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Relation of Quasi-coincidence for Neutrosophic Sets

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Abstract. We define the relation of quasi-coincidence between a neutrosophic point and a neutrosophic set as well as between two neutrosophic sets and investigate some properties based on that. We define the quasi-neighbourhood of a neutrosophic point and examine some properties. We also study the characterization of neutrosophic topological space in terms of quasi-neighbourhoods.

Keywords: Neutrosophic set ; Neutrosophic point ; Quasi-coincidence ; Quasi-neighbourhood.

1. Introduction

The notion of Fuzzy set was brought to light by Zadeh [38] in 1965 and Intuitionistic fuzzy set, a generalized version of fuzzy set, was introduced by Atanassov [1] in 1986. After a decade, a new branch of philosophy recognised as Neutrosophy was developed and studied by Florentin Smarandache [25–27]. Smarandache [27] proved that neutrosophic set was a generalization of intuitionistic fuzzy set. Like intuitionistic fuzzy set, an element in a neutrosophic set has the degree of membership and the degree of non-membership but it has another grade of membership known as the degree of indeterminacy and one very important point about neutrosophic set is that all the three neutrosophic components are independent of one another.

After Smarandache had brought the thought of neutrosophy, it was studied and taken ahead by many researchers [11, 31, 32, 35]. In the year 2002, Smarandache [26] added the thinking of neutrosophic topology on the non-standard interval and thereafter Lupiáñez [16–19] studied and investigated many properties of neutrosophic topological space. The author [17] also studied the relation between interval neutrosophic sets and topology. Salma et.al. [28–30] studied neutrosophic topological space, generalised neutrosophic topological space and neutrosophic continuous functions. In the year 2016, Karatas and Kuru [15] redefined the single valued neutrosophic set operations and introduced a new neutrosophic topology and then investigated some important properties of general topology on the redefined neutrosophic topological space. Later, various aspects of neutrosophic topology were developed by many researchers [2, 12, 14, 33].

Neutrosophy, due to the fact of its of flexibility and effectiveness, is attracting the researchers throughout the world and is very useful not only in the developement of science and technology but also in various other fields. For instance, Abdel-Basset et.al. [3–6] studied the applications of neutrosophic theory in a number of scientific fields. Pramanik and Roy [24] in 2014 studied on the conflict between India and Pakistan over Jammu-Kashmir through neutrosophic game theory. Works on medical diagnosis [7, 36], decision making problem [8, 37], image processing [10, 13], social issues [20, 23], educational problems [21, 22] were also done under neutrosophic environment.

In the year 1995 Coker and Demirci [9] introduced the idea of intuitionistic fuzzy points and their quasi-coincident relation. Very recently Ray and Dey [34] introduced the idea of neutrosophic point on single-valued neutrosophic sets and studied various properties. But the relation of quasi-coincidence in case of neutrosophic points or neutrosophic sets has not been studied so far. In this article, we define the relation of quasi-coincidence between a neutrosophic point and a neutrosophic set as well as between two neutrosophic sets and examine some properties based on the relation of quasi-coincidence. We then define neutrosophic quasi-neighbourhood of a neutrosophic point and investigate some properties. Lastly we study the characterization of neutrosophic topological space in terms of neutrosophic quasi-neighbourhoods.

2. Preliminaries

In this section we discuss some concepts related with neutrosophic sets.

2.1. Definition: [35]

Let X be the universe of discourse. A single valued neutrosophic set A over X is defined as $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\}$, where $\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A$ are functions from X to $[0, 1]$ and $0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \leq 3$.

The set of all single valued neutrosophic sets over X is denoted by $\mathcal{N}(X)$.

Throughout this article, a single valued neutrosophic set will simply be called a neutrosophic set (NS, for short).

2.2. Definition: [15]

Let $A, B \in \mathcal{N}(X)$. Then

(i) (Inclusion): If $\mathcal{T}_A(x) \leq \mathcal{T}_B(x), \mathcal{I}_A(x) \geq \mathcal{I}_B(x), \mathcal{F}_A(x) \geq \mathcal{F}_B(x)$ for all $x \in X$ then A is said to be a neutrosophic subset of B and which is denoted by $A \subseteq B$.

(ii) (Equality): If $A \subseteq B$ and $B \subseteq A$ then $A = B$.

(iii) (Intersection): The intersection of A and B, denoted by $A \cap B$, is defined as $A \cap B = \{\langle x, \mathcal{T}_A(x) \wedge \mathcal{T}_B(x), \mathcal{I}_A(x) \vee \mathcal{I}_B(x), \mathcal{F}_A(x) \vee \mathcal{F}_B(x)\rangle : x \in X\}$.

(iv) (Union): The union of A and B, denoted by $A \cup B$, is defined as $A \cup B = \{\langle x, \mathcal{T}_A(x) \vee \mathcal{T}_B(x), \mathcal{I}_A(x) \wedge \mathcal{I}_B(x), \mathcal{F}_A(x) \wedge \mathcal{F}_B(x)\rangle : x \in X\}$.

(v) (Complement): The complement of the NS A, denoted by A^c, is defined as $A^c = \{\langle x, \mathcal{F}_A(x), 1 - \mathcal{I}_A(x), \mathcal{T}_A(x)\rangle : x \in X\}$

(vi) (Universal Set): If $\mathcal{T}_A(x) = 1, \mathcal{I}_A(x) = 0, \mathcal{F}_A(x) = 0$ for all $x \in X$ then A is said to be neutrosophic universal set and which is denoted by $\tilde{X}$.

