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Pentapartitioned Neutrosophic Pythagorean Resolvable and Irresolvable Spaces

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Abstract: In this paper, the concepts of Pentapartitioned Neutrosophic Pythagorean resolvable , Pentapartitioned Neutrosophic Pythagorean irresolvable, Pentapartitioned Neutrosophic Pythagorean open hereditarily irresolvable and maximally Pentapartitioned Neutrosophic Pythagorean irresolvable spaces are introduced. Also we investigated several properties of the Pentapartitioned Neutrosophic Pythagorean open hereditarily irresolvable spaces besides giving characterization of these spaces by means of somewhat Pentapartitioned Neutrosophic Pythagorean continuous functions and somewhat Pentapartitioned Neutrosophic Pythagorean open functions.

Keywords: Pentapartitioned neutrosophic pythagorean resolvable, pentapartitioned neutrosophic pythagorean open hereditarily irresolvable, somewhat pentapartitioned neutrosophic pythagorean irresolvable, pentapartitioned neutrosophic pythagorean continuous and open functions.

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1. Introduction

Zadeh[16] introduced the important and useful concept of a fuzzy set which has invaded almost all branches of mathematics. The speculation of fuzzy topological space was studied and developed by C.L. Chang [4]. The paper of Chang sealed the approach for the following tremendous growth of the various fuzzy topological ideas. Since then a lot of attention has been paid to generalize the fundamental ideas of general topology in fuzzy setting and therefore a contemporary theory of fuzzy topology has been developed. Atanassov and plenty of researchers [1] worked on intuitionistic fuzzy sets within the literature. Florentin Smarandache [13] introduced the idea of Neutrosophic set in 1995 that provides the information of neutral thought by introducing the new issue referred to as uncertainty within the set. Thus neutrosophic set was framed and it includes the parts of truth membership function(T), indeterminacy membership function(I), and falsity membership function(F) severally. Neutrosophic sets deals with non normal interval of]−0 1+[. Pentapartitioned neutrosophic set and its properties were introduced by Rama Malik and Surpati Pramanik [12]. In this case, indeterminacy is divided into three components: contradiction, ignorance, and an unknown membership function. The concept of Pentapartitioned neutrosophic pythagorean sets was initiated by R. Radha and A. Stanis Arul Mary . The concept of neutrosophic fuzzy resolvable spaces and irresolvable spaces was introduced by M. Caldas et.al[3]. Now we extend the concepts to pentapartitioned neutrosophic pythagorean sets.

In this Paper we initiated the new concept of Pentapartitioned neutrosophic pythagorean resolvable, pentapartitioned neutrosophic pythagorean open hereditarily irresolvable, somewhat pentapartitioned neutrosophic pythagorean irresolvable, pentapartitioned neutrosophic pythagorean continuous and open functions and discussed some of its properties.

2. Preliminaries

2.1 Definition [13]

Let X be a universe. A Neutrosophic set A on X can be defined as follows:

$$A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\}$$

Where $T_A\ , I_A, F_A: U \to [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A\,(x) \leq 3$

Here, $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of inderminancy and $F_A(x)$ is the degree of non-membership.

2.2 Definition [7]

Let X be a universe. A Pentapartitioned neutrosophic pythagorean [PNP] set A with T, F, C and U as dependent neutrosophic components and I as independent component for A on X is an object of the form

$$A = \{< x, T_A, C_A, I_A, U_A, F_A >: x \in X\}$$

Where $T_A\ + F_A \leq 1, C_A\ + U_A \leq 1$ and

$$(T_A)^2 + (C_A)^2 + (I_A)^2 + (U_A)^2 + (F_A\,)^2 \leq 3$$

Here, $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership, $F_A(x)$ is the false membership and $I_A\,(x)$ is an unknown membership.

2.3 Definition [12]

Let P be a non-empty set. A Pentapartitioned neutrosophic set A over P characterizes each element p in P a truth -membership function T_A , a contradiction membership function C_A, an ignorance membership function G_A, unknown membership function U_A and a false membership function F_A , such that for each p in P

$T_A + C_A + G_A + U_A + F_A \leq 5.$

2.4 Definition [7]

The complement of a pentapartitioned neutrosophic pythagorean set A on R is denoted by A^C or A^* and is defined as

$A^C = \{< x, F_A(x), U_A\,(x), 1 - G_A(x), C_A(x), T_A(x) > : x \in X\}$

2.5 Definition [7]

Let A = $< x, T_A(x), C_A(x), G_A(x), U_A(x), F_A(x) >$ and B = $< x, T_B(x), C_B(x), G_B(x), U_B(x), F_B(x) >$ are pentapartitioned neutrosophic pythagorean sets. Then

