Neutrosophic Sets and Systems, Vol. 43, 2021
University of New Mexico

On Some Properties of Plithogenic Neutrosophic Hypersoft Almost Topological Group

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Abstract: The main objective of this study is to introduce the notion of plithogenic neutrosophic hypersoft almost topological group. We have defined some new concepts and investigated properties of regularly open set and regularly closed set and then we observed the definitions of plithogenic neutrosophic hypersoft closed mapping, open mapping and finally we have defined the definition of plithogenic neutrosophic hypersoft almost continuous mapping. By observing the definition of plithogenic neutrosophic hypersoft almost continuous mapping we have studied neutrosophic hypersoft topological group and plithogenic neutrosophic hypersoft almost topological group and some of their properties.

Keywords: Soft Set; Neutrosophic Hypersoft Set; Plithogenic Neutrosophic Hypersoft Set; Neutrosophic Hypersoft Topological Group; Plithogenic Neutrosophic Hypersoft Almost Topological Group.

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1. Introduction

In 1965, the fuzzy set (FS) theory concept was first defined by Zadeh [1]. With the help of FS, defined the concept of membership function and explained the idea of uncertainty. The concept of FS was generalized by Atanassov [2] and introduced the degree of non-membership as a component and proposed the intuitionistic fuzzy set (IFS). After that many researchers defined various new concepts on a generalization of FS. Smarandache [3] introduced neutrosophic set (NS) theory which are generalizations of IFS and FS and introduced the degree of indeterminacy as an independent component and discovered the neutrosophic set. Rana et. al. [16] discussed on plithogenic fuzzy whole hypersoft set of decision-making techniques.

The notion of soft set (SS) theory is one more fundamental set theory that was introduced by Molodtsov [3] in 1999. Now a day, SS theory is used in many branches of Science and Technology and SS has become one of the most popular branches in mathematics for its huge areas of applications in various research fields. Gradually, with the help of SS theory, many researchers have been introduced the notions of fuzzy SS [5], intuitionistic SS [6], neutrosophic SS [8] theory, etc. The concept of Hypersoft Set (HS) [14] theory was introduced by Smarandeche which is a generalization of SS theory. And also extended and introduced the concept of HS in the plithogenic environment and generalized it. Saqlain et. al [15] discussed the generalization of TOPSIS for Neutrosophic

Hypersoft set (NHS). Rahman et. al. [18] defined the development of Hybrids of HS with Complex FS, Complex IFS, and Complex NS, and also Rahman et. al. [19] discussed Convex and Concave HSs with their some properties. Saeed et. al [20] studied the fundamentals of HS theory and Abbas et. al. [21] discussed the basic operations on hypersoft sets and hypersoft points. Saqlain et. al. [22, 23] discussed aggregate operators of the neutrosophic hypersoft set and also single and multi-valued neutrosophic hypersoft set. Singh [24] worked on a plithogenic set (PS) for multi-variable data analysis and tried to develop new mathematical theories for precise representation through PS. Alkhazaleh [25] studied the concept of plithogenic soft set (PSS), also defined some properties of PSS. Zulqarnain et. al. [26] generalization of aggregated operators on NHS. Sankar et. al. [27] discussed Covid-19 by using PS. Khan et. al [28] studied the measures of linear and nonlinear interval-valued hexagonal fuzzy number. Haque et. al [29] discussed the multi-criteria group decision-making problems by exponential operational law in a generalised spherical fuzzy environment. Chakraborty et. al [30] studied the classification of trapezoidal bipolar neutrosophic numbers, de-bipolarization and implementation in cloud service based MCGDM problem. Zulqarnain et. al. [31] done work in solving decision-making problems using the TOPSIS method under an intuitionistic fuzzy hypersoft environment based on correlation coefficient and aggregation operators. Zulqarnain et. al. [32] discussed on operations of interval-valued NHS.

In this paper, we study the concept of the neutrosophic hypersoft topological group. Next, we introduce some definitions related to the neutrosophic hypersoft topological group and then we have introduced the definition of the Plithogenic Neutrosophic Hypersoft Almost Topological Group and discussed some related propositions.

2. Materials and Methods

2.1. Definition [3]

Let $\mathcal{U}$ be a universal set. A neutrosophic set (NS) A of $\mathcal{U}$ is denoted as $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in \mathcal{U}\}$, where $T_A(x), I_A(x), F_A(x) : \mathcal{U} \longrightarrow [0,1]$ are the corresponding degree of truth, indeterminacy, and falsity of any $x \in \mathcal{U}$. Note that $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

2.2. Definition [7, 8]

The component of neutrosophic set A is denoted by A^C and is defined as

$$A^c(x) = \{\langle \mathrm{x}, T_{A^c}(x) = F_A(x), I_{A^c}(x) = 1 - I_A(x), F_{A^c}(x) = T_A(x)\rangle : x \in \mathcal{U}\}.$$

2.3. Definition [7, 8]

Let $\mathcal{U}$ be a non-empty set and $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in \mathcal{U}\}$, $B = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle : x \in \mathcal{U}\}$, are neutrosophic sets. Then the neutrosophic set-theoretic operations are defined as follows:

(i) $A \cap B = \{\langle x, min(T_A(x), T_B(x)), min(I_A(x), I_B(x)), max(F_A(x), F_B(x))\rangle : x \in \mathcal{U}\}$

(ii) $A \cup B = \{\langle x, max(T_A(x), T_B(x)), max(I_A(x), I_B(x)), min(F_A(x), F_B(x))\rangle : x \in \mathcal{U}\}$

(iii) $A \leq B$ if for each $x \in X$, $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x)$, $F_A(x) \geq F_B(x)$.

2.4. Definition [3]

Let $\mathcal{U}$ be a crisp group (CG) and A be an NS of $\mathcal{U}$. Then A is said to be a Neutrosophic Subgroup (NSG) of $\mathcal{U}$ if and only if the following conditions are satisfied:

(i) $A(xy) \geq \min\{A(x), A(y)\}$

$i.e., T_A(xy) \geq T_A(x) \cap T_A(y)$, $I_A(xy) \geq I_A(x) \cap I_A(y)$, $F_A(xy) \geq F_A(x) \cap F_A(y)$

(ii) $A(x^{-1}) \geq A(x)$

$i.e., T_A(x^{-1}) \geq T_A(x)$, $I_A(x^{-1}) \geq I_A(x)$ and $F_A(x^{-1}) \leq F_A(x)$.

2.5. Definition [7]

Suppose X be a non-empty set and a neutrosophic topology is a family τ_N of neutrosophic subsets of X satisfying the following axioms:

(i) $0_N, 1_N \in \tau_N$

(ii) $G_{N_1} \cap G_{N_2} \in \tau_N$ for any $G_{N_1}, G_{N_2} \in \tau_N$

(iii) $\cup G_{N_i} \in \tau_N$; $\forall \{G_{N_i}: i \in J\} \subseteq \tau_N$

In this case, the pair (X, τ_N) is said to be a neutrosophic topological space and any neutrosophic set in τ_N is called a neutrosophic open set. The element of τ_N are known as open neutrosophic sets, a neutrosophic set F is a neutrosophic closed set if and only if it F^C is a neutrosophic open set.

