Reverse Subsystems of Interval Neutrosophic Automata

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Abstract. We introduce the concept of reverse subsystem and prove that the necessary and sufficient condition for R_{N_Q} to be a reverse subsystem. Also, we prove that the union and intersection of reverse subsystems of interval neutrosophic automata is reverse subsystem.

Keywords: Interval neutrosophic set, Interval neutrosophic automaton, Subsystem, Reverse Subsystem.

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1. Introduction

Fuzzy set theory which is a generalization of conventional set theory was proposed by Lofti A. Zadeh in 1965 with his seminal paper 'Fuzzy Sets' . Fuzzy set provides a simple mathematical tool to represent vagueness, uncertainty and imprecision inherently present in day to day life.

Fuzzy Logic provides a simple way to arrive at a definite conclusion based on vague, ambiguous, imprecise, noisy or missing input information. Since 1965, fuzzy set theory has witnessed enormous development by several researchers. Fuzzy logic based applications range from consumer products and industrial systems to biomedicine, decision analysis, information sciences and control engineering.

Fuzzy automata was introduced by W. G. Wee [17]. Subsequently, number of works have been contributed by many authors for development of generalizations of finite automata. General fuzzy automata was introduced by Doostfatemeh in [3]. It deals the problem of assigning membership values to the active states.

The neutrosophic set is the generalization of classical sets, fuzzy set [18], intuitionstic fuzzy set [1], interval valued intuitionistic fuzzy sets [2], vague set [4] and so on. Florentin Smarandache in 1998 [14] introduced the concept of neutrosophy and neutrosophic set. Single valued and interval valued neutrosophic sets were introuced by Wang $etal.$ in [15,16]. Recently, neutrosophic sets and systems have important applications in various fields especially in multicriteria decision making problems.

Tahir Mahmood et. al in [11, 12] were introduced single valued and interval neutrosophic finite automata. Consequently, J. Kavikumar et.al were introduced neutrosophic general finite automata and composite neutrosophic finite automata [9, 10].

Subsystems of finite fuzzy state machines was discussed in [13]. Later, Retrievability, subsystems, strong subsystems, and characterizations of submachines of Interval neutrosophic automata were discussed by V. Karthikeyan in [5–8]. In this paper, we introduce reverse subsystem (R.S) of interval neutrosophic automata and discuss their properties. We prove that the necessary and sufficient condition for R_{N_Q} to be a reverse subsystem, union and intersection of reverse subsystems of interval neutrosophic automata is reverse subsystems.

2. Preliminaries

Definition 2.1. [14] Let U be the universe of discourse. A neutrosophic set (NS) N in U is defined by a truth membership T_N, indeterminacy membership I_N and a falsity membership F_N, where T_N, I_N, and F_N are real standard or non-standard subsets of $]0^-, 1^+[$. That is
$N = \{\langle x, (T_N(x), I_N(x), F_N(x))\rangle, x \in U,\ T_N, I_N, F_N \in\ ]0^-, 1^+[\ \}$ and
$0^- \leq sup\ T_N(x) + sup\ I_N(x) + sup\ F_N(x) \leq 3^+$. We use the interval $[0, 1]$ instead of $]0^-, 1^+[$.

Definition 2.2. [16] Interval neutrosophic set (INS for short) is of the form $N = \{\langle \alpha_N(x), \beta_N(x), \gamma_N(x)\rangle \,|x \in U\}$
$= \{\langle x, [\inf\ \alpha_N(x), \sup\ \alpha_N(x)], [\inf\ \beta_N(x), \sup\ \beta_N(x)], [\inf\ \gamma_N(x), \sup\gamma_N(x)]\rangle\}$,
$x \in U$, $\alpha_N(x),\ \beta_N(x),\ \gamma_N(x) \subseteq [0, 1]$ and
$0 \leq \sup\ \alpha_N(x) +\ \sup\ \beta_N(x) +\ \sup\ \gamma_N(x) \leq\ 3.$

Definition 2.3. [16] An INS N is empty if $\inf\ \alpha_N(x) = \sup\ \alpha_N(x) = 0,\ \inf\ \beta_N(x) = \sup\ \beta_N(x) = 1,\ \inf\ \gamma_N(x) = \sup\ \gamma_N(x) = 1$ forall $x\ \in\ U.$

Definition 2.4. [11] Interval neutrosophic automaton $M = (Q,\ \Sigma,\ N)$ $(INAforshort)$, where Q and Σ are non-empty finite sets called the set of states and input symbols respectively, and $N = \{\langle \alpha_N(x), \beta_N(x), \gamma_N(x)\rangle\}$ is an INS in $Q \times \Sigma \times Q$.
The set of all words of finite length of Σ is denoted by Σ^*. The empty word is denoted by ϵ, and the length of each $x \in \Sigma^*$ is denoted by $|x|$.

