



# Pentapartitioned Neutrosophic Pythagorean Strongly Irresolvable Spaces

R. Radha <sup>1,\*</sup>, A. Stanis Arul Mary <sup>2</sup> and Said Broumi <sup>3</sup>

<sup>1</sup> Research Scholar, Department of Mathematics, Nirmala College for Women, Coimbatore, India (TN);  
radharmat2020@gmail.com

<sup>2</sup> Assistant Professor, Department of Mathematics, Nirmala College for Women, Coimbatore, India (TN);  
stanisarulmary@gmail.com

<sup>3</sup> Laboratory of Information Processing, Faculty of Science, Ben M' Sik, University Hassan II, Casablanca, Morocco;  
broumisaid78@gmail.com

\* Correspondence: radharmat2020@gmail.com

**Abstract:** The aim of this paper is to develop many characterizations of Pentapartitioned Neutrosophic Pythagorean (PNP) strongly irresolvable spaces and its properties is also studied. Several characterizations of Pentapartitioned Neutrosophic Pythagorean strongly irresolvable spaces are investigated in this study. Also examined are the conditions under which Pentapartitioned Neutrosophic Pythagorean strongly irresolvable spaces become Pentapartitioned Neutrosophic Pythagorean first category spaces, Pentapartitioned Neutrosophic Pythagorean Baire spaces, and Pentapartitioned Baire spaces.

**Keywords:** Pentapartitioned neutrosophic pythagorean set, pentapartitioned neutrosophic pythagorean resolvable space, pentapartitioned neutrosophic pythagorean irresolvable spaces, pentapartitioned neutrosophic pythagorean strongly irresolvable spaces.

## 1. Introduction

Zadeh [17] proposed the fuzzy set concept in 1965 as a new technique to modelling uncertainties. Researches revealed the value of the fuzzy concept and have effectively used it to all fields of mathematics. Topology provides the most natural framework for fuzzy set theories to flourish in mathematics. Chang [3] first suggested the method of fuzzy topological space in 1968. Chang's paper established the stage for the tremendous growth of several fuzzy topological concepts that followed. Several mathematicians have continued to integrate all of the key notions of general topology to fuzzy circumstances, resulting in the development of a current theory of fuzzy topology. Today, fuzzy topology has been firmly established as one of the basic disciplines of fuzzy mathematics. Atanassov and plenty of researchers [1] worked on intuitionistic fuzzy sets within the literature. Florentin Smarandache [14] introduced the idea of Neutrosophic set in 1995 that provides the information of neutral thought by introducing the new issue referred to as uncertainty within the set. thus neutrosophic set was framed and it includes the parts of truth membership function(T), indeterminacy membership function(I), and falsity membership function(F) severally. Neutrosophic sets deals with non normal interval of ]-0 1+[. Pentapartitioned neutrosophic set and its properties were introduced by Rama Malik and Surpati Pramanik [13]. In this case, indeterminacy is divided into three components: contradiction, ignorance, and an unknown membership function. The concept of Pentapartitioned neutrosophic pythagorean sets was initiated by R. Radha and A. Stanis Arul Mary[7]. The concept of intuitionistic fuzzy almost resolvable spaces and irresolvable spaces was introduced by Sharmila s [15].R. Radha and A.Stanis Arul Mary introduced Pentapartitioned

neutrosophic pythagorean resolvable and irresolvable spaces. Also we have studied the concept of Pentapartitioned neutrosophic pythagorean almost resolvable and irresolvable spaces. Now we extend the concepts to pentapartitioned neutrosophic pythagorean strongly irresolvable spaces and studied relations with other Pentapartitioned neutrosophic pythagorean baire spaces, first category set, second category set and hyper connected spaces.

**2. Preliminaries**

**2.1 Definition [14]**

Let X be a universe. A Neutrosophic set A on X can be defined as follows:

$$A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X \}$$

Where  $T_A, I_A, F_A: U \rightarrow [0,1]$  and  $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

Here,  $T_A(x)$  is the degree of membership,  $I_A(x)$  is the degree of indeterminacy and  $F_A(x)$  is the degree of non-membership.