2.6. Definition [9]

Let X be a group and $\mathcal{G}$ be a neutrosophic group on X. Let $\tau^{\mathcal{G}}$ be a neutrosophic topology on $\mathcal{G}$ and then $(\mathcal{G}, \tau^{\mathcal{G}})$ is said to be a neutrosophic topological group if the following conditions are satisfied:

(1) The mapping $\psi: (\mathcal{G}, \tau^{\mathcal{G}}) \times (\mathcal{G}, \tau^{\mathcal{G}}) \to (\mathcal{G}, \tau^{\mathcal{G}})$ defined by $\psi(x, y) = xy$, for all $x, y \in X$, is relatively neutrosophic continuous.

(2) The mapping $\mu: (\mathcal{G}, \tau^{\mathcal{G}}) \to (\mathcal{G}, \tau^{\mathcal{G}})$ defined by $\mu(x) = x^{-1}$, for all $x \in X$, is relatively neutrosophic continuous.

2.7. Definition: [5]

Let $\mathcal{U}$ be a universal set (US), let $\mathcal{P}(\mathcal{U})$ be the power set of $\mathcal{U}$ and E be the set of attributes values. Then the ordered pair of $(F, \mathcal{U})$ is said to be Soft Set (SS) over $\mathcal{U}$, where $F: E \to \mathcal{P}(\mathcal{U})$.

2.8. Definition: [4, 5]

Let $\mathcal{U}$ be a universal set (US) and $\mathcal{P}(\mathcal{U})$ be the power set of $\mathcal{U}$.

Let $a_1, a_2, a_3, \dots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attributes value are respectively the sets $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \dots, \mathcal{A}_n$, with $\mathcal{A}_i \cap \mathcal{A}_j = \phi$, for $i \neq j$ and $i, j \in \{1, 2, 3, \dots, n\}$. Let $E_\alpha = \mathcal{A}_1 \times \mathcal{A}_2, \times \dots \times \mathcal{A}_n$. Then the ordered pair (F, E_α) is called a Hypersoft Set (HS) of $\mathcal{U}$, where $F: E_\alpha \to \mathcal{P}(\mathcal{U})$.

2.9. Definition [4, 6]

Let $\mathcal{U}$ be a universal set (US) and P$\subseteq \mathcal{U}$. A plithogenic set (PS) is denoted by $P_r = (P, \alpha, E_\alpha, p, q)$ where α be an attribute, E_α is the respective range of attributes values, $p: P \times E_\alpha \to [0,1]^r$ is the degree of appurtenance function (DAF) and $q: E_\alpha \times E_\alpha \to [0,1]^s$ is the corresponding degree of contradiction function (DCF), where $r, s \in \{1,2,3\}$.

2.10. Definition [3, 4]

Let $\mathcal{U}_\mathcal{N}$ be the US termed as a neutrosophic universal set if for all $x \in \mathcal{U}_\mathcal{N}$, x has truth belongingness, indeterminacy belongingness, and falsity belongingness to $\mathcal{U}_\mathcal{N}$, i.e., membership of x belonging to $[0,1] \times [0,1] \times [0,1]$.

2.11. Definition [3, 4]

Let $\mathcal{U}_P$ be plithogenic universal set over an attribute value set α is termed as the plithogenic US if for all $x \in \mathcal{U}_P$, x belongs to $\mathcal{U}_P$ with some degree on the basis of each attribute value. This degree can be crisp, fuzzy, intuitionistic fuzzy or neutrosophic.

2.12. Definition [4, 5]

Let $\mathcal{U}_\mathcal{N}$ be a neutrosophic universal set and $\alpha = \{a_1, a_2, a_3, \dots, a_n\}$ be a set of attributes with attribute value sets respectively as $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \dots, \mathcal{A}_n$, with $\mathcal{A}_i \cap \mathcal{A}_j = \phi$, for $i \neq j$ and $i, j \in \{1, 2, 3, \dots, n\}$. Also, let $E_\alpha = \mathcal{A}_1 \times \mathcal{A}_2, \times \dots \times \mathcal{A}_n$. Then (F, E_α), where $F: E_\alpha \to \mathcal{P}(\mathcal{U}_\mathcal{N})$ is said to be a Neutrosophic Hypersoft Set (NHS) over $\mathcal{U}_\mathcal{N}$.

2.13. Definition [4]

Let $\mathcal{U}_P$ be a plithogenic universal set and $\alpha = \{a_1, a_2, a_3, \dots, a_n\}$ be a set of attributes with attribute value sets respectively as $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \dots, \mathcal{A}_n$, with $\mathcal{A}_i \cap \mathcal{A}_j = \phi$, for $i \neq j$ and $i, j \in \{1, 2, 3, \dots, n\}$. Also, let $E_\alpha = \mathcal{A}_1 \times \mathcal{A}_2, \times \dots \times \mathcal{A}_n$. Then (F, E_α), where $F: E_\alpha \to \mathcal{P}(\mathcal{U}_P)$ is said to be a plithogenic Hypersoft Set (PHS) over $\mathcal{U}_P$.

2.14. Definition [4]

The ordered pair (F, E_α) is said to be a plithogenic neutrosophic Hypersoft Set (PNHS) if for all $B \in$ range (F) and for all $i \in \{1,2,\dots,n\}$, there exists $f_{\mathcal{N}_i}: B \times R_i \to [0,1] \times [0,1] \times [0,1]$ such that for all $(b,r) \in B \times R_i$, $f_{\mathcal{N}_i}(b,r) \in [0,1] \times [0,1] \times [0,1]$.

A set of all the PNHSs over a set $\mathcal{U}$ is denoted by PNHS($\mathcal{U}$).

