



## Possibility Neutrosophic Hypersoft Set (PNHSS)

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**Abstract:** Soft set developed by Smarandache in 2018 to Hypersoft set (HSS) to deal with multi-argument approximate functions. The soft set cannot deal with cases when attributes are required to be further divided into disjoint attribute-valued sets. Neutrosophic Hypersoft Set (NHSS) is the most effective and useful method to handle the environment which involved more than one attribute. Neutrosophic Hypersoft Set introduced by combining Hypersoft Set and Neutrosophic Soft Set. In this paper, we first define the concept of Possibility Neutrosophic Hypersoft Set (PNHSS in short) which is combination of PNSS and HSS. Certain essential basic characteristics as subset, equal and complement are studied with illustrative examples. Basic operations such as: union, intersection and some properties such as commutative, associative, distributive low and De Morgan's law are discussing. Also, we introduce AND and OR operation of PNHSS with suitable examples and some propositions.

**Keywords:** Soft Set, Neutrosophic Soft Set, Hypersoft Set, Neutrosophic Hypersoft Set, Possibility Neutrosophic Set and Possibility Neutrosophic Hybersoft Set.

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### 1. Introduction

Fuzzy sets were developed by Zadeh [1] to solve problems which contain uncertain information. Some cases cannot deal with fuzzy set, so Turksen [2] introduced interval-valued fuzzy set. Atanassove [3] extended fuzzy set to Intuitionistic fuzzy set. Which more general than fuzzy set.

Neutrosophy introduced by Smarandache [4] which is a new tool for dealing with problems containing imprecise, indeterminacy and inconsistent data.

Neutrosophic sets which introduced by Smarandach in 2005 [5] is a generalization of the Intuitionistic fuzzy set.

Soft Set defined by Molodtsov [6] as another commonly used method in handling uncertainties in the data. Soft Set extended and introduced some of its operations and properties by Maji [8]. Sezgin et al. [11] were proved De Morgan's Law on Soft Set.

The concept of fuzzy soft set introduced by Maji [7]. Fuzzy soft set extended to Generalized fuzzy soft sets by Majumdar and Samanta in 2010 [9]. They joined the degree with the parameterization of fuzzy soft sets while defining a fuzzy soft set. Here for each parameter  $e_i$  and  $\forall i = 1, \dots, n$ ,  $F_{\mu}(e_i) = (F(e_i), \mu(e_i))$  indicates not only the degree of belongingness of the elements of  $U$  in  $F(e_i)$  but also the degree of possibility of such belongingness which is represented by  $\mu(e_i)$ . The concept of possibility fuzzy soft set introduced by Alkhazaleh et al. [10] by assigning a possibility degree to each number of fuzzy sets.

Neutrosophic Soft Set NSS with basic basic operation and properties proposed by Maji [12]. The new concept Generalised neutrosophic soft set GNSS which introduced by Sahin [13], was extension of the concept NSS defined by Maji [8]. NSS was also extended by Karaaslan [14] and

defined Possibility Neutrosophic Soft Set. NSS developed by Broumi [15] to Generalised Neutrosophic Soft Set with basic definitions and operations. He used this concept for solving decision making problems. Recently the researchers [16–20] extended the theory of neutrosophic soft set and developed it by discussion and applications in decision making.

In 2018, Soft Set developed to hypersoft Set by converting a single attribute- valued function to multi-attribute valued function by Smarandache [21]. In 2019, Saqlain et al. [22] extended this concept to deals with the Generalization of TOPSIS for NHSS, by using accuracy function.

In 2020, the concept of HSS was generalized and the fundamentals of HSS with some relations and operations on HSS by Saeed et al. [23, 24]. The concept of fuzzy plithogenic hypersoft set in matrix introduced with some basic operations and properties in [25]. The combination of two concepts: Plithogenic set and hypersoft set gave a new concept, which was Plithogenic hypersoft set introduced in [26].

The concept of hypersoft point defined in different environments such as; fuzzy hypersoft set, Intuitionistic fuzzy hypersoft set, neutrosophic hypersoft set and gave some basic operation of hypersoft points in these environments by Majahid et al. [27].

Aggregate operators of NHSS were discussed in some cases by Saqlain et al. [28] with examples.

Zulqarnain et al. [29] developed the Aggregate operators of NHSS with examples and properties.

The concept of Complex hypersoft set defined by Rahman et al. [30]. They generalized the hybrids of hypersoft set with complex fuzzy and its generalized structure. Rahman et al. [31] introduced the concept of Convex and Concave hypersoft Sets with some properties and suitable examples. In 2021, Rahman et al. [32] introduced an application in decision making based on fuzzy parametrized hypersoft set theory. They made the existing literature regarding fuzzy parametrized soft set in line with the need of multi-attribute function. Another application to solve problems in decision making based on neutrosophic parametrized hypersoft set theory introduced in [33]. Numerous researchers discussed the concept of Rough soft set which was combination between rough set and soft set. Rahman et al. [34] introduced development of rough hypersoft set with application in decision making for the best choice of chemical material. They proposed a new algorithm to solve decision making problems with illustrative examples. Saeed et al. [35] defined the concept of mapping hypersoft classes. They developed some properties of mapping on hypersoft set classes such as hypersoft images and hypersoft images.

In 2022, Debanath [36] presented the notion of Interval-valued intuitionistic fuzzy hypersoft sets (IVIFHSSs), which was combining interval-valued intuitionistic fuzzy sets (IVIFSS) and hypersoft sets (HSSs). He also, discussed some different operators of this concept such as complement, union, intersection, AND and OR. He introduced a new algorithm based on (IVIFHSSs). Finally, he introduced a numerical example to check the reliability and validity of the algorithm.

Florentin Smarandache [37] introduced for the first time the concept of IndetermSoft as extension of soft set, that deals with indeterminate data, where 'Indeterm' stands for 'Indeterminate'. Similarly, he extended hypersoft set to IndetermHypersoft set. At the end, he presented an application of the IndetermHyperSoft Set. Ihsan et. [38] defined expert set on Neutrosophic hypersoft set. This model solved the problem of dealing with one expert and solved the problem of different parametric-valued sets parallel to different characteristics. They discussed basic characteristics, aggregation operation, and results with examples. Finally, they presented an application to NHSES in decision making problem. Neutrosophic hypersoft set are developed and an application is discussed in decision making, which appear from [39]-[43].

