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P and R Order of Plithogenic Neutrosophic Cubic sets

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Abstract: The paper presents a new concept called *P*-Order (Union and Intersection) and *R*- Order (Union and Intersection) of the Plithogenic Neutrosophic Cubic Sets (PNCS). We derived some of the primary properties of the internal and external PNCS of *P* and *R*- Order. We also proved that *P*-Union and *P*- intersection of Truth (T) (resp. falsity (F), indeterminacy(I)) external PNCS may not be T (resp. F, I) external PNCS and R-Union and R-intersection of T (resp. F, I) internal PNCS may not be T (resp. F, I) internal PNCS with the numerical examples. This principle is extremely appropriate for analyzing problems that involve multi-attribute decision making since this PNCS is defined by many values of attribute and the reliability of the data is also so accurate.

Keywords: Cubic set, Neutrosophic set, Plithogenic set, Plithogenic neutrosophic cubic set, internal and external plithogenic neutrosophic cubic set

1. Introduction

The definition of fuzzy sets, proposed by Zadeh [19] in 1965, paved the way for fuzzy mathematics development. In many areas of mathematics, this concept has a wide range of applications, such as reasoning, set theory, number theory, fuzzy set theory, real analysis, metric theory, and topology. In 1975, Zadeh [19] developed the concepts of interval-valued fuzzy sets as an extension of fuzzy sets, that is, fuzzy sets with interval-valued membership functions.

A notable concept has been developed by Jun et al. [3], namely the cubic sets theory. An interval-valued fuzzy set and a fuzzy set constitute this structure. In addition, the definition of an internal cubic set and an external cubic set was also implemented by Jun et al. [3].

Smarandache [9, 10] proposed Neutrosophic sets (NSs), a generalisation of FS and IFS, which is highly helpful for dealing with inadequate, uncertain, and varying data that exists in the real life. NSs are characterised by functions of truth (T), indeterminacy (I) and falsity (F) belonging functions. This concept is very essential in several areas of application since indeterminacy is clearly enumerated and the truth, indeterminacy and falsity membership functions are independent.

Wang, Smarandache, Zhang and Sunderraman[18] anticipated the definition of an interval valued neutrosophic set (IVNS) as an extension of NS. The IVNS could reflect indeterminate, inaccurate, inadequate and unreliable data that occurs in the reality.

Plithogeny is the foundation, establishment, construction and development of new articles from the combination of consistent or inconsistent multiple old articles. Smarandache[13] introduced the plithogenic set as a generalisation of neutrosophy in 2017.

The elements of Plithogenic sets are denoted by one or many number of attributes and each of it have several values. Each values of attribute has its respective (fuzzy, intuitionistic fuzzy or neutrosophic) appurtenance degree for the component x (say) to the plithogenic set P (say) with respect to certain constraints. For the first time, Smarandache[12] introduced the inconsistency degree between each value of attribute and the dominant value of attribute which results in getting the better accuracy for the plithogenic aggregation operators(fuzzy, intuitionistic fuzzy or neutrosophic).

Priyadharshini et al [8] introduced the new concept called plithogenic cubic sets which has a wide range of application in multi criteria decision making problems. In particular, the ideology of Plithogenic neutrosophic cubic set helped in this paper to learn the P, R-union and the P, R-intersection of PNCS and derived some of its core properties.

We proved that the P-union and the P-intersection of T (resp. F, I) internal PNCS are also T (resp. F, I) internal PNCS. We provide examples to show that the P-union and the P-intersection of T (resp. F, I) external PNCS may not be T (resp. F, I) external PNCS, and the R-union and the R intersection of T (resp. F, I) internal PNCS may not be T (resp. F, I) internal PNCS. The conditions for the R-union and R-intersection of two T (resp. F, I) internal PNCS to be a T (resp. F, I) internal PNCS.

We believe that the suggested theorems and examples will be effective in resolving multi-attribute group decision-making concerns in a Plithogenic neutrosophic cubic set environment.

The rest of the article is as follows. Section 2 is concerned with the preliminary concepts and definitions that are absolutely vital for the proposed work. Section 3 provides a brief description of the P and R order of Plithogenic neutrosophic Cubic sets along with numerical examples. Section 4 summarizes the conclusion and the scope of future work.

2. Preliminaries

Definition 2.1 [9, 10] Let N be a non-void set. The set $A=\{\langle n,\lambda_A,\phi_A,\gamma_A\rangle n\in N\}$ is called a neutrosophic set (in short, NS) of N where the function $\lambda_A : N \to [0,1]$, $\phi_A : N \to [0,1]$ and $\gamma_A : N \to [0,1]$ denotes the membership degree (say $\lambda_A(n)$), indeterminacy degree (say $\phi_A(n)$), and non-membership degree (say $\gamma_A(n)$)of each element $n \in N$ to the set A and satisfies the constraint that $0 \le \lambda_A(n)+\phi_A(n)+\gamma_A(n) \le 3$.