2.15. Definition [4]

Let the ordered pair (F, E_α) be a plithogenic neutrosophic Hypersoft Set (PNHS) of a crisp group $\mathcal{U}$. where $E_\alpha = \mathcal{A}_1 \times \mathcal{A}_2, \times \dots \times \mathcal{A}_n$ and for all $i \in \{1,2,\dots,n\}$, R_i are crisp groups. Then (F, E_α) is said to be a plithogenic neutrosophic Hypersoft Subgroup (PNHSG) of $\mathcal{U}$ if and only if for all $B \in$ range (F); for all $(b_1, r_1), (b_2, r_2) \in B \times R_i$ and for all $f_{\mathcal{N}_i}: B \times R_i \to [0,1] \times [0,1] \times [0,1]$; with $f_{\mathcal{N}_i}(b,r) = \{< (b,R), f^T_{\mathcal{N}_i}(b,r), f^I_{\mathcal{N}_i}(b,r), f^F_{\mathcal{N}_i}(b,r) >: (b,r) \in B \times R_i\}$, the following subsequent conditions are satisfied:

(i) $f^T_{\mathcal{N}_i}((b_1,r_1).(b_2,r_2)^{-1}) \geq \min\{f^T_{\mathcal{N}_i}(b_1,r_1), f^T_{\mathcal{N}_i}(b_2,r_2)\}$

(ii) $f^T_{\mathcal{N}_i}(b_1,r_1)^{-1} \geq f^T_{\mathcal{N}_i}(b_1,r_1)$

(iii) $f^I_{\mathcal{N}_i}((b_1,r_1).(b_2,r_2)^{-1}) \geq \min\{f^I_{\mathcal{N}_i}(b_1,r_1), f^I_{\mathcal{N}_i}(b_2,r_2)\}$

(iv) $f^I_{\mathcal{N}_i}(b_1,r_1)^{-1} \geq f^I_{\mathcal{N}_i}(b_1,r_1)$

(v) $f^F_{\mathcal{N}_i}((b_1,r_1).(b_2,r_2)^{-1}) \leq \max\{f^F_{\mathcal{N}_i}(b_1,r_1), f^F_{\mathcal{N}_i}(b_2,r_2)\}$

(vi) $f^F_{\mathcal{N}_i}(b_1,r_1)^{-1} \leq f^F_{\mathcal{N}_i}(b_1,r_1)$.

A set of all the PNHSG of a crisp group $\mathcal{U}$ is denoted by PNHSG($\mathcal{U}$).

3. Main Results

3.1. Definition

Let $NHS(\mathcal{U}_\mathcal{N}, E) = N$ be the family of all NHS over $\mathcal{U}_\mathcal{N}$ via attributes in E and $\tau_{\mathcal{U}_\mathcal{N}} \subseteq NHS(\mathcal{U}_\mathcal{N}, E)$. Then $\tau_{\mathcal{U}_\mathcal{N}}$ is said to be neutrosophic hypersoft topology (NHT) on N if the following conditions hold:

(i) $\phi_{\mathcal{U}_\mathcal{N}}, 1_{\mathcal{U}_\mathcal{N}} \in \tau_{\mathcal{U}_\mathcal{N}}$

(ii) The intersection of any finite number of members of $\tau_{\mathcal{U}_\mathcal{N}}$ also belongs to $\tau_{\mathcal{U}_\mathcal{N}}$.

(iii) The union of any collection of members of $\tau_{\mathcal{U}_\mathcal{N}}$ belongs to $\tau_{\mathcal{U}_\mathcal{N}}$.

Then $(N, \tau_{\mathcal{U}_\mathcal{N}})$ is said to be neutrosophic hypersoft topological space (NHTS). Every member of $\tau_{\mathcal{U}_\mathcal{N}}$ is called $\tau_{\mathcal{U}_\mathcal{N}}$- open neutrosophic hypersoft set. An NHS is called $\tau_{\mathcal{U}_\mathcal{N}}$- closed if and only if its complement is called $\tau_{\mathcal{U}_\mathcal{N}}$- open.

3.2. Definition

Let the pair $(F, E_\alpha) = H$ be a neutrosophic hypersoft group (NHG) of a crisp group (CG) $\mathcal{U}$. Let $\tau_{\mathcal{U}_G}$ be the neutrosophic hypersoft topology on H then $(H, \tau_{\mathcal{U}_G})$ is said to be neutrosophic hypersoft topological group (NHTG) if the following conditions are satisfied:

(1) The mapping $\psi: (H, \tau_{\mathcal{U}_G}) \times (H, \tau_{\mathcal{U}_G}) \to (H, \tau_{\mathcal{U}_G})$ such that $\psi(x,y) = xy$, for all $x, y \in H = (F, E_\alpha)$, is relatively neutrosophic hypersoft continuous.

(2) The mapping $\mu: (H, \tau_{\mathcal{U}_G}) \to (H, \tau_{\mathcal{U}_G})$ such that $\mu(x) = x^{-1}$, for all $x \in H = (F, E_\alpha)$, is relatively neutrosophic hypersoft continuous.

where $x = (b_1, r_1)$ and $y = (b_2, r_2)$. Then the pair $(H, \tau_{\mathcal{U}_G})$ is known as NHTG.

3.3. Definition

Let the pair $(F, E_\alpha) = H$ be an NHG of a crisp group (CG) $\mathcal{U}$. Let $\tau_{\mathcal{U}_G}$ be the neutrosophic hypersoft topology group on H. Then for fixed $\sigma = (a_1, a_2) \in H$, the left translation $l_\sigma: (H, \tau_{\mathcal{U}_G}) \to (H, \tau_{\mathcal{U}_G})$ is defined by $l_\sigma(x) = \sigma x,\ \forall\, x \in H$,

$$\sigma x = \{\langle \sigma, T_{\mathcal{U}_G}(\sigma x), I_{\mathcal{U}_G}(\sigma x), F_{\mathcal{U}_G}(\sigma x)\rangle : x \in H = (F, E_\alpha)\}.$$

Similarly, the right translation $r_\sigma: (H, \tau_{\mathcal{U}_G}) \to (H, \tau_{\mathcal{U}_G})$ is defined by $r_\sigma(x) = x\sigma\ \ \forall\, x \in H$,

$$x\sigma = \{\langle \sigma, T_{\mathcal{U}_G}(x\sigma), I_{\mathcal{U}_G}(x\sigma), F_{\mathcal{U}_G}(x\sigma)\rangle : x \in H = (F, E_\alpha)\}.$$

3.1. Lemma

Suppose $(F, E_\alpha) = H$ be an NHG of a crisp group (CG) $\mathcal{U}$. Let $\tau_{\mathcal{U}_G}$ be an NHTG in H. Then for each $\sigma = (a_1, a_2) \in \mathcal{G}_{\mathrm{e}}$, the translations l_σ and r_σ respectively neutrosophic hypersoft homomorphism of $(H, \tau_{\mathcal{U}_G})$ into itself.