The organization of this paper as follows: Section 2 present the basic definitions of neutrosophic set, soft set, Neutrosophic soft set, Hypersoft set, Possibility neutrosophic soft set, Neutrosophic Hypersoft set and some relative definitions used in this work. Section 3 define the new concept of possibility neutrosophic hypersoft set with related definitions and suitable examples. Section 4 describes the basic operations of PNHSS. Section 5 discusses AND and OR operation. Section 6 presents conclude of this paper with suggested future work.

## 2. Preliminary

In this section, we present some definitions required in this paper.

### Definition 1 [5] Neutrosophic Set.

A neutrosophic set  $A$  on the universe of discourse  $X$  is defined as  $A = \{(x: T_A(x), I_A(x), F_A(x)); x \in X\}$  where  $T; I; F : X \rightarrow [0,1]$  and  $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ .

### Definition 2 [8] Soft Set.

Let  $U$  be a universe and  $E$  be a set of parameters. Let  $P(U)$  denote the power set of  $U$  and  $A \subseteq E$ . A pair  $(F, A)$  is called a soft set over  $U$ , where  $F$  is a mapping  $F: A \rightarrow P(U)$ .

### Definition 3 [7] Fuzzy soft set.

Let  $U$  be an initial universal set and let  $E$  be a set of parameters. Let  $I^U$  denote the power set of all fuzzy subsets of  $U$ . Let  $A \subseteq E$ . A pair  $(F, E)$  is called a fuzzy soft set over  $U$  where  $F$  is a mapping given by  $F : A \rightarrow I^U$ .

### Definition 4 [21] Neutrosophic Soft Set.

Let  $U$  be an initial universe set and  $E$  be a set of parameters. Consider  $A \subseteq E$ . Let  $P(U)$  denotes the set of all neutrosophic sets of  $U$ . The collection  $(F, A)$  is termed to be the soft neutrosophic set over  $U$ , where  $F$  is a mapping given by  $F: A \rightarrow P(U)$ .

### Definition 5 [21] Hypersoft Set.

Let  $U$  be a universal set and  $P(U)$  be the all neutrosophic subset of  $U$  and for  $n \geq 1$ , there are  $n$  distinct attributes such as  $\ell_1, \ell_2, \dots, \ell_n$  and  $L_1, L_2, \dots, L_n$  are sets for corresponding values attributes respectively with following conditions such as  $L_i \cap L_j = \emptyset$  for  $i \neq j$  and  $i, j \in \{1, 2, \dots, n\}$ .

Then the pair  $(\psi, L_1 \times L_1 \times \dots \times L_n)$  is said to be Hypersoft set over  $U$  where  $\psi$  is a mapping from  $L_1 \times L_1 \times \dots \times L_n$  to  $P(U)$ .

### Definition 6 [14] Possibility Neutrosophic Soft Set (GNSS).

Let  $U$  be an initial universe and  $E$  be a set of parameters. Let  $N(U)$  be the set of all neutrosophic sets Of  $U$  and  $I^U$  is collection of all fuzzy subset of  $U$ . A possibility neutrosophic soft set  $f_\mu$  over  $U$  is defined by the set of ordered pairs

$$f_\mu(e) = \left\{ \left( e_k, \left\{ \left( \frac{u_j}{f(e_k)(u_j)}, \mu(e_k)(u_j) \right) : u_j \in U \right\} \right) : e_k \in E \right\},$$

or a mapping defined by  $f_\mu: E \rightarrow N(U) \times I^U$  where  $\mu$  is a fuzzy set such that  $\mu: E \rightarrow I = [0,1]$  and  $f_\mu$  is a mapping defined by  $f_\mu: E \rightarrow N(U)$ .

### Definition 7 [27] Neutrosophic Hypersoft Set (NHSS).

Let  $U$  be a universal set and  $P(U)$  be a power set of  $U$  and for  $n \geq 1$ , there are  $n$  distinct attributes such as  $\ell_1, \ell_2, \dots, \ell_n$  and  $L_1, L_2, \dots, L_n$  are sets for corresponding values attributes respectively with following conditions such as  $L_i \cap L_j = \emptyset$  for  $i \neq j$  and  $i, j \in \{1, 2, \dots, n\}$ . Then the pair  $(\psi, \Lambda)$  is said to be NHSS over  $U$  if there exists a relation  $L_1 \times L_1 \times \dots \times L_n = \Lambda$ .  $\psi$  is a mapping from  $L_1 \times L_1 \times \dots \times L_n$  to  $P(U)$  and  $\psi_\Lambda(L_1 \times L_1 \times \dots \times L_n) = \{ \langle u, T_\Lambda(u), I_\Lambda(u), F_\Lambda(u) \rangle : u \in U \}$  where  $T, I, F$  are membership values for truthness, indeterminacy, and falsity respectively such that  $T, I, F: U \rightarrow [0,1]$  and  $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ .

### Definition 8 [28] Neutrosophic Hypersoft subset (NHSS).

For two Neutrosophic Hypersoft subsets (NHSs)  $\psi_{\Lambda_1}$  and  $\psi_{\Lambda_2}$  over  $U$ ,  $\psi_{\Lambda_1}$  is called a neutrosophic hypersoft subset of  $\psi_{\Lambda_2}$  if  $T(\psi_{\Lambda_1}) \leq T(\psi_{\Lambda_2}), I(\psi_{\Lambda_1}) \leq I(\psi_{\Lambda_2}), F(\psi_{\Lambda_1}) \geq F(\psi_{\Lambda_2})$ .

**Definition 9 [28] Neutrosophic Hypersoft set equal.**

Two Neutrosophic Hypersoft subsets (NHSs)  $\psi_{\Lambda_1}$  and  $\psi_{\Lambda_2}$  over  $U$ , are said to be equal if  $\psi_{\Lambda_1}$  is a NHSs of  $\psi_{\Lambda_2}$  and  $\psi_{\Lambda_2}$  is a NHSs of  $\psi_{\Lambda_1}$ .

**Definition 10 [28] Neutrosophic Hypersoft set complement.**

The complement of a Neutrosophic Hypersoft Set  $\psi_{\Lambda}$  is denoted by  $(\psi_{\Lambda})^c$  is defined by  $(\psi_{\Lambda})^c$  such that  $(\psi_{\Lambda})^c = \{ \langle u, T(\psi_{\Lambda}^c) = F(\psi_{\Lambda}), I(\psi_{\Lambda}^c) = 1 - I(\psi_{\Lambda}), F(\psi_{\Lambda}^c) = T(\psi_{\Lambda}) \rangle, u \in U \}$ .

