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Ranking of Interval Valued Neutrosophic Numbers by Qualitative and Quantitative Criteria

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Abstract. In Operations Research, making decisions based on multiple criteria is crucial. Neutrosophic numbers produce a more efficient conclusion when dealing with Fuzzy Multi Criteria Decision Making (MCDM) problems. In this paper, we determine qualitative and quantitative criteria for selecting the best building construction project. The major goal of this work is to show how to use interval valued neutrosophic sets in solving MCDM issues using the Max-Product formula. To calculate the weighted average for interval valued neutrosophic numbers, we offer a new technique in max product. Three approaches are used to rank the interval valued neutrosophic numbers, and their application is demonstrated numerically.

Keywords: Fuzzy sets; Multiple Criteria Decision Making; Neutrosophic Fuzzy sets; Interval Valued Neutrosophic Set; Interval Valued Neutrosophic Numbers.

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1. Introduction

L.A. Zadeh [1] introduced fuzzy sets, fuzzy membership functions, and fuzzy logic in 1965. K. Atanassov [2] introduced the Intuitionistic Fuzzy set in 1986. It's a fuzzy set generalization with a membership grade, non-membership grade, and degree of indeterminacy. MCDM is a very significant and rapidly increasing subject in operations research. Indeterminacy should be incorporated into the model formulation of difficulties because MCDM problems are well addressed in fuzzy and intuitionistic fuzzy. In the decision-making process, indeterminacy is very significant. As a result of the growth of the MCDM field in a fuzzy environment, the Neutrosophic Fuzzy MCDM was proposed, and it was used in SAW, AHP, GP, TOPSIS, and other applications. Neutrosophic set, the generalization of fuzzy set and intuitionstic fuzzy sets. In Multiple Criteria Decision Making, neutrosophic numbers are ranked to rate tough problems. Using Neutrosophic Sets in MCDM and ranking methodologies will provide the best possible solution to challenging situations. Smarandache [3] introduced Neutrosophic set in 1998. The membership functions of Neutrosophic sets are Truth, Indeterminacy, and False. Smarandache and Wang proposed interval valued neutrosophic sets in 2005, and they introduced single valued neutrosophic sets in 2010. It independently expresses truth, indeterminacy, and false membership degree.

The paper Interval Neutrosophic Sets was published by Haibin Wang, et al. in 2004 [4]. They introduce and verify the convexity of interval valued neutrosophic sets, as well as many features, operations, and relations of interval neutrosophic sets. Athar Kharal published a paper A Neutrosophic Multi-Criteria Decision Making Method [14] in 2014. This study presents a method of MCDM based on Neutrosophic sets. It is the first time that neutrosophic sets have been introduced to the MCDM community. In 2014, Based on Bhattacharya's distance, Broumi S and Smarandache F [7] define a novel cosine similarity between two Interval valued neutrosophic sets. They used the cosine similarity measure in pattern recognitionin this research. Jun Ye published a paper Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making [10] in 2014. The Hamming and Euclidean distances between interval neutrosophic sets (INSs) are described in this study, and similarity measures between INSs are provided based on the relationship between distances and similarity measures. The article, Interval neutrosophic sets and their application in MCDM problems was published by Zhang et al in 2014 [8]. They established Interval neutrosophic numbers operators and presented a comparativeapproach between INN and aggregation operators for INSs in this work. Saha and Broumi [15] presented New Operators on Interval Valued Neutrosophic Sets in 2019. They defined some new IVNS operators and examined their properties in this study. Theoperators are highly useful when dealing with two interval-valued neutrosophic sets. In the decision-making process, the similarity measure is essential in determining the degree of similarity between the ideal and each alternative. In 2019, Wang, et al. [13] proposed a multi-criteria decision-making system based on improved cosine similarity measures with interval neutrosophic sets. The purpose of this study is to develop an MCDM technique for INSs based on a similarity measure.

