Neutrosophic Sets and Systems, Vol. 35, 2020

University of New Mexico

Ranking of Pentagonal Neutrosophic Numbers and its Applications to solve Assignment Problem

K.Radhika[1] and K.Arun Prakash[2]

[1] Department of Mathematics, Kongu Engineering College, Perundurai; radhikavisu@gmail.com

[2] Department of Mathematics, Kongu Engineering College, Perundurai; arunfuzzy@gmail.com

Abstract: Introduction of Neutrosophic sets and Neutrosophic numbers paves a way to handle uncertainty more effectively. In this paper we propose a new approach for ranking neutrosophic number by using its magnitude. We develop an algorithm for the solution of neutrosophic assignment problems involving pentagonal neutrosophic number. The proposed method is easy to understand and to apply for finding solution of neutrosophic assignment problems occurring in real life situations. To show the proposed strategy numerical models are given and the acquired results are analyzed.

Keywords: Neutrosophic sets, Neutrosophic number, Pentagonal neutrosophic number, Neutrosophic Assignment Problem, Optimal Solution.

1. Introduction

This section gives a survey of research work carried out so far to handle uncertainity. The novelty of present work, motivation behind it and structure of the remaining sections were also provided.

1.1: Literature survey

Smarandache [1] introduced neutrosophic sets having three components truthiness, indeterminacies, and falseness. Wang et al [3] introduced a single valued neutrosophic set, which is a subclass of a neutrosophic set presented by Smarandache [1]. Introduction of neutrosophic measure, neutrosophic integral, and neutrosophic probability by Smarandache [2,4] gave notation and many examples for neutrosophic measure, and consequently, the neutrosophic integral and neutrosophic probability are also defined. Many researchers have applied the neutrosophic logic in various fields.

To develop an optimization problem and its solution procedure in uncertain environment, the study of fuzzy number, intuitionistic fuzzy number, neutrosophic number and their ranking is necessary. Several researchers paid attention to fuzzy and intuitionistic fuzzy optimization methods by adopting various ranking techniques. But ranking of neutrosophic number is a risk task. To handle optimization problems having indeterminacy, ranking of neutrosophic numbers plays a vital role. S.Subasri and K.Selvakumari [5] ranked triangular neutrosophic number and applied the same to solve travelling salesman problems. Avishek Chakraborty [6], [7] gave a new ranking method to rank pentagonal neutrosophic number. Chakraborty A, Mondal SP, Ahmadian A, Senu N, Alam S,Salahshour S in 2018 [8] formatted Different forms of triangular neutrosophic numbers, and introduced de-neutrosophication techniques, and applied in critical path analysis. Smarandache [9] in 2019 approached TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. Nabeeh NA, Abdel-Basset M, El-Ghareeb HA, Aboelfetouh A in 2019[10] developed multi-criteria decision making approach for IoT-based enterprises using neutrosophic numbers. Neutrosophic functions and neutrosophic calculus, was defined by Florentin Smarandache [11].Neutrosophic ordinary differential equation of first order via neutrosophic numbers is epitomized by Sumathi IR, MohanaPriya V [12], Differential equations in neutrosophic environment are explored, and solution of second-order linear differential equation with trapezoidal neutrosophic numbers as boundary conditions is discussed by R. Sumathi [13].Minimal spanning tree is one of the important fact in the field of graph theory. Single valued neutrosophic minimal spanning tree and clustering method was solved by Ye [14] in 2014. Mandal & Basu [15] solved similarity measure to find spanning tree related with neutrosophic arena. Mullai et.al [16] formulated minimum spanning tree problem in bipolar neutrosophic number. Broumi et.al [17] formulated shortest path problem on single valued neutrosophic graphs. Kandasamy [18] developed double-valued neutrosophic sets and their application in minimum spanning tree problems. Broumi et.al [19] formulated neutrosophic shortest path for solving Dijkstra's algorithm in graph theory. Mohamed Abdel-Basset [20] introduced bipolar neutrosophic number and applied in decission making problems he also proposed a model in [21] to evaluate the supply chain sustainability metrics based on a combination of quality function deployment and plithogenic aggregation operations. Assignment problems plays an important role in optimization. Many researchers have handled assignment problems in fuzzy and intuitionistic fuzzy environment but in neutrosophic environment, only few articles were published, that too involving other forms neutrosophic numbers. This was the first attempt to discuss assignment problems in neutrosophic environments involving pentagonal neutrosophic numbers.

