Pentapartitioned Neutrosophic Pythagorean Resolvable and Irresolvable Spaces

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Abstract: In this paper, the concepts of Pentapartitioned Neutrosophic Pythagorean resolvable, Pentapartitioned Neutrosophic Pythagorean irresolvable, Pentapartitioned Neutrosophic Pythagorean open hereditarily irresolvable and maximally Pentapartitioned Neutrosophic Pythagorean irresolvable spaces are introduced. Also we investigated several properties of the Pentapartitioned Neutrosophic Pythagorean open hereditarily irresolvable spaces besides giving characterization of these spaces by means of somewhat Pentapartitioned Neutrosophic Pythagorean continuous functions and somewhat Pentapartitioned Neutrosophic Pythagorean open functions.

Keywords: Pentapartitioned neutrosophic pythagorean resolvable, pentapartitioned neutrosophic pythagorean open hereditarily irresolvable, somewhat pentapartitioned neutrosophic pythagorean irresolvable, pentapartitioned neutrosophic pythagorean continuous and open functions.

1. Introduction

Zadeh[16] introduced the important and useful concept of a fuzzy set which has invaded almost all branches of mathematics. The speculation of fuzzy topological space was studied and developed by C.L. Chang [4]. The paper of Chang sealed the approach for the following tremendous growth of the various fuzzy topological ideas. Since then a lot of attention has been paid to generalize the fundamental ideas of general topology in fuzzy setting and therefore a contemporary theory of fuzzy topology has been developed. Atanassov and plenty of researchers [1] worked on intuitionistic fuzzy sets within the literature. Florentin Smarandache [13] introduced the idea of Neutrosophic set in 1995 that provides the information of neutral thought by introducing the new issue referred to as uncertainty within the set. Thus neutrosophic set was framed and it includes the parts of truth membership function(T), indeterminacy membership function(I), and falsity membership function(F) severally. Neutrosophic sets deals with non normal interval of ]−0 1+[. Pentapartitioned neutrosophic set and its properties were introduced by Rama Malik and Surpati Pramanik [12]. In this case, indeterminacy is divided into three components: contradiction, ignorance, and an unknown
membership function. The concept of Pentapartitioned neutrosophic pythagorean sets was initiated by R. Radha and A. Stanis Arul Mary. The concept of neutrosophic fuzzy resolvable spaces and irresolvable spaces was introduced by M. Caldas et.al[3]. Now we extend the concepts to pentapartitioned neutrosophic pythagorean sets.

In this Paper we initiated the new concept of Pentapartitioned neutrosophic pythagorean resolvable, pentapartitioned neutrosophic pythagorean open hereditarily irresolvable, somewhat pentapartitioned neutrosophic pythagorean irresolvable, pentapartitioned neutrosophic pythagorean continuous and open functions and discussed some of its properties.

2. Preliminaries

2.1 Definition [13]
Let X be a universe. A Neutrosophic set A on X can be defined as follows:

\[ A = \{ < x, T_A(x), I_A(x), F_A(x) > : x \in X \} \]

Where \( T_A, I_A, F_A : U \rightarrow [0,1] \) and \( 0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3 \)

Here, \( T_A(x) \) is the degree of membership, \( I_A(x) \) is the degree of indeterminacy and \( F_A(x) \) is the degree of non-membership.

2.2 Definition [7]
Let X be a universe. A Pentapartitioned neutrosophic pythagorean [PNP] set A with T, F, C and U as dependent neutrosophic components and I as independent component for A on X is an object of the form

\[ A = \{ < x, T_A, C_A, I_A, U_A, F_A > : x \in X \} \]

Where \( T_A + F_A \leq 1, C_A + U_A \leq 1 \) and \( (T_A)^2 + (C_A)^2 + (I_A)^2 + (U_A)^2 + (F_A)^2 \leq 3 \)

Here, \( T_A(x) \) is the truth membership, \( C_A(x) \) is contradiction membership, \( U_A(x) \) is ignorance membership, \( F_A(x) \) is the false membership and \( I_A(x) \) is an unknown membership.