**2.2 Definition [7]**

Let X be a universe. A Pentapartitioned neutrosophic pythagorean [PNP] set A with T, F, C and U as dependent neutrosophic components and I as independent component for A on X is an object of the form

$$A = \{ \langle x, T_A, C_A, I_A, U_A, F_A \rangle : x \in X \}$$

Where  $T_A + F_A \leq 1, C_A + U_A \leq 1$  and

$$(T_A)^2 + (C_A)^2 + (I_A)^2 + (U_A)^2 + (F_A)^2 \leq 3$$

Here,  $T_A(x)$  is the truth membership,  $C_A(x)$  is contradiction membership,  $U_A(x)$  is ignorance membership,

$F_A(x)$  is the false membership and  $I_A(x)$  is an unknown membership.

**2.3 Definition [13]**

Let P be a non-empty set. A Pentapartitioned neutrosophic set A over P characterizes each element p in P a truth -membership function  $T_A$ , a contradiction membership function  $C_A$ , an ignorance membership function  $G_A$ , unknown membership function  $U_A$  and a false membership function  $F_A$ , such that for each p in P

$$T_A + C_A + G_A + U_A + F_A \leq 5$$

**2.4 Definition [7]**

The complement of a pentapartitioned neutrosophic pythagorean set A on R Denoted by  $A^c$  or  $A^*$  and is defined as

$$A^c = \{ \langle x, F_A(x), U_A(x), 1 - G_A(x), C_A(x), T_A(x) \rangle : x \in X \}$$

**2.5 Definition [7]**

Let  $A = \langle x, T_A(x), C_A(x), G_A(x), U_A(x), F_A(x) \rangle$  and  $B = \langle x, T_B(x), C_B(x), G_B(x), U_B(x), F_B(x) \rangle$  are pentapartitioned neutrosophic pythagorean sets. Then

$$A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(C_A(x), C_B(x)), \min(G_A(x), G_B(x)), \min(U_A(x), U_B(x)), \min(F_A(x), F_B(x)), \rangle$$

$$A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(C_A(x), C_B(x)), \max(G_A(x), G_B(x)) \rangle$$

$$, \max(U_A(x), U_B(x)), \max(F_A(x), F_B(x)) >$$

## 2.6 Definition [7]

A PNP topology on a nonempty set  $R$  is a family of a PNP sets in  $R$  satisfying the following axioms

- 1)  $0, 1 \in \tau$
- 2)  $R_1 \cap R_2 \in \tau$  for any  $R_1, R_2 \in \tau$
- 3)  $\cup R_i \in \tau$  for any  $R_i: i \in I \subseteq \tau$

The complement  $R^*$  of PNP open set (PNPOS, in short) in PNP topological space [PNPTS]  $(R, \tau)$ , is called a PNP closed set [PNPCS].

## 2.7 Definition [7]

Let  $(R, \tau)$  be a PNPTS and  $L$  be a PNPTS in  $R$ . Then the PNP interior and PNP Closure of  $R$  denoted by

$$Cl(L) = \cap \{K: K \text{ is a PNPCS in } R \text{ and } L \subseteq K\}.$$

$$Int(L) = \cup \{G: G \text{ is a PNPOS in } R \text{ and } G \subseteq L\}.$$

## 2.8 Definition [11]

Let  $(R, \tau)$  be a PNPTS and  $K$  be a PNP set in  $(R, \tau)$ . Then the PNP closure operator satisfy the following properties.

$$1-PNPCI(K) = PNPInt(1-K)$$

$$1-PNPInt(K) = PNPCI(1-K)$$

## 2.9 Definition [11]

A PNP  $A$  in PNPTS  $(R, \tau)$  is called PNP dense if there exists no PNPCS  $L$  in  $(R, \tau)$  such that  $K \subseteq L \subseteq 1$ . That is  $PNPCI(K) = 1$ .

## 2.10 Definition [11]

A PNP  $A$  in PNPTS  $(R, \tau)$  is called PNP nowhere dense if there exists no nonzero PNPOS  $L$  in  $(R, \tau)$  such that  $L \subseteq PNPCI(K)$ . That is  $PNPInt(PNPCI(K)) = 0$ .

## 2.11 Definition [11]

A PNPTS  $(R, \tau)$  is called PNP resolvable if there exists a PNP dense set  $K$  in  $(R, \tau)$  such that  $PNPCI(1-K) = 1$ . Otherwise  $(R, \tau)$  is called PNP irresolvable.