In 2017, Deli and Subas [17] published the paper The concept of a single valued neutrosophic number (SVNN) is important for quantifying an unknown quantity, and the ranking of SVNNis atough problem in multi-attribute decision making problems. The goal of this work is to offer a methodology for using SVNNs to solve multi-attribute decision-making problems. They created a ranking approach based on the concept of values and uncertainties, which they used to multi attribute decision making issues where the ratings of alternatives on criteria are expressed as SVTN-numbers. Ranking methods of Single Valued Neutrosophic number and Its Applications to Multiple Criteria Decision Making [12] was published by D. Stanujkic, et al. in 2019. They demonstrate the utility of single-valued Neutrosophic sets in solving MCDM problems in this work. The proposed Ranking method's approach and numerical example were presented. Although single valued neutrosophic sets apply ranking methods, interval valued neutrosophic sets and numbers are also highly effective in ranking the alternatives.Ranking of Pentagonal Neutrosophic Numbers and its Applications to Solve Assignment Problem was published in 2020 by Radhika and Arun [18]. They suggest a new method for ranking neutrosophic numbers based on their magnitude in this work. Theyoffer a method for solving neutrosophic assignment issues with pentagonal neutrosophic numbers. The article, Ranking of single-valued neutrosophic numbers through the index of optimism and its reasonable properties was published by R. Chutia and F. Smarandache in 2021 [19]. The significance and vagueness of a single-valued neutrosophic number are used to construct a novel way of ranking neutrosophic numbers in this study. The method is unique inthe reasonable features of a ranking system.

There are many ranking methods that are applied in MCDM problems using the various types of neutrosophic numbers. The motive of our paper is to use Interval Valued Nutrosophic Numbers to build ranking techniques in MCDM. It gives better results when similarity measures, score function, and hamming distance are used to rank the interval valued neutrosophic numbers. The paper contains preliminaries and Basic elements of Interval Valued Neutrosophic sets, and ranking of IVNNs in section 2. The MCDM method based on Interval Valued Neutrosophic Numbers is provided in section 3. This proposed ranking approach is given numerical illustration in section 4. Finally, there is a ranking and a conclusion.

2. **Preliminaries**

Definition 2.1. Neutrosophic Set (NS) [3]

Let U be the universal set and every element x ∈ U has degree of True, Indeterminacy, False membership in S. Then the Neutrosophic set can be written as

$$S = \{\langle x, T_S(x), I_S(x), F_S(x)\rangle : x \in U\}$$

where, $0 \le T_S(x) + I_S(x) + F_S(x) \le 3$
and Truth Membership function $T_S : U \to [0, 1]$
Indeterminacy Membership function $I_S : U \to [0, 1]$
False Membership function $F_S : U \to [0, 1]$

Definition 2.2. Interval Valued Neutrosophic Set (IVNS) [8]

Let U be a nonempty set with generic elements in U denoted by x. The Interval Valued Neutrosophic set S in U is as follows

$$S = \{x : \langle x, T_S(x), I_S(x), F_S(x)\rangle ; x \in U\}$$

where, Interval Truth Membership Function $T_S(x) = [T_S^L, T_S^U]$
Interval Indeterminacy Membership Function $I_S(x) = [I_S^L, I_S^U]$
Interval False Membership Function $F_S(x) = [F_S^L, F_S^U]$
and for each point x ∈ U. $T_S(x), I_S(x), F_S(x) \in [0,1]$

Definition 2.3. Interval Valued Neutrosophic Number (IVNN)

For an IVNS S in U the triple interval $\langle [t_s^L, t_s^U], [i_s^L, i_s^U], [f_s^L, f_s^U] \rangle$ is called the Interval Valued Neutrosophic Number.