Definition 2.2 [11] Let R be a non-void set. An interval valued neutrosophic set (INS) A in R is described by the functions of the truth-value (A_T) , the indeterminacy (A_I) and the falsity-value (A_F) for each point $r \in R$, $A_T(r), A_I(r), A_F(r) \subseteq [0,1]$.

Definition 2.3 [3] Let E be a non-void set. By a cubic set in E, we construct a set which has the form $\Psi = \{< e, B(e), \mu(e) > | e \in E\}$ in which B is an interval valued fuzzy set (IVFS) in E and μ is a fuzzy set in E.

Definition 2.4 [3] Let E be a non-void set. If $B^{-}(e) \leq \mu(e) \leq B^{+}(e)$ for all $e \in E$ then the cubic set $\Psi =< B, \mu >$ in E is called an internal cubic set (briefly IPS).

Definition 2.5 [3] Let E be a non-void set. If $\mu(e) \notin (B^{-}(e), B^{+}(e))$ for all $e \in E$ then the cubic set $\Psi =< B, \mu >$ in E is called an external cubic set (briefly ECS).

Definition 2.6 [8] Let Ω be an universal set and Y be a non-void set. The structure $\Lambda =\{< y, B(y), \lambda(y) > | y \in Y\}$ is said to be Plithogenic Neutrosophic cubic set (PNCS) in Y, where $B = \left\{\left(B_{d_i}{}^{T}(y), B_{d_i}{}^{I}(y), B_{d_i}{}^{F}(y)\right)\right\}$ is an interval valued Plithogenic Neutrosophic set in Y and $\lambda = \{(\lambda_i{}^{T}(y), \lambda_i{}^{I}(y), \lambda_i{}^{F}(y)\}$ is a neutrosophic set in Y.

The pair $\Lambda =< B, \lambda >$ is called plithogenic neutrosophic cubic set over Ω where Λ is a mapping given by $\Lambda : B \to NC(\Omega)$ .The set of all plithogenic neutrosophic cubic sets(PNCS) over Ω will be denoted by $P_N{}^{\Omega}$

Definition 2.7 [8] For a non-void set Y, the plithogenic neutrosophic cubic set $\Lambda =< B, \lambda >$ in Y is called truth internal, indeterminacy internal, falsity internal respectively if the following equations hold

(i) $B_{d_i}{}^{-T}(y) \leq \lambda_i{}^{T}(y) \leq B_{d_i}{}^{+T}(y)$ (3.1)

(ii) $B_{d_i}{}^{-I}(y) \leq \lambda_i{}^{I}(y) \leq B_{d_i}{}^{+I}(y)$ (3.2)

(iii) $B_{d_i}{}^{-F}(y) \leq \lambda_i{}^{F}(y) \leq B_{d_i}{}^{+F}(y)$ (3.3)

Where for all $y \in Y$ and d_i represents the dissimilarity measure and their respective value of attributes.

If a PNCS in Y satisfies the above equations we conclude that Λ is an internal plithogenic neutrosophic cubic set (IPNCS) in Y.

Definition 2.8 [8] For a non-void set Y, the PNCS $\Lambda =< B, \lambda >$ in Y is called truth external, indeterminacy external, falsity external respectively if the following equations hold

(i) $\lambda_i{}^{T}(y) \notin (B_{d_i}{}^{-T}(y), B_{d_i}{}^{+T}(y))$ (3.4)

(ii) $\lambda_i^{I}(y) \notin (B_{d_i}^{-I}(y), B_{d_i}^{+I}(y))$ (3.5)

(iii) $\lambda_i^{F}(y) \notin (B_{d_i}^{-F}(y), B_{d_i}^{+F}(y))$ (3.6)

Where for all $y \in Y$ and d_i represents the dissimilarity measure and their respective value of attributes.

If a PNCS in Y satisfies the above equations, we conclude that Λ is an external plithogenic neutrosophic cubic set (EPNCS) in Y.

3. P and R Order of PNCS

Definition 3.1 Let $C = <\mathrm{K}, \Phi>$ and $D = <\mathrm{M}, \Omega>$ be a PNCS in a non-void set H where

$$\mathrm{K} = \{\langle h : K_{d_j}^{T}(h), K_{d_j}^{I}(h), K_{d_j}^{F}(h)\rangle \mid h \in H\},$$
$$\Phi = \{\langle h : \phi_{d_j}^{T}(h), \phi_{d_j}^{I}(h), \phi_{d_j}^{F}(h)\rangle \mid h \in H\},$$
$$\mathrm{M} = \{\langle h : \mathrm{M}_{d_j}^{T}(h), \mathrm{M}_{d_j}^{I}(h), \mathrm{M}_{d_j}^{F}(h)\rangle \mid h \in H\},$$
$$\Omega = \{\langle h : \delta_{d_j}^{T}(h), \delta_{d_j}^{I}(h), \delta_{d_j}^{F}(h)\rangle \mid h \in H\}.$$

Then we define the equality, P-order and R-order as follows:

(i) $C = D \Leftrightarrow K = M \text{ and } \Phi = \Omega.$ (Equality)

(ii) $C \subseteq_P D \Leftrightarrow K \subseteq M \text{ and } \Phi \le \Omega.$ (*P*-Order)

(iii) $C \subseteq_R D \Leftrightarrow K \subseteq M \text{ and } \Phi \ge \Omega.$ (*R*-Order)

The *P*-Union, *P*-Intersection, *R*-Union, *R*-Intersection of PNCS are described as follows.