2.3 Definition [12]
Let P be a non-empty set. A Pentapartitioned neutrosophic pythagorean set A over P characterizes each element p in P a truth -membership function \( T_A \), a contradiction membership function \( C_A \), an ignorance membership function \( G_A \), unknown membership function \( U_A \) and a false membership function \( F_A \), such that for each p in P

\[ T_A + C_A + G_A + U_A + F_A \leq 5. \]

2.4 Definition [7]
The complement of a pentapartitioned neutrosophic pythagorean set A on R is denoted by \( A^C \) or \( A^* \) and is defined as

\[ A^C = \{ < x, F_A(x), U_A(x), 1 - G_A(x), C_A(x), T_A(x) > : x \in X \} \]
2.5 Definition [7]

Let \( A = < x, T_A(x), C_A(x), U_A(x), F_A(x) > \) and \( B = < x, T_B(x), C_B(x), G_B(x), U_B(x), F_B(x) > \) are pentapartitioned neutrosophic pythagorean sets. Then

\[
A \cup B = < x, \max(T_A(x), T_B(x)), \max(C_A(x), C_B(x)), \min(G_A(x), G_B(x)), \min(U_A(x), U_B(x)), \min(F_A(x), F_B(x)) >
\]

\[
A \cap B = < x, \min(T_A(x), T_B(x)), \min(C_A(x), C_B(x)), \max(G_A(x), G_B(x)), \max(U_A(x), U_B(x)), \max(F_A(x), F_B(x)) >
\]

2.6 Definition [7]

A PNP topology \( \tau \) on a nonempty set \( R \) is a family of a PNP sets in \( R \) satisfying the following axioms

1) \( 0, 1 \in \tau \)
2) \( R_1 \cap R_2 \in \tau \) for any \( R_1, R_2 \in \tau \)
3) \( \bigcup R_i \in \tau \) for any \( R_i; i \in I \subseteq \tau \)

The complement \( R^c \) of PNP open set (PNPOS, in short) in PNP topological space [PNPTS] \((R, \tau)\), is called a PNP closed set [PNPCS].

2.7 Definition [7]

Let \((R, \tau)\) be a PNPTS and \( L \) be a PNPTS in \( R \). Then the PNP interior and PNP Closure of \( R \) denoted by

\[
\text{Cl}(L) = \bigcap \{ K : K \text{ is a PNPCS in } R \text{ and } L \subseteq K \}.
\]

\[
\text{Int}(L) = \bigcup \{ G : G \text{ is a PNPOS in } R \text{ and } G \subseteq L \}.
\]

3. Pentapartitioned Neutrosophic Pythagorean Resolvable and Irresolvable Spaces

3.1 Definition

A Pentapartitioned neutrosophic pythagorean (PNP) set \( P \) in Pentapartitioned neutrosophic pythagorean topological space (PNPTS) \((R, \tau)\) is called pentapartitioned neutrosophic pythagorean dense if there exists no pentapartitioned neutrosophic pythagorean closed set \( Q \) in \((R, \tau)\) such that \( P \subseteq Q \subset 1_R \)

\textbf{Note:} If \( P \) is a PNP open set, then the complement of PNP set \( P \) is a PNP closed set and it is denoted by \( P^c \).

3.2 Example

Let \( R = \{ e, f \} \) and define the pentapartitioned neutrosophic pythagorean set \( P \) as

\[
P = \{ (e, 0.4, 0.5, 0.7, 0.2, 0.3) \} \\
= \{ (f, 0.5, 0.3, 0.6, 0.1, 0.2) \}
\]
Then $\tau = \{0_R, 1_R, P\}$ is a pentapartitioned neutrosophic pythagorean topology on $R$. Hence $P$ is a PNP dense set in $(R, \tau)$.