Operations on IVNN

Let $s_1 = \langle [t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U] \rangle$ and $s_2 = \langle [t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U] \rangle$ be two IVNN and $\lambda > 0$, then the basic operations are defined as follows [5],

$$i) s_1 + s_2 = \langle [t_1^L + t_2^L - t_1^L t_2^L, t_1^U + t_2^U - t_1^U t_2^U], [i_1^L i_2^L, i_1^U i_2^U], [f_1^L f_2^L, f_1^U f_2^U] \rangle$$
$$ii) s_1 . s_2 = \langle [t_1^L t_2^L, t_1^U t_2^U], [i_1^L + i_2^L - i_1^L i_2^L, i_1^U + i_2^U - i_1^U i_2^U], [f_1^L + f_2^L - f_1^L f_2^L, f_1^U + f_2^U - f_1^U f_2^U] \rangle$$
$$iii) \lambda s_1 = \left\langle [1 - (1 - t_1^L)^\lambda, 1 - (1 - t_1^U)^\lambda], [(i_1^L)^\lambda, (i_1^U)^\lambda], [(f_1^L)^\lambda, (f_1^U)^\lambda] \right\rangle$$
$$iv) s_1^\lambda = \left\langle [(t_1^L)^\lambda, (t_1^U)^\lambda], [1 - (1 - i_1^L)^\lambda, 1 - (1 - i_1^U)^\lambda], [1 - (1 - f_1^L)^\lambda, 1 - (1 - f_1^U)^\lambda] \right\rangle$$

Definition 2.4. Score Function of IVNN

Let $s_1 = \langle [t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U] \rangle$ be an IVNN then a Score Funtion $S_{(s_1)}$ is [6]

$$S_{(s_1)} = \frac{1}{4}[2 + t_1^L + t_1^U - 2(i_1^L + i_1^U) - (f_1^L + f_1^U)]$$

Definition 2.5. Cosine Similarity Measure

Let $s_1 = \langle [t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U] \rangle$ and $s_2 = \langle [t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U] \rangle$ be two IVNN then a Cosine Similarity Measure $C_{(s)}$ between two IVNN s_1 and s_2 is as follows [7] ,

$$C(s_1, s_2) = \frac{\frac{1}{n}\sum_{i=1}^{n} [(t_1^L + t_1^U)(t_2^L + t_2^U) + (i_1^L + i_1^U)(i_2^L + i_2^U) + (f_1^L + f_1^U)(f_2^L + f_2^U)]}{\sqrt{(t_1^L + t_1^U)^2 + (i_1^L + i_1^U)^2 + (f_1^L + f_1^U)^2}\sqrt{(t_2^L + t_2^U)^2 + (i_2^L + i_2^U)^2 + (f_2^L + f_2^U)^2}}$$

where, n = 1

Definition 2.6. Hamming Distance

Let $s_1 = \langle [t_1^L, t_1^U], [i_1^L, i_1^U], [f_1^L, f_1^U] \rangle$ and $s_2 = \langle [t_2^L, t_2^U], [i_2^L, i_2^U], [f_2^L, f_2^U] \rangle$ be two IVNN then the Hamming Distance $H_{(s)}$ between two IVNN s_1 and s_2 is as follows [10],

$$D(s_1, s_2) = \frac{1}{6}\sum_{i=1}^{n}[|t_1^L - t_2^L| + |t_1^U - t_2^U| + |i_1^L - i_2^L| + |i_1^U - i_2^U| + |f_1^L - f_2^L| + |f_1^U - f_2^U|]$$

2.1. **Ranking of Interval Valued Neutrosophic numbers**

Let s_1 and s_2 be two IVNNs, then the ranking method for comparing two IVNS is defined as follows, [12]

(i) Score Function

If $S_{(s_1)} > S_{(s_2)}$ then $s_1 > s_2$

(ii) Cosine Similarity Measure

$C_{(s_1)} > C_{(s_2)}$ then $s_1 > s_2$

(iii) Hamming Distance

$H_{(s_1)} > H_{(s_2)}$ then $s_1 < s_2$

3. **A MCDM approach based on Interval Valued Neutrosophic Numbers**

In this section, we proposed a new max-product approach for determining the weighted average for interval-valued neutrosophic numbers. This formula can be applied to any order of matrices containing interval-valued neutrosophic numbers, as well as two or more matrices of the same order. The remaining part contains the suggested method's procedure and flowchart.