1.2. Motivation

For the past few years the ambiguous data were handled by fuzzy sets, intuitionistic fuzzy sets, interval valued fuzzy sets and many such structures. Recently, the introduction of neutrosophic sets proves to be more suited to handle vagueness than existing set theoretical structure. Fuzzy number can measure only uncertainty, intuitionistic and interval valued intuitionistic fuzzy number can measure uncertainty and vagueness not hesitation. Only neutrosophic number can measure all the three parameters effectively. Thus pentagonal neutrosophic number attracts more attention and paves path for new research.

1.3. Novelties

From its inception, a few research articles had just distributed in various journals in neutrosophic field. Only a countable amount of articles had dealt with pentagonal neutrosophic number in that other types neutrosophic number can be generalized from pentagonal neutrosophic number. Neutrosophic assignment problem is an area in which focus on the de-neutrosophication technique applied to solve neutrosophic assignment problem.

1.4. Contribution

In this research article, symmetric pentagonal neutrosophic fuzzy numbers are considered. These numbers are converted into crisp values by means of ranking approach by magnitude. There are many ranking procedures which rank uncertainty and vagueness separately. Here our ranking procedure converts all the three parts of pentagonal neutrosophic number into crisp number. Lastly, the proposed ranking was applied to solve neutrosophic assignment problem. Section-1 throws an introduction to neutrosophic number and literature survey in the field. Section-2 gives the preliminaries Section-3 covers representation, definition and ranking of pentagonal neutrosophic number. Section-4 provides mathematical formation of neutrosophic assignment problem, algorithm to solve it and numerical example illustrating the procedure. The last section gives the conclusion and scope of the future work.

2. Preliminaries

Definition 2.1: [1] Let X be a universe set. A neutrosophic set A on X is defined as $A=\{T_A(x), I_A(x), F_A(x) : x\in X\}$, where $T_A(x), I_A(x), F_A(x) : X \to -]0,1[+$ represents the degree of membership, degree of indeterministic, and degree of non-membership respectively of the element $x\in X$, such that $-0 \le T_A(x)+I_A(x)+F_A(x) \le 3+$.

Definition 2.2: [12] $(\alpha,\beta,\gamma)-$cut: The $(\alpha,\beta,\gamma)-$cut neutrosophic set is denoted by $F(\alpha,\beta,\gamma)$, where $\alpha,\beta,\gamma\in[0,1]$ and are fixed numbers, such that $\alpha+\beta+\gamma\le 3$ is defined as $F(\alpha,\beta,\gamma)=\{T_A(x), I_A(x), F_A(x) : x\in X, T_A(x)\ge\alpha, I_A(x)\le\beta, F_A(x)\le\gamma\}$.

Definition 2.3: [12] A neutrosophic set A defined on the universal set of real numbers R is said to be neutrosophic number if it has the following properties. (i) A is normal if there exist $x_0\in R$, such that $T_A(x_0)=1, I_A(x_0)=F_A(x_0)=0$. (ii) A is convex set for the truth function $T_A(x)$, i.e., $T_A(\mu x_1+(1-\mu)x_2)\ge \min(T_A(x_1), T_A(x_2)), \forall x_1,x_2\in R, \mu\in[0,1]$. (iii) A is concave set for the indeterministic function and false fuunction $I_A(x)$ and $F_A(x)$, i.e., $I_A(\mu x_1+(1-\mu)x_2)\ge \max(I_A(x_1), I_A(x_2)), \forall x_1,x_2\in R, \mu\in[0,1]$, $F_A(\mu x_1+(1-\mu)x_2)\ge \max(F_A(x_1), F_A(x_2)), \forall x_1,x_2\in R, \mu\in[0,1]$.

3 Ranking of pentagonal Neutrosophic number

This section gives the definition of symmetric pentagonal neutrosphic number and a method of ranking it by means of magnitude. Numerical examples were illustrated to explain the proposed ranking procedure.

Definition: Symmetric Pentagonal neutrosophic number: A is a subset of neutrosophic number in R with the following truth function, indeterministic function, and falsity function which is given by the following:

$$A=\{(a_1,a_2,a_3,a_4,a_5),(b_1,b_2,b_3,b_4,b_5),(c_1,c_2,c_3,c_4,c_5); p,q,r\}, \text{ where } p,q,r\in[0,1].$$