## 2.12 Definition [11]

A PNPTS  $(R, \tau)$  is called PNP submaximal if  $PNPCI(K) = 1$  for any non-zero PNP set  $K$  in  $(R, \tau)$ .

## 2.13 Definition [11]

A PNPTS  $(R, \tau)$  is called a PNP open hereditarily resolvable if  $PNPInt(PNPCI(K)) \neq \mathbf{0}$  for any PNP set  $K$  in  $(R, \tau)$ .

**2.14 Definition [11]**

PNPTS  $(R, \tau)$  is called PNP first category if  $\bigcup_{i=1}^{\infty} K_i$ , where  $K_i$ 's are PNP nowhere dense sets in  $(R, \tau)$ . A PNPTS which is not first category is said to be PNP second category.

**2.15 Definition [11]**

A PNPTS  $(R, \tau)$  is called a PNP baire space if  $\text{PNPInt}(\bigcup_{i=1}^{\infty} K_i) = \mathbf{0}$ , where  $K_i$ 's are PNP nowhere dense sets in  $(R, \tau)$ .

**2.16 Definition [12]**

A PNP  $K$  in a PNPTS  $(R, \tau)$  is called  $\text{PNPR}_1$  if  $K = \bigcap_{i=1}^{\infty} K_i$  where each  $K_i \in \tau$ .

**2.17 Definition [12]**

A PNP  $K$  in a PNPTS  $(R, \tau)$  is called  $\text{PNPR}_2$  if  $K = \bigcup_{i=1}^{\infty} K_i$  where each  $K_i \in \tau$ .

**2.18 Definition [12]**

A PNPTS  $(R, \tau)$  is called a PNP hyper-connected space if every PNP open set is PNP dense in  $(R, \tau)$ . That is  $\text{PNPCI}(K_i) = 1$  for all  $K_i \in \tau$ .

**2.19 Definition [12]**

A PNPTS  $(R, \tau)$  is called Pentapartitioned Neutrosophic Pythagorean nodec space, if every non-zero PNP nowhere dense set in  $(R, \tau)$  is PNP closed.

**3. Pentapartitioned Neutrosophic Pythagorean (PNP) Strongly Irresolvable Spaces****3.1 Definition**

A Pentapartitioned Neutrosophic Pythagorean topological space PNPTS  $(R, \tau)$  is called a Pentapartitioned Neutrosophic Pythagorean strongly irresolvable space if  $\text{PNPCI}(K) = 1$  for any non-zero Pentapartitioned neutrosophic pythagorean set  $K$  in  $(R, \tau)$  implies that  $\text{PNPCI}(\text{PNPInt}(K)) = 1$ .

**3.2 Example**

Let  $R = \{p\}$ . Let  $A$  and  $B$  be the PNP sets defined on  $R$  as follows.

$$A = \{p, 0.4, 0.3, 0.3, 0.5, 0.4\}$$

$$B = \{p, 0.5, 0.6, 0.5, 0.2, 0.3\}$$

Then, clearly  $\tau = \{0, A, 1\}$  is a PNP topology on  $R$ .

Then,  $\text{PNPCI}(A) = 1$  and  $\text{PNPCI}(\text{PNPInt}(A)) = 1$ ,

$\text{PNPCI}(B) = 1$  and  $\text{PNPCI}(\text{PNPInt}(B)) = 1$ .

Hence  $(R, \tau)$  is a PNP strongly irresolvable space.

**3.3 Theorem**

If  $(R, \tau)$  is a PNP strongly irresolvable space and if  $\text{PNPInt}(K) = \mathbf{0}$  for any non-zero PNP set  $K$  in  $(R, \tau)$ , then  $\text{PNPInt}(\text{PNPCI}(K)) = \mathbf{0}$ .

**Proof**

Let  $K$  be a non-zero PNP set in  $(R, \tau)$  such that  $\text{PNPInt}(K) = \mathbf{0}$ . Then  $1 - \text{PNPInt}(K) = 1$  which implies that  $\text{PNPCI}(1-K) = 1$ . Since  $(R, \tau)$  is PNP strongly irresolvable space, we have  $\text{PNPCI}(\text{PNPInt}(1-K)) = 1$  which implies that  $1 - \text{PNPInt}(\text{PNPCI}(K)) = 1$ . Therefore  $\text{PNPInt}(\text{PNPCI}(K)) = \mathbf{0}$ .