Definition 3.2 For any PNCS $C_{d_j} = <\mathrm{K}_{d_j}, \Phi_{d_j}>$ in a non-void set H where

$$K = \{\langle h : K_{d_j}^{T}(h), K_{d_j}^{I}(h), K_{d_j}^{F}(h)\rangle \mid h \in H\},$$
$$\Phi = \{\langle h : \phi_{d_j}^{T}(h), \phi_{d_j}^{I}(h), \phi_{d_j}^{F}(h)\rangle \mid h \in H\}.$$

For $j \in B$ and B is any index set, we define

(i) $\bigcup_{j \in B} {}_P K_{d_j} = (\bigcup_{j \in B} K_{d_j}, \bigvee_{j \in B} \Phi_{d_j})$ (*P*-Union)

(ii) $\bigcap_{j \in B} {}_P K_{d_j} = (\bigcap_{j \in B} K_{d_j}, \bigwedge_{j \in B} \Phi_{d_j})$ (*P*-Intersection)

(iii) $\bigcup_{j \in B} {}_R K_{d_j} = (\bigcup_{j \in B} K_{d_j}, \bigwedge_{j \in B} \Phi_{d_j})$ (*R*-Union)

(iv) $\bigcap_{j \in B} {}_R K_{d_j} = (\bigcap_{j \in B} K_{d_j}, \bigvee_{j \in B} \Phi_{d_j})$ (*R*-Intersection)

Where

$$\bigcup_{j\in B} K_{d_j} = \left\{ \left\langle h : \left(\bigcup_{j\in B} K_{d_j}{}^{T} \right)(h), \left(\bigcup_{j\in B} K_{d_j}{}^{I} \right)(h), \left(\bigcup_{j\in B} K_{d_j}{}^{F} \right)(h) \right\rangle \mid h \in H \right\},$$

$$\bigvee_{j\in B} \Phi_{d_j} = \left\{ \left\langle h : \left(\bigvee_{j\in B} \phi_{d_j}{}^{T} \right)(h), \left(\bigvee_{j\in B} \phi_{d_j}{}^{I} \right)(h), \left(\bigvee_{j\in B} \phi_{d_j}{}^{F} \right)(h) \right\rangle \mid h \in H \right\},$$

$$\bigcap_{j\in B} K_{d_j} = \left\{ \left\langle h : \left(\bigcap_{j\in B} K_{d_j}{}^{T} \right)(h), \left(\bigcap_{j\in B} K_{d_j}{}^{I} \right)(h), \left(\bigcap_{j\in B} K_{d_j}{}^{F} \right)(h) \right\rangle \mid h \in H \right\},$$

$$\bigwedge_{j\in B} \Phi_{d_j} = \left\{ \left\langle h : \left(\bigwedge_{j\in B} \phi_{d_j}{}^{T} \right)(h), \left(\bigwedge_{j\in B} \phi_{d_j}{}^{I} \right)(h), \left(\bigwedge_{j\in B} \phi_{d_j}{}^{F} \right)(h) \right\rangle \mid h \in H \right\}.$$

Remarks

(i) $\left(\bigcup_{P\, j\in B} C_{d_j} \right)' = \bigcap_{P\, j\in B} C'_{d_j}$

(ii) $\left(\bigcap_{P\, j\in B} C_{d_j} \right)' = \bigcup_{P\, j\in B} C'_{d_j}$

(iii) $\left(\bigcup_{R\, j\in B} C_{d_j} \right)' = \bigcap_{R\, j\in B} C'_{d_j}$

(iv) $\left(\bigcap_{R\, j\in B} C_{d_j} \right)' = \bigcup_{R\, j\in B} C'_{d_j}$

Proposition 3.3

For any PNCS $C = \langle K, \Phi \rangle$, $D = \langle M, \Omega \rangle$, $X = \langle L, \psi \rangle$ and $Y = \langle N, \Lambda \rangle$ in a non-void set H, we have

(i) If $C \subseteq_P D$ and $D \subseteq_P X$ then $C \subseteq_P X$

(ii) If $C \subseteq_P D$ then $D' \subseteq_P C'$

(iii) If $C \subseteq_P D$ and $C \subseteq_P X$ then $C \subseteq_P D \bigcap_P X$

(iv) If $C \subseteq_P D$ and $X \subseteq_P D$ then $C \bigcup_P X \subseteq_P D$

(v) If $C \subseteq_P D$ and $X \subseteq_P Y$ then $C \bigcup_P X \subseteq_P D \bigcup_P Y$ and $C \bigcap_P X \subseteq_P D \bigcap_P Y$

