}$ (W, M) = 1.

4. MADM-Strategy Based on SVPNWDSM under SVPNS Environment:

The main focus of this section is to propose a MADM-strategy using the SVPNWDSM between two SVPNSs under the SVPNS environment. Figure-1 represents the proposed MADM-strategy**.**

Let us consider a MADM-problem, where L = {L_1, L_2, …....,L_p} is a set of possible alternatives and A = {A_1, A_2, ..., A_q} is the family of attributes. Then, the decision maker can give their evaluation information for each alternative L_i(i = 1, 2,..., p)against the attribute A_j (j = 1, 2,..., q) by using SVPNS.

Then, the proposed MADM-strategy is designed in the following steps:

Step-1: Decision Matrix Formation using SVPNS.

Suppose, the decision maker gives their evaluation information by using the SVPNS E_{L_i}= {(A_j, $\Delta_{ij}(L_i, A_j)$, $\Gamma_{ij}(L_i, A_j)$, $\Pi_{ij}(L_i, A_j)$, $\Omega_{ij}(L_i, A_j)$, $\Phi_{ij}(L_i, A_j)$): $A_j \in A$} for each alternative L_i against the corresponding attributes A_j(j = 1, 2, ..., q), where ($\Delta_{ij}(L_i,A_j)$, $\Gamma_{ij}(L_i,A_j)$, $\Pi_{ij}(L_i,A_j)$, $\Omega_{ij}(L_i,A_j)$, $\Phi_{ij}(L_i,A_j)$) = (L_i,A_j)(i = 1, 2, ..., p and j = 1, 2, ..., q) is an SVPNN. By using all these evaluation information, a decision matrix (D^M) is billed as follows.

The decision matrix can be expressed as follows:

D^M	A_1	A_2	…….	……..	A_q

L_1	(L_1, A_1)	(L_1, A_1)	……..	…….	(L_1, A_q)
L_2	(L_2, A_1)	(L_2, A_2)	……	……..	(L_2, A_q)
……	……..	……..	…….	…….	………
L_p	(L_p, A_1)	(L_p, A_2)	………..	………	(L_p, A_q)

Step-2: Selection of the Positive Ideal Solution for the Decision Matrix.

The Positive Ideal Solution (PIS) for the decision matrix is defined as follows:

$$L^{+} = [(\Delta_1^{+}, \Gamma_1^{+}, \Pi_1^{+}, \Omega_1^{+}, \Phi_1^{+}), (\Delta_2^{+}, \Gamma_2^{+}, \Pi_2^{+}, \Omega_2^{+}, \Phi_2^{+}), \ldots\ldots\ldots, (\Delta_m^{+}, \Gamma_m^{+}, \Pi_m^{+}, \Omega_m^{+}, \Phi_m^{+})], \quad (3)$$

where Δ_j^{+}= max {$\Delta_{ij}(L_i, A_j)$: i=1, 2, 3, ….., n}, Γ_j^{+}= max {$\Gamma_{ij}(L_i, A_j)$: i=1, 2, 3, ….., n},Π_j^{+}= min {$\Pi_{ij}(L_i, A_j)$: i=1, 2, 3, ….., n}, Ω_j^{+}= min {$\Omega_{ij}(L_i, A_j)$: i=1, 2, 3, ….., n} and Φ_j^{+}= min {$\Phi_{ij}(L_i, A_j)$: i=1, 2, 3, ….., n}.

Step-3: Calculation of Attribute's Weight.

In any MADM problem, the decision maker can use the compromise function as tools for the calculation of the weight of each attribute those are completely unknown.

The compromise function is defined as follows:

$$\Psi_j=\sum_{i=1}^{p} (3 + \Delta_{ij}(L_i, A_j) + \Gamma_{ij}(L_i, A_j) - \Pi_{ij}(L_i, A_j) - \Omega_{ij}(L_i, A_j) - \Phi_{ij}(L_i, A_j)) / 5. \quad (4)$$

Then, the weight of the j-th attribute is defined by $w_j = \frac{\Psi_j}{\sum_{j=1}^{q} \Psi_j}$ (5)

Here, $\sum_{j=1}^{q} w_j = 1$.