(vii) (Empty Set): If $\mathcal{T}_A(x) = 0, \mathcal{I}_A(x) = 1, \mathcal{F}_A(x) = 1$ for all $x \in X$ then A is said to be neutrosophic empty set and which is denoted by $\tilde{\emptyset}$.

2.3. **Definition: [29]**

Let $\{A_i : i \in \triangle\} \subseteq \mathcal{N}(X)$, where $\triangle$ is an index set. Then

(i) $\cup_{i\in\triangle} A_i = \{\langle x, \vee_{i\in\triangle}\mathcal{T}_{A_i}(x), \wedge_{i\in\triangle}\mathcal{I}_{A_i}(x), \wedge_{i\in\triangle}\mathcal{F}_{A_i}(x)\rangle : x \in X\}$.

(ii) $\cap_{i\in\triangle} A_i = \{\langle x, \wedge_{i\in\triangle}\mathcal{T}_{A_i}(x), \vee_{i\in\triangle}\mathcal{I}_{A_i}(x), \vee_{i\in\triangle}\mathcal{F}_{A_i}(x)\rangle : x \in X\}$.

2.4. **Neutrosophic topological space:**

2.4.1. **Definition: [15]**

Let $\tau \subseteq \mathcal{N}(X)$. Then τ is called a neutrosophic topology on X if

(i) $\tilde{\emptyset}$ and $\tilde{X}$ belong to τ.

(ii) The union of any number of neutrosophic sets in τ belongs to τ.

(iii) The intersection of any two neutrosophic sets in τ belongs to τ.

If τ is a neutrosophic topology on X then the pair (X, τ) is called a neutrosophic topological space (NTS, for short) over X. The members of τ are called neutrosophic open sets in X. If for a neutrosophic set A, $A^c \in \tau$ then A is said to be a neutrosophic closed set in X.

2.4.2. **Theorem: [15]**

Let (X, τ) be a neutrosophic topological space over X. Then

(i) $\tilde{\emptyset}$ and $\tilde{X}$ are neutrosophic closed sets over X.

(ii) The intersection of any number of neutrosophic closed sets is a neutrosophic closed set over X.

(iii) The union of any two neutrosophic closed sets is a neutrosophic closed set over X.

2.5. **Definition: [34]**

Let $\mathcal{N}(X)$ be the set of all neutrosophic sets over X. A NS $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle : x \in X\}$ is called a neutrosophic point (NP, for short) iff for any element $y \in X$, $\mathcal{T}_P(y) = \alpha, \mathcal{I}_P(y) = \beta, \mathcal{F}_P(y) = \gamma$ for $y = x$ and $\mathcal{T}_P(y) = 0, \mathcal{I}_P(y) = 1, \mathcal{F}_P(y) = 1$ for $y \neq x$, where $0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1$.

A neutrosophic point $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle : x \in X\}$ will be denoted by $P^x_{\alpha,\beta,\gamma}$ or $P < x, \alpha, \beta, \gamma >$ or simply by $x_{\alpha,\beta,\gamma}$. For the NP $x_{\alpha,\beta,\gamma}$, x will be called its support.

The complement of the NP $P^x_{\alpha,\beta,\gamma}$ will be denoted by $(P^x_{\alpha,\beta,\gamma})^c$ or by $x^c_{\alpha,\beta,\gamma}$.

2.6. **Definition: [34]**

Let A be a neutrosophic set over X. Also let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha^/,\beta^/,\gamma^/}$ be two neutrosophic points in X. Then

(i) $x_{\alpha,\beta,\gamma}$ is said to be contained in A, denoted by $x_{\alpha,\beta,\gamma} \subseteq A$, iff $\alpha \leq \mathcal{T}_A(x), \beta \geq \mathcal{I}_A(x), \gamma \geq \mathcal{F}_A(x)$.

(ii) $x_{\alpha,\beta,\gamma}$ is said to belong to A, denoted by $x_{\alpha,\beta,\gamma} \in A$, iff $\alpha \leq \mathcal{T}_A(x), \beta \geq \mathcal{I}_A(x), \gamma \geq \mathcal{F}_A(x)$.

(iii) $x_{\alpha,\beta,\gamma}$ is said to be contained in $y_{\alpha^/,\beta^/,\gamma^/}$, denoted by $x_{\alpha,\beta,\gamma} \subseteq y_{\alpha^/,\beta^/,\gamma^/}$, iff $x = y$ and $\alpha \leq \alpha^/, \beta \geq \beta^/, \gamma \geq \gamma^/$.

(iv) $x_{\alpha,\beta,\gamma}$ is said to belong to $y_{\alpha^/,\beta^/,\gamma^/}$, denoted by $x_{\alpha,\beta,\gamma} \in y_{\alpha^/,\beta^/,\gamma^/}$, iff $x = y$ and $\alpha \leq \alpha^/, \beta \geq \beta^/, \gamma \geq \gamma^/$.

2.7. **Proposition: [34]**

Let $\{A_i : i \in \triangle\} \subseteq \mathcal{N}(X)$, where $\triangle$ is an index set. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha^/,\beta^/,\gamma^/}$ be any two neutrosophic points over X. Then the following hold good.

(i) $x_{\alpha,\beta,\gamma} \in \bigcap\{A_i : i \in \triangle\} \Longleftrightarrow x_{\alpha,\beta,\gamma} \in A_i \, \forall \, i \in \triangle$.

(ii) If $x_{\alpha,\beta,\gamma} \in A_i$ for some $i \in \triangle$ then $x_{\alpha,\beta,\gamma} \in \bigcup\{A_i : i \in \triangle\}$.