**3.4 Theorem**

If  $(R, \tau)$  is a PNP strongly irresolvable space and if  $\text{PNPInt}(\text{PNPCI}(K)) \neq 0$  for any non-zero PNP set  $K$  in  $(R, \tau)$  then  $\text{PNPInt}(K) \neq 0$ .

**Proof**

Let  $K$  be a non-zero PNP set in  $(R, \tau)$  such that  $\text{PNPInt}(\text{PNPCI}(K)) \neq 0$ . We claim that  $\text{PNPInt}(K) \neq 0$ . Suppose that  $\text{PNPInt}(K) = 0$ . Then  $1 - \text{PNPInt}(K) = 1$ . Now  $\text{PNPCI}(1 - K) = 1$ . Since  $(R, \tau)$  is a PNP strongly irresolvable space, we have  $\text{PNPCI}(\text{PNPInt}(1-K)) = 1$ . Hence  $1 - \text{PNPInt}(\text{PNPCI}(K)) = 1$  implies that  $\text{PNPInt}(\text{PNPCI}(K)) = 0$ , which is a contradiction. Hence we must have  $\text{PNPInt}(K) \neq 0$ .

**3.5 Theorem**

If  $(R, \tau)$  is a PNP strongly irresolvable space, then  $(R, \tau)$  is a PNP irresolvable space.

**Proof**

Let  $K$  be a non-zero PNP set in  $(R, \tau)$  such that  $\text{PNPCI}(K) = 1$ . We claim that  $\text{PNPInt}(K) \neq 0$ . Suppose that  $\text{PNPInt}(K) = 0$ , then  $1 - \text{PNPInt}(K) = 1$ , which implies that  $\text{PNPCI}(1-K) = 1$ . Then  $\text{PNPInt}(\text{PNPCI}(1-K)) = \text{PNPInt}(1) = 1$ . This implies that  $1 - \text{PNPCI}(\text{PNPInt}(K)) = 1$ . Then we have  $\text{PNPCI}(\text{PNPInt}(K)) = 0$  which is a contradiction to  $(R, \tau)$  is a PNP strongly irresolvable spaces. Hence our assumption  $\text{PNPInt}(K) = 0$  is wrong. Hence we must have  $\text{PNPInt}(K) \neq 0$  for all PNP dense sets  $K$  in  $(R, \tau)$ . Therefore  $(R, \tau)$  must be a PNP irresolvable space.

**3.6 Theorem**

If  $(R, \tau)$  is a PNP strongly irresolvable space, then  $\text{PNPInt}(K_1) \subseteq 1 - \text{PNPInt}(K_2)$  for any two dense sets  $K_1, K_2$  in  $(R, \tau)$ .

**Proof**

Let  $K_1$  and  $K_2$  be any two non-zero PNP dense sets in  $(R, \tau)$ . Then  $\text{PNPCI}(K_1) = 1$  and  $\text{PNPCI}(K_2) = 1$  which implies that  $\text{PNPInt}(\text{PNPCI}(K_1)) \neq 0$  and  $\text{PNPInt}(\text{PNPCI}(K_2)) \neq 0$ . Since  $(R, \tau)$  is a PNP strongly irresolvable space, by theorem 3.4, we have  $\text{PNPInt}(K_1) \neq 0$  and  $\text{PNPInt}(K_2) \neq 0$ . By theorem 3.5,  $(R, \tau)$  is a PNP irresolvable space, But  $(R, \tau)$  is PNP irresolvable if has a pair of dense sets,  $K_1$  &  $K_2$   $\exists K_1 \subseteq K_2$ . Now  $\text{PNPInt}(K_1) \subseteq K_1 \subseteq 1 - K_2 \subseteq 1 - \text{PNPInt}(K_2)$ . Therefore we have  $\text{PNPInt}(K_1) \subseteq 1 - \text{PNPInt}(K_2)$  for any two PNP dense sets  $K_1, K_2$  in  $(R, \tau)$ ,

**3.7 Theorem**

If a PNPTS  $(R, \tau)$  is a PNP submaximal space, then  $(R, \tau)$  is a PNP strongly irresolvable space.