The accuracy membership function $A_\mu(x): \mathbb{R}\to[0,1]$, the indeterminacy membership function $A_\nu(x): \mathbb{R}\to[0,1]$ and the falsity membership function $A_\varphi(x): \mathbb{R}\to[0,1]$ are defined as follows:

$$A_\mu(x)=\begin{cases}0, x\le a_1\\ p\left(\dfrac{x-a_1}{a_2-a_1}\right), a_1\le x\le a_2\\ 1+\dfrac{(p-1)(x-a_3)}{a_3-a_2}, a_2\le x\le a_3\\ 1,\ x=a_3\\ 1+\dfrac{(p-1)(x-a_3)}{a_3-a_2}, a_2\le x\le a_3\\ p\left(\dfrac{a_5-x}{a_5-a_4}\right), a_4\le x\le a_5\\ 0, x\ge a_5\end{cases}$$

$$A_\nu(x)=\begin{cases}1, x\le b_1\\ 1+\dfrac{(q-1)(x-b_1)}{b_2-b_1}, b_1\le x\le b_2\\ q\left(\dfrac{b_3-x}{b_3-b_2}\right), b_2\le x\le b_3\\ 0, x=b_3\\ q\left(\dfrac{x-b_3}{b_4-b_3}\right), b_3\le x\le b_4\\ q+\dfrac{(1-q)(x-b_4)}{b_5-b_4}, b_4\le x\le b_5\\ 1, x\ge b_5\end{cases}$$

$$A_\varphi(x)=\begin{cases}1, x\le c_1\\ 1+\dfrac{(r-1)(x-c_1)}{c_2-c_1}, c_1\le x\le c_2\\ r\left(\dfrac{c_3-x}{c_3-c_2}\right), c_2\le x\le c_3\\ 0, x=c_3\\ r\left(\dfrac{x-c_3}{c_4-c_3}\right), c_3\le x\le c_4\\ r+\dfrac{(1-r)(x-c_4)}{c_5-c_4}, c_4\le x\le c_5\\ 1, x\ge c_5\end{cases}$$

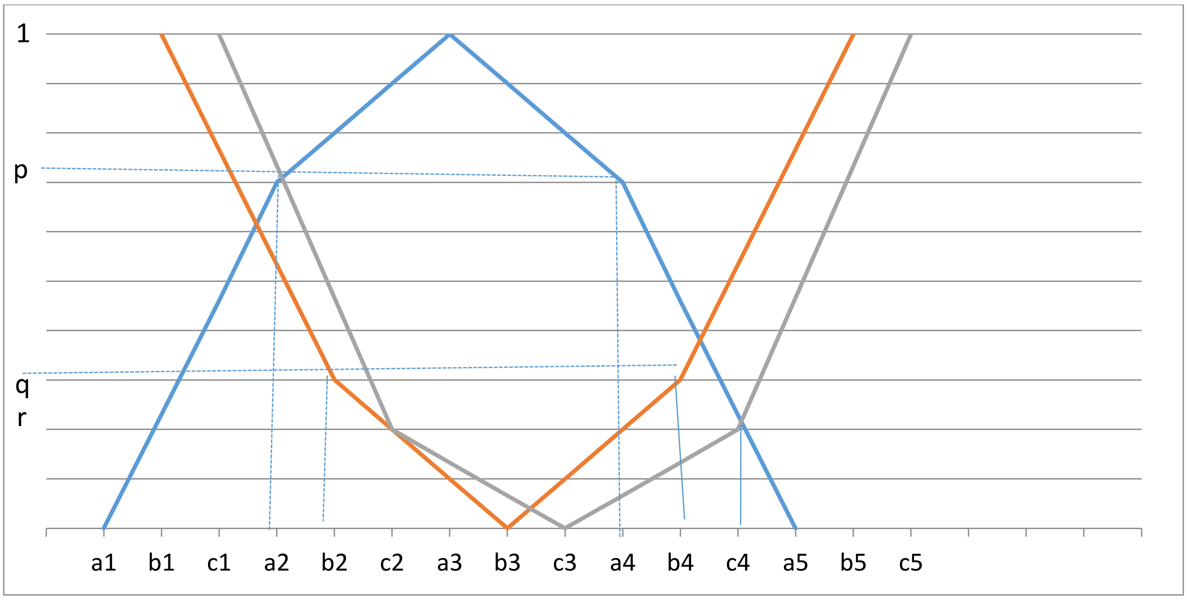

Figure 1.Representation of Symmetric Pentagonal neutrosophic number

3.1 Magnitude of a Pentagonal Neutrosophic Number

Let $A=\{(a_1,a_2,a_3,a_4,a_5),(b_1,b_2,b_3,b_4,b_5),(c_1,c_2,c_3,c_4,c_5);p,q,r\}$, where $p,q,r\in[0,1]$ be a symmetric pentagonal neutrosophic number whose accuracy membership function is given by

$$A_\mu(x)=\begin{cases}0, & x\le a_1\\ z_A^{L1}, & a_1\le x\le a_2\\ z_A^{L2}, & a_2\le x\le a_3\\ 1, & x=a_3\\ z_A^{R1}, & a_3\le x\le a_4\\ z_A^{R2}, & a_4\le x\le a_5\\ 0, & x\ge a_5\end{cases}$$