### 3.3 Definition

A PNPTS $(R, \tau)$ is called PNP resolvable if there exists a PNP dense set $P$ in $(R, \tau)$ such that $\text{PNPcl}(P^*) = 1_R$. Otherwise $(R, \tau)$ is called PNP irresolvable.

### 3.4 Example

Let $R = \{e, f\}$ and define the pentapartitioned neutrosophic pythagorean set $P$, $Q$ and $R$ as

$P = \{(e, 0.3, 0.4, 0.3, 0.3, 0.1), (f, 0.4, 0.2, 0.6, 0.5, 0.3)'\}$

$Q = \{(e, 0.4, 0.2, 0.7, 0.1, 0.3), (f, 0.6, 0.1, 0.3, 0.2, 0.2)\}$

$R = \{(e, 0.1, 0.2, 0.4, 0.3, 0.4), (f, 0.5, 0.4, 0.3, 0.2, 0.1)\}$.

It can be seen that $\tau = \{0_R, 1_R, P\}$ is a pentapartitioned neutrosophic pythagorean topology on $R$. Then $(R, \tau)$ is a pentapartitioned neutrosophic topological space. Now $\text{PNPInt}(Q) = 0_R$, $\text{PNPInt}(R) = 0_R$, $\text{PNPcl}(Q) = 1_R$ and $\text{PNPcl}(R) = 1_R$. Thus $P$ and $Q$ are PNP dense sets in $(R, \tau)$ such that $\text{PNPcl}(Q^*) = 1_R$ and $\text{PNPcl}(R^*) = 1_R$. Hence the PNP topological space $(R, \tau)$ is PNP resolvable.

### 3.5 Example

Let $R = \{e, f\}$ and define the pentapartitioned neutrosophic pythagorean set $P$, $Q$ and $R$ as

$P = \{(e, 0.2, 0.3, 0.5, 0.4, 0.5), (f, 0.1, 0.2, 0.5, 0.5, 0.3)'\}$

$Q = \{(e, 0.4, 0.5, 0.5, 0.4, 0.3), (f, 0.5, 0.4, 0.4, 0.3, 0.2)\}$

$R = \{(e, 0.3, 0.4, 0.4, 0.3, 0.2), (f, 0.2, 0.3, 0.5, 0.2, 0.1)\}$.

It can be seen that $\tau = \{0_R, 1_R, P\}$ is a pentapartitioned neutrosophic pythagorean topology on $R$. Then $(R, \tau)$ is a pentapartitioned neutrosophic topological space. Now $\text{PNPInt}(Q) = A_R$, $\text{PNPInt}(R) = A_R$, $\text{PNPcl}(Q) = 1_R$ and $\text{PNPcl}(R) = 1_R$. Thus $P$ and $Q$ are PNP dense sets in $(R, \tau)$ such that $\text{PNPcl}(Q^*) = P^*$ and $\text{PNPcl}(R^*) = P^*$. Hence the PNP topological space $(R, \tau)$ is PNP irresolvable.

### 3.6 Theorem

A PNPTS $(R, \tau)$ is a PNP resolvable space iff $(R, \tau)$ has a pair of PNP dense set $K_i$ and $K_j$ such that $K_i \subseteq K_j^*$.

**Proof**

Let $(R, \tau)$ be a PNPTS and $(R, \tau)$ be PNP resolvable space. Suppose that for all PNP dense sets $K_i$ and $K_j$, we have $K_i \subseteq K_j^*$. Then $K_i \supseteq K_j^*$. Then $\text{PNPcl}(K_i) \supseteq \text{PNPcl}(K_j^*)$ which implies that $1_R \supseteq \text{PNPcl}(K_j^*)$. Then $\text{PNPcl}(K_j^*) = 1_R$. Also $K_j \supseteq K_j^*$, then $\text{PNPcl}(K_j) \supseteq \text{PNPcl}(K_j^*)$ which implies that...
1$\subseteq$ PNPCI($K'_1$). Therefore PNPCI ($K'_1$) $\neq$ 1$\subseteq$ RN. Hence PNPCI($K'_1$) = 1$\subseteq$ RN but PNPCI($K'_1$) $\neq$ 1$\subseteq$ RN for all PNP set $K'_1$ in (R, $\tau$) which is a contradiction. Hence (R, $\tau$) has a pair of PNP dense set $K_1$ and $K_2$ such that $K_1 \subseteq K_2$.