(vi) If $C \subseteq_R D$ and $D \subseteq_R X$ then $C \subseteq_R X$

(vii) If $C \subseteq_R D$ then $D' \subseteq_R C'$

(viii) If $C \subseteq_R D$ and $C \subseteq_R X$ then $C \subseteq_R D \bigcap_R X$

(ix) If $C \subseteq_R D$ and $X \subseteq_R D$ then $C \bigcup_R X \subseteq_R D$

(x) If $C \subseteq_R D$ and $X \subseteq_R Y$ then $C \bigcup_R X \subseteq_R D \bigcup_R Y$ and $C \bigcap_R X \subseteq_R D \bigcap_R Y$

Proposition 3.4

Let $\{C_{d_j} = < \mathrm{K}_{d_j}, \Phi_{d_j} > | \, j \in B\}$ be a family of F- IPNCS in a non-void set H, then the *P*-Order of

$\{C_{d_j} = < \mathrm{K}_{d_j}, \Phi_{d_j} > | \, j \in B\}$ are F-IPNCS in H.

Proof

Since $C_{d_j} = < \mathrm{K}_{d_j}, \Phi_{d_j} >$ is an F-IPNCS in a non-void set H, We have

$$(K_{d_j}{}^F)^-(h) \le \phi_{d_j}{}^F(h) \le (K_{d_j}{}^F)^+(h) \, for \; j \in B$$

It follows that

$$(\bigcup_{j\in B} K_{d_j}{}^F)^-(h) \le (\bigvee_{j\in B} \phi_{d_j}{}^F(h) \le (\bigcup_{j\in B} K_{d_j}{}^F)^+(h)$$

and

$$(\bigcap_{j\in B} K_{d_j}{}^F)^-(h) \le (\bigwedge_{j\in B} \phi_{d_j}{}^F(h) \le (\bigcap_{j\in B} K_{d_j}{}^F)^+(h)$$

Therefore

$\bigcup_{j\in B} {}_P C_{d_j} = (\bigcup_{j\in B} K_{d_j}, \bigvee_{j\in B} \phi_{d_j})$ and $\bigcap_{j\in B} {}_P C_{d_j} = (\bigcap_{j\in B} K_{d_j}, \bigwedge_{j\in B} \phi_{d_j})$ are F-INPCS in H.

Correspondingly the following propositions holds.

Proposition 3.5

Let $\{C_{d_j} = < \mathrm{K}_{d_j}, \Phi_{d_j} > | \, j \in B\}$ be a family of T-IPNCS in a non-void set H, then the *P*-Order of

$\{C_{d_j} = < \mathrm{K}_{d_j}, \Phi_{d_j} > | \, j \in B\}$ are T-IPNCS in H.

Proposition 3.6

Let $\{C_{d_j} = < \mathrm{K}_{d_j}, \Phi_{d_j} > | \, j \in B\}$ be a family of I-IPNCS in a non-void set H, then the *P*-Order of

$\{C_{d_j} = < \mathrm{K}_{d_j}, \Phi_{d_j} > | \, j \in B\}$ are I-IPNCS in H.

Corollary 3.7

Let $\{C_{d_j} = < \mathrm{K}_{d_j}, \Phi_{d_j} > | \, j \in B\}$ be a family of IPNCS in a non-void set H, then the *P*-Order of

$\{C_{d_j} = < \mathrm{K}_{d_j}, \Phi_{d_j} > | \, j \in B\}$ are IPNCS in H.

The subsequent illustration indicates that the P -Union and P- intersection of T, I -IPNCS may not be T, I-IPNCS.

Example 3.8

Let us consider the attribute values National Electronic Fund Transfer (NEFT), Real Time Gross Settlement (RTGS), Immediate Payment Service (IMPS), Unified Payment Interface (UPI) for transferring money and $C = <\mathrm{K}, \Phi>$ and $D = <\mathrm{M}, \Omega>$ be the PNCS in H with the Table values 1 & 2 correspondingly.

Table 1. $C = <\mathrm{K}, \Phi>$

Dissimilarity Measure	Value of attributes	Appurtenance Measure $K(h)$	$\Phi(h)$
0	NEFT	([0.35, 0.4], [0.7, 0.9], [0.2, 0.5])	(0.2, 0.65, 0.4)
0.5	RTGS	([0.6, 0.8], [0.5, 0.7], [0.3, 0.7])	(0.2, 0.85, 0.6)
0.75	IMPS	([0.45, 0.6], [0.2, 0.4], [0.1, 0.3])	(0.7, 0.6, 0.2)
1	UPI	([0.2, 0.3], [0.1, 0.3], [0.6, 0.9])	(0.1, 0.4, 0.95)

Table 2. $D = <\mathrm{M}, \Omega>$

Dissimilarity Measure	Value of attributes	Appurtenance Measure $M(h)$	$\Omega(h)$
0	NEFT	([0.1, 0.3], [0.7, 0.8], [0.6, 0.9])	(0.7, 0.92, 0.7)
0.5	RTGS	([0.6, 0.7], [0.5, 0.6], [0.2, 0.4])	(0.9, 0.2, 0.3)
0.75	IMPS	([0.45, 0.8], [0.2, 0.5], [0.5, 0.7])	(0.35, 0.1, 0.6)
1	UPI	([0.5, 0.6], [0.4, 0.5], [0.4, 0.7])	(0.45, 0.3, 0.45)