Proof: From Proposition 3.11 [10], we have $l_\sigma[H] = \mathcal{G}$ and $r_\sigma[H] = H$, for all $\sigma \in \mathrm{H}_{\mathrm{e}}$ and let $\pi: (H, \tau_{\mathcal{U}_G}) \to (H, \tau_{\mathcal{U}_G}) \times (H, \tau_{\mathcal{U}_G})$ defined by $\pi(x) = (\sigma, x)$ for each $x \in H$. Then $r_\sigma: \beta o \pi$. Since $\sigma \in H_{\mathrm{e}}$, $T_{\mathcal{U}_G}(\sigma) = T_{\mathcal{U}_G}(e)$, $I_{\mathcal{U}_G}(\sigma) = I_{\mathcal{U}_G}(e)$ and $F_{\mathcal{U}_G}(\sigma) = F_{\mathcal{U}_G}(e)$. Thus $T_{\mathcal{U}_G}(\sigma) \supseteq T_{\mathcal{U}_G}(x)$, $I_{\mathcal{U}_G}(\sigma) \supseteq I_{\mathcal{U}_G}(x)$ and $F_{\mathcal{U}_G}(\sigma) \subseteq F_{\mathcal{U}_G}(x)$, for each $x \in H$. It follows from Proposition 3.34 [11] that $\pi: (H, \tau_{\mathcal{U}_G}) \to (H, \tau_{\mathcal{U}_G}) \times (H, \tau_{\mathcal{U}_G})$ is relatively neutrosophic hypersoft continuous. By the hypothesis, β is relatively neutrosophic hypersoft continuous. So, r_σ is relatively neutrosophic hypersoft continuous. Moreover ${r_\sigma}^{-1} = r_{\sigma^{-1}}$. Similarly, we are shown the relatively neutrosophic hypersoft continuous of ${l_\sigma}^{-1} = l_{\sigma^{-1}}$.

3.4. Definition

Let $PNHS(\mathcal{U}_{\mathcal{P}}, E) = P$ be the family of all PNHS over $\mathcal{U}_{\mathcal{P}}$ via attributes in E and $\tau_{\mathcal{U}_{\mathcal{P}}} \subseteq PNHS(\mathcal{U}_{\mathcal{P}}, E)$. Then $\tau_{\mathcal{U}_{\mathcal{P}}}$ is said to be plithogenic neutrosophic hypersoft topology (PNHT) on P if the following conditions are satisfied:

(i) $\phi_{\mathcal{U}_{\mathcal{P}}}, 1_{\mathcal{U}_{\mathcal{P}}} \in \tau_{\mathcal{U}_{\mathcal{P}}}$

(ii) The intersection of any two neutrosophic hypersoft sets in $\tau_{\mathcal{U}_{\mathcal{P}}}$ belongs to $\tau_{\mathcal{U}_{\mathcal{P}}}$.

(iii) The union of neutrosophic hypersoft sets in $\tau_{\mathcal{U}_{\mathcal{P}}}$ belongs to $\tau_{\mathcal{U}_{\mathcal{P}}}$.

Then $(P, \tau_{\mathcal{U}_{\mathcal{P}}})$ is said to be plithogenic neutrosophic hypersoft topological space (PNHTS).

3.5. Definition

The complement $\mathcal{A}^c$ of a plithogenic neutrosophic hypersoft open set (PNHOS) in an NHTS $(P, \tau_{\mathcal{U}_{\mathcal{P}}})$ is said to be plithogenic neutrosophic hypersoft closed set (PNHCoS) in $(P, \tau_{\mathcal{U}_{\mathcal{P}}})$.

3.6. Definition

Let the pair $(F, E_\alpha) = M$ be a PNHS of a crisp group (CG) $\mathcal{U}$. Let $\tau_{\mathcal{U}_G}$ [from definition 2.15] be the plithogenic neutrosophic hypersoft topology on M then $(M, \tau_{\mathcal{U}_G})$ is said to be plithogenic neutrosophic hypersoft topological group (PNHTG) if the following conditions are satisfied:

(1) The mapping $\psi: (M, \tau_{\mathcal{U}_G}) \times (M, \tau_{\mathcal{U}_G}) \to (M, \tau_{\mathcal{U}_G})$ such that $\psi(x, y) = xy$, for all $x, y \in M = (F, E_\alpha)$, is relatively plithogenic neutrosophic hypersoft continuous.

(2) The mapping $\mu: (M, \tau_{\mathcal{U}_G}) \to (M, \tau_{\mathcal{U}_G})$ such that $\mu(x) = x^{-1}$, for all $x \in M = (F, E_\alpha)$, is relatively plithogenic neutrosophic hypersoft continuous.

where $x = (b_1, r_1)$ and $y = (b_2, r_2)$. Then the pair $(M, \tau_{\mathcal{U}_G})$ is called a PNHTG.

3.7. Definition

Let the pair (F, E_α) be a PNHS of a crisp group (CG) $\mathcal{U}$, where $E_\alpha = \mathcal{A}_1 \times \mathcal{A}_2, \times \ldots \times \mathcal{A}_n$ and $i = \{1, 2, \ldots, n\}$, $\mathcal{A}_i$ are crisp groups. Let U, V be two PNHS in (F, E_α). We define the product of UV PNHS U, V and V^{-1} of V as follows:

$$UV(z) = \{< z, T_{UV}(z), I_{UV}(z), F_{UV}(z) >: z = (b, r) \in (F, E_\alpha)\}$$

where

$$T_{UV}(z) = \sup\{\min\{T_U(x), T_V(y)\}\}$$

$$I_{UV}(z) = \sup\{\min\{I_U(x), I_V(y)\}\}$$

$F_{UV}(z) = \sup\{\min\{F_U(x), F_V(y)\}\}$

where $z = x.y$ and $x = (b_1, r_1)$; $y = (b_2, r_2)$ and for $V = \{< z, T_V(z), I_V(z), F_V(z) >: z = (b, r) \in (F, E_\alpha)\}$,

we have $V^{-1} = \{< z, T_V(z^{-1}), I_V(z^{-1}), F_V(z^{-1}) >: z = (b, r) \in (F, E_\alpha)\}$.

3.8. Definition

Let the ordered pair (F, E_α) be a plithogenic neutrosophic hypersoft set, where $E_\alpha = \mathcal{A}_1 \times \mathcal{A}_2, \times \ldots \times \mathcal{A}_n$. Let $(P, \tau_{\mathcal{U}_\mathcal{P}})$ be a PNHTS and $\mathcal{A} = \{\langle x, T_\mathcal{A}(x), I_\mathcal{A}(x), F_\mathcal{A}(x)\rangle : x \in (F, E_\alpha)\}$ be a PNHS in $(P, \tau_{\mathcal{U}_\mathcal{P}})$, then the plithogenic neutrosophic hypersoft interior of $\mathcal{A}$ is defined as

$$PNH - int(\mathcal{A}) = \cup\{G: G \text{ is an } PNHOS \text{ in } X \text{ and } G \subseteq \mathcal{A}\}.$$

3.9. Definition

Let the ordered pair (F, E_α) be a plithogenic neutrosophic hypersoft set, where $E_\alpha = \mathcal{A}_1 \times \mathcal{A}_2, \times \ldots \times \mathcal{A}_n$. Let $(P, \tau_{\mathcal{U}_\mathcal{P}})$ be a PNHTS and $\mathcal{A} = \{\langle x, T_\mathcal{A}(x), I_\mathcal{A}(x), F_\mathcal{A}(x)\rangle : x \in (F, E_\alpha)\}$, be a PNHS in $(P, \tau_{\mathcal{U}_\mathcal{P}})$, then the plithogenic neutrosophic hypersoft closure of $\mathcal{A}$ is defined as

$$PNH - cl(\mathcal{A}) = \cap\{K: K \text{ is an } PNHCoS \text{ in } X \text{ and } K \supseteq \mathcal{A}\}.$$

3.10. Definition

A mapping $\phi: (P, \tau_{\mathcal{U}_{\mathcal{P}1}}) \longrightarrow (K, \tau_{\mathcal{U}_{\mathcal{P}2}})$ is a plithogenic neutrosophic hypersoft continuous if the pre-image of each open plithogenic neutrosophic hypersoft set in $(K, \tau_{\mathcal{U}_{\mathcal{P}2}})$ is open plithogenic neutrosophic hypersoft set in $(P, \tau_{\mathcal{U}_{\mathcal{P}1}})$.