(iii) If $x_{\alpha,\beta,\gamma} \in \bigcup\{A_i : i \in \triangle\}$ then there exists a NS $A(x_{\alpha,\beta,\gamma})$ such that $x_{\alpha,\beta,\gamma} \in A(x_{\alpha,\beta,\gamma}) \subseteq \bigcup\{A_i : i \in \triangle\}$.

For other definitions and results concerning neutrosophic points used in this article, please see [34]

3. Main Results

3.1. Definition:

A NP $x_{\alpha,\beta,\gamma} \in \mathcal{N}(X)$ is said to be quasi-coincident with a NS $A \in \mathcal{N}(X)$ or $x_{\alpha,\beta,\gamma} \in \mathcal{N}(X)$ quasi-coincides with a NS $A \in \mathcal{N}(X)$, denoted by $x_{\alpha,\beta,\gamma} q A$, iff $\alpha > \mathcal{T}_{A^c}(x)$ or $\beta < \mathcal{I}_{A^c}(x)$ or $\gamma < \mathcal{F}_{A^c}(x)$, i.e., $\alpha > \mathcal{F}_A(x)$ or $\beta < 1 - \mathcal{I}_A(x)$ or $\gamma < \mathcal{T}_A(x)$.

A NS A is said to be quasi-coincident with a NS B at $x \in X$ or A quasi-coincides with B at $x \in X$, denoted by AqB at x, iff $\mathcal{T}_A(x) > \mathcal{T}_{B^c}(x)$ or $\mathcal{I}_A(x) < \mathcal{I}_{B^c}(x)$ or $\mathcal{F}_A(x) < \mathcal{F}_{B^c}(x)$. We say A quasi-coincides with B or A is quasi-coincident with B, denoted by AqB, iff A quasi-coincides with B at some point $x \in X$. Thus A quasi-coincides with B or A is quasi-coincident with B iff there exists an element $x \in X$ such that $\mathcal{T}_A(x) > \mathcal{T}_{B^c}(x)$ or $\mathcal{I}_A(x) < \mathcal{I}_{B^c}(x)$ or $\mathcal{F}_A(x) < \mathcal{F}_{B^c}(x)$, i.e., $\mathcal{T}_A(x) > \mathcal{F}_B(x)$ or $\mathcal{I}_A(x) < 1 - \mathcal{I}_B(x)$ or $\mathcal{F}_A(x) < \mathcal{T}_B(x)$.

If the NP $x_{\alpha,\beta,\gamma}$ is not quasi-coincident with a NS A, we shall denote it by $x_{\alpha,\beta,\gamma}\hat{q}A$. Similarly if the NS A is not quasi-coincident with the NS B, we shall denote it by $A\hat{q}B$.

The set of all the points in X, at which AqB, will be denoted by $A\Omega B$, i.e., $A\Omega B = \{x \in X : AqB \text{ at } x\}$.

Before proceeding to the results connected to quasi-coincident relation we first prove a simple result on neutrosophic sets.

3.2. Proposition:

Let $A, B \in \mathcal{N}(X)$. Then $A \subseteq B \Leftrightarrow B^c \subseteq A^c$.

Proof:

$$
\begin{aligned}
& A \subseteq B \\
\Leftrightarrow & \mathcal{T}_A(x) \leq \mathcal{T}_B(x), \mathcal{I}_A(x) \geq \mathcal{I}_B(x), \mathcal{F}_A(x) \geq \mathcal{F}_B(x) \text{ for all } x \in X \\
\Leftrightarrow & \mathcal{F}_B(x) \leq \mathcal{F}_A(x), 1 - \mathcal{I}_A(x) \leq 1 - \mathcal{I}_B(x), \mathcal{T}_B(x) \geq \mathcal{T}_A(x) \text{ for all } x \in X \\
\Leftrightarrow & \mathcal{T}_{B^c}(x) \leq \mathcal{T}_{A^c}(x), \mathcal{I}_{B^c}(x) \geq \mathcal{I}_{A^c}(x), \mathcal{F}_{B^c}(x) \geq \mathcal{F}_{A^c}(x) \text{ for all } x \in X \\
\Leftrightarrow & B^c \subseteq A^c
\end{aligned}
$$

3.3. Proposition:

Let A, B, C be three neutrosophic sets and $x_{\alpha,\beta,\gamma}$ be a neutrosophic point in X. Then

(i) $x_{\alpha,\beta,\gamma}\hat{q}\tilde{\emptyset}$.

(ii) $x_{\alpha,\beta,\gamma} q \tilde{X}$.

(iii) $x_{\alpha,\beta,\gamma} \in A \Leftrightarrow x_{\alpha,\beta,\gamma}\hat{q}A^c$.

(iv) $x_{\alpha,\beta,\gamma} q A \Leftrightarrow x_{\alpha,\beta,\gamma} \notin A^c$.

(v) $A \subseteq B \Leftrightarrow A\hat{q}B^c$.

(vi) $AqB \Leftrightarrow A \nsubseteq B^c$

(vii) $x_{\alpha,\beta,\gamma}qA$ and $A \subseteq B$ then $x_{\alpha,\beta,\gamma}qB$.

(viii) CqA and $A \subseteq B$ then CqB.

(ix) AqB at $x \Leftrightarrow BqA$ at x.

(x) $AqB \Leftrightarrow BqA$.

Proofs:

(i) Very obvious.

(ii) Very obvious.