Conversely, suppose that the PNP topological space (R, $\tau$) has a pair of PNP dense set $K_1$ and $K_2$ such that $K_1 \subseteq K_2$. Suppose that (R, $\tau$) is a PNP irresolvable space, then for all PNP dense sets $K_1$ and $K_2$ in (R, $\tau$), we have PNPCI($K'_1$) $\neq$ 1$\subseteq$ RN. Then PNPCI($K'_2$) $\neq$ 1$\subseteq$ RN implies that there exists a PNP closed set L in (R, $\tau$) such that $K'_2 \subseteq L \subseteq 1$$\subseteq$ RN. Then $K_1 \subseteq K'_2 \subseteq L \subseteq 1$$\subseteq$ RN implies that $K_1 \subseteq L \subseteq 1$$\subseteq$ RN. But this is a contradiction. Hence (R, $\tau$) is a PNP resolvable space.

3.7 Theorem
If (R, $\tau$) is a PNP irresolvable space iff PNInt(P) $\neq$ 0 for all PNP dense set P in (R, $\tau$).

Proof
Since (R, $\tau$) is PNP irresolvable space for all PNP dense set P in (R, $\tau$), PNPCI(P') $\neq$ 1$\subseteq$ RN. Then (PNPCI(P')) $\neq$ 1$\subseteq$ RN which implies PNInt(P) $\neq$ 0$\subseteq$ RN.

Conversely PNInt(P) $\neq$ 0$\subseteq$ RN for all PNP dense set P in (R, $\tau$). Suppose that (R, $\tau$) is PNP resolvable. Then there exists a PNP dense set P in (R, $\tau$) such that PNPCI(P') = 1$\subseteq$ RN. This implies that (PNInt(P')) $= 1$ $\subseteq$ RN which again implies PNInt(P) = 0$\subseteq$ RN. But this is a contradiction. Hence (R, $\tau$) is PNP resolvable space.

3.8 Definition
A PNP topological space (R, $\tau$) is called a PNP submaximal space if for each PNP set P in (R, $\tau$), PNPCI(P) = 1$\subseteq$ RN.

3.9 Proposition
If the PNP topological space (R, $\tau$) is PNP submaximal, then (R, $\tau$) is PNP irresolvable.

Proof. Let (R, $\tau$) be a PNP submaximal space. Assume that (R, $\tau$) is a PNP resolvable space. Let P be a PNP dense set in (R, $\tau$). Then PNPCI(P') = 1$\subseteq$ RN. Hence (PNInt(P')) $= 1$ $\subseteq$ RN which implies that PNInt(P) = 0$\subseteq$ RN. Then P $\notin$ $\tau$. This is a contradiction. Hence (R, $\tau$) is PNP irresolvable space.

The converse of the above theorem is not true, which can be shown by the following example. See example 3.5.

3.10 Definition
A PNP topological space (R, $\tau$) is called a maximal PNP irresolvable space if (R, $\tau$) is PNP irresolvable and every PNP dense set P of (R, $\tau$) is PNP open.