A $\cup$ B = $< x, max(T_A(x), T_B(x)), max(C_A(x), C_B(x)), \min(G_A(x), G_B(x)), \min(U_A(x), U_B(x)), \min(F_A(x), F_B(x)), >$

A $\cap$ B = $< x, min(T_A(x), T_B(x)), min(C_A(x), C_B(x)), \max(G_A(x), G_B(x)), max(U_A(x), U_B(x)), max(F_A(x), F_B(x)) >$

2.6 Definition[7]

A PNP topology τ on a nonempty set R is a family of a PNP sets in R satisfying the following axioms

1) $0,1 \in \tau$
2) $R_1 \cap R_2 \in \tau$ for any $R_1, R_2 \in \tau$
3) $\cup R_i \in \tau$ for any $R_i : i \in I \subseteq \tau$

The complement R* of PNP open set (PNPOS, in short) in PNP topological space [PNPTS] (R,$\boldsymbol{\tau}$), is called a PNP closed set [PNPCS].

2.7 Definition [7]

Let (R,$\boldsymbol{\tau}$) be a PNPTS and L be a PNPTS in R. Then the PNP interior and PNP Closure of R denoted by

Cl(L) = $\cap\{\boldsymbol{K}$: K is a PNPCS in R and L$\subseteq$ K}.

Int(L) = $\cup$\{G: G is a PNPOS in R and G$\subseteq$ L}.

3. Pentapartitioned Neutrosophic Pythagorean Resolvable and Irresolvable Spaces

3.1 Definition

A Pentapartitioned neutrosophic pythagorean (PNP) set P in Pentapartitioned neutrosophic pythagorean topological space (PNPTS) (R, τ) is called pentapartitioned neutrosophic pythagorean dense if there exists no pentapartitioned neutrosophic pythagorean closed set Q in (R, τ) such that

$P \subset Q \subset 1_R$

Note: If P is a PNP open set , then the complement of PNP set P is a PNP closed set and it is denoted by P^*.

3.2 Example

Let R = { e, f} and define the pentapartitioned neutrosophic pythagorean set P as

$$P = \begin{cases} \{e, 0.4, 0.5, 0.7, 0.2, 0.3\} \\ \{f, 0.5, 0.3, 0.6, 0.1, 0.2\} \end{cases}$$

Then $\tau = \{0_R, 1_R, P\}$ is a pentapartitioned neutrosophic pythagorean topology on R. Hence P is a PNP dense set in (R, τ}.

3.3 Definition

A PNPTS (R, τ) is called PNP resolvable if there exists a PNP dense set P in (R, τ) such that PNPCl (P^*) = 1_R. Otherwise (R, τ) is called PNP irresolvable.

3.4 Example

Let R = { e, f} and define the pentapartitioned neutrosophic pythagorean set P, Q and R as

P = $\begin{cases}\{e, 0.3,0.4,0.3,0.3,0.1\} \\ \{f, 0.4,0.2,0.6,0.5,0.3\}\end{cases}$,

Q = $\begin{cases}\{e, 0.4,0.2,0.7,0.1,0.3\} \\ \{f, 0.6,0.1,0.3,0.2,0.2\}\end{cases}$ and

R = $\begin{cases}\{e, 0.1,0.2,0.4,0.3,0.4\} \\ \{f, 0.5,0.4,0.3,0.2,0.1\}\end{cases}$.

It can be seen that $\tau = \{0_R, 1_R, P\}$ is a pentapartitioned neutrosophic pythagorean topology on R. Then (R, τ) is a pentapartitioned neutrosophic topological space. Now PNPInt(Q) = 0_R, PNPInt(R) = 0_R, PNPCl (Q) = 1_R and PNPCl(R) = 1_R . Thus P and Q are PNP dense sets in (R, τ) such that PNPCl (Q^*) = 1_R and PNPCl(R^*) = 1_R. Hence the PNP topological space (R, τ) is PNP resolvable.

3.5 Example

Let R = { e, f} and define the pentapartitioned neutrosophic pythagorean set P, Q and R as

P = $\begin{cases}\{e, 0.2,0.3,0.5,0.4,0.5\} \\ \{f, 0.1,0.2,0.5,0.5,0.3\}\end{cases}$,

Q = $\begin{cases}\{e, 0.4,0.5,0.5,0.4,0.3\} \\ \{f, 0.5,0.4,0.4,0.3,0.2\}\end{cases}$ and

R = $\begin{cases}\{e, 0.3,0.4,0.4,0.3,0.2\} \\ \{f, 0.2,0.3,0.5 \ ,0.2,0.1\}\end{cases}$.