Step-4: Determination of the SVPNWDSM between PIS and E_{L_i} (i = 1, 2,..., p).

In this step, the SVPNWDSM between the decision elements from the decision matrix and the PIS is calculated by using eq. (2).

Step-5: Ranking Order of the Alternatives.

Finally, the ranking order of alternatives is determined based on the ascending order of SVPNWDSM between the PIS and the decision elements from the decision matrix. The alternative associated with the highest SVPNWDSM value is the most suitable alternatives.

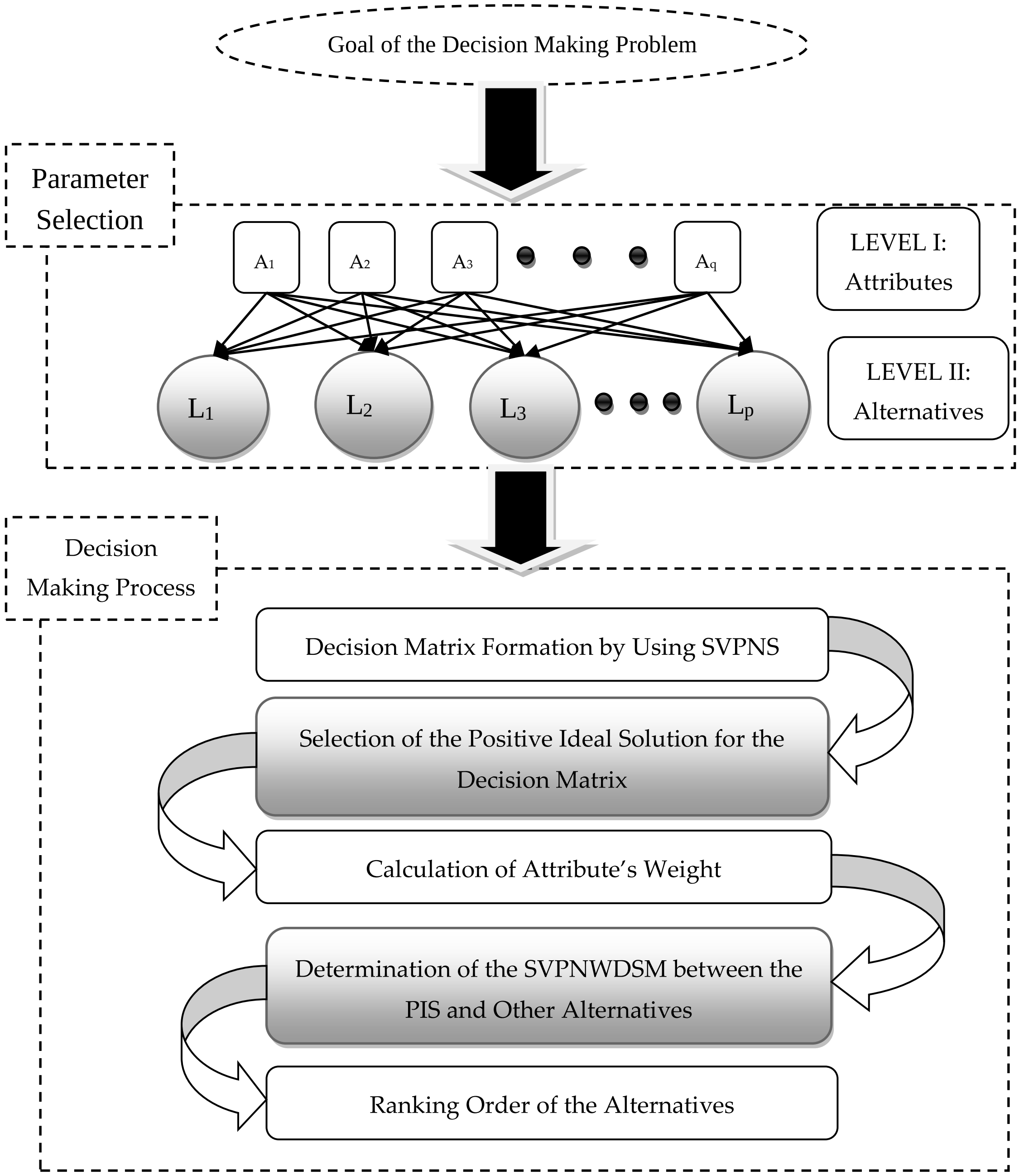

Figure-1: Proposed MADM-Strategy

5. Application of the Proposed MADM Strategy in the Selection of Suitable Metal Oxide Nano-Additive for Biodiesel Blend on Environmental Aspect under the SVPNS Environment:

The search for a potential alternative fuel has flourished due to the global demand for fossil fuels and environmental problems. Among all alternative fuels, biodiesel has become more popular in many nations. But main problem with biodiesel have their low calorific value and low heat value, which result low engine performance. In different research studies, nano-additives have been proposed for improving the performance and emission characteristics of biodiesel. Metal oxide nano-additives are basically used frequently for the improvement of combustion quality.

The proposed research work focused on the selection of suitable metal oxide nano-additive for biodiesel blend on environmental aspect under the SVPNS environment.

For the current research work five attributes namely (*i*) CO emission, (*ii*) HC emission (*iii*) SO_2 emission, (*iv*) NO_x emission and (*v*) Smoke emission, and six alternatives namely (*i*) GO, (*ii*) SiO_2.,(*iii*) CuO, (*iv*) Al_2O_3, (*v*) Fe_2O_3 and (*vi*) TiO_2 are chosen from different literature [1, 16, 31, 39, 41, 44]. Best alternative among them was chosen with the proposed MADM strategy under the SVPNS environment.