3.11 Example
Let R = {e, f} and define the pentapartitioned neutrosophic pythagorean set Q and R as

\[ Q = \{\{e, 0.3, 0.4, 0.3, 0.3, 0.1\} \]
\[ \{\{f, 0.4, 0.2, 0.6, 0.5, 0.3\}\} \]

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\[ R = \{(e, 0.1, 0.2, 0.4, 0.3, 0.4), (f, 0.5, 0.4, 0.3, 0.2, 0.1)\}. \]

It can be seen that \( \tau = \{0_R, 1_R, P\} \) is a pentapartitioned neutrosophic pythagorean topology on \( R \). Then \((R, \tau)\) is a pentapartitioned neutrosophic topological space. Now \( \text{PNPInt}(Q') = 0_R \) and \( \text{PNPInt}(R^*) = 0_R \). Thus \( P \) and \( Q \) are PNP dense sets in \((R, \tau)\) such that \( \text{PNPCL}(Q') = Q' \) and \( \text{PNPCL}(R^*) = R^* \). Thus \((R, \tau)\) is PNP irresolvable and every PNP dense set of \((R, \tau)\) is PNP open. Therefore PNP topological space \((R, \tau)\) is maximally PNP irresolvable.

4 PNP open hereditarily Irresolvable space

4.1 Definition

A PNP topological space \((R, \tau)\) is said to be PNP open hereditarily irresolvable if \( \text{PNPInt}(\text{PNPCL}(P)) \neq 0_R \) and \( \text{PNPInt}(P) \neq 0_R \), for any PNP set \( P \) in \((R, \tau)\).

4.2 Example

Let \( R = \{e, f\} \) and define the pentapartitioned neutrosophic pythagorean set \( P, Q, R \) and \( S \) as

\[
P = \{(e, 0.2, 0.1, 0.5, 0.4, 0.5), (f, 0.1, 0.2, 0.6, 0.5, 0.4)\}
\]

It can be seen that \( \tau = \{0_R, 1_R, P\} \) is a pentapartitioned neutrosophic pythagorean topology on \( R \). Then \((R, \tau)\) is a pentapartitioned neutrosophic topological space. Now \( \text{PNPInt}(P) \neq 0_R \) and \( \text{PNPInt}(\text{PNPCL}(P)) \neq 0_R \). Thus \((R, \tau)\) is PNP open hereditarily irresolvable space.

4.3 Theorem

Let \((R, \tau)\) be a PNP topological space. If \((R, \tau)\) is PNP open hereditarily irresolvable, then \((R, \tau)\) is PNP irresolvable.

Proof

Let \( P \) be a PNP dense set in \((R, \tau)\). Then \( \text{PNPCL}(P) = 1_R \) which implies that \( \text{PNPInt}(\text{PNPCL}(P)) = 1_R \). Since \((R, \tau)\) is PNP open hereditarily irresolvable, we have \( \text{PNPInt}(P) \neq 0_R \). Therefore by theorem 3.7, \( \text{PNPInt}(P) \neq 0_R \) for all PNP dense set in \((R, \tau)\) implies that \((R, \tau)\) is PNP irresolvable.

The converse of the above theorem is not true. See Example 4.4.

4.4 Example

Let \( R = \{e, f\} \) and define the pentapartitioned neutrosophic pythagorean set \( P, Q, R \) and \( S \) as

\[
P = \{(e, 0.1, 0.5, 0.5, 0.2, 0.6), (f, 0.2, 0.3, 0.6, 0.3, 0.3)\}
\]

\[
Q = \{(e, 0.4, 0.5, 0.1, 0.2, 0.4), (f, 0.3, 0.2, 0.7, 0.2, 0.1)\}
\]
\[
R = \{ \{ e, 0.4, 0.5, 0.1, 0.2, 0.4 \} \text{ and } \}
\]
\[
\{ f, 0.3, 0.3, 0.6, 0.2, 0.1 \} \}
\]
\[
S = \{ \{ e, 0.2, 0.1, 0.7, 0.6, 0.2 \} \}
\]
\[
\{ f, 0.1, 0.2, 0.6, 0.5, 0.4 \} \}.
\]