3.11. Definition

Let $\mathcal{A}$ be a PNHS of a PNHTS $(P, \tau_{\mathcal{U}_\mathcal{P}})$, then $\mathcal{A}$ is called a plithogenic neutrosophic hypersoft semi-open set (PNHSOS) of $(P, \tau_{\mathcal{U}_\mathcal{P}})$ if there exists a $\mathcal{B} \in \tau_{\mathcal{U}_\mathcal{P}}$ such that $\mathcal{A} \subseteq PNH - Cl(\mathcal{B})$.

3.12. Definition

Let $\mathcal{A}$ be a PNHS of a PNHTS $(P, \tau_{\mathcal{U}_\mathcal{P}})$, then $\mathcal{A}$ is called a plithogenic neutrosophic hypersoft semi-closed set (PNHSCoS) of $(P, \tau_{\mathcal{U}_\mathcal{P}})$ if there exists a $\mathcal{B}^c \in \tau_{\mathcal{U}_\mathcal{P}}$ such that $PNH - Int(\mathcal{B}) \subseteq \mathcal{A}$.

3.13. Definition

A PNHS $\mathcal{A}$ of a PNHTS $(P, \tau_{\mathcal{U}_\mathcal{P}})$ is said to be a plithogenic neutrosophic hypersoft regularly open set (PNHROS) of $(P, \tau_{\mathcal{U}_\mathcal{P}})$ if $PNH - int(PNH - cl(\mathcal{A})) = \mathcal{A}$.

3.14. Definition

A PNHS $\mathcal{A}$ of a PNHTS $(P, \tau_{\mathcal{U}_\mathcal{P}})$ is said to be a plithogenic neutrosophic hypersoft regularly closed set (PNHRCoS) of $(P, \tau_{\mathcal{U}_\mathcal{P}})$ if $PNH - cl(PNH - int(\mathcal{A})) = \mathcal{A}$.

3.1. Theorem: (i) The intersection of any two PNHROSs is a PNHROS, and
(ii) The union of any two PNHRCoSs is a PNHRCoS.

Proof:

(i) Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be any two PNHROSs of a PNHTS $(P, \tau_{\mathcal{U}_\mathcal{P}})$. Since $\mathcal{A}_1 \cap \mathcal{A}_2$ is PNHOS, we have $\mathcal{A}_1 \cap \mathcal{A}_2 \subseteq PNH - int(PNH - cl(\mathcal{A}_1 \cap \mathcal{A}_2)$. Now, $PNH - int(PNH - cl(\mathcal{A}_1 \cap \mathcal{A}_2)) \subseteq PNH - int(PNH - cl(\mathcal{A}_1)) = \mathcal{A}_1$ and $PNH - int(PNH - cl(\mathcal{A}_1 \cap \mathcal{A}_2)) \subseteq PNH - int(PNH - cl(\mathcal{A}_2)) = \mathcal{A}_2$ implies that $PNH - int(PNH - cl(\mathcal{A}_1 \cap \mathcal{A}_2)) \subseteq \mathcal{A}_1 \cap \mathcal{A}_2$. Hence the theorem.

(ii) Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be any two PNHROSs of a PNHTS $(P, \tau_{\mathcal{U}_\mathcal{P}})$. Since $\mathcal{A}_1 \cup \mathcal{A}_2$ is PNHOS, we have $\mathcal{A}_1 \cup \mathcal{A}_2 \supseteq PNH - cl(PNH - int(\mathcal{A}_1 \cup \mathcal{A}_2))$. Now, $PNH - cl(PNH - int(\mathcal{A}_1 \cup \mathcal{A}_2)) \supseteq PNH - cl(PNH - int(\mathcal{A}_1)) = \mathcal{A}_1$ and $PNH - cl(PNH - int(\mathcal{A}_1 \cup \mathcal{A}_2)) \supseteq PNH - cl(PNH - int(\mathcal{A}_2)) = \mathcal{A}_2$ implies that $\mathcal{A}_1 \cup \mathcal{A}_2 \subseteq PNH - cl(PNH - int(\mathcal{A}_1 \cup \mathcal{A}_2))$. Hence the theorem.

3.15. Definition

Let $\phi: (P, \tau_{\mathcal{U}_{\mathcal{P}1}}) \longrightarrow (K, \tau_{\mathcal{U}_{\mathcal{P}2}})$ be a mapping from a PNHTS $(P, \tau_{\mathcal{U}_{\mathcal{P}1}})$ to another PNHTS $(K, \tau_{\mathcal{U}_{\mathcal{P}2}})$, then ϕ is called a Plithogenic neutrosophic hypersoft continuous mapping (PNHCM), if $\phi^{-1}(\mathcal{A}) \in \tau_{\mathcal{U}_{\mathcal{P}1}}$ for each $\mathcal{A} \in \tau_{\mathcal{U}_{\mathcal{P}2}}$; or equivalently $\phi^{-1}(\mathcal{B})$ is a PNHCoS of $(P, \tau_{\mathcal{U}_{\mathcal{P}1}})$ for each PNHCoS $\mathcal{B}$ of $(K, \tau_{\mathcal{U}_{\mathcal{P}2}})$.

3.16. Definition

Let $\phi: (P, \tau_{\mathcal{U}_{\mathcal{P}1}}) \longrightarrow (K, \tau_{\mathcal{U}_{\mathcal{P}2}})$ be a mapping from a PNHTS $(P, \tau_{\mathcal{U}_{\mathcal{P}1}})$ to another PNHTS $(K, \tau_{\mathcal{U}_{\mathcal{P}2}})$, then ϕ is called a plithogenic neutrosophic hypersoft open mapping (PNHOM), if $\phi(\mathcal{A}) \in \tau_{\mathcal{U}_{\mathcal{P}2}}$ for each $\mathcal{A} \in \tau_{\mathcal{U}_{\mathcal{P}1}}$.