(iii)

$$\begin{aligned}
& x_{\alpha,\beta,\gamma} \in A \\
\Leftrightarrow & \alpha \leq \mathcal{T}_A(x), \beta \geq \mathcal{I}_A(x), \gamma \geq \mathcal{F}_A(x) \\
\Leftrightarrow & \alpha \ngtr \mathcal{T}_A(x), \beta \nless \mathcal{I}_A(x), \gamma \nless \mathcal{F}_A(x) \\
\Leftrightarrow & \alpha \ngtr \mathcal{T}_{(A^c)^c}(x), \beta \nless \mathcal{I}_{(A^c)^c}(x), \gamma \nless \mathcal{F}_{(A^c)^c}(x) \\
\Leftrightarrow & x_{\alpha,\beta,\gamma}\hat{q}A^c
\end{aligned}$$

(iv)

$$\begin{aligned}
& x_{\alpha,\beta,\gamma}qA \\
\Leftrightarrow & \alpha > \mathcal{T}_{A^c}(x) \text{ or } \beta < \mathcal{I}_{A^c}(x) \text{ or } \gamma < \mathcal{F}_{A^c}(x) \\
\Leftrightarrow & \alpha \nleq \mathcal{T}_{A^c}(x) \text{ or } \beta \ngeq \mathcal{I}_{A^c}(x) \text{ or } \gamma \ngeq \mathcal{F}_{A^c}(x) \\
\Leftrightarrow & x_{\alpha,\beta,\gamma} \notin A^c
\end{aligned}$$

(v)

$$\begin{aligned}
& A \subseteq B \\
\Leftrightarrow & \mathcal{T}_A(x) \leq \mathcal{T}_B(x), \mathcal{I}_A(x) \geq \mathcal{I}_B(x), \mathcal{F}_A(x) \geq \mathcal{F}_B(x) \, \forall \, x \in X \\
\Leftrightarrow & \mathcal{T}_A(x) \ngtr \mathcal{T}_B(x), \mathcal{I}_A(x) \nless \mathcal{I}_B(x), \mathcal{F}_A(x) \nless \mathcal{F}_B(x) \, \forall \, x \in X \\
\Leftrightarrow & \mathcal{T}_A(x) \ngtr \mathcal{T}_{(B^c)^c}(x), \mathcal{I}_A(x) \nless \mathcal{I}_{(B^c)^c}(x), \mathcal{F}_A(x) \nless \mathcal{F}_{(B^c)^c}(x) \, \forall \, x \in X \\
\Leftrightarrow & A\hat{q}B^c
\end{aligned}$$

(vi)

$$\begin{aligned}
&AqB\\
\Leftrightarrow&\mathcal{T}_A(x) > \mathcal{T}_{B^c}(x)\,\text{or}\,\mathcal{I}_A(x) < \mathcal{I}_{B^c}(x)\,\text{or}\,\mathcal{F}_A(x) < \mathcal{F}_{B^c}(x)\,\text{for some}\,x \in X\\
\Leftrightarrow&\mathcal{T}_A(x) \nleq \mathcal{T}_{B^c}(x)\,\text{or}\,\mathcal{I}_A(x) \ngeq \mathcal{I}_{B^c}(x)\,\text{or}\,\mathcal{F}_A(x) \ngeq \mathcal{F}_{B^c}(x)\,\text{for some}\,x \in X\\
\Leftrightarrow&A \nsubseteq B^c
\end{aligned}$$

(vii) Since $x_{\alpha,\beta,\gamma}qA$, so $\alpha > \mathcal{T}_{A^c}(x)\,\text{or}\,\beta < \mathcal{I}_{A^c}(x)\,\text{or}\,\gamma < \mathcal{F}_{A^c}(x)$. Now

$$\begin{aligned}
&A \subseteq B\\
\Rightarrow&B^c \subseteq A^c\\
\Rightarrow&\mathcal{T}_{B^c}(x) \leq \mathcal{T}_{A^c}(x), \mathcal{I}_{B^c}(x) \geq \mathcal{I}_{A^c}(x), \mathcal{F}_{B^c}(x) \geq \mathcal{F}_{A^c}(x) \text{ for all } x \in X\\
\Rightarrow&\mathcal{T}_{A^c}(x) \geq \mathcal{T}_{B^c}(x), \mathcal{I}_{A^c}(x) \leq \mathcal{I}_{B^c}(x), \mathcal{F}_{A^c}(x) \leq \mathcal{F}_{B^c}(x) \text{ for all } x \in X\\
\Rightarrow&\alpha > \mathcal{T}_{B^c}(x) \text{ or } \beta < \mathcal{I}_{B^c}(x) \text{ or } \gamma < \mathcal{F}_{B^c}(x)\\
\Rightarrow&x_{\alpha,\beta,\gamma}qB
\end{aligned}$$

(viii) $CqA \Rightarrow C \nsubseteq A^c \Rightarrow C \nsubseteq B^c$ [$\because A \subseteq B \Rightarrow B^c \subseteq A^c$]$\Rightarrow CqB$.