**Definition 11 [29] Neutrosophic Hypersoft set union.**

The union of two NHSs  $\psi_{\Lambda_1}$  and  $\psi_{\Lambda_2}$  over the common universe  $U$ . denoted by  $\psi_{\Lambda_1} \cup \psi_{\Lambda_2}$  is the NHS and is given as follows:

$$T(\psi_{\Lambda_1} \cup \psi_{\Lambda_2}) = \begin{cases} T(\psi_{\Lambda_1}) & \text{if } u \in \Lambda_1 - \Lambda_2, \\ T(\psi_{\Lambda_2}) & \text{if } u \in \Lambda_2 - \Lambda_1, \\ \max(T(\psi_{\Lambda_1}), T(\psi_{\Lambda_2})) & \text{if } u \in \Lambda_1 \cap \Lambda_2, \end{cases}$$

$$I(\psi_{\Lambda_1} \cup \psi_{\Lambda_2}) = \begin{cases} I(\psi_{\Lambda_1}) & \text{if } u \in \Lambda_1 - \Lambda_2, \\ I(\psi_{\Lambda_2}) & \text{if } u \in \Lambda_2 - \Lambda_1, \\ \min(I(\psi_{\Lambda_1}), I(\psi_{\Lambda_2})) & \text{if } u \in \Lambda_1 \cap \Lambda_2, \end{cases}$$

$$F(\psi_{\Lambda_1} \cup \psi_{\Lambda_2}) = \begin{cases} F(\psi_{\Lambda_1}) & \text{if } u \in \Lambda_1 - \Lambda_2, \\ F(\psi_{\Lambda_2}) & \text{if } u \in \Lambda_2 - \Lambda_1, \\ \min(F(\psi_{\Lambda_1}), F(\psi_{\Lambda_2})) & \text{if } u \in \Lambda_1 \cap \Lambda_2. \end{cases}$$

**Definition 12 [29] Neutrosophic Hypersoft intersection.**

The intersection of two NHSs  $\psi_{\Lambda_1}$  and  $\psi_{\Lambda_2}$  over the common universe  $U$ . denoted by  $\psi_{\Lambda_1} \cap \psi_{\Lambda_2}$  is the NHS and is given as follows:

$$T(\psi_{\Lambda_1} \cap \psi_{\Lambda_2}) = \begin{cases} T(\psi_{\Lambda_1}) & \text{if } u \in \Lambda_1 - \Lambda_2, \\ T(\psi_{\Lambda_2}) & \text{if } u \in \Lambda_2 - \Lambda_1, \\ \min(T(\psi_{\Lambda_1}), T(\psi_{\Lambda_2})) & \text{if } u \in \Lambda_1 \cap \Lambda_2, \end{cases}$$

$$I(\psi_{\Lambda_1} \cap \psi_{\Lambda_2}) = \begin{cases} I(\psi_{\Lambda_1}) & \text{if } u \in \Lambda_1 - \Lambda_2, \\ I(\psi_{\Lambda_2}) & \text{if } u \in \Lambda_2 - \Lambda_1, \\ \max(I(\psi_{\Lambda_1}), I(\psi_{\Lambda_2})) & \text{if } u \in \Lambda_1 \cap \Lambda_2, \end{cases}$$

$$F(\psi_{\Lambda_1} \cap \psi_{\Lambda_2}) = \begin{cases} F(\psi_{\Lambda_1}) & \text{if } u \in \Lambda_1 - \Lambda_2, \\ F(\psi_{\Lambda_2}) & \text{if } u \in \Lambda_2 - \Lambda_1, \\ \max(F(\psi_{\Lambda_1}), F(\psi_{\Lambda_2})) & \text{if } u \in \Lambda_1 \cap \Lambda_2. \end{cases}$$

**Definition 13 [29] AND-Operation of Two Neutrosophic Hypersoft Set.**

Let  $\psi_{\Lambda_1}$  and  $\psi_{\Lambda_2}$  be two NHSs over the common universe  $U$ , then  $\psi_{\Lambda_1} \wedge \psi_{\Lambda_2} = \psi_{\Lambda_1 \times \Lambda_2}$  is given as follows:

$$T(\psi_{\Lambda_1 \times \Lambda_2}) = \min(T(\psi_{\Lambda_1}), T(\psi_{\Lambda_2})),$$

$$I(\psi_{\Lambda_1 \times \Lambda_2}) = \max(I(\psi_{\Lambda_1}), I(\psi_{\Lambda_2})),$$

$$F(\psi_{\Lambda_1 \times \Lambda_2}) = \max(I(\psi_{\Lambda_1}), I(\psi_{\Lambda_2})).$$

**Definition 14 [29] OR-Operation of Two Neutrosophic Hypersoft Set.**

Let  $\psi_{\Lambda_1}$  and  $\psi_{\Lambda_2}$  be two NHSs over the common universe  $U$ , then  $\psi_{\Lambda_1} \vee \psi_{\Lambda_2} = \psi_{\Lambda_1 \times \Lambda_2}$  is given as follows:

$$T(\psi_{\Lambda_1 \times \Lambda_2}) = \max(T(\psi_{\Lambda_1}), T(\psi_{\Lambda_2})),$$

$$I(\psi_{\Lambda_1 \times \Lambda_2}) = \min(I(\psi_{\Lambda_1}), I(\psi_{\Lambda_2})),$$

$$F(\psi_{\Lambda_1 \times \Lambda_2}) = \min(I(\psi_{\Lambda_1}), I(\psi_{\Lambda_2})).$$

### 3.Fundamental of Possibility Neutrosophic Hypersoft Set

**Definition 15 Possibility Neutrosophic Hypersoft Set (PNHSS)**

Let  $\mathfrak{S}$  be the universal set and  $N(\mathfrak{S})$  be set of all neutrosophic subset of  $\mathfrak{S}$ . For  $n \geq 1$ , let  $\ell_1, \ell_2, \dots, \ell_n$  be  $n$  well-defined attributes, whose corresponding attributive are respectively the set  $L_1, L_2, \dots, L_n$  with  $L_i \cap L_j = \emptyset$  for  $i \neq j$  and  $i, j \in \{1, 2, \dots, n\}$  and their relation  $L_1 \times L_2 \times \dots \times L_n = \Lambda$ . The pair  $(\psi^\mu, \Lambda)$  is said to be possibility neutrosophic hypersoft set over  $\mathfrak{S}$  where