**Proof**

Let  $(R, \tau)$  be a PNP submaximal space and  $K$  be a PNP dense set in  $(R, \tau)$ . Since  $K$  is a PNP dense set in  $(R, \tau)$ ,  $\text{PNPCI}(K) = 1$ , which implies  $\text{PNPInt}(1-K) = 1-1 = 0$ . Therefore  $\text{PNPCI}(\text{PNPInt}(1-K)) = 0$ . That is  $1 - \text{PNPCI}(\text{PNPInt}(K)) = 1$ , which implies  $1 - \text{PNPInt}(\text{PNPCI}(1-K)) = 1$ . Hence  $\text{PNPCI}(\text{PNPInt}(K)) = 1$ . Therefore  $(R, \tau)$  is a strongly irresolvable space

**3.8 Theorem**

If  $K$  is a PNP nowhere dense set in a PNP topological space  $(R, \tau)$ , then  $(1 - K)$  is a PNP dense set in  $(R, \tau)$ .

**Proof**

Let  $K$  be a PNP nowhere dense set in  $(R, \tau)$ . Then we have  $\text{PNPInt}(\text{PNPCI}(K)) = 0$ . Now  $1 - \text{PNPInt}(\text{PNPCI}(K)) = 1 - 0 = 1$ . Then  $\text{PNPCI}(1 - \text{PNPInt}(\text{PNPCI}(K))) = 1$ , which implies that  $\text{PNPCI}(1 - \text{PNPInt}(1-K)) = 1$ . But  $\text{PNPCI}(1 - \text{PNPInt}(1-K)) \subseteq \text{PNPCI}(\text{PNPCI}(1-K))$ . Hence  $1 \subseteq \text{PNPCI}(\text{PNPCI}(1-K))$ . Therefore  $\text{PNPCI}(1 - \text{PNPInt}(1-K)) = 1$ . Also  $1 - \text{PNPInt}(\text{PNPCI}(K)) = 1 - 0 = 1$ . Then we have  $\text{PNPCI}(1 - \text{PNPInt}(K)) = 1$ , which implies that  $\text{PNPCI}(\text{PNPInt}(1-K)) = 1$ . But  $\text{PNPCI}(\text{PNPInt}(1-K)) \subseteq \text{PNPCI}(\text{PNPCI}(1-K))$ . Hence  $1 \subseteq \text{PNPCI}(\text{PNPCI}(1-K))$ . That is  $\text{PNPCI}(\text{PNPCI}(1-K)) = 1$ . Therefore  $1 - K$  is a PNP dense set in  $(R, \tau)$ .

**3.9 Theorem**

If a PNPTS  $(R, \tau)$  is a PNP submaximal space, then  $(R, \tau)$  is a PNP nodec space.

**Proof**

Let  $(R, \tau)$  be a PNP submaximal space and  $K$  be a PNP nowhere dense set in  $(R, \tau)$ . Then by theorem 3.8,  $1-K$  is a PNP dense set in  $(R, \tau)$ . Since  $(R, \tau)$  is a PNP submaximal space,  $1-K$  is a PNP open set in  $(R, \tau)$ . This implies that  $K$  is a PNP closed set in  $(R, \tau)$ . Hence each PNP nowhere dense set is a PNP closed set in  $(R, \tau)$  and therefore  $(R, \tau)$  is a PNP nodec space.

**3.10 Theorem**

If  $(R, \tau)$  is a PNP strongly irresolvable then  $(R, \tau)$  is a PNP Baire space if and only if  $\text{PNPCL}(\bigcap_{i=1}^{\infty} K_i) = 1$ .

**Proof**

Let  $(R, \tau)$  be a PNP strongly irresolvable space. Suppose that  $K_i$ 's are PNP dense set in  $(R, \tau)$ , then  $\text{PNPCL}(\text{PNPInt}(K_i)) = 1$ . Now  $1 - \text{PNPCL}(\text{PNPInt}(K_i)) = 1-1 = 0$ . Then we have  $\text{PNPInt}(\text{PNPCL}(1-K_i)) = 0$ . Hence  $(1-K_i)$ 's are PNP nowhere dense sets in  $(R, \tau)$ . Now  $\text{PNPCL}(\bigcap_{i=1}^{\infty} K_i) = 1$  implies that  $1 - \text{PNPCL}(\bigcap_{i=1}^{\infty} K_i) = 0$  and hence  $\text{PNPInt}(1 - (\bigcap_{i=1}^{\infty} K_i)) = 0$  and hence  $\text{PNPInt}(\bigcup_{i=1}^{\infty} (1-K_i)) = 0$ , where  $(1-K_i)$ 's are PNP nowhere dense sets in  $(R, \tau)$  and therefore  $(R, \tau)$  is a PNP baire space.