Table 3. $C \cup_P D = (K \cup M, \Phi \vee \Omega)$

Dissimilarity Measure	Value of attributes	Appurtenance Measure $(K \cup M)(h)$	$(\Phi \vee \Omega)(h)$
0	NEFT	([0.35, 0.4], [0.7, 0.9], [0.6, 0.9])	(0.7, 0.92, 0.7)
0.5	RTGS	([0.6, 0.8], [0.5, 0.7], [0.3, 0.7])	(0.9, 0.85, 0.6)
0.75	IMPS	([0.45, 0.8], [0.2, 0.5], [0.5, 0.7])	(0.7, 0.6, 0.6)
1	UPI	([0.5, 0.6], [0.4, 0.5], [0.6, 0.9])	(0.45, 0.4, 0.95)

Table 4. $C \cap_P D = (K \cap M, \Phi \wedge \Omega)$

Dissimilarity	Value of	Appurtenance Measure	$(\Phi \wedge \Omega)(h)$

Measure	attributes	$(K \cap M)(h)$	
0	NEFT	([0.1, 0.3], [0.7, 0.8], [0.2, 0.5])	(0.2, 0.65, 0.4)
0.5	RTGS	([0.6, 0.7], [0.5, 0.6], [0.2, 0.4])	(0.2, 0.2, 0.3)
0.75	IMPS	([0.45, 0.6], [0.2, 0.4], [0.1, 0.3])	(0.35, 0.1, 0.2)
1	UPI	([0.2, 0.3], [0.1, 0.3], [0.4, 0.7])	(0.1, 0.3, 0.45)

Then $C = < K, \Phi >$ and $D = < M, \Omega >$ are both T-external and I-external PNCS in H. $C \cup_P D = (K \cup M, \Phi \vee \Omega)$ and $C \cap_P D = (K \cap M, \Phi \wedge \Omega)$ are given by Tables 3 & 4 correspondingly.

Then $C \cup_P D = (K \cup M, \Phi \vee \Omega)$ is neither an I-EPNCS nor T-EPNCS in H since

$$((\Phi^T \vee \Omega^T)(IMPS) = 0.7 \in (0.45, 0.8) = ((C^T \cup D^T)^-(IMPS), (C^T \cup D^T)^+(IMPS)))$$

And

$$(\Phi^I \vee \Omega^I)(UPI) = 0.4 \in (0.4, 0.5) = ((C^I \cup D^I)^-(UPI), (C^I \cup D^I)^+(UPI))$$

Also $C \cap_P D = (K \cap M, \Phi \wedge \Omega)$ is neither an I-EPNCS nor T-EPNCS in H since

$$(\Phi^T \wedge \Omega^T)(NEFT) = 0.2 \in (0.1, 0.3) = ((C^T \cap D^T)^-(NEFT), (C^T \cap D^T)^+(NEFT))$$

$$(\Phi^I \wedge \Omega^I)(UPI) = 0.3 \in (0.1, 0.3) = ((C^I \cap D^I)^-(UPI), (C^I \cap D^I)^+(UPI))$$

We provide conditions for the *R*-Union of two T-internal (resp. I- internal and F-Internal) PNCS to be a T- internal (resp. I- internal and F-Internal) PNCS.

Proposition 3.9

If $C = < K, \Phi >$ and $D = < M, \Omega >$ be T-IPNCS in a non-void set H such that

$$(\forall z \in Z)(\max\{(K^T)^-(h), (M^T)^-(h)\} \leq (\phi^T \wedge \delta^T)(h)). \quad (3.1)$$

Then the *R*-Union of $C = < K, \Phi >$ and $D = < M, \Omega >$ is a T-INPCS in H.

Proof

Let $C = < K, \Phi >$ and $D = < M, \Omega >$ be T-IPNCS in a non-void set H which satisfies the constraints (3.1). Then

$$(K^T)^-(h) \leq \phi^T(h) \leq (K^T)^+(h) \text{ and } (M^T)^-(h) \leq \delta^T(h) \leq (M^T)^+(h),$$

and so $(\phi^T \wedge \delta^T)(h) \leq (K^T \cup M^T)^+(h)$.

It follows from (3.1) that

$$(K^T \cup M^T)^-(h) = \max\{(K^T)^-(h), (M^T)^-(h)\} \leq (\phi^T \wedge \delta^T)(h) \leq (K^T \cup M^T)^+(h)$$

Hence $C \cup_R D = (K \cup M, \phi \cup \delta)$ is a T-INPCS in Z.

Correspondingly the following propositions hold.

Proposition 3.10

If $C=<\mathrm{K},\Phi>$ and $D=<\mathrm{M},\Omega>$ be I-IPNCS in a non-void set H such that

$$(\forall h\in H)(\max\{(K^{I})^{-}(h),(M^{I})^{-}(h)\}\leq(\phi^{I}\wedge\delta^{I})(h)). \qquad (3.2)$$

Then the *R*-Union of $C=<\mathrm{K},\Phi>$ and $D=<\mathrm{M},\Omega>$ is a I-INPCS in H.