$\psi_\Lambda^\mu(e) = \{(x, (\psi_\Lambda(e)(x), \mu(e))) : x \in \mathfrak{S}, \psi_\Lambda(e)(x) \in N(\mathfrak{S}) \text{ and } \mu(e) \in I = [0,1]\}$ . Where  $\psi_\Lambda$  is a mapping given by  $\psi_\Lambda: L_1 \times L_2 \times \dots \times L_n \rightarrow N(\mathfrak{S})$  and  $\mu$  is a fuzzy set such that  $\mu: \Lambda \rightarrow I$ . Here  $\psi_\Lambda^\mu$  is a mapping defined

$$\psi_\Lambda^\mu: L_1 \times L_2 \times \dots \times L_n \rightarrow N(\mathfrak{S}) \times I.$$

**Example 1**

Let  $\mathfrak{S}$  be the set of decision makers to decide best car given as  $\mathfrak{S} = \{d_1, d_2, d_3, d_4\}$  and a set

$M = \{d_1, d_2\} \subset \mathfrak{S}$ . Also consider the set of attributes as

$L_1 = \text{Car type}, L_2 = \text{Engine capacity}, L_3 = \text{Saftey}, L_4 = \text{Performace}$  and their respective attributes are given as follows:

$$L_1 = \text{Car type} = \{\text{Mercedes – Benz}, \text{BMW}, \text{Volvo}, \text{Ford}\}$$

$$L_2 = \text{Engine capacity} = \{1500\text{cc}, 1800\text{cc}, 2000\text{cc}, 2500\text{cc}\}$$

$$L_3 = \text{Saftey} = \{\text{APS}, \text{Air bag}\}$$

$$L_4 = \text{Performace} = \{\text{car torque}, \text{speeds}\}$$

$$\text{Let } \psi_\Lambda^\mu: L_1 \times L_2 \times L_3 \times L_4 \rightarrow N(\mathfrak{S}) \times I$$

And  $\mu: \Lambda \rightarrow I$ . Assume that the customer concentrate on type of car is BMW with engine capacity which provide air bag and speed. Then PNHSS is defined as follows:

$$\psi_\Lambda^\mu(L_1 \times L_2 \times L_3 \times L_4) = \psi_\Lambda^\mu(\text{BMW}, 2000\text{cc}, \text{Air bag}, \text{Speed}) = \{d_1, d_2\}$$

Then the relation of above PNHSS is given as

$$\psi_\Lambda^\mu(\text{BMW}, 2000\text{cc}, \text{Air Bag}, \text{Speed}) = \{(d_1, (\langle 0.6, 0.4, 0.5 \rangle, \langle 0.8, 0.3, 0.4 \rangle, \langle 0.8, 0.1, 0.2 \rangle, \langle 0.4, 0.7, 0.5 \rangle, (0.2))), \\ , \langle d_2, (\langle 0.4, 0.3, 0.2 \rangle, \langle 0.9, 0.4, 0.1 \rangle, \langle 0.3, 0.7, 0.6 \rangle, \langle 0.7, 0.4, 0.2 \rangle, (0.5))\}.$$

**Definition 16**

Let  $\psi_{\Lambda_1}^\mu$  &  $\psi_{\Lambda_2}^\eta$  be two PNHSS over  $\mathfrak{S}$ . Then  $\psi_{\Lambda_1}^\mu$  is the GNHS subset of  $\psi_{\Lambda_2}^\eta$  if:

- 1)  $\mu$  is fuzzy subset of  $\eta$
- 2)  $\Lambda_1$  is a subset of  $\Lambda_2$ .
- 3)  $\forall e \in \Lambda_1 \cap \Lambda_2, \psi_{\Lambda_1}(e)$  is a NHSS  $\psi_{\Lambda_2}(e)$ .

**Example 2**

Consider the two PNHSS  $\psi_{\Lambda_1}^\mu$  &  $\psi_{\Lambda_2}^\eta$  over the same universe  $\mathfrak{S} = \{d_1, d_2, d_3, d_4, d_5\}$ .

Then  $(\psi^\eta, \Lambda_2) \subset (\psi^\mu, \Lambda_1)$ .

Where,  $(\psi^\eta, \Lambda_2) = \{ \langle d_1, (\langle 0.4, 0.3, 0.7 \rangle, \langle 0.7, 0.2, 0.5 \rangle, \langle 0.6, 0.0, 0.4 \rangle, \langle 0.2, 0.3, 0.7 \rangle, (0.1)) \rangle \}$

is a GNHS subset of  $(\psi^\mu, \Lambda_1) = \{ \langle d_1, (\langle 0.6, 0.4, 0.5 \rangle, \langle 0.8, 0.3, 0.4 \rangle, \langle 0.8, 0.1, 0.2 \rangle, \langle 0.4, 0.7, 0.5 \rangle, (0.2)) \rangle, \langle d_2, (\langle 0.4, 0.3, 0.2 \rangle, \langle 0.9, 0.4, 0.1 \rangle, \langle 0.3, 0.7, 0.6 \rangle, \langle 0.7, 0.4, 0.2 \rangle), (0.5)) \rangle \}$ .

**Definition 17**

Let  $\psi_{\Lambda_1}^\mu$  &  $\psi_{\Lambda_2}^\eta$  be two PNHSS over  $\mathfrak{S}$ . Then  $\psi_{\Lambda_1}^\mu$  &  $\psi_{\Lambda_2}^\eta$  are called GNHS equal, denoted by  $\psi_{\Lambda_1}^\mu = \psi_{\Lambda_2}^\eta$  if  $\psi_{\Lambda_1}$  is a GNHS subset of  $\psi_{\Lambda_2}$  &  $\psi_{\Lambda_2}$  is a GNHS subset of  $\psi_{\Lambda_1}$ .