Conversely, Let  $K_i$ 's be PNP nowhere dense sets in a PNP strongly irresolvable space and PNP baire space  $(R, \tau)$ . Since  $(R, \tau)$  is a PNP baire space,  $\text{PNPInt}(\bigcup_{i=1}^{\infty} K_i) = 0$ . Then  $1 - \text{PNPInt}(\bigcup_{i=1}^{\infty} K_i) = 1$ . This implies that

$$\text{PNPCL}(\bigcap_{i=1}^{\infty} (1 - K_i)) = 1 \tag{1}$$

Since  $K_i$ 's be PNP nowhere dense sets in a PNP strongly irresolvable space then by theorem 3.8,  $(1-K_i)$ 's are PNP dense sets in . Let  $B_i = 1-K_i$ . Then from(1),  $\text{PNPCL}(\bigcap_{i=1}^{\infty} K_i) = 1$ , where  $K_i$ 's are PNP nowhere dense sets in  $(R, \tau)$ .

**3.11 Theorem**

If  $(R, \tau)$  is a PNP strongly irresolvable and  $K = \bigcap_{i=1}^{\infty} K_i$  be a PNP dense set in  $(R, \tau)$ . Then  $1 - K$  is a PNP first category set in  $(R, \tau)$ ,

**Proof**

Let  $K =$  be a PNP dense set in  $(R, \tau)$ . Then  $\text{PNPCL}(\bigcap_{i=1}^{\infty} K_i) = 1$ . But  $\text{PNPCL}(\bigcap_{i=1}^{\infty} K_i) \subseteq \bigcap_{i=1}^{\infty} \text{PNPCL}(K_i)$ . Thus  $1 \subseteq \text{PNPCL}(\bigcap_{i=1}^{\infty} K_i) \subseteq \bigcap_{i=1}^{\infty} \text{PNPCL}(K_i)$  . Then  $\bigcap_{i=1}^{\infty} (\text{PNPCL}(K_i)) = 1$ . This implies that  $\text{PNPCL}(K_i) = 1$ . Thus  $K_i$ 's are PNP dense set in  $(R, \tau)$ . Since  $(R, \tau)$  is PNP strongly irresolvable, by theorem 3.8,  $(1- K_i)$ 's are PNP dense sets in  $(R, \tau)$ . Therefore , we have  $1 - K = \bigcup_{i=1}^{\infty} (1-K_i)$ , where  $(1-K_i)$  's are PNP nowhere dense sets. Hence  $1 - K$  is a PNP first category set in  $(R, \tau)$ .

**3.12 Theorem**

If is a PNP strongly irresolvable space and  $K = \bigcap_{i=1}^{\infty} K_i$  be a PNP dense set in  $(R, \tau)$ . Then  $K$  is a PNP residual set in  $(R, \tau)$ .

**Proof**

Let  $K = \bigcap_{i=1}^{\infty} K_i$  be a PNP dense set in  $(R, \tau)$ . Since  $(R, \tau)$  is a PNP strongly irresolvable space, by theorem 3.11,  $1 - K$  is a PNP first category set in  $(R, \tau)$ . Therefore  $K$  is a PNP residual set in  $(R, \tau)$ .

**3.13 Theorem**

Let  $(R, \tau)$  be a PNP strongly irresolvable space. If  $K$  is a PNP dense set in  $(R, \tau)$ , then  $1 - K$  is a PNP nowhere dense set.

**Proof**

Let  $K$  be a PNP dense set in  $(R, \tau)$ . Since  $(R, \tau)$  is a PNP strongly irresolvable space,  $\text{PNPCL}(\text{PNPInt}(K)) = 1$ . This implies that  $1 - \text{PNPCL}(\text{PNPInt}(K)) = 0$ . Therefore  $\text{PNPInt}(\text{PNPCL}(1-K)) = 0$  and hence  $1 - K$  is a PNP nowhere dense set in  $(R, \tau)$ .