Proposition 3.11

If $C=<\mathrm{K},\Phi>$ and $D=<\mathrm{M},\Omega>$ be F-IPNCS in a non-void set H such that

$$(\forall h\in H)(\max\{(K^{F})^{-}(h),(M^{F})^{-}(h)\}\leq(\phi^{F}\wedge\delta^{F})(h)). \qquad (3.3)$$

Then the *R*-Union of $C=<\mathrm{K},\Phi>$ and $D=<\mathrm{M},\Omega>$ is a F-INPCS in H.

Corollary 3.12

If two INPCS $C=<\mathrm{K},\Phi>$ and $D=<\mathrm{M},\Omega>$ satisfies the constraints (3.1), (3.2), (3.3), then the *R*-Union of $C=<\mathrm{K},\Phi>$ and $D=<\mathrm{M},\Omega>$ is an INPCS in H.

We provide conditions for the *R*-Intersection of two T-internal (resp. I- internal and F-Internal) PNCS to be a T- internal (resp. I- internal and F-Internal) PNCS.

Proposition 3.13

If $C=<\mathrm{K},\Phi>$ and $D=<\mathrm{M},\Omega>$ be T-IPNCS in a non-void set H such that

$$(\forall h\in H)((\phi^{T}\vee\delta^{T})(h)\leq\min\{(K^{T})^{+}(h),(M^{T})^{+}(h)\}). \qquad (3.4)$$

Then the *R*-Intersection of $C=<\mathrm{K},\Phi>$ and $D=<\mathrm{M},\Omega>$ is an T-INPCS in H.

Proof

Assume that the condition (4.4) is valid. Then

$$(K^{T})^{-}(h)\leq\phi^{T}(h)\leq(K^{T})^{+}(h) \text{ and } (M^{T})^{-}(h)\leq\delta^{T}(h)\leq(M^{T})^{+}(h) \text{ for all } h\in H.$$

It follows from (4.4) that

$$(K^{T}\cap M^{T})(h)\leq(\phi^{T}\vee\delta^{T})(h)\leq\min\{(K^{T})^{+},(M^{T})^{+}(h)\}=(A^{T}\cap B^{T})^{+}(h) \text{ for all } h\in H.$$

Therefore $C\cap_{R}D=(K\cap M,\phi\vee\delta)$ is a T-INPCS.

Correspondingly the subsequent propositions hold.

Proposition 3.14

If $C=<\mathrm{K},\Phi>$ and $D=<\mathrm{M},\Omega>$ be I-IPNCS in a non-void set H such that

$(\forall h \in H)((\phi^I \vee \delta^I)(h) \leq \min\{(K^I)^+(h), (M^I)^+(h)\}).$ (3.5)

Then the *R*-Intersection of $C = < \mathrm{K}, \Phi >$ and $D = < \mathrm{M}, \Omega >$ is an I-INPCS in H.

Proposition 3.15

If $C = < \mathrm{K}, \Phi >$ and $D = < \mathrm{M}, \Omega >$ be F-IPNCS in a non-void set H such that

$(\forall h \in H)((\phi^F \vee \delta^F)(h) \leq \min\{(K^F)^+(h), (M^F)^+(h)\}).$ (3.6)

Then the *R*-Intersection of $C = < \mathrm{K}, \Phi >$ and $D = < \mathrm{M}, \Omega >$ is an F-INPCS in H.

Corollary 3.16

If two INPCS $C = < \mathrm{K}, \Phi >$ and $D = < \mathrm{M}, \Omega >$ satisfies the constraints (3.4), (3.5), (3.6), then the *R*-Intersection of $C = < \mathrm{K}, \Phi >$ and $D = < \mathrm{M}, \Omega >$ is an INPCS in H.

The subsequent illustration indicates that the R -Union and R- intersection of T, F -EPNCS may not be T, F-EPNCS

Example 3.17

Let us consider the attribute values of life insurance policies 'Whole life Insurance (WLI), Term Life Insurance(TLI),Universal Life Insurance(ULI) and Variable Life Insurance (VLI) and $C = < \mathrm{K}, \Phi >$ and $D = < \mathrm{M}, \Omega >$ be the PNCS in H with the Table values 5 & 6 respectively.