It can be seen that \( \tau = \{ 0_R, 1_R, P, Q, R \} \) is a pentapartitioned neutrosophic pythagorean topology on \( R \). Then \((R, \tau)\) is a pentapartitioned neutrosophic topological space. Now \( \text{PNPcI}(P) = 1_R, \text{PNPCI}(Q) = 1_R, \) \( \text{PNPCI}(R) = 1_R \) and \( \text{PNPCI}(S) = 1_R \). Thus \( P, Q, R \) and \( S \) are PNP dense sets in \((R, \tau)\) such that \( \text{PNPCI}(P^*) = P^*, \text{PNPCI}(Q^*) = Q^* \) and \( \text{PNPCI}(R^*) = R^* \) and \( \text{PNPCI}(S^*) = P^*. \) Hence the PNP topological space \((R, \tau)\) is PNP irresolvable. But \( \text{PNPInt}(\text{PNPCI}(S^*)) = \text{PNPInt}(P^*) = 0_R. \) Therefore \((R, \tau)\) is not a PNP open hereditarily irresolvable space.

4.5 Theorem
Let \((R, \tau)\) be a PNP open hereditarily irresolvable. Then \( \text{PNPInt}(P) \not\subset \text{PNPInt}(Q)^* \) for any two PNP dense sets \( P \) and \( Q \) in \((R, \tau)\).

Proof.
Let \( P \) and \( Q \) be any two PNP dense sets in \((R, \tau)\). Then \( \text{PNPCI}(P) = 1_R \) and \( \text{PNPCI}(Q) = 1_R \) implies that \( \text{PNPInt}(\text{PNPCI}(P)) \neq 0_R \) and \( \text{PNPInt}(\text{PNPCI}(Q)) \neq 0_R \). Since \((R, \tau)\) is PNP open hereditarily irresolvable, \( \text{PNPInt}(P) \neq 0_R \) and \( \text{PNPInt}(Q) \neq 0_R \). Hence by theorem 3.6, \( P \not\subset Q^* \). Therefore \( \text{PNPInt}(P) \not\subset P \not\subset Q^* \subset (\text{PNPInt}(Q))^* \). Hence we have \( \text{PNPInt}(P) \not\subset (\text{PNPInt}(Q))^* \) for any two PNP dense sets \( P \) and \( Q \) in \((R, \tau)\).

4.6 Theorem
Let \((R, \tau)\) be a PNP topological space. If \((R, \tau)\) is PNP open hereditarily irresolvable, then \( \text{PNPcI}(P) = 0_R \) for any nonzero PNP dense set \( P \) in \((R, \tau)\) which implies that \( \text{PNPInt}(\text{PNPCI}(P)) = 0_R. \)

Proof:
Let \( P \) be a PNP set in \((R, \tau)\) such that \( \text{PNPInt}(P) = 0_R \). We claim that \( \text{PNPInt}(\text{PNPCI}(P)) = 0_R \). Suppose that \( \text{PNPInt}(\text{PNPCI}(P)) = 0_R \). Since \((R, \tau)\) is PNP open hereditarily irresolvable, we have \( \text{PNPInt}(P) \neq 0_R \) which is a contradiction to \( \text{PNPInt}(P) = 0_R. \) Hence \( \text{PNPInt}(\text{PNPCI}(P)) = 0_R. \)

4.7 Theorem
Let \((R, \tau)\) be a PNP topological space. If \((R, \tau)\) is PNP open hereditarily irresolvable, then \( \text{PNPCI}(P) = 1_R \) for any nonzero PNP dense set \( P \) in \((R, \tau)\) which implies that \( \text{PNPCI}(\text{PNPInt}(P)) = 0_R. \)

Proof
Let \( P \) be a PNP set in \((R, \tau)\) such that \( \text{PNPCI}(P) = 1_R \). Then we have \( (\text{PNPCI}(P))^* = 0_R \) which implies that \( \text{PNPInt}(P^*) = 0_R \). Since \((R, \tau)\) is PNP open hereditarily irresolvable by theorem 4.6. We have \( \text{PNPInt}(\text{PNPCI}(P^*)) = 0_R \). Therefore \( (\text{PNPCI}(\text{PNPInt}(P))^*) = 0_R \) implies that \( \text{PNPCI}(\text{PNPInt}(P)) = 1_R. \)
5 Somewhat PNP Continuous and PNP Somewhat PNP open