indeterminacy membership function is given by

$$A_\nu(x)=\begin{cases}1, & x\le b_1\\ k_A^{L1}, & b_1\le x\le b_2\\ k_A^{L2}, & b_2\le x\le b_3\\ 0, & x=b_3\\ k_A^{R1}, & b_3\le x\le b_4\\ k_A^{R2}, & b_4\le x\le b_5\\ 1, & x=b_5\end{cases}$$ and

and falsity membership function is given by

$$A_\phi(x)=\begin{cases}1, & x\le c_1\\ m_A^{L1}, & c_1\le x\le c_2\\ m_A^{L2}, & c_2\le x\le c_3\\ 0, & x=c_3\\ m_A^{R1}, & c_3\le x\le c_4\\ m_A^{R2}, & c_4\le x\le c_5\\ 1, & x\ge c_5\end{cases}$$

Here $z_A^{L1}(x):[a_1,a_2]\to[0,\mathrm{p}], z_A^{L2}(x):[a_2,a_3]\to[\mathrm{p},1], z_A^{R1}(x):[a_3,a_4]\to[\mathrm{p},1], \mathrm{z}_A^{R2}(\mathrm{x}):[\mathrm{a}_4,\mathrm{a}_5]\to[0,p]$,

where $z_A^{L1}(x), z_A^{L2}(\mathrm{x})$ are non-decreasing left accuracy functions and $z_A^{R1}(x),\ z_A^{R2}(x)$ are non-increasing right accuracy functions of symmetric Pentagonal neutrosophic number. Also $k_A^{L1}(x):[b_1,b_2]\to[q,1], k_A^{L2}(x):[b_2,b_3]\to[0,\mathrm{q}], k_A^{R1}(x):[b_3,b_4]\to[0,\mathrm{q}], k_A^{R2}(x):[b_4,b_5]\to[\mathrm{q},1]$, where $k_A^{L1}(x),\ \mathrm{k}_A^{L2}(\mathrm{x})$ are non-increasing left indeterminacy membership functions and $k_A^{R1}(x),\ \mathrm{k}_A^{R2}(x)$ are non-decreasing right indeterminacy membership functions of symmetric pentagonal neutrosophic number. Similarly the functions that occur in falsity membership function were defined as follows:

$m_A^{L1}(x):[\mathrm{c}_1, c_2]\to[\mathrm{r},1], \mathrm{m}_A^{L2}(x):[\mathrm{c}_2, c_3]\to[0,\mathrm{r}], \mathrm{m}_A^{R1}(x):[\mathrm{c}_3, c_4]\to[0,\mathrm{r}], \mathrm{m}_A^{R2}(x):[\mathrm{c}_4, c_5]\to[\mathrm{r},1]$, where $m_A^{L1}(x),\ \mathrm{m}_A^{L2}(\mathrm{x})$ are non-increasing left falsity membership function and $m_A^{R1}(x),\ \mathrm{m}_A^{R2}(x)$ are non-decreasing right falsity membership function of symmetric Pentagonal neutrosophic number. It is clear that $z_A^{L1}(x), z_A^{L2}(\mathrm{x})$ $z_A^{R1}(x), z_A^{R2}(x)$, $k_A^{L1}(x), \mathrm{k}_A^{L2}(\mathrm{x})$, $k_A^{R1}(x), \mathrm{k}_A^{R2}(x)$, $m_A^{L1}(x), \mathrm{m}_A^{L2}(\mathrm{x})$, $m_A^{R1}(x),\ \mathrm{m}_A^{R2}(x)$ are one to one and inverse exist.

The inverse functions of left and right accuracy, indeterminacy and falsity functions are defined as follows:

$f_A^{L1}:[0,p]\to R,\ f_A^{L2}:[p,1]\to R,\ f_A^{R1}:[p,1]\to R,\ f_A^{R2}:[0,p]\to R, g_A^{L1}:[1,q]\to R, g_A^{L2}:[0,q]\to R,$
$g_A^{R1}:[0,q]\to R, g_A^{R2}:[1,q]\to R, h_A^{L1}:[1,q]\to R, h_A^{L2}:[0,q]\to R, h_A^{R1}:[0,q]\to R, h_A^{R2}:[1,q]\to R,$ where

$$f_A^{L1}(y)=a_1+y\frac{(a_2-a_1)}{p},\ \ 0\le y\le p$$
$$f_A^{L2}(y)=a_2+\frac{(a_3-a_2)(y-p)}{1-p},\ \ p\le y\le 1$$
$$f_A^{R1}(y)=a_3+\frac{(a_4-a_3)(y-1)}{p-1},\ \ p\le y\le 1$$
$$f_A^{R2}(y)=a_5+y\frac{(a_4-a_5)}{p},\ \ 0\le y\le p$$