3.17. Definition

Let $\phi: (P, \tau_{\mathcal{U}_{\mathcal{P}1}}) \longrightarrow (K, \tau_{\mathcal{U}_{\mathcal{P}2}})$ be a mapping from a PNHTS $(P, \tau_{\mathcal{U}_{\mathcal{P}1}})$ to another PNHTS $(K, \tau_{\mathcal{U}_{\mathcal{P}2}})$, then ϕ is called a plithogenic neutrosophic hypersoft closed mapping (PNHCoM) if $\phi(\mathcal{B})$ is a PNHCoS of $(K, \tau_{\mathcal{U}_{\mathcal{P}2}})$ for each PNHCoS $\mathcal{B}$ of $(P, \tau_{\mathcal{U}_{\mathcal{P}1}})$.

3.18. Definition

Let $\phi: (H, \tau_{\mathcal{U}_{\mathcal{P}1}}) \longrightarrow (K, \tau_{\mathcal{U}_{\mathcal{P}2}})$ be a mapping from a PNHTS $(H, \tau_{\mathcal{U}_{\mathcal{P}1}})$ to another PNHTS $(K, \tau_{\mathcal{U}_{\mathcal{P}2}})$, then ϕ is called a plithogenic neutrosophic hypersoft Semi-Continuous Mapping (PNHSCM), if $\phi^{-1}(\mathcal{A})$ is a plithogenic neutrosophic hypersoft semi-open set of $(H, \tau_{\mathcal{U}_{\mathcal{P}1}})$, for each $\mathcal{A} \in \tau_{\mathcal{U}_{\mathcal{P}2}}$.

3.19. Definition

Let $\phi: (P, \tau_{\mathcal{U}_{\mathcal{P}1}}) \longrightarrow (K, \tau_{\mathcal{U}_{\mathcal{P}2}})$ be a mapping from a PNHTS $(P, \tau_{\mathcal{U}_{\mathcal{P}1}})$ to another PNHTS $(K, \tau_{\mathcal{U}_{\mathcal{P}2}})$, then ϕ is called a Plithogenic neutrosophic hypersoft semi-open mapping (PNHSOM) if $\phi(\mathcal{A})$ is a PNHSOS for each $\mathcal{A} \in \tau_{\mathcal{U}_{\mathcal{P}1}}$.

3.20. Definition

Let $\phi: (P, \tau_{\mathcal{U}_{\mathcal{P}1}}) \longrightarrow (K, \tau_{\mathcal{U}_{\mathcal{P}2}})$ be a mapping from a PNHTS $(P, \tau_{\mathcal{U}_{\mathcal{P}1}})$ to another PNHTS $(K, \tau_{\mathcal{U}_{\mathcal{P}2}})$, then ϕ is called a Plithogenic neutrosophic hypersoft semi-closed mapping (PNHSCoM) if $\phi(\mathcal{B})$ is a PNHSCoS for each PNHCoS $\mathcal{B}$ of $(P, \tau_{\mathcal{U}_{\mathcal{P}1}})$.

3.21. Definition

A mapping $\phi: (M, \tau_{\mathcal{U}_{\mathcal{P}1}}) \longrightarrow (K, \tau_{\mathcal{U}_{\mathcal{P}2}})$ is said to be a plithogenic neutrosophic hypersoft almost continuous mapping (PNHACM), if $\phi^{-1}(\mathcal{A}) \in (M, \tau_{\mathcal{U}_{\mathcal{P}1}})$ for each plithogenic neutrosophic hypersoft regularly open set $\mathcal{A}$ of $(K, \tau_{\mathcal{U}_{\mathcal{P}2}})$.

3.22. Definition

Let the pair $(F, E_\alpha) = M$ be a PNHS of a crisp group (CG) $\mathcal{U}$. Let $\tau_{\mathcal{U}_G}$ [from definition 2.15] be the plithogenic neutrosophic hypersoft topology on M then $(M, \tau_{\mathcal{U}_G})$ is said to be plithogenic neutrosophic hypersoft almost topological group (PNHATG) if the following conditions are satisfied:

(1) The mapping $\psi: (M, \tau_{\mathcal{U}_G}) \times (M, \tau_{\mathcal{U}_G}) \rightarrow (M, \tau_{\mathcal{U}_G})$ such that $\psi(x, y) = xy$, for all $x, y \in M = (F, E_\alpha)$, is relatively plithogenic neutrosophic hypersoft almost continuous.

(2) The mapping $\mu: (M, \tau_{\mathcal{U}_G}) \rightarrow (M, \tau_{\mathcal{U}_G})$ such that $\mu(x) = x^{-1}$, for all $x \in M = (F, E_\alpha)$, is relatively plithogenic neutrosophic hypersoft almost continuous.

where $x = (b_1, r_1)$ and $y = (b_2, r_2)$. Then the pair $(M, \tau_{\mathcal{U}_G})$ is known as PNHATG.

3.2. Theorem:

Let $(M, \tau_{\mathcal{P}_G})$ be a PNHATG and let $\sigma = (a_1, a_2) \in M$ be any element. Then

(i) A mapping $g_\sigma: (M, \tau_{\mathcal{U}_G}) \longrightarrow (H, \tau_{\mathcal{U}_G})$ such that $g_\sigma(x) = \sigma x$, for all $x \in M$, is PNHACM;

(ii) A mapping $h_\sigma: (M, \tau_{\mathcal{U}_G}) \longrightarrow (M, \tau_{\mathcal{U}_G})$ such that $h_\sigma(x) = x\sigma$, for all $x \in M$, is PNHACM.

Proof:

(i) Let $\delta = (a_3, a_4) \in M$ and let W be a PNHROS containing $\sigma\delta$ in M. From Definition 3.22, $\exists$ plithogenic neutrosophic hypersoft open nbds $\mathcal{U}, \mathcal{V}$ of σ, δ in M so that $\mathcal{U}\mathcal{V} \subseteq W$. Especially, $\sigma\mathcal{V} \subseteq W$ that is $g_\sigma(\mathcal{V}) \subseteq W$. This shows that g_σ is PNHACM at δ and therefore g_σ is PNHACM.

(ii) Suppose $\delta = (a_3, a_4) \in M$ and $W \in PNHROS(M)$ containing $\delta\sigma$. Then $\exists$ PNHOSs $\delta \in \mathcal{U}$ and $\sigma \in \mathcal{V}$ in M so that $\mathcal{U}\mathcal{V} \subseteq W$. This shows $\mathcal{U}_\sigma \subseteq W$, i.e., $h_\sigma(\mathcal{U}) \subseteq W$. This implies h_σ is PNHACM at δ. As arbitrary element δ is in M, therefore h_σ is PNHACM.

3.3. Theorem:

Let U be PNHROS in a PNHATG $(M, \tau_{\mathcal{U}_G})$. Then the following conditions hold good, where $\sigma = (a_1, a_2)$

(1) $\sigma U \in PNHROS(M)$, for all $\sigma \in M$.