Result 3.1. Let $A_{x\times y}$ and $B_{x\times y}$ be two matrix with an interval valued neutrosophic numbers. Then the Max-product for A and B is defined as follows,

$$\left\langle max\left(\prod_{m=1}^{n} m^L_{txy_1}, \prod_{m=1}^{n} m^L_{txy_2}, \dots\right), max\left(\prod_{m=1}^{n} m^U_{txy_1}, \prod_{m=1}^{n} m^U_{txy_2}, \dots\right)\right\rangle$$

$$\left\langle max\left(\prod_{m=1}^{n} m^L_{ixy_1}, \prod_{m=1}^{n} m^L_{ixy_2}, \dots\right), max\left(\prod_{m=1}^{n} m^U_{ixy_1}, \prod_{m=1}^{n} m^U_{ixy_2}, \dots\right)\right\rangle$$

$$\left\langle max\left(\prod_{m=1}^{n} m^L_{fxy_1}, \prod_{m=1}^{n} m^L_{fxy_2}, \dots\right), max\left(\prod_{m=1}^{n} m^U_{fxy_1}, \prod_{m=1}^{n} m^U_{fxy_2}, \dots\right)\right\rangle$$

Where m denotes the number of matrices.

We know, the fuzzy max-product composition,

Let A and B be x × y and y × z matrices respectively. The Fuzzy Max product composition of A and B is defined by,

$$\mu_{A\circ B} = max[\mu_A(x,y).\mu_B(y,z)]$$

From this we can extend the concept of Interval valued fuzzy number and Interval valued neutrosophic number.

Let A and B be $x \times y$ matrices with an interval valued fuzzy numbers. For A and B matrices, we should find the maximum product. The lower and upper limits are independent in this case. As a result, we calculate the max-product separately for the lower and upper limit values.

Assume that $A_{1\times 2}$ and $B_{1\times 2}$ be two matix with interval valued numbers.
we take A = $\left((a_1^L, a_1^U) \quad (a_2^L, a_2^U)\right)$ and B = $\left((b_1^L, b_1^U) \quad (b_2^L, b_2^U)\right)$

Max-product of A and B = $\left(max[(a_1^L.b_1^L),(a_2^L.b_2^L)] \quad max[(a_1^U.b_1^U),(a_2^U.b_2^U)]\right)$

Let A and B be two matrices with interval valued neutrosophic numbers.
we take A = $\left[\left\langle (a_{t1}^L, a_{t1}^U),(a_{i1}^L, a_{i1}^U),(a_{f1}^L, a_{f1}^U)\right\rangle \quad \left\langle (a_{t2}^L, a_{t2}^U),(a_{i2}^L, a_{i2}^U),(a_{f2}^L, a_{f2}^U)\right\rangle\right]$

B = $\left[\left\langle (b_{t1}^L, b_{t1}^U),(b_{i1}^L, b_{i1}^U),(b_{f1}^L, b_{f1}^U)\right\rangle \quad \left\langle (b_{t2}^L, b_{t2}^U),(b_{i2}^L, b_{i2}^U),(b_{f2}^L, b_{f2}^U)\right\rangle\right]$

Max-product of A and B =

$$[\langle max[(a_{t1}^L.b_{t1}^L),(a_{t2}^L.b_{t2}^L)], max[(a_{t1}^U.b_{t1}^U),(a_{t2}^U.b_{t2}^U)]\rangle$$
$$\langle max[(a_{i1}^L.b_{i1}^L),(a_{i2}^L.b_{i2}^L)], max[(a_{i1}^U.b_{i1}^U),(a_{i2}^U.b_{i2}^U)]\rangle$$
$$\langle max[(a_{f1}^L.b_{f1}^L),(a_{f2}^L.b_{f2}^L)], max[(a_{f1}^U.b_{f1}^U),(a_{f2}^U.b_{f2}^U)]\rangle]$$

This equation represents the max product value of 1×2 matrices, and we calculate the values for $x \times y$ matrices in the same way.