**Definition 18**

The complement of a PNHSS  $\psi_\Lambda^\mu$  is denoted by  $(\psi_\Lambda^\mu)^c$  and define

$$(\psi_\Lambda^\mu)^c = \{ \langle x, \psi_\Lambda^c(e)(x), \mu^{(c)}(e) \rangle : x \in \mathfrak{S}, \psi_\Lambda(e)(x) \in N(\mathfrak{S}) \text{ and } \mu(e) \in I = [0,1] \}$$

where,  $\mu^{(c)}(e) = 1 - \mu(e)$  and  $\psi_\Lambda^c =$  neutrosophic soft complement with

$$T_\Lambda^{(c)}(e) = F_\Lambda(e), I_\Lambda^{(c)}(e) = 1 - I_\Lambda(e), F_\Lambda^{(c)}(e) = T_\Lambda(e)$$

**Example 3**

Let  $\psi_\Lambda^\mu = \{ \langle d_1, (\langle 0.6, 0.4, 0.5 \rangle, \langle 0.8, 0.3, 0.4 \rangle, \langle 0.8, 0.1, 0.2 \rangle, \langle 0.4, 0.7, 0.5 \rangle, (0.2)) \rangle, \langle d_2, (\langle 0.4, 0.3, 0.2 \rangle, \langle 0.9, 0.4, 0.1 \rangle, \langle 0.3, 0.7, 0.6 \rangle, \langle 0.7, 0.4, 0.2 \rangle), (0.5)) \rangle \}$

By using the PNHSS complement, we obtain the complement given by

$(\psi_\Lambda^\mu)^c = \{ \langle d_1, (\langle 0.5, 0.6, 0.6 \rangle, \langle 0.4, 0.7, 0.8 \rangle, \langle 0.2, 0.9, 0.8 \rangle, \langle 0.5, 0.3, 0.4 \rangle, (0.8)) \rangle, \langle d_2, (\langle 0.2, 0.7, 0.4 \rangle, \langle 0.1, 0.6, 0.9 \rangle, \langle 0.6, 0.3, 0.3 \rangle, \langle 0.2, 0.6, 0.7 \rangle), (0.5)) \rangle \}$ .

**Proposition 1**

Let  $\psi_\Lambda^\mu$  be PNHSS, then  $((\psi_\Lambda^\mu)^c)^c = \psi_\Lambda^\mu$ .

Proof. Let  $(\psi_\Lambda^\mu)^c = \{ \langle x, \psi_\Lambda^c(e)(x), \mu^{(c)}(e) \rangle : x \in \mathfrak{S} \}$

$$= \{ \langle x, (F_\Lambda(e), I_\Lambda^{(c)}(e), T_\Lambda(e)), 1 - \mu(e) \rangle : x \in \mathfrak{S} \}$$

$$= \{ \langle x, (F_\Lambda(e), 1 - I_\Lambda(e), T_\Lambda(e)), 1 - \mu(e) \rangle : x \in \mathfrak{S} \}$$

Then,  $((\psi_\Lambda^\mu)^c)^c = [\{ \langle x, (F_\Lambda(e), 1 - I_\Lambda(e), T_\Lambda(e)), 1 - \mu(e) \rangle : x \in \mathfrak{S} \}]^c$

$$= \{ \langle x, (T_\Lambda(e), 1 - (1 - I_\Lambda(e)), F_\Lambda(e)), 1 - (1 - \mu(e)) \rangle : x \in \mathfrak{S} \}$$

$$= \{ \langle x, (T_\Lambda(e), I_\Lambda(e), F_\Lambda(e)), 1 - \mu(e) \rangle : x \in \mathfrak{S} \} = \psi_\Lambda^\mu, \forall e \in \Lambda, \mu(e) \in [0,1].$$

OR

$$\begin{aligned} & ((\psi_\Lambda^\mu)^c)^c = \left[ \langle u, (T_\Lambda^{(c)}(e) = F_\Lambda(e), I_\Lambda^{(c)}(e) = 1 - I_\Lambda(e), F_\Lambda^{(c)}(e) = T_\Lambda(e), \mu^{(c)}(e) = 1 - \mu(e)) \rangle : u \in \mathfrak{S} \right]^c \\ & = \left[ \langle u, (T_\Lambda(e) = F_\Lambda^{(c)}(e), I_\Lambda(e) = 1 - I_\Lambda^{(c)}(e), T_\Lambda(e) = F_\Lambda^{(c)}(e), \mu(e) = 1 - \mu^{(c)}(e)) \rangle : u \in \mathfrak{S} \right] \\ & = \left[ \langle u, (T_\Lambda(e) = F_\Lambda^{(c)}(e), I_\Lambda(e) = 1 - [1 - I_\Lambda(e)], T_\Lambda(e) = F_\Lambda^{(c)}(e), \mu(e) = 1 - (1 - \mu(e))) \rangle : u \in \mathfrak{S} \right] \\ & = \left[ \langle u, (T_\Lambda(e) = F_\Lambda^{(c)}(e), I_\Lambda(e) = I_\Lambda(e), T_\Lambda(e) = F_\Lambda^{(c)}(e), \mu(e) = \mu(e)) \rangle : u \in \mathfrak{S} \right] \\ & = \psi_\Lambda^\mu, \forall e \in \Lambda, \mu(e) \in [0,1]. \end{aligned}$$

### 4. Basic Operations

In this section, we present some basic operation with illustrative examples and propositions.

#### Definition 19

The union of two PNHSS  $\psi_{\Lambda_1}^\mu$  &  $\psi_{\Lambda_2}^\eta$  over  $\mathfrak{S}$  is a PNHSS  $\psi_\Lambda^\lambda$  defined as  $\psi(\Lambda, \lambda)$  where  $\Lambda = \Lambda_1 \cup \Lambda_2$  and  $\lambda(e) = \max(\mu(e), \eta(e))$  and  $\forall e \in \Lambda$  we have the follow:

$$\psi_\Lambda^\lambda = \psi_{\Lambda_1}^\mu \hat{\cup} \psi_{\Lambda_2}^\eta \quad \text{where } e \in \Lambda_1 \cap \Lambda_2$$

Where  $\hat{\cup}$  is a NHSS union.