In addition of TiO_2 nano-particles the values of CO, HC and smoke opacity emission reduced, while emission of CO_2 and NO are increased. This happen due to the intensified combustion process as compared to Bio diesel blends without TiO_2 [31]. CuO nano-particles shows good impact in reduction of CO, HC and smoke emission though in addition of CuO nano-additive CO_2 emission increase while NO_x emission increase slightly [39]. A comparative study was done by Tomar and Kumar [41] between Al_2O_3 and Fe_2O_3. Reduction of all kind emission was observed with both nano-additives though Fe_2O_3 is slightly more effective in reduction of CO emission but in the case of SO_2 and NOx emission reduction, Al_2O_3 is more effective. Effect of GO was studied by Hoseini et al. [16]. In addition of GO, the emission of HC and CO decrease with a penalty of increased NO_x emission. The effect of Al_2O_3 nano-particles was studied separately and it was observed that all the emission i.e., HC, CO, smoke and NO_x emission reduced significantly at different loading condition. Ağbuluta et al. [1] have done a comparative study among three nano-particles metallic oxide namely Al_2O_3, TiO_2 and SiO_2. In [1], the authors reported, emission of CO, HC and NO_x were reduced in the presence of three nano-additives with blend though the highest reduction of CO emission observed with Al_2O_3 nano-particles and highest NO_x emission with TiO_2 nano-particles. Table-1 represents the list of nano-Particles added to biodiesel and their corresponding engine emissions.

Table-1: List of nano-particles added to biodiesel and their corresponding engine emissions

Nano-particles & Dosage	**Operating Condition**	**CO**	**HC**	**NO_x**	**CO_2**	**Smoke**