(ix)

$$\begin{aligned}
&AqB \text{ at } x\\
\Leftrightarrow&\mathcal{T}_A(x) > \mathcal{T}_{B^c}(x) \text{ or } \mathcal{I}_A(x) < \mathcal{I}_{B^c}(x) \text{ or } \mathcal{F}_A(x) < \mathcal{F}_{B^c}(x)\\
\Leftrightarrow&\mathcal{T}_A(x) > \mathcal{F}_B(x) \text{ or } \mathcal{I}_A(x) < 1 - \mathcal{I}_B(x) \text{ or } \mathcal{F}_A(x) < \mathcal{T}_B(x)\\
\Leftrightarrow&\mathcal{T}_B(x) > \mathcal{F}_A(x) \text{ or } \mathcal{I}_B(x) < 1 - \mathcal{I}_A(x) \text{ or } \mathcal{F}_B(x) < \mathcal{T}_A(x)\\
\Leftrightarrow&\mathcal{T}_B(x) > \mathcal{T}_{A^c}(x) \text{ or } \mathcal{I}_B(x) < \mathcal{I}_{A^c}(x) \text{ or } \mathcal{F}_B(x) < \mathcal{F}_{A^c}(x)\\
\Leftrightarrow&BqA \text{ at } x
\end{aligned}$$

(x) Obvious from (ix).

3.4. **Proposition:**

Let $x_{\alpha,\beta,\gamma}$ be a NP in X, $A \in \mathcal{N}(X)$ and $\{A_i : i \in \triangle\} \subseteq \mathcal{N}(X)$, $\triangle$ is an index set. Then

(i) $x_{\alpha,\beta,\gamma}q \cup_{i\in\triangle} A_i \Leftrightarrow x_{\alpha,\beta,\gamma}qA_j$ for some $j \in \triangle$.

(ii) $Aq \cup_{i\in\triangle} A_i \Leftrightarrow AqA_j$ for some $j \in \triangle$.

(iii) $x_{\alpha,\beta,\gamma}q \cap_{i\in\triangle} A_i \Rightarrow x_{\alpha,\beta,\gamma}qA_i$ for all $i \in \triangle$. Converse is not true.

(iv) $Aq \cap_{i\in\triangle} A_i \Rightarrow AqA_i$ for all $i \in \triangle$. Converse is not true.

Proofs: (i)

$$x_{\alpha,\beta,\gamma} q \cup_{i\in\triangle} A_i$$
$$\Leftrightarrow x_{\alpha,\beta,\gamma} \notin (\cup_{i\in\triangle} A_i)^c$$
$$\Leftrightarrow x_{\alpha,\beta,\gamma} \notin \cap_{i\in\triangle} A_i^c$$
$$\Leftrightarrow x_{\alpha,\beta,\gamma} \notin A_j^c \text{ for some } j \in\triangle$$
$$\Leftrightarrow x_{\alpha,\beta,\gamma} q A_j \text{ for some } j \in\triangle$$

(ii)

$$A q \cup_{i\in\triangle} A_i$$
$$\Leftrightarrow A \nsubseteq (\cup_{i\in\triangle} A_i)^c$$
$$\Leftrightarrow A \nsubseteq \cap_{i\in\triangle} A_i^c$$
$$\Leftrightarrow A \nsubseteq A_j^c \text{ for some } j \in\triangle$$
$$\Leftrightarrow A q A_j \text{ for some } j \in\triangle$$

(iii)

$$x_{\alpha,\beta,\gamma} q \cap_{i\in\triangle} A_i$$
$$\Rightarrow x_{\alpha,\beta,\gamma} \notin (\cap_{i\in\triangle} A_i)^c$$
$$\Rightarrow x_{\alpha,\beta,\gamma} \notin \cup_{i\in\triangle} A_i^c$$
$$\Rightarrow x_{\alpha,\beta,\gamma} \notin A_i^c \text{ for all } i \in\triangle$$
$$\Rightarrow x_{\alpha,\beta,\gamma} q A_i \text{ for all } i \in\triangle$$

Converse is not true. We establish it by the following counter example.

Let $X = \{x, y\}$. Also let $A = \{\langle x, 0.3, 0.6, 0.2\rangle, \langle y, 0.6, 0.7, 0.7\rangle\}, B = \{\langle x, 0.3, 0.5, 0.6\rangle, \langle y, 0.3, 0.8, 0.7\rangle\}$ and $C = \{\langle x, 0.4, 0.5, 0.7\rangle, \langle y, 0.6, 0.1, 0.7\rangle\}$ be three neutrosophic sets over X. Then $A \cap B \cap C = \{\langle x, 0.3, 0.6, 0.7\rangle, \langle y, 0.3, 0.8, 0.7\rangle\}$ Let us consider the neutrosophic point $x_{0.3,0.4,0.8}$. Clearly $x_{0.3,0.4,0.8} qA$, $x_{0.3,0.4,0.8} qB$ and $x_{0.3,0.4,0.8} qC$ but $x_{0.3,0.4,0.8}$ is not quasi-coincident with $A \cap B \cap C$.

(iv)

$$A q \cap_{i\in\triangle} A_i$$
$$\Rightarrow A \nsubseteq (\cap_{i\in\triangle} A_i)^c$$
$$\Rightarrow A \nsubseteq \cup_{i\in\triangle} A_i^c$$
$$\Rightarrow A \nsubseteq A_i^c \text{ for all } i \in\triangle$$
$$\Rightarrow A q A_i \text{ for all } i \in\triangle$$

Converse is not true. We establish it by the following counter example.

Let $X = \{x, y\}$. Also let $A = \{\langle x, 0.3, 0.6, 0.2\rangle, \langle y, 0.6, 0.7, 0.7\rangle\}, B = \{\langle x, 0.3, 0.5, 0.6\rangle, \langle y, 0.3, 0.8, 0.7\rangle\}$ and $C = \{\langle x, 0.4, 0.5, 0.7\rangle, \langle y, 0.6, 0.1, 0.7\rangle\}$ be three neutrosophic sets over X. Then $A \cap B \cap C = \{\langle x, 0.3, 0.6, 0.7\rangle, \langle y, 0.3, 0.8, 0.7\rangle\}$ Let us consider the neutrosophic set $D = \{\langle x, 0.3, 0.4, 0.8\rangle, \langle y, 0.5, 0.7, 0.7\rangle\}$. Clearly DqA , DqB and DqC but D is not quasi-coincident with $A \cap B \cap C$.