$$g_A^{L1}(y) = b_1 + \frac{(b_2 - b_1)(y-1)}{q-1}, \quad q \le y \le 1$$

$$g_A^{L2}(y) = b_3 - \frac{(b_3 - b_2)y}{q}, \quad 0 \le y \le q$$

$$g_A^{R1}(y) = b_3 + \frac{(b_4 - b_3)y}{q}, \quad 0 \le y \le q$$

$$g_A^{R2}(y) = b_4 + \frac{(b_5 - b_4)(y-q)}{1-q}, \quad q \le y \le 1$$

$$h_A^{L1}(y) = c_1 + \frac{(c_2 - c_1)(y-1)}{r-1}, \quad r \le y \le 1$$

$$h_A^{L2}(y) = c_3 - \frac{(c_3 - c_2)y}{r}, \quad 0 \le y \le r$$

$$h_A^{R1}(y) = c_3 + \frac{(c_4 - c_3)y}{r}, \quad 0 \le y \le r$$

$$h_A^{R2}(y) = c_4 + \frac{(c_5 - c_4)(y-r)}{1-r}, \quad r \le y \le 1$$

The magnitude denoted by Mag(A) of a symmetric pentagonal neutrosophic number $A = \{(a_1, a_2, a_3, a_4, a_5), (b_1, b_2, b_3, b_4, b_5), (c_1, c_2, c_3, c_4, c_5); p, q, r\}$, is determined as follows:

$$Mag(A) = \frac{1}{2}\int (f_A^{L1}(\alpha) + f_A^{L2}(\alpha) + f_A^{R1}(\alpha) + f_A^{R2}(\alpha) + 2\,\mathrm{a}_3 + g_A^{L1}(\alpha) + g_A^{L2}(\alpha) + g_A^{R1}(\alpha) + g_A^{R2}(\alpha) + 2\,\mathrm{b}_3 + h_A^{L1}(\alpha) + h_A^{L2}(\alpha) + h_A^{R1}(\alpha) + h_A^{R2}(\alpha) + 2\,\mathrm{c}_3)\, t(\alpha)\, d\alpha.$$

$$= \frac{1}{12}[p^2(a_1 + a_5) + (1+p)(a_2 + a_4) - (2p^2 + 2p - 10)a_3 - (q^2 + q - 2)(b_1 + b_5) + (1+q)(b_2 + b_4) + (2q^2 + 6)b_3 - (r^2 + r - 2)(c_1 + c_5) + (1+r)(c_2 + c_4) + (2r^2 + 6)c_3] \quad (1)$$

where the function $t(\alpha)$is a weighted function and is a non-negative and increasing function on [0,1] with t(0)=0,t(1)=1and $\int_0^1 t(\alpha)d\alpha = \frac{1}{2}$ we choose $t(\alpha) = \alpha$.The scalar value Mag(A) is used to rank Pentagonal neutrosophic number.

Remark:

When $p = 0, q = 1, r = 1$ pentagonal neutrosophic number becomes triangular neutrosophic number. Then the magnitude of A defined in equation (1) will be transformed into

$$Mag(A) = \frac{1}{12}[(a_2 + a_4) + 10a_3 + 2(b_2 + b_4) + 8b_3 + 2(c_2 + c_4) + 8c_3]$$

3.2 Ranking Procedure

Using the magnitude of symmetric pentagonal neutrosophic number defined above, the ordering of pentagonal neutrosophic numbers is explained in this section.

Let $A=\{(a_1,a_2,a_3,a_4,a_5),(b_1,b_2,b_3,b_4,b_5),(c_1,c_2,c_3,c_4,c_5);p,q,r\}$, and

$B=\{(d_1,d_2,d_3,d_4,d_5),(e_1,e_2,e_3,e_4,e_5),(i_1,i_2,i_3,i_4,i_5);u,v,w\}$ be any two arbitrary Pentagonal neutrosophic numbers. Then the ranking procedure is as follows:

Step 1: Compute Mag(A) , Mag(B),any one of the following cases prevail.

Step 2:

(i) If $Mag(A) > \text{Mag(B)}$, then A > B

(ii) If $Mag(A) < \text{Mag(B)}$, then A < B

(iii) If $Mag(A) = \text{Mag(B)}$, then A = B

3.3 Numerical examples

The ordering procedure in the previous section is illustrated by numerical examples. Consider the following sets of Pentagonal neutrosophic numbers.