TiO_2 0.01% by mass [31]	1000 rpm, 1500 rpm, 2000rpm, 2500 rpm, 3000 rpm	B_20+TiO_2 reduce the 25.56% CO emission compared to Diesel	B_20+TiO_2 reduce HC emission around 34.12% at 3000 rpm	TiO_2 dramatically increasing the in cylinder pressure and temperature since the rising of NO emission	--	Average reduction 25.07%
CuO, 25, 50, and 75 ppm[39]	Different load condition	Reduced	Reduced	Increase slightly	Increase	Reduced
Fe_2O_3,Al_2O_3, 30;60;90 ppm[41]	1800 rpm and at 50% load condition	Reduced		Reduced(up to 24%Reduction found with Al_2O_3)		10-15% lower at 300 ppm
GO, 30;60;90 ppm [16]	2100 rpm and different load condition	Reduced	Reduced	Increase	Increase	
Al_2O_3, 25;50 ppm [44]	1500 rpm and different load condition	Reduced (Maximum Reduction found with 50 ppm Al_2O_3)	Reduced (Maximum Reduction found with 50 ppm Al_2O_3)	Reduced (Maximum Reduction found with 50 ppm Al_2O_3)	Reduced (Maximum Reduction found with 50 ppm Al_2O_3)	Reduced (Maximum Reduction found with 50 ppm Al_2O_3)
Al_2O_3;TiO_2,SiO_2ppm [1]	2000 rpm and different load condition	Reduced (Maximum Reduction found with Al_2O_3)	Reduced (Maximum Reduction found with Al_2O_3)	Increase (Maximum increment found with TiO_2)		

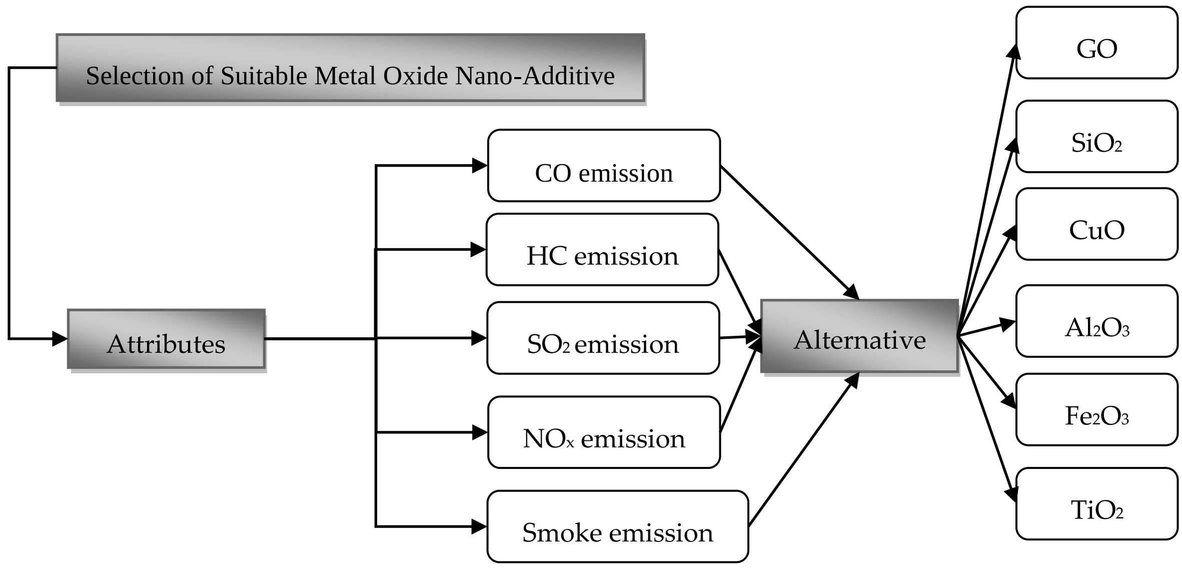

Figure-2: Decision Hierarchy of the Current MADM Problem

Figure-2 represents decision hierarchy of the current MADM problem and steps involve in the current MADM problem is presented as follows:

By using the evaluation information for all alternatives given by the decision makers, prepare the decision matrix in Table-2.