5.1 Definition

Let \((R, \tau)\) and \((M, \sigma)\) be any two PNP topological spaces. A function \(f: (R, \tau) \rightarrow (M, \sigma)\) is called somewhat PNP continuous if for a \(P \in \sigma\) and \(f^{-1}(P) \neq 0_R\), there exists a \(Q \in \tau\) such that \(Q \neq 0_R\) and \(Q \subseteq f^{-1}(P)\).

5.2 Definition

Let \((R, \tau)\) and \((M, \sigma)\) be any two PNP topological spaces. A function \(f: (R, \tau) \rightarrow (M, \sigma)\) is called somewhat PNP open if for a \(P \in \sigma\) and \(P \neq 0_R\), there exists a \(Q \in \tau\) such that \(Q \neq 0_R\) and \(Q \subseteq f(P)\).

5.3 Theorem

Let \((R, \tau)\) and \((M, \sigma)\) be any two PNP topological spaces. A function \(f: (R, \tau) \rightarrow (M, \sigma)\) is called somewhat PNP continuous and injective. If \(\text{PNPInt}(P) = 0_R\) for any non-zero PNP set \(P\) in \((R, \tau)\), then \(\text{PNPInt}(f(P)) = 0_M\) in \((M, \sigma)\).

Proof

Let \(P\) be a non-zero PNP set in \((R, \tau)\) such that \(\text{PNPInt}(P) = 0_R\). Now we prove that \(\text{PNPInt}(f(P)) = 0_M\). Suppose that \(\text{PNPInt}(f(P)) \neq 0_M\) in \((M, \sigma)\). Then there exists a non-zero PNP set \(Q\) in \((M, \sigma)\) such that \(Q \subseteq f(P)\). Thus, we have \(f^{-1}(Q) \subseteq f^{-1}(f(P))\). Since \(f\) is somewhat PNP continuous, there exists a \(S \in \tau\) such that \(S \neq 0_R\) and \(S \subseteq f^{-1}(Q)\). Hence \(S \subseteq f^{-1}(Q) \subseteq P\) which implies that \(\text{PNPInt}(P) \neq 0_R\). This is a contradiction. Hence \(\text{PNPInt}(f(P)) = 0_M\) in \((M, \sigma)\).

5.4 Theorem

Let \((R, \tau)\) and \((M, \sigma)\) be any two PNP topological spaces. A function \(f: (R, \tau) \rightarrow (M, \sigma)\) is called somewhat PNP continuous, injective and \(\text{PNPInt}(\text{PNPCL}(P)) = 0_R\) for any non-zero PNP set \(P\) in \((R, \tau)\), then \(\text{PNPInt}(\text{PNPCL}(f(P))) = 0_M\) in \((M, \sigma)\).

Proof

Let \(P\) be a non-zero PNP set in \((R, \tau)\) such that \(\text{PNPInt}(\text{PNPCL}(P)) = 0_R\). Now we claim that \(\text{PNPInt}(\text{PNPCL}(f(P))) = 0_M\). Suppose that \(\text{PNPInt}(\text{PNPCL}(f(P))) \neq 0_M\) in \((M, \sigma)\). Then \(\text{PNPCL}(f(P)) \neq 0_M\) and \(\text{PNPCL}(f(P))^* \neq 0_M\). Now \(\text{PNPCL}(f(P))^* \neq 0_M \in M\). Since \(f\) is somewhat PNP continuous, there exists a \(Q \in \tau\) such that \(Q \neq 0_R\) and \(Q \subseteq f^{-1}(\text{PNPCL}(f(P))^*)\). Observe that \(Q \subseteq f^{-1}(\text{PNPCL}(f(P)))\) which implies that \(f^{-1}(\text{PNPCL}(f(P))) \subseteq Q^*\).