Table 5. $C = < \mathrm{K}, \Phi >$

Dissimilarity Measure	Value of attributes	Appurtenance Measure $K(h)$	$\Phi(h)$
0	WLI	([0.8, 0.9], [0.2, 0.4], [0.3, 0.5])	(0.8, 0.25, 0.5)
0.5	TLI	([0.7, 0.9], [0.3, 0.6], [0.7, 0.8])	(0.8, 0.5, 0.75)
0.75	ULI	([0.5, 0.8], [0.3, 0.5], [0.1, 0.6])	(0.7, 0.4, 0.1)
1	VLI	([0.1, 0.2], [0.1, 0.6], [0.4, 0.8])	(0.2, 0.7, 0.6)

Table 6. $D = < \mathrm{M}, \Omega >$

Dissimilarity Measure	Value of attributes	Appurtenance Measure $M(h)$	$\Omega(h)$
0	WLI	([0.4, 0.7], [0.3, 0.6], [0.4, 0.6])	(0.6, 0.45, 0.6)
0.5	TLI	([0.1, 0.5], [0.3, 0.4], [0.6, 0.8])	(0.3, 0.3, 0.7)
0.75	ULI	([0.6, 0.8], [0.6, 0.7], [0.2, 0.9])	(0.6, 0.7, 0.9)
1	VLI	([0.5, 0.7], [0.1, 0.3], [0.2, 0.5])	(0.65, 0.2, 0.4)

Table 7. $C \cup_R D = (K \cup M, \Phi \wedge \Omega)$

Dissimilarity Measure	Value of attributes	Appurtenance Measure $(K \cup M)(h)$	$(\Phi \wedge \Omega)(h)$
0	WLI	([0.8, 0.9], [0.3, 0.6], [0.4, 0.6])	(0.6, 0.25, 0.5)
0.5	TLI	([0.7, 0.9], [0.3, 0.6], [0.6, 0.8])	(0.3, 0.3, 0.7)
0.75	ULI	([0.6, 0.8], [0.6, 0.7], [0.2, 0.9])	(0.6, 0.4, 0.1)
1	VLI	([0.5, 0.7], [0.1, 0.6], [0.4, 0.8])	(0.2, 0.2, 0.4)

Table 8. $C \cap_R D = (K \cap M, \Phi \vee \Omega)$

Dissimilarity Measure	Value of attributes	Appurtenance Measure $(K \cap M)(h)$	$(\Phi \vee \Omega)(h)$
0	WLI	([0.4, 0.7], [0.2, 0.4], [0.3, 0.5])	(0.8, 0.45, 0.6)
0.5	TLI	([0.1, 0.5], [0.3, 0.4], [0.7, 0.8])	(0.8, 0.5, 0.75)
0.75	ULI	([0.5, 0.8], [0.3, 0.5], [0.1, 0.6])	(0.7, 0.7, 0.9)
1	VLI	([0.1, 0.2], [0.1, 0.3], [0.2, 0.5])	(0.65, 0.7, 0.6)

Then $C = <\mathrm{K}, \Phi>$ and $D = <\mathrm{M}, \Omega>$ are both T-internal and F-internal PNCS in H. $C \cup_R D = (K \cup M, \Phi \wedge \Omega)$ and $C \cap_R D = (K \cap M, \Phi \vee \Omega)$ are given by Tables 7 & 8 respectively.

Then $C \cup_R D = (K \cup M, \Phi \wedge \Omega)$ is neither an I-IPNCS nor T-IPNCS in H since

$$(\Phi^T \wedge \Omega^T)(WLI) = 0.6 \notin (0.8, 0.9) = ((C^T \cup D^T)^-(WLI), (C^T \cup D^T)^+(WLI));$$

$$(\Phi^T \wedge \Omega^T)(TLI) = 0.3 \notin (0.7, 0.9) = ((C^T \cup D^T)^-(TLI), (C^T \cup D^T)^+(TLI));$$

$$(\Phi^T \wedge \Omega^T)(VLI) = 0.2 \notin (0.5, 0.7) = ((C^T \cup D^T)^-(VLI), (C^T \cup D^T)^+(VLI)).$$

And

$$(\Phi^F \wedge \Omega^F)(ULI) = 0.1 \notin (0.2, 0.9) = ((C^F \cup D^F)^-(ULI), (C^F \cup D^F)^+(ULI)).$$

Also $C \cap_R D = (K \cap M, \Phi \vee \Omega)$ is neither an I-IPNCS nor T-IPNCS in H since

$$(\Phi^T \vee \Omega^T)(WLI) = 0.8 \notin (0.4, 0.7) = ((C^T \cap D^T)^-(WLI), (C^T \cap D^T)^+(WLI));$$

$$(\Phi^T \vee \Omega^T)(TLI) = 0.8 \notin (0.1, 0.5) = ((C^T \cap D^T)^-(TLI), (C^T \cap D^T)^+(TLI));$$

$$(\Phi^T \vee \Omega^T)(VLI) = 0.65 \notin (0.1, 0.2) = ((C^T \cap D^T)^-(VLI), (C^T \cap D^T)^+(VLI)).$$

And

$$(\Phi^F \vee \Omega^F)(WLI) = 0.6 \notin (0.3, 0.5) = ((C^F \cap D^F)^-(WLI), (C^F \cap D^F)^+(WLI));$$

$$(\Phi^F \vee \Omega^F)(ULI) = 0.9 \notin (0.1, 0.6) = ((C^F \cap D^F)^-(ULI), (C^F \cap D^F)^+(ULI));$$

$$(\Phi^F \vee \Omega^F)(VLI) = 0.6 \notin (0.2, 0.5) = ((C^F \cap D^F)^-(VLI), (C^F \cap D^F)^+(VLI)).$$

4. Conclusion and Future Work

In this paper we studied the *P*-Order and *R*- Order (Union and Intersection) of the PNCS. The theorems and results we derived will be useful in the multi criteria decision making problems. Applications in various engineering, technical, medical, and so on areas of learning P and R-Order should be assessed. In future we can extend the concept to plithogenic neutrosophic soft sets which may help abundantly in the areas related with decision making.