Max-product of m matrices =

$$\left\langle max\left(\prod_{m=1}^{n} m_{txy_1}^L, \prod_{m=1}^{n} m_{txy_2}^L, \dots\right), max\left(\prod_{m=1}^{n} m_{txy_1}^U, \prod_{m=1}^{n} m_{txy_2}^U, \dots\right)\right\rangle$$
$$\left\langle max\left(\prod_{m=1}^{n} m_{ixy_1}^L, \prod_{m=1}^{n} m_{ixy_2}^L, \dots\right), max\left(\prod_{m=1}^{n} m_{ixy_1}^U, \prod_{m=1}^{n} m_{ixy_2}^U, \dots\right)\right\rangle$$
$$\left\langle max\left(\prod_{m=1}^{n} m_{fxy_1}^L, \prod_{m=1}^{n} m_{fxy_2}^L, \dots\right), max\left(\prod_{m=1}^{n} m_{fxy_1}^U, \prod_{m=1}^{n} m_{fxy_2}^U, \dots\right)\right\rangle \quad (1)$$

Here m denotes the number of matrices. The max product of more than two matrices with x rows and y columns is represented by Equation (1). For Interval valued neutrosophic numbers, this equation is used as the weighted average max product formula.

3.1. **Procedure and Flowchart for the proposed method**

The ranking of interval-valued neutrosophic numbers is used to solve some difficult problems. Here we use score function, cosine similarity function, and hamming distance for ranking the values and we use two types of criteria which as qualitative and quantitative that are used for the more accurate outcome. The method's procedure is as follows, when we take k alternatives over m criteria by n experts.

Step 1: Define an available alternatives based on selected problem.

Step 2: Define a set of qualitative and quantitative criteria for evaluating the alternatives.

Step 3: The performance of the alternatives are evaluted by the group of experts. These performance are taken into interval valued neutrosophic numbers.

Step 4: Calculated overall ratings for qualitative and quantitative criteria separately by using weighted average max product formula given in Equation 1.

Step 5: Calculate the score function, cosine similarity function and hamming distance between qualitative and quantitative values.

Step 6: Rank the alternatives using the ranking of IVNNs and select the best one among those alternatives.

We take 4 alternatives over 3 qualitative criteria and 3 quantitative criteria by 3 experts. By these expressions, the following flowchart is the steps for solving the problem.

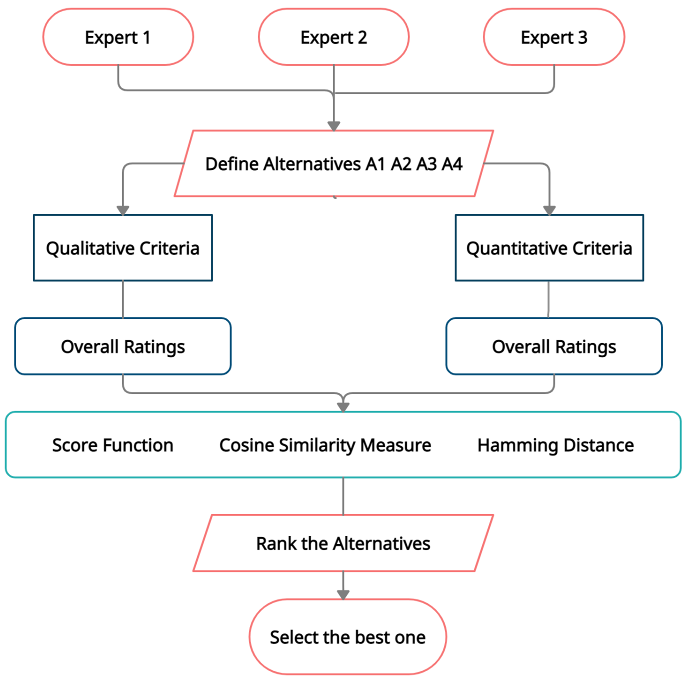


FIGURE 1. Flowchart

4. Numerical Illustration

An example of choosing the optimal construction for a building project to show how IVNNs may be used to solve MCDM challenges. Assume the manager is tasked with choosing the best tender construction for their structure. As a result, a group of three experts (E_1, E_2, E_3) was formed. On the basis of the following Qualitative and Quantitative criteria, the experts choose the best option out of four (A_1, A_2, A_3, A_4) alternatives.