3.5. **Proposition:**

(i) $A\Omega B = B\Omega A$.
(ii) $AqB \Leftrightarrow A\Omega B \neq \emptyset$.
(iii) $A \subseteq B \Rightarrow A\Omega C \subseteq B\Omega C$.
(iv) $A\Omega(\cup_{i\in\triangle} A_i) = \cup_{i\in\triangle}(A\Omega A_i)$.
(v) $A\Omega(\cap_{i\in\triangle} A_i) \subseteq \cap_{i\in\triangle}(A\Omega A_i)$. Converse is not true.

Proofs:

(i) $A\Omega B = \{x \in X : AqB \,\text{at}\, x\} = \{x \in X : BqA \,\text{at}\, x\} = B\Omega A$.

(ii)$AqB \Leftrightarrow AqB$ at some $x \in X \Leftrightarrow x \in A\Omega B$. Therefore $AqB \Leftrightarrow A\Omega B \neq \emptyset$.

(iii)$A \subseteq B \Rightarrow \mathcal{T}_A(x) \leq \mathcal{T}_B(x), \mathcal{I}_A(x) \geq \mathcal{I}_B(x), \mathcal{F}_A(x) \geq \mathcal{F}_B(x)$ for all $x \in X$. Now

$$\begin{aligned}
& x \in A\Omega C \\
\Rightarrow & AqC \text{ at } x \in X \\
\Rightarrow & \mathcal{T}_A(x) > \mathcal{T}_{C^c}(x) \text{ or } \mathcal{I}_A(x) < \mathcal{I}_{C^c}(x) \text{ or } \mathcal{F}_A(x) < \mathcal{F}_{C^c}(x) \\
\Rightarrow & \mathcal{T}_B(x) > \mathcal{T}_{C^c}(x) \text{ or } \mathcal{I}_B(x) < \mathcal{I}_{C^c}(x) \text{ or } \mathcal{F}_B(x) < \mathcal{F}_{C^c}(x) \\
\Rightarrow & BqC \text{ at } x \in X \\
\Rightarrow & x \in B\Omega C \\
\therefore & A\Omega C \subseteq B\Omega C.
\end{aligned}$$

(iv)

$$\begin{aligned}
& x \in A\Omega(\cup_{i\in\triangle} A_i) \\
\Rightarrow & Aq(\cup_{i\in\triangle} A_i) \text{ at } x \in X \\
\Rightarrow & \exists\, j \in\triangle \text{ such that } AqA_j \text{ at } x \in X \\
\Rightarrow & \exists\, j \in\triangle \text{ such that } x \in A\Omega A_j \\
\Rightarrow & x \in \cup_{i\in\triangle}(A\Omega A_i) \\
\therefore & A\Omega(\cup_{i\in\triangle} A_i) \subseteq \cup_{i\in\triangle}(A\Omega A_i).
\end{aligned}$$

Again

$$
\begin{aligned}
& x \in \cup_{i\in\triangle}(A\Omega A_i) \\
\Rightarrow & \bigvee_{i\in\triangle}(AqA_i \text{ at } x \in X) \\
\Rightarrow & \bigvee_{i\in\triangle}(A_iqA \text{ at } x \in X) \\
\Rightarrow & \bigvee_{i\in\triangle}[\mathcal{T}_{A_i}(x) > \mathcal{T}_{A^c}(x) \text{ or } \mathcal{I}_{A_i}(x) < \mathcal{I}_{A^c}(x) \text{ or } \mathcal{F}_{A_i}(x) < \mathcal{F}_{A^c}(x)] \\
\Rightarrow & \sup_{i\in\triangle}\mathcal{T}_{A_i}(x) > \mathcal{T}_{A^c}(x) \text{ or } \inf_{i\in\triangle}\mathcal{I}_{A_i}(x) < \mathcal{I}_{A^c}(x) \text{ or } \inf_{i\in\triangle}\mathcal{F}_{A_i}(x) < \mathcal{F}_{A^c}(x) \\
\Rightarrow & \mathcal{T}_{\cup A_i}(x) > \mathcal{T}_{A^c}(x) \text{ or } \mathcal{I}_{\cup A_i}(x) < \mathcal{I}_{A^c}(x) \text{ or } \mathcal{F}_{\cup A_i}(x) < \mathcal{F}_{A^c}(x) \\
\Rightarrow & (\cup_{i\in\triangle}A_i)qA \text{ at } x \in X \\
\Rightarrow & Aq(\cup_{i\in\triangle}A_i) \text{ at } x \in X \\
\Rightarrow & x \in A\Omega(\cup_{i\in\triangle}A_i) \\
\therefore & \cup_{i\in\triangle}(A\Omega A_i) \subseteq A\Omega(\cup_{i\in\triangle}A_i)
\end{aligned}
$$

Hence $A\Omega(\cup_{i\in\triangle}A_i) = \cup_{i\in\triangle}(A\Omega A_i)$.