It can be seen that $\tau = \{0_R, 1_R, P\}$ is a pentapartitioned neutrosophic pythagorean topology on R. Then (R, τ) is a pentapartitioned neutrosophic topological space. Now PNPInt(Q) = A, PNPInt(R) = A, PNPCl (Q) = 1_R and PNPCl(R) = 1_R . Thus P and Q are PNP dense sets in (R, τ) such that PNPCl (Q^*) = P^* and PNPCl(R^*) =P^*. Hence the PNP topological space (R, τ) is PNP irresolvable.

3.6 Theorem

A PNPTS (R, τ) is a PNP resolvable space iff (R, τ) has a pair of PNP dense set K_1 and K_2 such that $K_1 \subseteq K_2^*$.

Proof

Let (R, τ) be a PNPTS and (R, τ) be PNP resolvable space. Suppose that for all PNP dense sets K_i and K_j, we have $K_i \nsubseteq K_j^*$. Then $K_i \supset K_j^*$. Then PNPCl(K_i) $\supset$ PNPCl(K_j^*) which implies that $1_R \supset$ PNPCl(K_j^*).Then PNPCl(K_j^*) $\neq 1_R$. Also $K_j \supset K_i^*$, then PNPCl(K_j) $\supset$ PNPCl(K_i^*) which implies that

$1_R \supset$ PNPCl(K_i^*). Therefore PNPCl (K_i^*) $\neq$ 1_R. Hence PNPCl(K_i^*) = 1_R, but PNPCl(K_i) $\neq$ 1_R for all PNP set K_i in (R, τ) which is a contradiction. Hence (R, τ) has a pair of PNP dense set K_1 and K_2 such that $K_1 \subseteq K_2^*$.

Conversely, suppose that the PNP topological space (R, τ) has a pair of PNP dense set K_1 and K_2 such that $K_1 \subseteq K_2^*$. Suppose that (R, τ) is a PNP irresolvable space, Then for all PNP dense sets K_1 and K_2 in (R, τ), we have PNPCl(K_1^*) $\neq 1_R$. Then PNPCl(K_2^*) $\neq 1_R$ implies that there exists a PNP closed set L in (R, τ) such that $K_2^* \subset$ L $\subset 1_R$. Then $K_1 \subset K_2^* \subset$ L $\subset 1_R$ implies that $K_1 \subset$ L $\subset 1_R$. But this is a contradiction. Hence (R, τ) is a PNP resolvable space.

3.7 Theorem

If (R, τ) is a PNP irresolvable space iff PNInt(P) $\neq$ 0 for all PNP dense set P in (R, τ).

Proof

Since (R, τ) is PNP irresolvable space for all PNP dense set P in (R, τ), PNPCl(P^*) $\neq 1_R$.Then (PNPInt(P)* $\neq 1_R$ which implies PNPInt(P) $\neq 0_R$.

Conversely PNPInt(P) $\neq 0_R$, for all PNP dense set P in (R, τ). Suppose that (R, τ) is PNP resolvable. Then there exists a PNP dense set P in (R, τ) such that PNPCl(P^*) = 1_R. This implies that (PNPInt(P)* = 1_R which again implies PNPInt(P) = 0_R. But this is a contradiction. Hence (R, τ) is PNP resolvable space.

3.8 Definition

A PNP topological space (R, τ) is called a PNP submaximal space if for each PNP set P in (R, τ), PNPCl(P) = 1_R.

3.9 Proposition

If the PNP topological space (R, τ) is PNP submaximal, then (R, τ) is PNP irresolvable.

Proof. Let (R, τ) be a PNP submaximal space. Assume that (R, τ) is a PNP resolvable space. Let P be a PNP dense set in (R, τ). Then PNPCl(P^*) = 1_R . Hence (PNPInt(P)* = 1_R which implies that PNPInt(P) = 0_R . Then P $\notin$ τ. This is a contradiction. Hence (R, τ) is PNP irresolvable space.

The converse of the above theorem is not true, which can be shown by the following example. See example 3.5.

3.10 Definition

A PNP topological space (R, τ) is called a maximal PNP irresolvable space if (R, τ) is PNP irresolvable and every PNP dense set P of (R, τ) is PNP open.