Qualitative : C_1- Technical skills, C_2- Architectural Design, C_3- Reliability

Quantitative : C_4- Performance, C_5- Price, C_6- Period of work

The experts give the rating values to each alternative for the given criteria. The values are taken as Interval-valued neutrosophic numbers. When the alternatives have good criteria it has a high value in truth membership value. From this concept, experts have directly rated the alternatives in Interval values. In some critical situations, we use linguistic variables for collecting ratings from experts. Tables 1, 2 and 3 illustrate the ratings given by the three experts for qualitative criteria. Tables 4, 5, and 6 provide the ratings for qualitative criteria.

TABLE 1. Qulitative ratings by Expert 1

	C_1	C_2	C_3
A_1	[(.5,.7),(.2,.4),(.6,.7)]	[(.6,.8),(.3,.5),(.4,.6)]	[(.7,.8),(.4,.5),(.3,.5)]
A_2	[(.4,.5),(.3,.6),(.3,.4)]	[(.4,.5),(.2,.3),(.3,.4)]	[(.3,.5),(.2,.4),(.5,.6)]
A_3	[(.5,.6),(.1,.3),(.4,.5)]	[(.4,.5),(.2,.4),(.4,.5)]	[(.6,.8),(.3,.5),(.4,.5)]
A_4	[(.6,.7),(.3,.4),(.2,.3)]	[(.7,.8),(.3,.4),(.6,.7)]	[(.5,.7),(.4,.5),(.6,.7)]

TABLE 2. Qulitative ratings by Expert 2

	C_1	C_2	C_3
A_1	[(.3,.4),(.5,.6),(.6,.7)]	[(.6,.7),(.2,.3),(.4,.5)]	[(.4,.5),(.1,.3),(.5,.6)]
A_2	[(.6,.7),(.2,.3),(.3,.4)]	[(.4,.6),(.2,.4),(.3,.4)]	[(.5,.7),(.3,.5),(.3,.4)]
A_3	[(.6,.8),(.2,.4),(.4,.5)]	[(.5,.7),(.3,.4),(.4,.5)]	[(.4,.6),(.3,.4),(.4,.5)]
A_4	[(.3,.5),(.2,.4),(.6,.8)]	[(.4,.6),(.3,.5),(.5,.7)]	[(.3,.4),(.5,.6),(.5,.7)]

TABLE 3. Qulitative ratings by Expert 3

	C_1	C_2	C_3
A_1	[(.5,.6),(.2,.3),(.4,.5)]	[(.6,.8),(.1,.3),(.5,.6)]	[(.7,.8),(.3,.4),(.4,.5)]
A_2	[(.3,.4),(.3,.5),(.5,.7)]	[(.4,.6),(.3,.4),(.5,.6)]	[(.3,.5),(.1,.3),(.4,.5)]
A_3	[(.4,.5),(.2,.3),(.5,.7)]	[(.5,.6),(.2,.4),(.6,.7)]	[(.6,.7),(.3,.5),(.3,.4)]
A_4	[(.4,.6),(.2,.3),(.5,.6)]	[(.6,.7),(.2,.3),(.6,.8)]	[(.3,.4),(.5,.6),(.5,.7)]

TABLE 4. Quantitative ratings by Expert 1

	C_4	C_5	C_6
A_1	[(.3,.4),(.2,.3),(.6,.7)]	[(.4,.5),(.3,.4),(.5,.6)]	[(.3,.5),(.3,.4),(5,.6)]
A_2	[(.4,.5),(.3,.5),(.5,.6)]	[(.6,.7),(.3,.5),(.4,.5)]	[(.4,.5),(.2,.3),(.6,.7)]
A_3	[(.7,.8),(.2,.4),(.3,.5)]	[(.6,.7),(.3,.4),(.4,.6)]	[(.5,.7),(.2,.4),(.3,.4)]
A_4	[(.7,.8),(.2,.3),(.4,.5)]	[(.5,.7),(.3,.5),(.2,.4)]	[(.6,.8),(.3,.4),(.5,.6)]