Table-2: Decision Matrix

	A_1	A_2	A_3	A_4	A_5
L_1	(1.0,0.2,0.2,0.0,0.0)	(0.8,0.1,0.1,0.1,0.1)	(0.9,0.0,0.2,0.1,0.1)	(1.0,0.0,0.2,0.1,0.1)	(0.9,0.0,0.0,0.0,0.1)
L_2	(1.0,0.1,0.2,0.1,0.2)	(0.9,0.1,0.2,0.2,0.1)	(0.8,0.1,0.0,0.0,0.1)	(0.9,0.0,0.0,0.1,0.1)	(0.9,0.1,0.0,0.2,0.0)
L_3	(1.0,0.1,0.0,0.0,0.1)	(0.9,0.0,0.1,0.1,0.2)	(0.9,0.1,0.1,0.1,0.1)	(0.8,0.1,0.2,0.1,0.1)	(1.0,0.2,0.0,0.0,0.1)
L_4	(0.9,0.2,0.1,0.1,0.0)	(1.0,0.1,0.0,0.0,0.1)	(0.9,0.1,0.1,0.1,0.1)	(1.0,0.1,0.0,0.0,0.1)	(0.8,0.1,0.1,0.2,0.1)
L_5	(1.0,0.2,0.1,0.1,0.1)	(0.8,0.1,0.0,0.0,0.1)	(0.7,0.2,0.0,0.1,0.1)	(0.8,0.1,0.0,0.0,0.1)	(1.0,0.1,0.0,0.0,0.1)
L_6	(0.8,0.1,0.1,0.2,0.1)	(1.0,0.0,0.1,0.2,0.1)	(0.9,0.0,0.2,0.1,0.1)	(0.8,0.1,0.1,0.2,0.1)	(1.0,0.1,0.0,0.0,0.1)

Now, by using the eq. (3), the PIS (L^{+}) is formed for the decision matrix in Table-3.

Table-3: Positive Ideal Solution

	A_1	A_2	A_3	A_4	A_5
L^+	(1.0,0.2,0.0,0.0,0.0)	(1.0,0.1,0.0,0.0,0.1)	(0.9,0.2,0.0,0.0,0.1)	(1.0,0.1,0.0,0.0,0.1)	(1.0,0.2,0.0,0.0,0.0)

Weights of the attributes are determined by using the eq. (4) & eq. (5). The weights of the attribute are w_1=0.2042819, w_2=0.1962533, w_3=0.1953613, w_4=0.1971454, w_5=0.2069581.

By using the eq. (2), obtained SVPNWDSM of similarities between the PIS and the decision elements from the decision matrix as follows:

$D_{SVPNWDSM}(L_1, L^+)$ =0.966693;

$D_{SVPNWDSM}(L_2, L^+)$ = 0.968999;

$D_{SVPNWDSM}(L_3, L^+)$ = 0.978151;

$D_{SVPNWDSM}(L_4, L^+)$ = 0.980355;

$D_{SVPNWDSM}(L_5, L^+)$ = 0.978792;

$D_{SVPNWDSM}(L_6, L^+)$ = 0.95907.

The ascending order of the SVPNWDSM between the PIS and the decision elements from the decision matrix is as follows:

$D_{SVPNWDSM}(L_6, L^+) < D_{SVPNWDSM}(L_1, L^+) < D_{SVPNWDSM}(L_2, L^+) < D_{SVPNWDSM}(L_3, L^+) < D_{SVPNWDSM}(L_5, L^+) < D_{SVPNWDSM}(L_4, L^+)$.

Hence, the alternative L_4 i.e., Al_2O_3 is the most suitable metal oxide nano-additive for the biodiesel blend on environmental aspect under the SVPNS environment.

6. Comparative Study:

To verify the proposed result based on the SVPNWDSM, an investigation has been conducted for the purpose of comparison with the existing MADM techniques [13, 23].

From the comparative table (see Table-4) it is observed that the existing methods support the same performance as per the proposed method for best attribute. According to the Table-4 it is clear that the weighted values of all attribute are much closed for two existing methods. In case of proposed technique the weighted values of all attribute is not closed compare to existing tool, it helps to take better decision for considering attributes. So the proposed method is more effective compare to considering MADM methods.