Set 1:A={(.5,1.5,2.5,3.5,4.5)(0.3,1.3,2.3,3.3,4.3)(1.8,2.8,3.8,4.8,5.8);0.5,0.5,0.5}

B={(.7,1.7,2.5,3.5,4.7)(.5,1.5,2.2,3.2,4)(1.7,2.7,3.,4.7,5.7);0.5,0.5,0.5}

C={(1,4,7,10,13)(0.5,3.5,6.5,9.5,12.5)(4.5,7.5,9,12,14.5);0.5,0.5,0.5}

Set 2:A={(10,15,20,25,30)(0,3,5,7,10)(0,1,2,3,4,);0.5,0.5,0.5}

B={(5,10,15,20,25)(1,2,3,4,5)(1,1.5,2,2.5,3);0.5,0.5,0.5}

C={(10,20,30,40,50)(1,4,7,8,10)(1,1.5,2,2.5,3);0.5,0.5,0.5}

The table.1 gives the comparison of proposed ranking of Pentagonal neutrosophic numbers with the existing methods.

Author name and method	**Set 1**	**Set 2**
Proposed method	A=8.6 B=8.4 C=22.79 Result: C > A > B	A=27 B=20 C=38.5 Result: C>A>B
Avishek Chakraborty's De-Neutrosophication value [6]	A=2.86 B=2.66 C=7.66 Result: C > A = B	A=9 B=6.66 C=12.70 Result: C>A>B
Avishek Chakraborty's accuracy function value [7]	A= -.533 B= -.45 C= -2.33 Result: B > A >C	A=5 B=4 C=8 Result: C>A>B

Table.1 Comparison table for ranking Pentagonal neutrosophic numbers

The table 2 gives the numerical example of the De-Neutrosophication value of Triangular neutrosophic numbers.

Sl.No	Triangular neutrosophic numbers	proposed method of ranking
1	A={(1,2,3)(0.5,1.5,2.5)(1.2,2.7,3.5)}	6.083
	B={(.5,1.5,2.5)(.3,1.3,2.2)(.7,1.7,2.2}	3.723
	Result	A > B

Table.2 De-Neutrosophication value Triangular neutrosophic numbers

4. Application of Ranking of Pentagonal Neutrosophic Number in solving Neutosphic Assignment Problem

In this section neutrosophic assignment problem with pentagonal neutrosophic numbers as parameters was formulated, algorithm for identifying the optimal solution to neutrosophic assignment problem was stated. Finally a numerical example was produced to explain the proposed algorithm.

Need for Pentagonal neutrosophic numbers

Suppose there are n facilities and n jobs it is clear that in this case, there will be n assignments. Each facility or say worker can perform each job, one at a time. But there should be certain procedure by which assignment should be made so that the profit is maximized or the cost or time is minimized. But in our real life applications the times taken to complete the job undergo uncertainty, hesitation and vagueness. In such cases we cannot have the parameter as a real value. So we have to use some other representation of the parameter with which the uncertainty, hesitation and vagueness can be measured. The below discussion justify the need for selecting the cost parameter in the terms of Pentagonal neutrosophic number.

- **If the parameter is a real value -** uncertainty hesitation and vagueness cannot be handled
- **If the parameter is a fuzzy value-** uncertainty but hesitation and vagueness cannot be handled
- **If the parameter is an Intuitionistic Fuzzy value -** uncertainty and hesitation can be handled but vagueness cannot be handled.
- **If the parameter is a Pentagonal neutrosophic value -** uncertainty, hesitation and vagueness (i.e) all the components can be handled.

From the above discussion, it is clear that only pentagonal neutrosophic environment can tackle the impreciseness, hesitation and truthiness in a membership function of an uncertain number, which is more reliable, logical and realistic for a decision maker. Pentagonal neutrosophic numbers enabled to meet the imprecise parameters as well, which is approvingly the advantageous for the decision makers to analyze the result in a more precise manner. Moreover Pentagonal neutrosophic numbers generalize other types of neutrosophic numbers.

Pentagonal neutrosophic assignment problem may be formulated as follows:

Consider the assignment problem with cost function as Pentagonal neutrosophic number.

Minimize $z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij}$, subject to

$\sum_{i=1}^{n} x_{ij} = 1, j = 1,2,3,......n$ and $\sum_{j=1}^{n} x_{ij} = 1, j = 1,2,3,......n,$ $x_{ij} = 1$ or 0 for al $i, j,$ where c_{ij} is a Pentagonal neutrosophic number and the total cost for performing all the activity is given by $\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij}$.