#### Example 4

Assume that two PNHSS  $\psi_{\Lambda_1}^\mu$  &  $\psi_{\Lambda_2}^\eta$  over the same universe  $\mathfrak{S} = \{d_1, d_2, d_3, d_4\}$  are defined as follows:

$$\begin{aligned} \psi_{\Lambda_1}^\mu &= \{ \langle d_1, (\langle 0.6, 0.4, 0.5 \rangle, \langle 0.8, 0.3, 0.4 \rangle, \langle 0.8, 0.1, 0.2 \rangle, \langle 0.4, 0.7, 0.5 \rangle, (0.2)) \rangle, \\ & \quad \langle d_2, (\langle 0.4, 0.3, 0.2 \rangle, \langle 0.9, 0.4, 0.1 \rangle, \langle 0.3, 0.7, 0.6 \rangle, \langle 0.7, 0.4, 0.2 \rangle), (0.5) \rangle \} \\ \psi_{\Lambda_2}^\eta &= \{ \langle d_1, (\langle 0.8, 0.6, 0.3 \rangle, \langle 0.9, 0.5, 0.2 \rangle, \langle 0.3, 0.2, 0.4 \rangle, \langle 0.3, 0.2, 0.7 \rangle), (0.3) \rangle, \\ & \quad \langle d_3, (\langle 0.5, 0.3, 0.4 \rangle, \langle 0.2, 0.5, 0.7 \rangle, \langle 0.7, 0.1, 0.5 \rangle, \langle 0.4, 0.5, 0.2 \rangle), (0.4) \rangle \}. \end{aligned}$$

Then,

$$\begin{aligned} \psi_\Lambda^\lambda = \psi_{\Lambda_1}^\mu \hat{\cup} \psi_{\Lambda_2}^\eta &= \{ \langle d_1, (\langle 0.8, 0.4, 0.3 \rangle, \langle 0.9, 0.3, 0.2 \rangle, \langle 0.8, 0.1, 0.2 \rangle, \langle 0.4, 0.2, 0.5 \rangle), (0.3) \rangle \} \\ & \quad \langle d_2, (\langle 0.4, 0.3, 0.2 \rangle, \langle 0.9, 0.4, 0.1 \rangle, \langle 0.3, 0.7, 0.6 \rangle, \langle 0.7, 0.4, 0.2 \rangle), (0.5) \rangle \\ & \quad \langle d_3, (\langle 0.5, 0.3, 0.4 \rangle, \langle 0.2, 0.5, 0.7 \rangle, \langle 0.7, 0.1, 0.5 \rangle, \langle 0.4, 0.5, 0.2 \rangle), (0.4) \rangle \}. \end{aligned}$$

#### Proposition 2

Let  $\psi_{\Lambda_1}^\mu, \psi_{\Lambda_2}^\eta$  &  $\psi_{\Lambda_3}^\delta$  are PNHSS over  $\mathfrak{S}$ . Then

- 1)  $\psi_{\Lambda_1}^\mu \hat{\cup} \psi_{\Lambda_2}^\eta = \psi_{\Lambda_2}^\eta \hat{\cup} \psi_{\Lambda_1}^\mu$  (Commutative law)
- 2)  $(\psi_{\Lambda_1}^\mu \hat{\cup} \psi_{\Lambda_2}^\eta) \hat{\cup} \psi_{\Lambda_3}^\delta = \psi_{\Lambda_1}^\mu \hat{\cup} (\psi_{\Lambda_2}^\eta \hat{\cup} \psi_{\Lambda_3}^\delta)$  (Associative law)

Proof. In the following proof first two cases are trivial, we consider only the third case.

$$\begin{aligned} & 1) \quad \psi_{\Lambda_1}^\mu \hat{\cup} \psi_{\Lambda_2}^\eta \\ & = \{ \langle x, (\max\{T(\psi_{\Lambda_1}^\mu), T(\psi_{\Lambda_2}^\eta)\}, \min\{I(\psi_{\Lambda_1}^\mu), I(\psi_{\Lambda_2}^\eta)\}, \min\{F(\psi_{\Lambda_1}^\mu), F(\psi_{\Lambda_2}^\eta)\}, \max\{\mu(e), \eta(e)\}) \rangle \} \end{aligned}$$

$$= \{ \langle x, ( \max\{T(\psi_{\Lambda_2}^\eta), T(\psi_{\Lambda_1}^\mu)\}, \min\{I(\psi_{\Lambda_2}^\eta), I(\psi_{\Lambda_1}^\mu)\}, \min\{F(\psi_{\Lambda_2}^\eta), F(\psi_{\Lambda_1}^\mu)\}, \max\{\eta(e), \mu(e)\} \rangle \}$$

$$= \psi_{\Lambda_2}^\eta \hat{\cup} \psi_{\Lambda_1}^\mu.$$

$$2) \psi_{\Lambda_1}^\mu \hat{\cup} \psi_{\Lambda_2}^\eta$$

$$= \{ \langle x, ( \max\{T(\psi_{\Lambda_1}^\mu), T(\psi_{\Lambda_2}^\eta)\}, \min\{I(\psi_{\Lambda_1}^\mu), I(\psi_{\Lambda_2}^\eta)\}, \min\{F(\psi_{\Lambda_1}^\mu), F(\psi_{\Lambda_2}^\eta)\}, \max\{\mu(e), \eta(e)\} \rangle \}$$

Then  $(\psi_{\Lambda_1}^\mu \hat{\cup} \psi_{\Lambda_2}^\eta) \hat{\cup} \psi_{\Lambda_3}^\delta$

$$= \{ \langle x, ( \max\{ \max\{T(\psi_{\Lambda_1}^\mu), T(\psi_{\Lambda_2}^\eta)\}, T(\psi_{\Lambda_3}^\delta) \}, \min\{ \min\{I(\psi_{\Lambda_1}^\mu), I(\psi_{\Lambda_2}^\eta)\}, I(\psi_{\Lambda_3}^\delta) \}, \min\{F(\psi_{\Lambda_1}^\mu), F(\psi_{\Lambda_2}^\eta)\}, F(\psi_{\Lambda_3}^\delta) \}, \max\{ \max\{\mu(e), \eta(e)\}, \delta(e) \} \rangle \}$$

$$= \{ \langle x, ( \max\{T(\psi_{\Lambda_1}^\mu), T(\psi_{\Lambda_2}^\eta), T(\psi_{\Lambda_3}^\delta)\}, \min\{ \min\{I(\psi_{\Lambda_1}^\mu), I(\psi_{\Lambda_2}^\eta), I(\psi_{\Lambda_3}^\delta)\}, \min\{F(\psi_{\Lambda_1}^\mu), F(\psi_{\Lambda_2}^\eta), F(\psi_{\Lambda_3}^\delta)\}, \max\{\mu(e), \eta(e), \delta(e)\} \rangle \}$$

$$= \{ \langle x, ( \max\{T(\psi_{\Lambda_1}^\mu), \max\{T(\psi_{\Lambda_2}^\eta), T(\psi_{\Lambda_3}^\delta)\}\}, ( \min\{I(\psi_{\Lambda_1}^\mu), \min\{I(\psi_{\Lambda_2}^\eta), I(\psi_{\Lambda_3}^\delta)\}\} ), ( \min\{F(\psi_{\Lambda_1}^\mu), \min\{F(\psi_{\Lambda_2}^\eta), F(\psi_{\Lambda_3}^\delta)\}\} ), ( \max\{\mu(e), \max\{F\eta(e), \delta(e)\}\} \rangle \}$$

$$= \psi_{\Lambda_1}^\mu \hat{\cup} (\psi_{\Lambda_2}^\eta \hat{\cup} \psi_{\Lambda_3}^\delta).$$