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References

1. R.Anita Cruz and F.Nirmala Irudayam, Neutrosophic Soft Cubic set, *International Journal of Mathematics trends and Technology*, 2017, 46 (2), 88f-94.
2. K.T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy sets and Systems*, 1986, 20, 87- 96.
3. M. Gulistan, N.Yaqoob, Z. Rashid, F. Smarandache, H. A. Wahab, A Study on Neutrosophic Cubic Graphs with Real Life Applications in Industries,*Symmetry*,2018, 10(6), 1-22.
4. M. Gulistan, M. Ali, M.Azhar, S. Rho, S.Kadry, Novel neutrosophic Cubic Graphs Structures wth Application in Decision making problems, *IEEE access*, 2019, 7, 94757-94778.
5. Y.B.Jun, C.S.Kim and K.O.Yang, Cubic sets, *Annals of Fuzzy Mathematics and Informatics*, 2012, 4(3), 83-98.
6. Y.B. Jun, F.Smarandache and C.S. Kim, Neutrosophic Cubic set, *New Mathematics and Natural Computation*, 2017, 13 (1), 41-54.
7. Majid Khan, Ismat Beg and Muhammad Gulistan, "Exponential Laws and Aggregation Operators on Neutrosophic Cubic Sets", *International Journal of Neutrosophic Science*,2020, 4(1), 47-71.
8. Nivetha Martin and Florentin Smarandache, Plithogenic Cognitive Maps in Decision Making, *International Journal of Neutrosophic Science*, 2020,9(1), 09-21.
9. Prem Kumar Singh, "Plithogenic set for multi-variable data analysis", *International Journal of Neutrosophic Science*, 2020, 1(2), 81-89.
10. S.P.Priyadharshini, F. Nirmala Irudayam, F.Smarandache, Plithogenic Cubic Set, *International Journal of Neutrosophic Science,* 2020,11(1), 30-38.
11. F.Smarandache, A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic, Rehoboth: American Research Press, 1999.
12. F.Smarandache, Neutrosophic set, a generalisation of the intuitionistic fuzzy sets, *Inteer J. PuBre Appl. Math*, 2005, 24, 287-297.
13. F.Smarandache, Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy,NeutrosophyLogic, Set, Probability and Statistics, University of New Mexico,Gallup,NM, 87301,USA, 2002.
14. F.Smarandache Neutrosophy, A new Branch of Philosophy, *Multiple Valued Logic /An International Journal*, USA, ISSN 1023-6627,2002, 8(3), 297-384.

15. F.Smarandache, Plithogeny, PlithogenicSet, Logic,Probability, and Statistics, 141 pages, Pons Editions,Brussels,Belgium,2017,arXiv.org(CornellUniversity),Computer Science-Artificial Intelligence,03Bxx:abstract:https://arxiv.org/abs/1808.03948, book: https://arxiv.org/ftp/arxiv/papers/1808/1808.03948.pdf,HarvardSAO/NASAADS http://adsabs.harvard.edu/cgi-bin/bib_query?arXiv:1808.03948.
16. I.B. Turksen, Interval valued fuzzy sets based on normal forms, *Fuzzy sets and Systems*, 1968, 20,191-210.
17. I.B. Turksen, Interval-valued fuzzy sets and compensatory, *Fuzzy Sets and* Systems, 1992, 51, 295–307.
18. I. B. Turksen, Interval-valued strict preference with Zadeh triples, *Fuzzy Sets and Systems*, 1996, 78, 183–195.
19. Victor Christianto , Robert N. Boyd and Florentin Smarandache, Three Possible Applications of Neutrosophic Logic in Fundamental and Applied Sciences, International Journal of Neutrosophic Science ,2020, 1(2), 90-95.
20. Wang, H.; Smarandache, F.; Zhang, Y. Q.; Sunderraman, R. "Interval Neutrosophic Sets and logic: Theory and Applications in Computing"; Hexis, Phoenix, Ariz, USA, 2005.
21. L.A Zadeh, *Fuzzy sets*, Inform and Control 1965, 8, 338-353.
22. J. Zhan, M. Khan, M.Gulistan, A. Ali, Applications of neutrosophic cubic sets in multi-criteria decision making, *Internatinal Journal for Uncertainty Quantificattion*, 2017, 7(5), 377-394.
23. R. M. Zulqarnain, X. L. Xin, B. Ali, S. Broumi, S. Abdal, M. I. Ahamad, Decision-Making Approach Based on Correlation Coefficient with its Properties Under Interval-Valued Neutrosophic hypersoft set environment, *Neutrosophic Sets and Systems*, 2021,41,12-28.

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