Fundamental Theorems of a Pentagonal neutrosophic Assignment Problem

The solution of a Pentagonal neutrosophic assignment problem is fundamentally based on the following two theorems:

Theorem 1:

In a Pentagonal neutrosophic assignment problem, if we add or subtract an Pentagonal neutrosophic number to every element of any row (or column) of the Pentagonal neutrosophic parameter matrix $[c_{ij}]$, then an assignment that minimizes the total Pentagonal neutrosophic parameter on one matrix also minimizes the total Pentagonal neutrosophic parameter on the other matrix.

Minimize $Z = \sum^{n}\sum^{n} c_{ij}x_{ij}$ with $\sum^{n} x_{ij} = 1, j = 12....n, \sum^{n} x_{ij} = 1, i = 1,2..n$ $x_{ij} = 0 \text{ or } 1 \text{ for every } i, j$ then x_{ij}^{*} also minimize $Z^{*} = \sum^{n}\sum c_{ij}^{*}x_{ij}$ where $c_{ij}^{*} = c_{ij} - u_i - v_j$ *for all* $i, j = 1,2...n$ for all i, j=1, 2,……n are some real valued Pentagonal neutrosophic number.

Proof:

$$Z^{*} = \sum\sum c_{ij}^{*}x_{ij}$$

$$= \sum_{j}^{n}\sum_{i}^{n}(c_{ij} - u_i - v_j)x_{ij}$$

$$= \sum_{i}^{n}\sum_{j}^{n} c_{ij}x_{ij} - \sum_{i}^{n} u_i \sum_{j}^{n} x_{ij} - \sum_{j}^{n} v_j - \sum_{i}^{n} x_{ij}$$

$$= Z - \sum_{i}^{n} u_i - \sum_{j}^{n} v_j \quad \text{since} \sum_{i}^{n} x_{ij} = 1 \, and \sum_{j}^{n} x_{ij} = 1$$

This shows that the minimization of the new objective function Z^{*}yields the same solution as the minimization of original objective function Z

Theorem 2:

In a pentagonal neutrosophic assignment problem with parameter function c_{ij} if all $c_{ij} \geq 0$ the feasible solution which x_{ij} satisfies $\sum_{i}^{n}\sum_{j}^{n} c_{ij}x_{ij} = 0$ is an optimal solution.

Proof: Since all $c_{ij} \geq 0$ all $x_{ij} \geq 0$. The objective function $Z = \sum^{n} \sum^{n} c_{ij} x_{ij}$ cannot be negative the minimum possible that Z can have is 0.Therefore any feasible solution x_{ij} obtained that satisfies $Z = \sum^{n} \sum^{n} c_{ij} x_{ij} = 0$ will be optimal.

Algorithm to solve Pentagonal neutrosophic assignment problem:

We, now introduce a new algorithm called the Pentagonal neutrosophic Hungarian method for finding a Pentagonal neutrosophic optimal assignment for Pentagonal neutrosophic assignment problem.

Step 1: Determine the Pentagonal neutrosophic parameter table from the given problem.

Step 2: Convert the given Pentagonal neutrosophic assignment matrix to crisp by using the magnitude method.

Step 3: Subtract the row minimum from each row entry of that row. Subtract the column minimum of the resulting matrix from each column entry of that column. Each column and row now has at least one zero.

Step 4: In the modified assignment table obtained in step 3, search for optimal assignment as follows.

Examine the rows successively until a row with a single zero is found. Assign the zero and cross off all other zeros in its column. Continue this for all the rows. Repeat the procedure for each column of reduced assignment table. If a row and / or column have two or more zeros assign arbitrary any one of these zeros and cross off all other zeros of that row/column. Repeat the above process successively until the chain of assigning or cross ends.

Step 5: If the number of assignments is equal to n, the order of the parameter matrix, optimal solution is reached. If the number of assignments is less than n, parameter matrix, go to the step 6.

Step 6: Draw the minimum number of horizontal and / or vertical lines to cover all the zeros of the reduced assignment matrix. This can be done by using the following:

(i)Mark rows that do not have any assigned zero. (ii)Mark columns that have zeros in the marked rows. (iii)Mark rows that do have zeros in the marked columns. Repeat (ii) and (iii) of the above until the chain of marking is completed. Draw lines through all the unmarked rows and marked columns. This gives the desired minimum number of lines.

Step 7: Develop the new revised reduced parameter matrix as follows: Find the smallest entry of the reduced matrix not covered by any of the lines. Subtract this entry from all the uncovered entries and add the same to all the entries lying at the intersection of any two lines.

Step 8: Repeat step 5 to step 7 until optimal solution to the given assignment problem is attained.