3.11 Example

Let R = { e, f} and define the pentapartitioned neutrosophic pythagorean set Q and R as

Q = $\begin{cases}\{e, 0.3,0.4,0.3,0.3,0.1\} \\ \{f, 0.4,0.2,0.6,0.5,0.3\}\end{cases}$ and

R = $\begin{cases}\{e, 0.1,0.2,0.4,0.3,0.4\}\\ \{f, 0.5,0.4,0.3,0.2,0.1\}\end{cases}$.

It can be seen that $\tau = \{0_R, 1_R, P\}$ is a pentapartitioned neutrosophic pythagorean topology on R. Then (R, τ) is a pentapartitioned neutrosophic topological space. Now PNPInt(Q^*) = 0_R, PNPInt(R^*) = 0_R, PNPCl (Q) = 1_R and PNPCl(R) = 1_R . Thus P and Q are PNP dense sets in (R, τ) such that PNPCl (Q^*) = Q^* and PNPCl(R^*) =R^*.Thus (R, τ) is PNP irresolvable and every PNP dense set of (R, τ) is PNP open. Therefore PNP topological space (R, τ) is maximally PNP irresolvable.

4 PNP open hereditarily Irresolvable space

4.1 Definition

A PNP topological space (R, τ) is said to be PNP open hereditarily irresolvable if PNPInt(PNPCl(P))≠ 0_R and PNPInt(P)≠ 0_R , for any PNP set P in (R, τ).

4.2 Example

Let R = { e, f} and define the pentapartitioned neutrosophic pythagorean set Q as

P = $\begin{cases}\{e, 0.2,0.1,0.5,0.4,0.5\}\\ \{f, 0.1,0.2,0.6,0.5,0.4\}\end{cases}$

It can be seen that $\tau = \{0_R, 1_R, P\}$ is a pentapartitioned neutrosophic pythagorean topology on R. Then (R, τ) is a pentapartitioned neutrosophic topological space. Now PNPInt(P) = P $\neq 0_R$ and PNPInt(PNPCl(P)) =PNPInt(P^*) = P $\neq 0_R$. Thus (R, τ) is PNP open hereditarily irresolvable space.

4.3 Theorem

Let (R, τ) be a PNP topological space. If (R, τ) is PNP open hereditarily irresolvable, then (R, τ) is PNP irresolvable.

Proof

Let P be a PNP dense set in (R, τ). Then PNPCl(P) = 1_R which implies that PNPInt(PNPCl(P) = 1_R $\neq 0_R$. Since (R, τ) is PNP open hereditarily irresolvable, we have PNPInt(P) ≠ 0_R. Therefore by theorem 3.7, PNPInt(P) ≠ 0_R for all PNP dense set in (R, τ) implies that (R, τ) is PNP irresolvable.

The converse of the above theorem is not true. See Example 4.4.

□

4.4 Example

Let R = { e, f} and define the pentapartitioned neutrosophic pythagorean set P, Q , R and S as

P = $\begin{cases}\{e, 0.1,0.5,0.5,0.2,0.6\}\\ \{f, 0.2,0.3,0.6,0.3,0.3\}'\end{cases}$

Q = $\begin{cases}\{e, 0.4,0.5,0.1,0.2,0.4\}\\ \{f, 0.3,0.2,0.7,0.2,0.1\}\end{cases}$

R = $\begin{cases} \{e, 0.4,0.5,0.1,0.2,0.4\} \\ \{f, 0.3,0.3,0.6 \ ,0.2,0.1\} \end{cases}$ and

S = $\begin{cases} \{e, 0.2,0.1,0.7,0.6,0.2\} \\ \{f, 0.1,0.2,0.6 \ ,0.5,0.4\} \end{cases}$.

It can be seen that $\tau = \{0_R, 1_R, P, Q, R\}$ is a pentapartitioned neutrosophic pythagorean topology on R. Then (R, τ) is a pentapartitioned neutrosophic topological space. Now PNPCl(P) = 1_R, PNPCl(Q) = 1_R, PNPCl(R) = 1_R and PNPCl (S) = 1_R . Thus P , Q,R and S are PNP dense sets in (R, τ) such that PNPCl (P^*) = P^* , PNPCl (Q^*) = Q^* and PNPCl(R^*) =R^* and PNPCl (S^*) = P^*. Hence the PNP topological space (R, τ) is PNP irresolvable. But PNPInt(PNPCl(S^*) = PNPInt(P^*) = 0_R. Therefore (R, τ) is not a PNP open hereditarily irresolvable space.