(2) $U\sigma \in PNHROS(M)$, for all $\sigma \in M$.

(3) $U^{-1} \in PNHROS(M)$.

Proof:

(1) First, we have to prove that $\sigma U \in \tau_{\mathcal{U}_G}$. Let $\delta = (a_3, a_4) \in \sigma U$. Then from Definition 3.22 of PNHATGs, $\exists$ PNHOSs $\sigma^{-1} \in W_1$ and $\delta \in W_2$in M so that $W_1W_2 \subseteq U$. Especially, $\sigma^{-1}W_2 \subseteq U$. i.e., equivalently, $W_2 \subseteq \sigma U$. This shows that $\delta \in PNH - int(\sigma U)$ and thus, $PNH - int(\sigma U) = \sigma U$. i.e., $\sigma U \in \tau_{\mathcal{U}_G}$. Consequently, $\sigma U \subseteq PNH - int(PNH - cl(\sigma U))$.

Now, we have to prove that $PNH - int(PNH - cl(\sigma U)) \subseteq \sigma U$. Since U is PNHOS, $PNH - cl(U) \in PNHRCoS(M)$. From Theorem 3.2, $g_{\sigma^{-1}}: (M, \tau_{\mathcal{U}_G}) \longrightarrow (M, \tau_{\mathcal{U}_G})$ is PNHACM and therefore, $\sigma PNH - cl(U)$ is PNHCoS. Thus, $PNH - int\big(PNH - cl(\sigma U)\big) \subseteq PNH - cl(\sigma U) \subseteq \sigma PNH - cl(U)$. i.e., $\sigma^{-1}PNH - int(PNH - cl(\sigma U)) \subseteq PNH - cl(U)$. Since $PNH - int(PNH - cl(\sigma U))$ is PNHROS, it follows that $\sigma^{-1}PNH - int\big(PNH - cl(\sigma U)\big) \subseteq PNH - int\big(PNH - cl(U)\big) = U$, i.e., $PNH - int(PNH - cl(\sigma U)) \subseteq \sigma U$. Thus $\sigma U = PNH - int(PNH - cl(\sigma U))$. This shows that $\sigma U \in PNHROS(M)$.

(2) Following Theorem 3.3 (1), the proof is straightforward.

(3) Let $x \in U^{-1}$, then $\exists$ PNHOS $\delta \in W$ in H so that $W^{-1} \subseteq U \Rightarrow W \subseteq U^{-1}$. Therefore U^{-1} has interior-point δ. Thus, U^{-1} is PNHOS. i.e., $U^{-1} \subseteq PNH - int\big(PNH - cl(U^{-1})\big)$. Now we have to prove that $PNH - int\big(PNH - cl(U^{-1})\big) \subseteq U^{-1}$. Since U is PNHOS, $PNH - cl(U)$ is PNHRCoS and hence $PNH - cl(U)^{-1}$ is PNHCoS in M. Therefore, $PNH - int\big(PNH - cl(U^{-1})\big) \subseteq PNH - cl(U^{-1}) \subseteq PNH - cl(U)^{-1} \Rightarrow PNH - int\big(PNH - cl(U^{-1})\big) \subseteq (PNH - cl(U))^{-1} \subseteq U^{-1}$. Thus, $U^{-1} = PNH - int\big(PNH - cl(U^{-1})\big)$. This shows that $U^{-1} \in PNHROS(H)$.

3.1. Corollary

Let Q be any PNHRCoS in a PNHATG in M. Then

(1) $\sigma Q \in PNHRCoS(M)$, for each $\sigma \in M$.

(2) $Q^{-1} \in PNHRCoS(M)$.

3.4. Theorem:

Let U be any PNHROS in a PNHATG M. Then

(1) $PNH - cl(U\sigma) = PNH - cl(U)\sigma$, for each $\sigma \in M$, where $\sigma = (a_1, a_2)$

(2) $PNH - cl(\sigma U) = \sigma PNH - cl(U)$, for each $\sigma \in M$.

(3) $PNH - cl(U^{-1}) = PNH - cl(U)^{-1}$.

Proof:

(1) Taking $\delta = (a_3, a_4) \in PNH - cl(U\sigma)$ and consider $q = \delta\sigma^{-1}$. Let $q \in W$ be PNHOS in M. Then $\exists$ PNHOSs $\sigma^{-1} \in V_1$ and $\delta \in V_2$ in M, so that $V_1V_2 \subseteq PNH - int(PNH - cl(W))$. By assumption, there is $g \in U\sigma \cap V_2 \Rightarrow g\sigma^{-1} \in U \cap V_1V_2 \subseteq U \cap PNH - int(PNH - cl(W)) \Rightarrow U \cap PNH -$

$int(PNH - cl(W)) \neq \phi_{\mathcal{U}_{\mathcal{P}}} \Rightarrow U \cap (PNH - cl(W)) \neq \phi_{\mathcal{U}_{\mathcal{P}}}$. Since U is PNHOS, $U \cap W \neq \phi_{\mathcal{U}_{\mathcal{P}}}$. i.e., $x \in PNH - cl(U)\sigma$.

Conversely, let $q \in PNH - cl(U)\sigma$. Then $q = \delta g$ for some $\delta \in PNH - cl(U)$.

To prove $PNH - cl(U)a \subseteq PNH - cl(Ua)$.

Let $\delta g \in W$ be an PNHOS in M. Then $\exists$ PNHOSs $\sigma \in V_1$ in M and $\delta \in V_2$ in M so that $V_1V_2 \subseteq PNH - int(PNH - cl(W))$. Since $\delta \in PNH - cl(U)$, $U \cap V_2 \neq \phi_{\mathcal{U}_{\mathcal{P}}}$. There is $g \in U \cap V_2$. This gives $g\sigma \in (U\sigma) \cap PNH - int(PNH - cl(W)) \Rightarrow (U\sigma) \cap (PNH - cl(W)) \neq \phi_{\mathcal{U}_{\mathcal{P}}}$. From Theorem 3.2, $U\sigma$ is PNHOS and thus $(U\sigma) \cap W \neq \phi_{\mathcal{U}_{\mathcal{P}}}$, therefore $q \in PNH - cl(U\sigma)$.

Therefore $PNH - cl(U\sigma) = PNH - cl(U)\sigma$.

(2) Following Theorem 3.4 (1), prove is straightforward.