Numerical example:

Suppose we want to assign jobs A, B, C, D to machine M1, M2, M3, and M4. Our aim is to find the minimum time so that the job is completed so that each machine is assigned only one job, The time parameter may not be a real value since the time taken to complete a job depend on the facts such as (i) working condition of the machine (ii) climatic condition and so on. So we represent the time parameter as Pentagonal neutrosophic number. The problem can be considered as follows.

Minimize $Z = \sum_{j}^{4} \sum_{i}^{4} c_{ij} x_{ij}$

Where

c_{11} = {(8,13,19,24,30)(7,10,15,22,27)(10,16,23,25,32);0.5,0.5,0.5}

c_{12} = {(7,12,18,24,30)(6,10,14,20,25)(10,15,20,25,35);0.6,0.4,0.3}

c_{13} = {(3,8,14,20,26)(2,7,12,18,22)(5,10,15,24,30);0.4,0.3,0.4}

c_{14} = {(10,15,20,26,32)(7,12,18,22,26)(12,16,22,28,35);0.6,0.4,0.3}

c_{21} = {(8,14,20,26,32)(6,12,18,22,28)(10,18,24,28,35);0.4,0.5,0.4}

c_{22} = {(6,10,15,20,25)(4,8,12,18,22)(8,14,20,24,30);0.6,0.6,0.5}

c_{23} = {(9,14,20,25,30)(6,12,16,21,24)(12,15,23,28,35);0.5,0.4,0.3}

c_{24} = {(11,15,19,23,27)(8,12,17,21,24)(14,18,22,26,30);0.6,0.6,0.4}

c_{31} = {(7,10,13,16,20)(6,8,12,15,18)(10,14,18,22,25);0.7,0.4,0.5}

c_{32} = {(12,15,18,24,26)(5,9,13,17,21)(10,14,18,22,26);0.8,0.6,0.2}

c_{33} = {(6,11,15,18,24,26)(9,13,17,20,23)(14,18,22,25,29);0.6,0.4,0.3}

c_{34} = {(7,11,14,17,26)(4,8,13,19,23)(9,14,19,24,28);0.6,0.5,0.4}

c_{41} = {(4,9,15,21,27)(3,8,13,19,23)(6,11,16,25,31);0.6,0.6,0.4}

c_{42} = {(11,14,17,23,25)(7,11,16,20,23)(13,17,21,25,29);0.5,0.3,0.4}

c_{43} = {(6,11,15,18,24,26)(7,11,15,18,20)(13,17,21,24.27);0.7,0.3,0.4}

c_{44} = {(10,14,18,22,26)(5,9,13,19,23)(9,15,21,25,31);0.5,0.4,0.3}

Subject to $\sum_{j=1}^{4} x_{ij} = 1, \; i = 1, 2, 3, 4. \; \sum_{i}^{4} x_{ij} = 1, \; j = 1, 2, 3, 4., \; x_{ij} = 1 \text{ or } 0, \text{ for all } i, j.$

Pentagonal neutrosophic assignment matrix in the crisp form

C_{ij} =

	A	B	C	D
M1	56.5	53.52	42.18	60.0
M2	60.9	47.0	58.54	57.69
M3	42.82	49.55	54.12	46.25
M4	45.23	54.98	49.99	51.98

Applying step 3, 4 the following time parameter matrix is obtained is

C_{ij} =

	A	B	C	D
M1	14.32	11.34	0	17.82
M2	13.9	0	11.54	10.69
M3	0	6.73	11.3	3.43
M4	0	9.75	4.76	6.75

Applying step 5 we the following result

		A	B	C	D
C_{ij} =	M1	14.32	11.34	0	14.39
	M2	13.9	0	11.54	6.26
	M3	0	6.73	11.3	0
	M4	0	8.81	4.76	3.32

Number of assignment is equal to the order of the matrix. Therefore the optimal assignment is $A \to M4, B \to M2, C \to M1, D \to M3$ the minimum time to complete the job is $45.23 + 47 + 42.18 + 46.25 = 180.66.$.

5. Conclusions

In this research article the de-Neutrosophication Pentagonal neutrosophic number into a real number has been introduced by means of magnitude approach. The resulted ranking has been applied to solve neutrosophic assignment problems. The algorithm stated in this paper is simple to use and applicable to solve neutrosophic assignment problems in short time. Also it produces accurate result. There is much scope for future work in this field. This ranking can be applied to solve linear, non-linear and transportation problems involving pentagonal neutrosophic number. Further image processing multi-criteria decision making problems can also make use of this ranking method for smart computation.

Funding: “This research received no external funding”

Acknowledgments: In this section we acknowledge the Chief Editor and reviewers for their valuable suggestions.

Conflicts of Interest: “The authors declare no conflict of interest.”

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Received: Apr 10, 2020. Accepted: July 13 2020