4.5 Theorem

Let (R, τ) be a PNP open hereditarily irresolvable. Then PNPInt(P) $\nsubseteq$ PNPInt(Q)* for any two PNP dense sets P and Q in (R, τ).

Proof.

Let P and Q be any two PNP dense sets in (R, τ). Then PNPCl(P) = 1_R and PNPCl(Q) =1_R implies that PNPInt(PNPCl(P))$\neq 0_R$ and PNPInt(PNPCl(Q)$\neq$ 0_R. Since (R, τ) is PNP open hereditarily irresolvable, PNPInt(P) $\neq 0_R$ and PNPInt(Q) $\neq$ 0_R. Hence by theorem 3.6, P $\nsubseteq Q^*$. Therefore PNPInt(P) $\subseteq P \nsubseteq Q^* \subseteq (PNPInt(Q))^*$. Hence we have PNPInt(P) $\subseteq (PNPInt(Q))^*$ for any two PNP dense sets P and Q in (R, τ).

4.6 Theorem

Let (R, τ) be a PNP topological space. If (R, τ) is PNP open hereditarily irresolvable, then PNPInt(P) = 0_R for any nonzero PNP dense set P in (R, τ) which implies that PNPInt(PNPCl(P)) = 0_R.

Proof:

Let P be a PNP set in (R, τ) such that PNPInt(P) = 0_R . We claim that PNPInt(PNPcl(P)) = 0_R. Suppose that PNPInt(PNPCl(P)) = 0_R. Since (R, τ) is PNP open hereditarily irresolvable, we have PNPInt(P) $\neq$ 0_R which is a contradiction to PNPInt(P) = 0_R. Hence PNPInt(PNPCl(P) = 0_R.

4.7 Theorem

Let (R, τ) be a PNP topological space. If (R, τ) is PNP open hereditarily irresolvable, then PNPCl(P) = 1_R for any nonzero PNP dense set P in (R, τ) which implies that PNPCl(PNPInt(P) = 0_R.

Proof

Let P be a PNP set in (R, τ) such that PNPCl(P) = 1_R.Then we have (PNPCl(P))* = 0_R which implies that PNPInt(P^*) = 0_R.Since (R, τ) is PNP open hereditarily irresolvable by theorem 4.6. We have PNPInt(PNPCl(P^*)) = 0_R. Therefore (PNPCl(PNPInt(P))* = 0_R implies that PNPCl(PNPInt(P)) = 1_R.

5 Somewhat PNP Continuous and PNP Somewhat PNP open

5.1 Definition

Let (R, τ) and (M, σ) be any two PNP topological spaces. A function f: (R, τ) → (M, σ) is called somewhat PNP continuous if for a P ∈ σ and $f^{-1}(P) \neq 0_R$, there exists a Q ∈ τ such that Q≠ 0_R and Q ⊆ $f^{-1}(P)$.

5.2 Definition

Let (R, τ) and (M, σ) be any two PNP topological spaces. A function f: (R, τ) → (M, σ) is called somewhat PNP open if for a P ∈ σ and $P \neq 0_R$, there exists a Q ∈ τ such that Q≠ 0_R and Q⊆ $f(P)$.

5.3 Theorem

Let (R, τ) and (M, σ) be any two PNP topological spaces. A function f: (R, τ) → (M, σ) is called somewhat PNP continuous and injective. If PNPInt(P) = 0_R for any non-zero PNP set P in (R, τ), then PNPInt(f(P)) = 0_M in (M, σ) .

Proof

Let P be a non-zero PNP set in (R, τ) such that PNPInt(P) = 0_R. Now we prove that PNPInt(f(P)) = 0_M . Suppose that PNPInt(f(P)) $\neq 0_M$ in (M, σ) . Then there exists a nonzero PNP set Q in (M, σ) such that Q ⊆ f(P). Thus , we have $f^{-1}(Q) \subseteq f^{-1}(f(P))$. Since f is somewhat PNP continuous, there exists a S ∈ τ such that S $\neq 0_R$ and S ⊆ $f^{-1}(Q)$. Hence S ⊆ $f^{-1}(Q) \subseteq P$ which implies that PNPInt(P) $\neq 0_R$ .This is a contradiction. Hence PNPInt(f(P)) = 0_M in (M, σ).

5.4 Theorem

Let (R, τ) and (M, σ) be any two PNP topological spaces. A function f: (R, τ) → (M, σ) is called somewhat PNP continuous , injective and PNPInt(PNPCl(P)) = 0_R for any non-zero PNP set P in (R, τ), then PNPInt(PNPCl(f(P))) = 0_M in (M, σ) .