(3) Since $PNH - cl(U)$ is PNHRCoS, $PNH - cl(U)^{-1}$ is PNHCoS in M. So, $U^{-1} \subseteq PNH - cl(U)^{-1}$ this implies $PNH - cl(U^{-1}) \subseteq PNH - cl(U)^{-1}$. Next, let $q \in PNH - cl(U)^{-1}$. Then $q = \delta^{-1}$, for some $\delta \in PNH - cl(U)$. Let $q \in V$ be any PNHOS in M. Then $\exists$ PNHOS U in M so that $\delta \in U$ with $U^{-1} \subseteq PNH - int(PNH - cl(V))$. Also, there is $\sigma \in \mathcal{A} \cap U$ which implies $\sigma^{-1} \in \mathcal{A}^{-1} \cap PNH - int(PNH - cl(V))$. That is, $\mathcal{A}^{-1} \cap PNH - int(PNH - cl(V)) \neq \phi_{\mathcal{U}_{\mathcal{P}}} \Rightarrow U^{-1} \cap PNH - cl(V) \neq \phi_{\mathcal{U}_{\mathcal{P}}} \Rightarrow \mathcal{A}^{-1} \cap V \neq \phi_{\mathcal{U}_{\mathcal{P}}}$, since U^{-1} is PNHOS. Therefore, $q \in PNH - cl(U)^{-1}$. Hence $PNH - cl(U^{-1}) \subseteq PNH - cl(U)^{-1}$.

3.5. Theorem:

Let Q be PNHRCo subset in a PNHATG M. Then the following statements are satisfied:

(1) $PNH - int(\sigma Q) = \sigma PNH - int(Q)$, for all $\sigma \in M$, where $\sigma = (a_1, a_2)$

(2) $PNH - int(Q\sigma) = PNH - int(Q)\sigma$, for all $\sigma \in M$.

(3) $PNH - int(Q^{-1}) = PNH - int(Q)^{-1}$.

Proof:

(1) Since Q is PNHRCoS, $PNH - int(Q)$ is PNHROS in M. Consequently, $\sigma PNH - int(Q) \subseteq PNH - int(\sigma Q)$. Conversely, let q be an arbitrary element of $PNH - int(\sigma Q)$. Assume that $q = \sigma\delta$, for some $\delta = (a_3, a_4) \in Q$. By assumption, this shows σQ is PNHCoS and that is $PNH - int(\sigma Q)$ is PNHROS in M. Suppose $\sigma \in U$ and $\delta \in V$ be PNHOSs in M, so that $UV \subseteq PNH - int(\sigma Q)$. Then $\sigma V \subseteq \sigma Q$, which it follows that $\sigma V \subseteq \sigma PNH - int(Q)$. Thus, $PNH - int(\sigma Q) \subseteq \sigma PNH - int(Q)$. Hence the statement follows.

(2) Following Theorem 3.5 (1), prove is straightforward.

(3) Since $PNH - int(Q)$ is PNHROS, so $PNH - int(Q)^{-1}$ is PNHOS in M. Therefore, $Q^{-1} \subseteq PNH - int(Q)^{-1}$ implies that $PNH - int(Q^{-1}) \subseteq PNH - int(Q)^{-1}$. Next, let q be an arbitrary element of $PNH - int(Q)^{-1}$. Then $q = \delta^{-1}$, for some $\delta \in PNH - int(Q)$. Let $q \in V$ be PNHOS in M. Then $\exists$ PNHOS U is in M so that $\delta \in U$ with $U^{-1} \subseteq PNH - cl(PNH - int(V))$. Also, there is $g \in Q \cap U$ which implies $g^{-1} \in Q^{-1} \cap PNH - cl(PNH - int(V))$. That is $Q^{-1} \cap PNH - cl(PNH - int(V)) \neq \phi_{\mathcal{U}_{\mathcal{P}}} \Rightarrow Q^{-1} \cap PNH - int(V) \neq \phi_{\mathcal{U}_{\mathcal{P}}} \Rightarrow Q^{-1} \cap V \neq \phi_{\mathcal{U}_{\mathcal{P}}}$, since Q^{-1} is PNHCoS. Hence $PNH - int(Q^{-1}) = PNH - int(Q)^{-1}$.

3.6. Theorem:

Let $\mathcal{A}$ be any PNHSOS in a PNHATG M. Then

(1) $PNH - cl(\sigma\mathcal{A}) \subseteq \sigma PNH - cl(\mathcal{A})$, for all $\sigma \in M$, where $\sigma = (a_1, a_2)$

(2) $PNH - cl(\mathcal{A}\sigma) \subseteq PNH - cl(\mathcal{A})\sigma$, for all $\sigma \in M$.

(3) $PNH - cl(\mathcal{A}^{-1}) \subseteq PNH - cl(\mathcal{A})^{-1}$.

Proof:

(1) As $\mathcal{A}$ is PNHSOS, $PNH - cl(\mathcal{A})$ is PNHRCoS. From Theorem 3.2, $g_{\sigma^{-1}}: (M, \tau_{\mathcal{U}_G}) \longrightarrow (M, \tau_{\mathcal{U}_G})$ is PNHACM. So, $\sigma PNH - cl(\mathcal{A})$ is PNHCoS. Hence $PNH - cl(\sigma\mathcal{A}) \subseteq \sigma PNH - cl(\mathcal{A})$.

(2) As $\mathcal{A}$ is PNHSOS, $PNH - cl(\mathcal{A})$ is PNHRCoS. From Theorem 3.2, $h_{\sigma^{-1}}: (M, \tau_{\mathcal{U}_G}) \longrightarrow (M, \tau_{\mathcal{U}_G})$ is PNHACM. So, $PNH - cl(\mathcal{A})\sigma$ is PNHCoS. Thus, $PNH - cl(\mathcal{A}\sigma) \subseteq PNH - cl(\mathcal{A})\sigma$.

(3) Since $\mathcal{A}$ is PNHSOS, so, $PNH - cl(\mathcal{A})$ is PNHRCoS and hence $PNH - cl(\mathcal{A})^{-1}$ is PNHCoS. Consequently, $PNH - cl(\mathcal{A}) \subseteq PNH - cl(\mathcal{A})^{-1}$.

4. **Limitation:** Every Plithogenic Neutrosophic Topological Group is Plithogenic Neutrosophic Hypersoft Almost Topological Group but the converse is not true.

5. Conclusion

In this paper, we have studied the concept of the Plithogenic Neutrosophic Hypersoft Almost Topological Group (PNHATG). To study PNHATG we have introduced some definitions related to PNHATG such as regularly open set and regularly closed set and then we observed the definitions of plithogenic neutrosophic hypersoft closed mapping, open mapping and finally, we have defined the definition of plithogenic neutrosophic hypersoft almost continuous mapping and then we have defined PNHATG and proved some theorems on PNHATG. We hope our work will encourage the reader for future work. In the future, we try to extend our work to study closed subgroups of Plithogenic Interval-valued Neutrosophic Hypersoft Almost Topological Group.

Acknowledgments: The Authors are very much thankful to the honorable referees for their valuable comments and suggestions.

Conflicts of Interest: The authors declare that they have no conflict of interest.

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Received: March 3, 2021. Accepted: Jun 2, 2021