Since \(f\) is injective, thus \(P \subseteq f^{-1}(f(P)) \subseteq f^{-1}(\text{PNPCL}(f(P))) \subseteq Q^*\) which implies that \(P \subseteq Q^*\). Therefore \(Q \subseteq P^*\). This implies that \(\text{PNPInt}(P^*) \neq 0_R\). Let \(\text{PNPInt}(P^*) = S \neq 0_R\). Then we have \(\text{PNPCL}(\text{PNPInt}(P^*)) = \text{PNPCL}(S) \neq 1_R\) which implies that \(\text{PNPInt}(\text{PNPCL}(P)) \neq 0_R\). This is a contradiction. Hence \(\text{PNPInt}(\text{PNPCL}(f(P))) = 0_M\) in \((M, \sigma)\).

5.5 Theorem
Let \( (R, \tau) \) and \( (M, \sigma) \) be any two PNP topological spaces. If the function \( f: (R, \tau) \rightarrow (M, \sigma) \) is somewhat PNP open and \( \text{PNPInt}(P) = 0_R \) for any non-zero PNP set \( P \) in \( (M, \sigma) \), then \( \text{PNPInt}(f^{-1}(P)) = 0_R \) in \( (R, \tau) \).

**Proof**

Let \( P \) be a non-zero PNP set in \( (M, \sigma) \) such that \( \text{PNPInt}(P) = 0_R \). Now we claim that \( \text{PNPInt}(f^{-1}(P)) = 0_R \) in \( (R, \tau) \). Suppose that \( \text{PNPInt}(f^{-1}(P)) \neq 0_R \) in \( (R, \tau) \). Since \( f \) is somewhat PNP open, there exists a non-zero PNP open set \( Q \) in \( (R, \tau) \) such that \( Q \subseteq f^{-1}(P) \). Thus we have \( Q \subseteq f(f^{-1}(P)) \subseteq P \). This implies that \( f(Q) \subseteq P \).

5.6 Theorem

Let \( (R, \tau) \) and \( (M, \sigma) \) be any two PNP topological spaces Let \( (R, \tau) \) be a PNP open hereditarily irresolvable space. If the function \( f: (R, \tau) \rightarrow (M, \sigma) \) is somewhat PNP open, somewhat PNP continuous and a bijective function, then \( (M, \sigma) \) is a PNP open hereditarily irresolvable space.

**Proof**

Let \( P \) be a non-zero PNP set in \( (M, \sigma) \) such that \( \text{PNPInt}(P) = 0_R \). Now \( \text{PNPInt}(P) = 0_R \) and \( f \) is somewhat PNP open which implies \( \text{PNPInt}(f^{-1}(P)) = 0_R \) in \( (R, \tau) \) by theorem 5.5. Since \((R, \tau) \) is a PNP open hereditarily irresolvable, we have Suppose that \( \text{PNPInt}(\text{PNPCI}(f^{-1}(P))) = 0_R \) in \((R, \tau) \) by theorem 4.6. Since \( \text{PNPInt}(\text{PNPCI}(f^{-1}(P))) = 0_R \) and \( f \) is somewhat PNP continuous by theorem 5.4, we have that \( \text{PNPInt}(\text{PNPCI}(f^{-1}(P))) = 0_R \). Since \( f \) is onto, thus \( \text{PNPInt}(\text{PNPCI}(P)) = 0_R \). Hence, by theorem 4.6, \((M, \sigma) \) is a PNP open hereditarily irresolvable space.

5. Conclusion

In this paper we have proposed Pentapartitioned neutrosophic pythagorean resolvable and irresolvable spaces and studied some of its properties. Furthermore we also characterized Pentapartitioned Neutrosophic Pythagorean open hereditarily spaces and open functions in Pentapartitioned neutrosophic pythagorean topological spaces. In the future work, we extend the concept to Pentapartitioned Pythagorean almost resolvable and irresolvable spaces.

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**References**

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