Table-4: Comparative Study

Methods	L_1	L_2	L_3	L_4	L_5	L_6	Ranking Order
MADM Strategy Based on Tangent Similarity Measure under SVPNS Environment [13]	0.976292	0.978664	0.981409	0.985021	0.982482	0.975305	$L_6 < L_1 < L_2 < L_3 < L_5 < L_4$

MADM Strategy Based on Cosine Similarity Measure under SVPNS Environment [23]	0.834713	0.834963	0.834974	0.836300	0.835798	0.834743	$L_1 < L_6 < L_2 < L_3 < L_5 < L_4$
Proposed MADM Strategy	0.966693	0.968999	0.978151	0.980355	0.978792	0.959070	$L_6 < L_1 < L_2 < L_3 < L_5 < L_4$

From the above comparison Table-4, it is clear that L_4 is the most appropriate alternative in all the MADM strategies.

7. Conclusions:

In this article, a novel MADM is proposed for selecting suitable nano-additives for biodiesel to enhance performance and emissions characteristics of internal combustion engines. Using this method, a ranking among the alternatives is generated. The ranking order $L_6 < L_1 < L_2 < L_3 < L_5 < L_4$ is derived by the proposed method. It is obvious from the ranking order generated by the new method that alternative L_4 is the best among all alternatives. A comparison of the results obtained by the new MADM method is performed using different existing methods. Based on all methods, alternative L_4 is the best, and therefore, it is concluded that the proposed method is well suited for solving such a problem.

In a future study, the nano-additive selected from this present work will be applied to biodiesel in different concentrations and its performance and emission characteristics will be examined experimentally. Further, it is hoped that, the proposed MADM-strategy can also be used to deal with the other real life problems such as Data Mining [30], Medical Diagnosis [35-36], Fault Diagnosis [46-47], and decision-making problems such as Tender Selection [7], Electronic Goods Selection [12], Plot Selection [13], Weaver Selection [15], Brick Selection [25, 29], Logistic Center Location Selection [32-33], Teacher Selection [37], etc.

Conflict of Interest: The authors declare that they have no conflict of interest.

Authors Contribution: The authors declare that all the authors have equal contribution for the preparation of this article.

References:

1. Ağbulut, Ü., Karagöz, M., Sarıdemir, S., & Öztürk, A. (2020). Impact of various metal-oxide based nanoparticles and biodiesel blends on the combustion, performance, emission, vibration and noise characteristics of a CI engine. *Fuel*, 270, 117521.
2. Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20, 87-96.

3. Biswas, P., Pramanik, S., & Giri, B.C. (2014). Entropy based grey relational analysis method for multi-attribute decision making under single valued neutrosophic assessments. *Neutrosophic Sets and Systems*, 2, 102-110.
4. Das, S. (2021). Neutrosophic Supra Simply Open Set and Neutrosophic Supra Simply Compact Space. *Neutrosophic Sets and Systems*, 43, 105-113.
5. Das, S., Das, R., & Granados, C. (2021). Topology on QuadripartitionedNeutrosophic Sets. *Neutrosophic Sets and Systems*, 45, 54-61.
6. Das, S., Das, R., Granados, C., & Mukherjee, A. (2021). PentapartitionedNeutrosophic*Q*-Ideals of *Q*-Algebra. *Neutrosophic Sets and Systems*, 41, 52-63.
7. Das, S., Das, R., & Tripathy, B.C. (2020). Multi criteria group decision making model using single-valued neutrosophic set. *LogForum*, 16(3), 421-429.
8. Das, S., & Pramanik, S. (2020). Generalized neutrosophic *b*-open sets in neutrosophic topological space. *Neutrosophic Sets and Systems*, 35, 522-530.
9. Das, S., & Pramanik, S. (2020). Neutrosophic Φ-open sets and neutrosophic Φ-continuous functions. *Neutrosophic Sets and Systems*,38, 355-367.
10. Das, S., & Pramanik, S. (2020). Neutrosophic simply soft open set in neutrosophic soft topological space. *Neutrosophic Sets and Systems*, 38, 235-243.
11. Das, S., & Pramanik, S. (2021). Neutrosophic Tri-Topological Space. *Neutrosophic Sets and Systems*, 45, 366-377.
12. Das, S., Shil, B., & Pramanik, S. (In Press). SVPNS-MADM strategy based on GRA in SVPNS Environment. *Neutrosophic Sets and Systems*.
13. Das, S., Shil, B., & Tripathy, B.C. (2021). Tangent Similarity Measure Based MADM-Strategy under SVPNS-Environment. *Neutrosophic Sets and Systems*, 43, 93-104.
14. Das, S., & Tripathy, B.C. (2021). Pentapartitioned Neutrosophic Topological Space. *Neutrosophic Sets and Systems*, 45, 121-132.
15. Dey, P.P., Pramanik, S., & Giri, B.C. (2015). An extended grey relational analysis based interval neutrosophic multi attribute decision making for weaver selection. *Journal of New Theory*, 9, 82-93.
16. Hoseini, S.S., Najafi, G., Ghobadian, B., Ebadi, M.T., Mamat, R., & Yusaf, T. (2020). Performance and emission characteristics of a CI engine using graphene oxide (GO) nano-particles additives in biodiesel-diesel blends. *Renewable Energy*, 145, 458-465.
17. Kamaci, H. (2020). Neutrosophic Cubic Hamacher Aggregation Operators and Their Applications in Decision Making. *Neutrosophic Sets and Systems*, 33, 234-255.
18. Kamaci, H. (2021). Simplified Neutrosophic Multiplicative Refined Sets and Their Correlation Coefficients with Application in Medical Pattern Recognition. *Neutrosophic Sets and Systems*, 41, 270-285.