**Definition 20**

The intersection of two PNHSS  $\psi_{\Lambda_1}^\mu$  &  $\psi_{\Lambda_2}^\eta$  over  $\mathfrak{S}$  is a PNHSS  $\psi_\Lambda^\lambda$  defined as  $\psi(\varepsilon, \lambda)$  where  $\Lambda = \Lambda_1 \cap \Lambda_2$  and  $\varepsilon(e) = \min(\mu(e), \eta(e))$  and  $\forall e \in \Lambda$  we have the follow:

$$\psi_\Lambda^\varepsilon = \psi_{\Lambda_1}^\mu \hat{\cap} \psi_{\Lambda_2}^\eta \quad \text{where } e \in \Lambda_1 \cap \Lambda_2$$

Where  $\hat{\cap}$  is a NHSS intersection.

**Example 5**

Consider example 4. By using basic neutrosophic intersection we can easily verify that  $\psi_\Lambda^\varepsilon = \psi_{\Lambda_1}^\mu \hat{\cap} \psi_{\Lambda_2}^\eta$ , where

$$\psi_\Lambda^\varepsilon = \psi_{\Lambda_1}^\mu \hat{\cap} \psi_{\Lambda_2}^\eta = \{ \langle d_1, (\langle 0.6, 0.6, 0.5 \rangle, \langle 0.8, 0.5, 0.4 \rangle, \langle 0.3, 0.2, 0.2 \rangle, \langle 0.3, 0.7, 0.7 \rangle, (0.2)) \rangle \}.$$

**Proposition 3**

Let  $\psi_{\Lambda_1}^\mu, \psi_{\Lambda_2}^\eta$  &  $\psi_{\Lambda_3}^\delta$  are PNHSS over  $\mathfrak{S}$ . Then

- 1)  $\psi_{\Lambda_1}^\mu \hat{\cap} \psi_{\Lambda_2}^\eta = \psi_{\Lambda_2}^\eta \hat{\cap} \psi_{\Lambda_1}^\mu$  (Commutative law)
- 2)  $(\psi_{\Lambda_1}^\mu \hat{\cap} \psi_{\Lambda_2}^\eta) \hat{\cap} \psi_{\Lambda_3}^\delta = \psi_{\Lambda_1}^\mu \hat{\cap} (\psi_{\Lambda_2}^\eta \hat{\cap} \psi_{\Lambda_3}^\delta)$  (Associative law)

Proof. Similar to proposition 2.

**Proposition 4**

Let  $\psi_{\Lambda_1}^\mu$  &  $\psi_{\Lambda_2}^\eta$  are PNHSS over  $\mathfrak{S}$ . Then

- 1)  $(\psi_{\Lambda_1}^\mu \hat{\cup} \psi_{\Lambda_2}^\eta)^c = (\psi_{\Lambda_2}^\eta)^c \hat{\cap} (\psi_{\Lambda_1}^\mu)^c$ .
- 2)  $(\psi_{\Lambda_1}^\mu \hat{\cap} \psi_{\Lambda_2}^\eta)^c = (\psi_{\Lambda_2}^\eta)^c \hat{\cup} (\psi_{\Lambda_1}^\mu)^c$ .

Proof. The proof is straightforward from Definitions 18 and 19.

**Proposition 5**

Let  $\psi_{\Lambda_1}^\mu$  &  $\psi_{\Lambda_2}^\eta$  are PNHSS over  $\mathfrak{S}$ . Then

- 1)  $\psi_{\Lambda_1}^\mu$  GNHS subset of  $\psi_{\Lambda_1}^\mu \hat{\cup} \psi_{\Lambda_2}^\eta$
- 2)  $\psi_{\Lambda_1}^\mu \hat{\cap} \psi_{\Lambda_2}^\eta$  GNHS subset of  $\psi_{\Lambda_1}^\mu$

Proof. It's clear from definition.

**Proposition 6**

Let  $\psi_{\Lambda_1}^\mu, \psi_{\Lambda_2}^\eta$  &  $\psi_{\Lambda_3}^\delta$  are PNHSS over  $\mathfrak{S}$ . Then

- 1)  $(\psi_{\Lambda_1}^\mu \hat{\cup} \psi_{\Lambda_2}^\eta) \hat{\cap} \psi_{\Lambda_3}^\delta = (\psi_{\Lambda_1}^\mu \hat{\cap} \psi_{\Lambda_3}^\delta) \hat{\cup} (\psi_{\Lambda_2}^\eta \hat{\cap} \psi_{\Lambda_3}^\delta)$ .
- 2)  $(\psi_{\Lambda_1}^\mu \hat{\cap} \psi_{\Lambda_2}^\eta) \hat{\cup} \psi_{\Lambda_3}^\delta = (\psi_{\Lambda_1}^\mu \hat{\cup} \psi_{\Lambda_3}^\delta) \hat{\cap} (\psi_{\Lambda_2}^\eta \hat{\cup} \psi_{\Lambda_3}^\delta)$ .

Proof. The proof can be easily obtained from relative definitions.

**5. AND and OR Operation.**

In this section, we introduce the definitions of AND and OR operations for Possibility neutrosophic hypersoft set, derive their properties, and give some examples.