Table 5. Quantitative ratings by Expert 2

	C_4	C_5	C_6
A_1	[(.2,.4),(.3,.4),(.6,.7)]	[(.3,.4),(.2,.3),(.5,.6)]	[(.4,.5),(.3,.4),(.6,.8)]
A_2	[(.4,.6),(.2,.4),(.6,.8)]	[(.5,.6),(.1,.2),(.7,.8)]	[(.6,.7),(.2,.3),(.7,.8)]
A_3	[(.7,.8),(.3,.5),(.4,.5)]	[(.6,.8),(.2,.3),(.4,.5)]	[(.8,.9),(.3,.5),(.5,.6)]
A_4	[(.5,.6),(.3,.4),(.2,.3)]	[(.4,.6),(.1,.3),(.5,.6)]	[(.5,.6),(.2,.3),(.3,.5)]

Table 6. Quantitative ratings by Expert 3

	C_4	C_5	C_6
A_1	[(.4,.5),(.2,.3),(.5,.7)]	[(.4,.5),(.2,.4),(.6,.7)]	[(.7,.8),(.3,.4),(.6,.7)]
A_2	[(.7,.9),(.4,.5),(.3,.4)]	[(.6,.7),(.3,.4),(.4,.5)]	[(.5,.6),(.4,.5),(.3,.4)]
A_3	[(.4,.6),(.2,.4),(.4,.5)]	[(.4,.5),(.2,.3),(.6,.7)]	[(.6,.7),(.3,.4),(.4,.5)]
A_4	[(.6,.8),(.4,.5),(.3,.4)]	[(.7,.8),(.3,.4),(.4,.5)]	[(.6,.7),(.4,.5),(.3,.4)]

We generate the overall rating values for qualitative and quantitative criteria using the weighted average max product formula.

$$\begin{aligned} A_1(T^L) &= max\{(.5 \times .3 \times .5), (.6 \times .6 \times .6), (.7 \times .4 \times .7)\} \\ &= max\{0.075, 0.216, 0.112\} = 0.216 \\ A_1(T^U) &= max\{(.7 \times .4 \times .6), (.8 \times .7 \times .8), (.8 \times .5 \times .8)\} \\ &= max\{0.168, 0.448, 0.32\} = 0.448 \end{aligned}$$

Similarly, the interminacy and false values are calculated for A_1 and the remaining alternatives are calculated in the same way.

Overall ratings for qualitative criteria by three experts

$$\begin{array}{ll} A_1 & [(0.216, 0.448), (0.02, 0.072), (0.144, 0.245)] \\ A_2 & [(0.072, 0.18), (0.018, 0.09), (0.06, 0.12)] \\ A_3 & [(0.15, 0.336), (0.027, 0.1), (0.08, 0.175)] \\ A_4 & [(0.168, 0.336), (0.1, 0.18), (0.18, 0.392)] \end{array}$$

Overall ratings for quantitative criteria by three experts

$$\begin{array}{ll} A_1 & [(0.084, 0.2), (0.027, 0.064), (0.18, 0.343)] \\ A_1 & [(0.18, 0.294), (0.024, 0.1), (0.126, 0.224)] \\ A_1 & [(0.24, 0.441), (0.018, 0.08), (0.096, 0.21)] \\ A_1 & [(0.21, 0.384), (0.024, 0.06), (0.045, 0.12)] \end{array}$$

Score Function

$$Qualitative\ A_1 = \frac{1}{4}[2 + 0.216 + 0.448 - 2(0.02 + 0.072) - (0.144 + 0.245)]$$
$$= 0.5227$$
$$Quantitative\ A_1 = \frac{1}{4}[2 + 0.084 + 0.2 - 2(0.027 + 0.064) - (0.18 + 0.343)]$$
$$= 0.3948$$
$$Average\ S(A_1) = \frac{A_1 + A_1}{2}$$
$$S(A_1) = 0.4587$$

Cosine Similarity Measure

$$C(A_1) = \frac{(0.664)(0.284) + (0.092)(0.091) + (0.389)(0.523)}{\sqrt{(0.664)^2 + (0.092)^2 + (0.389)^2}\sqrt{(0.284)^2 + (0.091)^2 + (0.523)^2}}$$
$$C(A_1) = 0.8581$$