**3.14 Theorem**

If  $(R, \tau)$  is a PNP strongly irresolvable and PNP nodec space, then  $(R, \tau)$  is a PNP submaximal space,

**Proof**

Let  $(R, \tau)$  be a PNP strongly and PNP nodec space. Let  $K$  be a PNP dense set in  $(R, \tau)$ . Since  $(R, \tau)$  is a PNP strongly irresolvable, by theorem 3.13,  $1-K$  is a PNP nowhere dense set in  $(R, \tau)$ . Since  $(R, \tau)$  is a PNP nodec space,  $1-K$  is a PNP closed set in  $(R, \tau)$ . Then  $K$  is a PNP open set in  $(R, \tau)$ . Hence every PNP dense set is PNP open set in  $(R, \tau)$ . Therefore  $(R, \tau)$  is a PNP submaximal space.

### 3.15 Theorem

If  $(R, \tau)$  is a PNP strongly irresolvable and PNP second category space, then  $\bigcap_{i=1}^{\infty} K_i \neq \emptyset$  where  $K_i$ 's are PNP dense sets in  $(R, \tau)$ .

#### Proof

Let  $(R, \tau)$  be a PNP second category space. Let us assume that  $\bigcap_{i=1}^{\infty} K_i = \emptyset$ . Since  $K_i$ 's are PNP dense sets in  $(R, \tau)$ , by theorem 3.12,  $(1-K_i)$ 's are PNP nowhere dense sets in  $(R, \tau)$ . Now  $1 - \bigcap_{i=1}^{\infty} K_i = 1$ , implies that  $\bigcup_{i=1}^{\infty} (1 - K_i)$  and  $(1 - K_i)$ 's are PNP nowhere dense sets in  $(R, \tau)$ . Hence  $(R, \tau)$  is a PNP first category space, which is a contradiction. Therefore  $\bigcap_{i=1}^{\infty} K_i \neq \emptyset$ , where  $K_i$ 's are PNP dense sets in  $(R, \tau)$ .

### 3.16 Theorem

If  $(R, \tau)$  is a PNP submaximal space and  $K$  is a PNP first category set, then  $1 - K$  is a  $PNPR_2$  set in  $(R, \tau)$ .

#### Proof

Let  $K$  be a PNP first category set in  $(R, \tau)$ . Then  $K = \bigcup_{i=1}^{\infty} K_i$ , where  $K_i$ 's are PNP nowhere dense sets in  $(R, \tau)$ . Therefore, by theorem 3.8,  $(1-K_i)$ 's are PNP dense sets in  $(R, \tau)$ . Since  $(R, \tau)$  is a PNP submaximal space,  $(1-K_i)$ 's are open set in  $(R, \tau)$ . Also  $1-K = 1-(\bigcup_{i=1}^{\infty} K_i) = \bigcap_{i=1}^{\infty} (1-K_i)$ , where  $(1-K_i)$ 's are PNP open sets in  $(R, \tau)$ . Therefore  $1-K$  is a  $PNPR_2$  set in  $(R, \tau)$ .

### 3.17 Theorem

If  $(R, \tau)$  is a PNP submaximal space, then every PNP first category set is a  $PNPR_1$  set in  $(R, \tau)$ .

#### Proof

Let  $K$  be a PNP first category set in  $(R, \tau)$ . Since  $(R, \tau)$  is a PNP submaximal space, by theorem 3.16,  $1-K$  is a  $PNPR_2$  set in  $(R, \tau)$  and hence  $K$  is a  $PNPR_1$  set in  $(R, \tau)$ .

### 3.18 Theorem

If  $(R, \tau)$  is a PNP submaximal space, then every PNP residual set is a  $PNPR_1$  set in  $(R, \tau)$ .

#### Proof

Let  $K$  be a PNP residual set in  $(R, \tau)$ . Then  $1-K$  is a PNP first category set in **3.17 Theorem**

If  $(R, \tau)$  is a PNP submaximal space, then every PNP first category set is a  $PNPR_1$  set in  $(R, \tau)$ .