**Definition 21**

Let  $\psi_{\Lambda_1}^\mu$  &  $\psi_{\Lambda_2}^\eta$  are PNHSS over  $\mathfrak{S}$ . Then  $\psi_{\Lambda_1}^\mu$  AND  $\psi_{\Lambda_2}^\eta$  denoted by  $\psi_{\Lambda_1}^\mu \hat{\wedge} \psi_{\Lambda_2}^\eta$  is given as

$$\psi_{\Lambda_1}^\mu \hat{\wedge} \psi_{\Lambda_2}^\eta = \psi_{\Lambda_1 \times \Lambda_2}^\varepsilon$$

such that  $\psi_{\Lambda_1 \times \Lambda_2}^\varepsilon(\alpha, \beta) = \psi_{\Lambda_1}^\mu(\alpha) \hat{\cap} \psi_{\Lambda_2}^\eta(\beta), \forall(\alpha, \beta) \in \Lambda_1 \times \Lambda_2$

Where  $\hat{\cap}$  is a NHSS intersection and  $\varepsilon(e) = \min(\mu(e), \eta(e))$ .

**Example 6**

Consider example 4. Then we can easily verify  $\psi_{\Lambda_1}^\mu \hat{\wedge} \psi_{\Lambda_2}^\eta = \psi_{\Lambda_1 \times \Lambda_2}^\varepsilon$  where

$$\psi_{\Lambda_1 \times \Lambda_2}^\varepsilon = \{ \langle (d_1, d_1), (\langle 0.6, 0.6, 0.5 \rangle, \langle 0.8, 0.5, 0.4 \rangle, \langle 0.3, 0.2, 0.4 \rangle, \langle 0.3, 0.7, 0.7 \rangle, (0.2)) \rangle, \langle (d_1, d_3), (\langle 0.5, 0.4, 0.5 \rangle, \langle 0.2, 0.5, 0.7 \rangle, \langle 0.7, 0.1, 0.5 \rangle, \langle 0.4, 0.7, 0.5 \rangle, (0.2)) \rangle, \langle (d_2, d_1), (\langle 0.4, 0.6, 0.3 \rangle, \langle 0.9, 0.5, 0.2 \rangle, \langle 0.3, 0.7, 0.6 \rangle, \langle 0.3, 0.4, 0.7 \rangle, (0.3)) \rangle, \langle (d_2, d_3), (\langle 0.4, 0.3, 0.4 \rangle, \langle 0.2, 0.5, 0.7 \rangle, \langle 0.3, 0.7, 0.6 \rangle, \langle 0.4, 0.5, 0.2 \rangle, (0.4)) \rangle \}$$

**Definition 22**

Let  $\psi_{\Lambda_1}^\mu$  &  $\psi_{\Lambda_2}^\eta$  are PNHSS over  $\mathfrak{S}$ . Then  $\psi_{\Lambda_1}^\mu$  OR  $\psi_{\Lambda_2}^\eta$  denoted by  $\psi_{\Lambda_1}^\mu \hat{\vee} \psi_{\Lambda_2}^\eta$  is given as

$$\psi_{\Lambda_1}^\mu \hat{\vee} \psi_{\Lambda_2}^\eta = \psi_{\Lambda_1 \times \Lambda_2}^\lambda$$

such that  $\psi_{\Lambda_1 \times \Lambda_2}^\lambda(\alpha, \beta) = \psi_{\Lambda_1}^\mu(\alpha) \hat{\cup} \psi_{\Lambda_2}^\eta(\beta), \forall(\alpha, \beta) \in \Lambda_1 \times \Lambda_2$

Where  $\hat{\cup}$  is a GNSS union and  $\lambda(e) = \max(\mu(e), \eta(e))$ .

**Example 7**

Consider example 4. Then we can easily verify  $\psi_{\Lambda_1}^\mu \hat{\vee} \psi_{\Lambda_2}^\eta = \psi_{\Lambda_1 \times \Lambda_2}^\lambda$  where

$$\psi_{\Lambda_1 \times \Lambda_2}^\lambda = \{ \langle (d_1, d_1), (\langle 0.8, 0.4, 0.3 \rangle, \langle 0.9, 0.3, 0.2 \rangle, \langle 0.8, 0.1, 0.4 \rangle, \langle 0.4, 0.2, 0.5 \rangle, (0.3)) \rangle, \dots \}$$

$\langle (d_1, d_3), (\langle 0.6, 0.3, 0.4 \rangle, \langle 0.8, 0.3, 0.4 \rangle, \langle 0.8, 0.1, 0.2 \rangle, \langle 0.4, 0.5, 0.2 \rangle, (0.4)) \rangle,$   
 $\langle (d_2, d_1), (\langle 0.8, 0.3, 0.2 \rangle, \langle 0.9, 0.4, 0.1 \rangle, \langle 0.3, 0.2, 0.4 \rangle, \langle 0.7, 0.2, 0.2 \rangle, (0.5)) \rangle,$   
 $\langle (d_2, d_3), (\langle 0.5, 0.3, 0.2 \rangle, \langle 0.9, 0.4, 0.1 \rangle, \langle 0.7, 0.1, 0.5 \rangle, \langle 0.7, 0.4, 0.2 \rangle, (0.5)) \rangle \}.$

### Proposition 7

Let  $\psi_{\Lambda_1}^\mu$  &  $\psi_{\Lambda_2}^\eta$  are PNHSS, then

- 1)  $(\psi_{\Lambda_1}^\mu \hat{\vee} \psi_{\Lambda_2}^\eta)^c = (\psi_{\Lambda_1}^\mu)^c \hat{\wedge} (\psi_{\Lambda_2}^\eta)^c.$
- 2)  $(\psi_{\Lambda_1}^\mu \hat{\wedge} \psi_{\Lambda_2}^\eta)^c = (\psi_{\Lambda_1}^\mu)^c \hat{\vee} (\psi_{\Lambda_2}^\eta)^c.$

Proof. The proof is straightforward from Definitions 18, 21 and 22.

### 4. Conclusions

In this paper we have introduced the concept of Possibility Neutrosophic Hypersoft Set and studied some of its properties like: subset, equal, complement with detailed examples. Basic operation of PNHSS are established like: union, intersection with illustrative examples. Some basic laws such as commutative, associative, distributive and De Morgens low are discussed. AND and OR operation of PNHSS are defined with suitable examples and some propositions.

In the future we use the new concept of PNHSS in decision making problem and in medical diagnosis. Also, the authors may extend this Possibility Neutrosophic Hypersoft Set to algebraic structure such as group, ring and field and their generalizations may be studied.

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