Proof

Let P be a non-zero PNP set in (R, τ) such that PNPInt(PNPCl(P)) = 0_R. Now we claim that (PNPCl(f(P))) = 0_M . Suppose that PNPInt(PNPCl(f(P))) $\neq 0_M$ in (M, σ) .Then PNPCl(f(P) $\neq 0_M$ and PNPCl(f(P))* ≠ 0_M. Now PNPCl(f(P))* ≠ 0_M ∈ M. Since f is somewhat PNP continuous, there exists a Q ∈ τ such that Q $\neq 0_R$ and Q ⊆ f^{-1}((PNPCl(f(P)))*). Observe that Q ⊆ f^{-1}((PNPCl(f(P)))*) which implies that f^{-1}(PNPCl(f(P))) ⊆ Q^*.

Since f is injective, thus P ⊆ f^{-1}(f(P) ⊆ f^{-1}(PNPCl(f(P))) ⊆ Q^* which implies that P ⊆ Q^* . Therefore Q ⊆ P^* . This implies that PNPInt(P^*) ≠ 0_R.Let PNPInt(P^*) = $S \neq 0_R$. Then we have PNPCl(PNPInt(P^*)) = PNPCl(S) ≠ 1_R which implies that PNPInt(PNPCl(P)) ≠ 0_R .This is a contradiction. Hence PNPInt(PNPCl(f(P))) = 0_M in (M, σ).

5.5 Theorem

Let (R, τ) and (M, σ) be any two PNP topological spaces. If the function f: (R, τ) $\rightarrow (\mathrm{M}, \sigma)$ is somewhat PNP open and PNPInt((P)) = 0_R for any non-zero PNP set P in (M, σ), then PNPInt(f^{-1}(P)) = 0_R in (R, τ).

Proof

Let P be a non-zero PNP set in (M, σ) such that PNPInt(P) = 0_R. Now we claim that PNPInt(f^{-1}(P)) = 0_R in (R, τ). Suppose that (PNPInt(f^{-1}(P)) $\neq 0_R$ in (R, τ). Then there exists a non-zero PNP open set Q in (R, τ) such that Q $\subseteq f^{-1}(P)$. Thus we have f(Q) $\subseteq f\left(f^{-1}(P)\right) \subseteq P$.This implies that f(Q) $\subseteq P$. Since f is somewhat PNP open, there exists a S$\in \tau$ such that S $\neq 0_R$ and S $\subseteq f(Q)$. Therefore S $\subseteq f(Q) \subseteq P$ which implies that S $\subseteq A$. Hence PNPInt((P) $\neq 0_R$ which is a contradiction. Hence PNPInt(f^{-1}(P)) = 0_R in (R, τ).

5.6 Theorem

Let (R, τ) and (M, σ) be any two PNP topological spaces Let (R, τ) be a PNP open hereditarily irresolvable space. If the function f: (R, τ) $\rightarrow (\mathrm{M}, \sigma)$ is somewhat PNP open, somewhat PNP continuous and a bijective function, then (M, σ) is a PNP open hereditarily irresolvable space.

Proof

Let P be a non-zero PNP set in (M, σ) such that PNPInt(P) = 0_R. Now PNPInt(P) = 0_R and f is somewhat PNP open which implies PNPInt(f^{-1}(P)) = 0_R in (R, τ) by theorem 5.5. Since (R, τ) is a PNP open hereditarily irresolvable ,we have Suppose that PNPInt(PNPCl(f^{-1}(P))) = 0_R in (R, τ) by theorem 4.6.Since PNPInt(PNPCl(f^{-1}(P))) = 0_R and f is somewhat PNP continuous by theorem 5.4, we have that PNPInt(PNPCl(f(f^{-1}(P)))) = 0_R. Since f is onto, thus PNPInt(PNPCl(P) = 0_R. Hence, by theorem 4.6, (M, σ) is a PNP open hereditarily irresolvable space.

5. Conclusion

In this paper we have proposed Pentapartitioned neutrosophic pythagorean resolvable and irresolvable spaces and studied some of its properties. Furthermore we also characterized Pentapartitioned Neutrosophic Pythagorean open hereditarily spaces and open functions in Pentapartitioned neutrosophic pythagorean topological spaces. In the future work, we extend the concept to Pentapartitioned Pythagorean almost resolvable and irresolvable spaces.

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