(v)

$$
\begin{aligned}
& x \in A\Omega(\cap_{i\in\triangle}A_i) \\
\Rightarrow & Aq(\cap_{i\in\triangle}A_i) \text{ at } x \in X \\
\Rightarrow & AqA_i \text{ at } x \in X \text{for all} i \in\triangle \\
\Rightarrow & x \in A\Omega A_i \text{for all} i \in\triangle \\
\Rightarrow & x \in \cap_{i\in\triangle}(A\Omega A_i) \\
\therefore & A\Omega(\cap_{i\in\triangle}A_i) \subseteq \cap_{i\in\triangle}(A\Omega A_i).
\end{aligned}
$$

Converse is not true We establish it by the following counter example.

Let $X = \{x, y\}$. Also let $A = \{\langle x, 0.3, 0.6, 0.2\rangle, \langle y, 0.6, 0.7, 0.7\rangle\}, B = \{\langle x, 0.3, 0.5, 0.6\rangle, \langle y, 0.3, 0.8, 0.7\rangle\}$ and $C = \{\langle x, 0.4, 0.5, 0.7\rangle, \langle y, 0.6, 0.1, 0.7\rangle\}$ be three neutrosophic sets over X. Then $A \cap B \cap C = \{\langle x, 0.3, 0.6, 0.7\rangle, \langle y, 0.3, 0.8, 0.7\rangle\}$ Let us consider the neutrosophic set $D = \{\langle x, 0.3, 0.4, 0.8\rangle, \langle y, 0.5, 0.7, 0.7\rangle\}$. Clearly $D\Omega A = \{x\}$, $D\Omega B = \{x\}$, $D\Omega C = \{x, y\}$ and $D\Omega(A \cap B \cap C) = \emptyset$. Therefore $(D\Omega A) \cap (D\Omega B) \cap (D\Omega C) = \{x\} \nsubseteq D\Omega(A \cap B \cap C)$.

3.6. **Definition:**

Let (X, τ) be a NTS. A neutrosophic set A is called a neutrosophic quasi-neighbourhood or simply Q-neighbourhood (Q-nhbd, for short) of a neutrosophic point $x_{\alpha,\beta,\gamma}$ iff there exists a NS $B \in \tau$ such that $x_{\alpha,\beta,\gamma}qB \subseteq A$.

The family consisting of all the Q-neighbourhoods of the NP $x_{\alpha,\beta,\gamma}$ is called the system of Q-neighbourhoods or Q-neighbourhood system of $x_{\alpha,\beta,\gamma}$. This family is denoted by $\mathbf{N_Q}(x_{\alpha,\beta,\gamma})$.

3.7. **Proposition:**

Every neutrosophic open set A in a NTS (X,τ) is a Q-nhbd of every NP quasi-coincident with A.

Proofs: Obvious because for every NP $x_{\alpha,\beta,\gamma}qA$, we have $x_{\alpha,\beta,\gamma}qA \subseteq A$.

3.8. **Properties of Neutrosophic Q-neighbourhoods :**

Let $\mathbf{N_Q}(x_{\alpha,\beta,\gamma})$ be the collection of all Q-neighbourhoods of the NP $x_{\alpha,\beta,\gamma}$ in a NTS (X,τ). Then

N1) $\mathbf{N_Q}(x_{\alpha,\beta,\gamma}) \neq \emptyset$ for every NP $x_{\alpha,\beta,\gamma} \in \mathcal{N}(X)$.

N2) $P \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma}) \Rightarrow x_{\alpha,\beta,\gamma}qP$.

N3) $P \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma}), P \subseteq Q \Rightarrow Q \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma})$.

N4) $P \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma}) \Rightarrow$ there exists a $Q \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma})$ such that $Q \subseteq P$ and $Q \in \mathbf{N_Q}(y_{\alpha/,\beta/,\gamma/})$ for every NP $y_{\alpha/,\beta/,\gamma/}$ quasi-coincident with Q.

Proofs:

N1) Obviously $\tilde{X}$ is a Q-nhbd of every NP $x_{\alpha,\beta,\gamma} \in \mathcal{N}(X)$. Thus there exists at least one Q-nhbd for every NP $x_{\alpha,\beta,\gamma} \in \mathcal{N}(X)$. Therefore $\mathbf{N_Q}(x_{\alpha,\beta,\gamma}) \neq \emptyset$ for every NP $x_{\alpha,\beta,\gamma} \in \mathcal{N}(X)$.

N2) $P \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma}) \Rightarrow P$ is a Q-nhbd of $x_{\alpha,\beta,\gamma} \Rightarrow \exists$ a $S \in \tau$ such that $x_{\alpha,\beta,\gamma}qS \subseteq P$. Therefore $x_{\alpha,\beta,\gamma}qP$.

N3) $P \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma}) \Rightarrow P$ is a Q-nhbd of $x_{\alpha,\beta,\gamma} \Rightarrow \exists$ an open set G such that $x_{\alpha,\beta,\gamma}qG \subseteq P \Rightarrow \exists$ an open set G such that $x_{\alpha,\beta,\gamma}qG \subseteq Q \Rightarrow Q$ is a Q-nhbd of $x_{\alpha,\beta,\gamma} \Rightarrow Q \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma})$

N4) Since $P \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma})$, so there exists a τ-open set Q such that $x_{\alpha,\beta,\gamma}qQ \subseteq P$. Since Q is an open set, so $Q \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma})$. Thus $Q \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma})$ and $Q \subseteq P$.

Again since Q is an open set, so Q is a Q-nhbd of every NP quasi-coincident with Q. Therefore $Q \in \mathbf{N_Q}(y_{\alpha/,\beta/,\gamma/})$ for every NP $y_{\alpha/,\beta/,\gamma/}$ quasi-coincident with Q .

Hence proved.