#### Proof

Let  $K$  be a PNP first category set in  $(R, \tau)$ . Since  $(R, \tau)$  is a PNP submaximal space, by theorem 3.16,  $1-K$  is a  $PNPR_2$  set in  $(R, \tau)$  and hence  $K$  is a  $PNPR_1$  set in  $(R, \tau)$ . Since  $(R, \tau)$  is a PNP submaximal space, by theorem 3.17,  $1-K$  is a  $PNPR_1$  set in  $(R, \tau)$  and hence  $K$  is a  $PNPR_2$  set in  $(R, \tau)$ .

## 5. Conclusion

In this paper, it is established that in PNP strongly irresolvable spaces, the condition under which PNP topological spaces become PNP strongly irresolvable spaces is obtained by means of the PNP denseness of PNP open sets. It is proved that PNP first category sets are PNP closed sets in a PNP Baire, PNP nodec and PNP strongly irresolvable spaces. It is established that PNP resolvable and PNP irresolvable spaces are not PNP strongly irresolvable spaces. The conditions under which PNP strongly irresolvable spaces become PNP Baire spaces are also obtained. In future study, we can study about filters and ultra filters in PNP irresolvable space.

**Funding:** "This research received no external funding" .

**Acknowledgments:** I specially thank S. P. Rhea and R. Kathiresan for their endless support and constant guidance throughout this paper.

**Conflicts of Interest:** "The authors declare no conflict of interest."

## References

1. K. Atanassov, Intuitionistic Fuzzy Sets, *Fuzzy Sets and Systems*.1986,volume 20 87-96.
2. Broumi S, Smarandache F (2014) Rough Neutrosophic sets, *Ital J Pure Appl Math* ,2014, volume 32:493-502.
3. C.L. Chang Fuzzy topological spaces, *J. Math. Anal. Appl*, 1984, volume 24, 182-190.
4. D.H. Hong, Fuzzy measures for a correlation coefficient of fuzzy numbers under Tw (the weakest tnorm)-based fuzzy arithmetic operations, *Information Sciences* 2006 volume 176,150-160.
5. Rajashi Chatterjee, P. Majumdar and S. K. Samanta, On some similarity measures and entropy on quadripartitioned single valued neutrosophic sets, *Journal of Intelligent and Fuzzy Systems* ,2016, volume 302475-2485.
6. R. Radha, A. Stanis Arul Mary. Pentapartitioned Neutrosophic pythagorean Soft set, *IRJMETS*, 2021 , Volume 3(2),905-914.
7. R. Radha, A. Stanis Arul Mary. Pentapartitioned Neutrosophic Pythagorean Set, *IRJASH*, 2021, volume 3, 62-82.
8. R. Radha, A. Stanis Arul Mary. Heptapartitioned neutrosophic sets, *IRJCT*, 2021 ,volume 2,222-230.
9. R. Radha, A. Stanis Arul Mary, F. Smarandache. Quadripartitioned Neutrosophic Pythagorean soft set, *International journal of Neutrosophic Science*, 2021, volume14(1),9-23.
10. R. Radha, A. Stanis Arul Mary, F. Smarandache. Neutrosophic Pythagorean soft set, *Neutrosophic sets and systems*, 2021,vol 42,65-78.
11. R. Radha ,A.Stanis Arul Mary, Pentapartitioned neutrosophic pythagorean resolvable and irresolvable spaces(Communicated)
12. R. Radha, A.Stanis Arul Mary , Pentapartitioned neutrosophic pythagorean almost resolvable and irresolvable spaces(Communicated)
13. Rama Malik, Surapati Pramanik. Pentapartitioned Neutrosophic set and its properties, *Neutrosophic Sets and Systems*, 2020, Vol 36,184-192,2020
14. F.Smarandache, A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic, *American Research Press, Rehoboth*.
15. S. Sharmila, I. Arockiarani, On Intuitionistic Fuzzy almost resolvable and irresolvable spaces, *Asian Journal of Applied Sciences*,2015, Volume 3 Issue 6,918-928
16. Wang H, Smarandache F, Zhang YQ, Sunderraman R ,Single valued neutrosophic sets, *Multispace Multistruct* ,2010 ,volume 4:410-413.
17. L. Zadeh , Fuzzy sets, *Information and Control* 1965, volume 8, 87-96.

Received: Dec. 5, 2021. Accepted: April 3, 2022.