19. Kamaci, H. (2021). Linguistic single-valued neutrosophic soft sets with applications in game theory. *International Journal of Intelligent Systems*, 36(8), 3917-3960.

20. Kamaci, H., Garg, H., & Petchimuthu, S. (2021). Bipolar trapezoidal neutrosophic sets and their Dombi operators with applications in multicriteria decision making. *Soft Computing*, 25(13), 8417-8440.

21. Liu, D., Chen, X., & Peng, D. (2018). The Intuitionistic Fuzzy Linguistic Cosine Similarity Measure and Its Application in Pattern Recognition. *Complexity*, 2018, Article ID 9073597, 1-11. https://doi.org/10.1155/2018/9073597

22. Majumder, P., Biswas, P., & Majumder, S. (2020). Application of New TOPSIS Approach to Identify the Most Significant Risk Factor and Continuous Monitoring of Death of COVID-19. *Electronic Journal of General Medicine*, 17(6).

23. Majumder, P., Das, S., Das, R., & Tripathy, B.C. (2021). Identification of the Most Significant Risk Factor of COVID-19 in Economy Using Cosine Similarity Measure under SVPNS-Environment. *Neutrosophic Sets and Systems*, 46, 112-127.

24. Mallick, R., & Pramanik, S. (2020). Pentapartitioned neutrosophic set and its properties. *Neutrosophic Sets and Systems*, 36, 184-192.

25. Mondal, K., & Pramanik, S. (2015). Neutrosophic decision making model for clay-brick selection in construction field based on grey relational analysis. *Neutrosophic Sets and Systems*, 9, 72-79.

26. Mondal, K., & Pramanik, S. (2015). Neutrosophic tangent similarity measure and its application to multiple attribute decision making. *NeutrosophicSets and Systems*, 9, 80-87.

27. Mondal, K., Pramanik, S., & Giri, B.C. (2018). Interval neutrosophic tangent similarity measure based MAMD strategy and its application to MAMD problems. *Neutrosophic Sets and Systems*, 19, 47-53.

28. Mondal, K., Pramanik, S., & Giri, B.C. (2018). Single valued neutrosophic hyperbolic sine similarity measure based MADM strategy. *Neutrosophic Sets and Systems*, 20, 3-11.

29. Mondal, K., Pramanik, S., & Smarandache, F. (2014). Intuitionistic fuzzy multi-criteria group decision making approach to quality-brick selection problem. *Journal of Applied Quantitative Methods*, 9(2), 35-50.

30. Mondal, K., Pramanik, S., & Smarandache, F. (2016). Role of neutrosophic logic in data mining. *New Trends in Neutrosophic Theory and Application*. *Pons Editions*, *Brussels*, 15-23.