Hamming Distance

$$D(A_1) = \frac{1}{6}[|0.216 - 0.084| + |0.448 - 0.2| + |0.02 - 0.027| + |0.072 - 0.064| + |0.144 - 0.18| + |0.245 - 0.343|]$$

$D(A_1) = 0.0882$

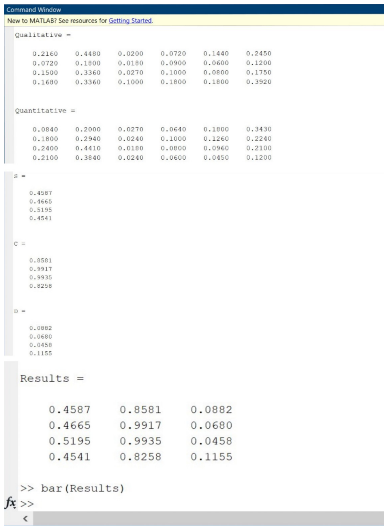
The remaining values of score function, cosine similarity measure and hamming distance are calculated for each alternatives in the same way.

Finally, the following table shows the ranking of the score function, cosine similarity function, and hamming distance.

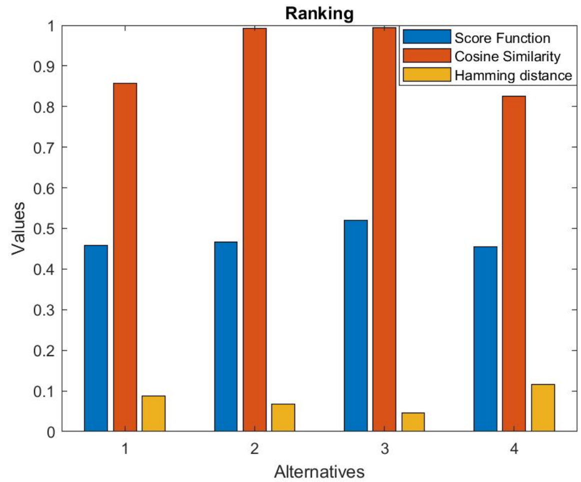
TABLE 7. The ranking results of three approaches

	$S(A)$	Rank	$C(A)$	Rank	$D(A)$	Rank
A_1	0.4587	III	0.8581	III	0.0882	III
A_2	0.4665	II	0.9917	II	0.0680	II
A_3	0.5195	I	0.9935	I	0.0458	I
A_4	0.4541	IV	0.8258	IV	0.1155	IV

The values are calculated manually and then ran through MATLAB R2020a. We can solve a large number of matrices using Matlab code. This helps to solve the difficult situations in multiple criteria. Figure (a) shows the final output results, and Figure (b) shows the final ranking as a bar chart.



(A) Ouput Result



(B) Ranking Bar chart

FIGURE 2. MATLAB R2020a

The colour blue represents the score function and the colour orange represents the cosine similarity measure in this bar chart, and A_3 being the highest value in both. The hamming distance is represented by the colour yellow. When the distance between two points is small, the value receives the highest ranking. Clearly, A_3 is the highest. In comparison to the other alternatives, A_3 is the best one.

Ranking order of the alternatives : $A_3 > A_2 > A_1 > A_4$

5. Conclusions

Ranking methods always give a good result in decision-making. Particularly comparison of neutrosophic numbers uses to rank the values very easily. In that situvation, single-valued neutrosophic numbers and interval-valued neutrosophic numbers play the most part. In this paper, the basic concepts of interval valued neutrosophic sets and ranking of IVNNs are presented. We proposed a new technique using max product to calculate the weighted average for interval valued neutrosophic numbers, which achieved a very efficient result. The ranking values of three approaches produced an ideal outcome for choosing the best option which is established in Numerical example. When compared to other alternatives, A_3 is the best choice in terms of both qualitative and quantitative criteria. The output result verified through Matlab. This work can further be developed to solve more complex multi criteria decision making problems using many types of Neutrosophic numbers such as bi-polar, m-polar neutrosophic numbers.

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