3.9. **Characterization of NTS in terms of Neutrosophic Q-neighbourhoods:**

Let X be a non-empty set and let $x \in X$. Let $\mathbf{N_Q}(x_{\alpha,\beta,\gamma})$ be a family of all neutrosophic sets over X satisfying the following conditions :

N1) $P \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma}) \Rightarrow x_{\alpha,\beta,\gamma}qP$.

N2) $P, Q \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma}) \Rightarrow P \cap Q \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma})$.

N3) $P \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma}), P \subseteq Q \Rightarrow Q \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma})$.

Then there exists a neutrosophic topology τ on X. If, in addition to that, the following condition (N4) is also satisfied then $\mathbf{N_Q}(x_{\alpha,\beta,\gamma})$ is exactly the Q-neighbourhood system of $x_{\alpha,\beta,\gamma}$ in the NTS (X, τ).

N4) $P \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma}) \Rightarrow$ there exists a $Q \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma})$ such that $Q \subseteq P$ and $Q \in \mathbf{N_Q}(y_{\alpha^/,\beta^/,\gamma^/})$ for every NP $y_{\alpha^/,\beta^/,\gamma^/}$ quasi-coincident with Q.

Proof: We define τ as follows :

A NS $G \in \tau$ iff $G \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma})$ whenever $x_{\alpha,\beta,\gamma} q G$.

We claim that τ is a neutrosophic topology on X.

T1) $\tilde{\emptyset} \in \tau$ as no NP is quasi-coincident with $\tilde{\emptyset}$. By (N3), $\tilde{X} \in \tau$. Thus $\tilde{\emptyset}, \tilde{X} \in \tau$.

T2) Suppose $G_1, G_2 \in \tau$ and $x_{\alpha,\beta,\gamma} q(G_1 \cap G_2)$. Since $x_{\alpha,\beta,\gamma} q(G_1 \cap G_2)$, so $x_{\alpha,\beta,\gamma} q G_1$ and $x_{\alpha,\beta,\gamma} q G_2$. Therefore $G_1, G_2 \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma})$ and so, by (N2), $G_1 \cap G_2 \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma})$.

T3) Suppose $\{G_i : i \in \triangle\} \subseteq \tau$ and $x_{\alpha,\beta,\gamma} q(\cup_{i\in\triangle} G_i)$. We show that $\cup\{G_i : i \in\triangle\} \in \tau$. Now $x_{\alpha,\beta,\gamma} q(\cup_{i\in\triangle} G_i) \Rightarrow \exists\, \text{a}\, j \in\triangle$ such that $x_{\alpha,\beta,\gamma} q G_j \Rightarrow \exists\, \text{a}\, j \in\triangle$ such that $G_j \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma}) \Rightarrow \cup\{G_i : i \in\triangle\} \in \mathbf{N}(x_{\alpha,\beta,\gamma})$[by (N3)] $\Rightarrow \cup\{G_i : i \in\triangle\} \in \tau$.

Therefore τ is a neutrosophic topology on X.

Let the condition (N4) be satisfied. Suppose that $\mathbf{N^*_Q}(x_{\alpha,\beta,\gamma})$, is the family of all Q-neighbourhoods of the NP $x_{\alpha,\beta,\gamma}$ in (X, τ). We show that $\mathbf{N^*_Q}(x_{\alpha,\beta,\gamma}) = \mathbf{N_Q}(x_{\alpha,\beta,\gamma})$.

Let $N \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma})$. Then by (N4) there exists a $M \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma})$ such that $M \subseteq N$ and $M \in \mathbf{N_Q}(y_{\alpha^/,\beta^/,\gamma^/})$ for every NP $y_{\alpha^/,\beta^/,\gamma^/}$ quasi-coincident with M. Now $M \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma}) \Rightarrow x_{\alpha,\beta,\gamma} q M$ [by (N1)]. Therefore $M \in \tau$. Thus M is a τ-open set such that $x_{\alpha,\beta,\gamma} q M \subseteq N$. Therefore $N \in \mathbf{N^*_Q}(x_{\alpha,\beta,\gamma})$ and so $\mathbf{N_Q}(x_{\alpha,\beta,\gamma}) \subseteq \mathbf{N^*_Q}(x_{\alpha,\beta,\gamma})$. Conversely let $N \in \mathbf{N^*_Q}(x_{\alpha,\beta,\gamma})$ so that N is a Q-nhbd of $x_{\alpha,\beta,\gamma}$. Then there exists a τ-open set G such that $x_{\alpha,\beta,\gamma} q G \subseteq N$. Therefore $G \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma})$. But $G \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma})$ and $G \subseteq N$ together imply by (N3) that $N \in \mathbf{N_Q}(x_{\alpha,\beta,\gamma})$. Therefore $\mathbf{N^*_Q}(x_{\alpha,\beta,\gamma}) \subseteq \mathbf{N_Q}(x_{\alpha,\beta,\gamma})$.Thus $\mathbf{N_Q}(x_{\alpha,\beta,\gamma}) = \mathbf{N^*_Q}(x_{\alpha,\beta,\gamma})$.

Hence proved.

4. Conclusion

In this article we have introduced the notion of quasi-coincident relation and established some vital properties based on that. We have also defined the quasi-neighbourhood of a neutrosophic point and studied some properties. At last we have thrown light on the characterization of neutrosophic topological space through the the quasi-neighbourhoods of the neutrosophic points. Hope that the findings in this article will assist the research fraternity to move forward for the development of different aspects of neutrosophic topology.

5. Conflict of Interest

We certify that there is no actual or potential conflict of interest in relation to this article.

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