31. Örs, I., Sarıkoç, S., Atabani, A.E., Ünalan, S.E.B.A.H.A.T.T.İ.N., & Akansu, S.O. (2018). The effects on performance, combustion and emission characteristics of DICI engine fuelled with TiO2 nanoparticles addition in diesel/biodiesel/n-butanol blends. Fuel, 234, 177-188.

32. Pramanik, S., Dalapati, S., & Roy, T.K. (2016). Logistics center location selection approach based on neutrosophic multi criteria decision making. *New Trends in Neutrosophic Theory and Application*. *Pons Editions*, *Brussels*, 161-174.

33. Pramanik, S., Dalapati, S., & Roy, T.K. (2018). Neutrosophic multi-attribute group decision making strategy for logistic center location selection. *Neutrosophic Operational Research*, Vol. III. *Pons Asbl*, *Brussels*, 13-32.

34. Pramanik, S., Dey, P.P., & Giri, B.C. (2015). TOPSIS for single valued neutrosophic soft expert set based multi-attribute decision making problems. *Neutrosophic Sets and Systems*, 10, 88-95.

35. Pramanik, S., & Mondal, K. (2015). Cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. *Global Journal of Advanced Research*, 2(1), 212-220.

36. Pramanik, S., & Mondal, K. (2015). Cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis. *Journal of New Theory*, 4, 464-471.

37. Pramanik, S., & Mukhopadhyaya, D. (2011). Grey relational analysis based intuitionistic fuzzy multi criteria group decision-making approach for teacher selection in higher education. *International Journal of Computer Applications*, 34(10), 21-29.

38. Poonia, M., & Bajaj, R.K. (2021). On Measures of Similarity for Neutrosophic Sets with Applications in Classification and Evaluation Processes. *Neutrosophic Sets and Systems*, 93, 86-100.

39. Rastogi, P.M., Sharma, A., & Kumar, N. (2021). Effect of CuO nanoparticles concentration on the performance and emission characteristics of the diesel engine running on jojoba (SimmondsiaChinensis) biodiesel. *Fuel*, 286, 119358.

40. Smarandache, F. (1998). A unifying field in logics, neutrosophy: neutrosophic probability, set and logic. *Rehoboth*, *American Research Press*.

41. Tomar, M., & Kumar, N. (2020). Influence of nanoadditives on the performance and emission characteristics of a CI engine fuelled with diesel, biodiesel, and blends–a review. *Energy Sources*, *Part A: Recovery*, *Utilization*, *and Environmental Effects*, 42(23), 2944-2961.

42. Tripathy, B.C., & Das, S. (2021). Pairwise Neutrosophic*b*-Continuous Function in NeutrosophicBitopological Spaces. *Neutrosophic Sets and Systems*, 43, 82-92.

43. Ulucay, V., Deli, I., & Sahin, M. (2016). Similarity measures of bipolar neutrosophic sets and their application to multiple criteria decision making. *Neural Computing& Application*, DOI 10.1007/s00521-016-2479-1

44. Venu, H., & Appavu, P. (2020). Al_2O_3 nano additives blended Polanga biodiesel as a potential alternative fuel for existing unmodified DI diesel engine. *Fuel*, 279, 118518.

45. Wang, H., Smarandache, F., Zhang, Y.Q., & Sunderraman, R. (2010). Single valued neutrosophic sets. *Multispace and Multistructure*, 4, 410-413.

46. Ye, J. (2016). Fault diagnoses of steam turbine using the exponential similarity measure of neutrosophic numbers. *Journal of Intelligent and Fuzzy Systems*, 30(4), 1927-1934.
47. Ye, J. (2017). Single-valued neutrosophic similarity measures based on cotangent function and their application in the fault diagnosis of steam turbine. *Soft Computing*, 21 (3), 817-825.
48. Ye, J., & Zhang, Q.S. (2014). Single valued neutrosophic similarity measures for multiple attribute decision making. *Neutrosophic Sets and Systems*, 2, 48-54.
49. Zadeh, L.A. (1965). Fuzzy sets. *Information and Control*, 8, 338-353.

Received: Nov 22, 2021. Accepted: Feb 2, 2022