Definition 2.5. [11] Let $M = (Q,\ \Sigma,\ N)$ be an interval neutrosophic automaton and extended interval neutrosophic set is defined as $N^* = \{\langle \alpha_{N^*}(x), \beta_{N^*}(x), \gamma_{N^*}(x)\rangle\}$ in $Q \times \Sigma^* \times Q$ by

$$\alpha_{N^*}(q_i,\ \epsilon,\ q_j) = \begin{cases} [1,1] & \text{if } q_i = q_j \\ [0,0] & \text{if } q_i \neq q_j \end{cases}$$

$$\beta_{N^*}(q_i,\ \epsilon,\ q_j) = \begin{cases} [0,0] & \text{if } q_i = q_j \\ [1,1] & \text{if } q_i \neq q_j \end{cases}$$

$$\gamma_{N^*}(q_i,\ \epsilon,\ q_j) = \begin{cases} [0,0] & \text{if } q_i = q_j \\ [1,1] & \text{if } q_i \neq q_j \end{cases}$$

$\alpha_{N^*}(q_i, w, q_j) = \alpha_{N^*}(q_i, xy, q_j) = \vee_{q_r \in Q}[\alpha_{N^*}(q_i, x, q_r) \wedge \alpha_{N^*}(q_r, y, q_j)]$,
$\beta_{N^*}(q_i, w, q_j) = \beta_{N^*}(q_i, xy, q_j) = \wedge_{q_r \in Q}[\beta_{N^*}(q_i, x, q_r) \vee \beta_{N^*}(q_r, y, q_j)]$,
$\gamma_{N^*}(q_i, w, q_j) = \gamma_{N^*}(q_i, xy, q_j) = \wedge_{q_r \in Q}[\gamma_{N^*}(q_i, x, q_r) \vee \gamma_{N^*}(q_r, y, q_j)], \forall q_i, q_j \in Q$,
$w = xy, x \in \Sigma^*$ and $y \in \Sigma$.

3. Reverse Subsystems of Interval Neutrosophic Automata

Definition 3.1. Let $M = (Q,\ \Sigma,\ N)$ be an interval neutrosophic automaton and R_{N_Q} be an interval neutrosophic set of Q. Let $q_i \in Q$, and R_{N_Q} is defined as $R_{N_Q} = \left\{\left\langle \alpha_{R_{N_Q}}(q_i),\ \beta_{R_{N_Q}}(q_i),\ \gamma_{R_{N_Q}}(q_i)\right\rangle\right\}$
$= \left\{\left\langle q_i, [\inf\alpha_{R_{N_Q}}(q_i), \sup\alpha_{R_{N_Q}}(q_i)], [\inf\beta_{R_{N_Q}}(q_i), \sup\beta_{R_{N_Q}}(q_i)], [\inf\gamma_{R_{N_Q}}(q_i), \sup\gamma_{R_{N_Q}}(q_i)]\right\rangle\right\}$.
Here, $\alpha_{R_{N_Q}}(q_i),\ \beta_{R_{N_Q}}(q_i), \gamma_{R_{N_Q}}(q_i) \subseteq [0,1]$.
Then (Q, R_{N_Q}, Σ, N) is said to be an reverse subsystem of M if $\forall q_i, q_j \in Q$ and $x \in \Sigma$ such that $\alpha_{R_{N_Q}}(q_j) \leq \vee_{q_i \in Q}\{\alpha_{R_{N_Q}}(q_i) \wedge \alpha_N(q_i, x, q_j)\}$,
$\beta_{R_{N_Q}}(q_j) \geq \wedge_{q_i \in Q}\{\beta_{R_{N_Q}}(q_i) \vee \beta_N(q_i, x, q_j)\}$ and
$\gamma_{R_{N_Q}}(q_j) \geq \wedge_{q_i \in Q}\{\gamma_{R_{N_Q}}(q_i) \vee \gamma_N(q_i, x, q_j)\}$.
In this case, the reverse subsystem (Q, R_{N_Q}, Σ, N) of M is denoted by R_{N_Q}.

Example 3.2. Let $M = (Q,\ \Sigma,\ N)$ be an interval neutrosophic automaton, where $Q = \{q_1, q_2, q_3, q_4, q_5\},\ \Sigma = \{x\}$, and $N,\ N_Q(q_i),\ i = 1, 2, 3, 4, 5$ are defined as below.

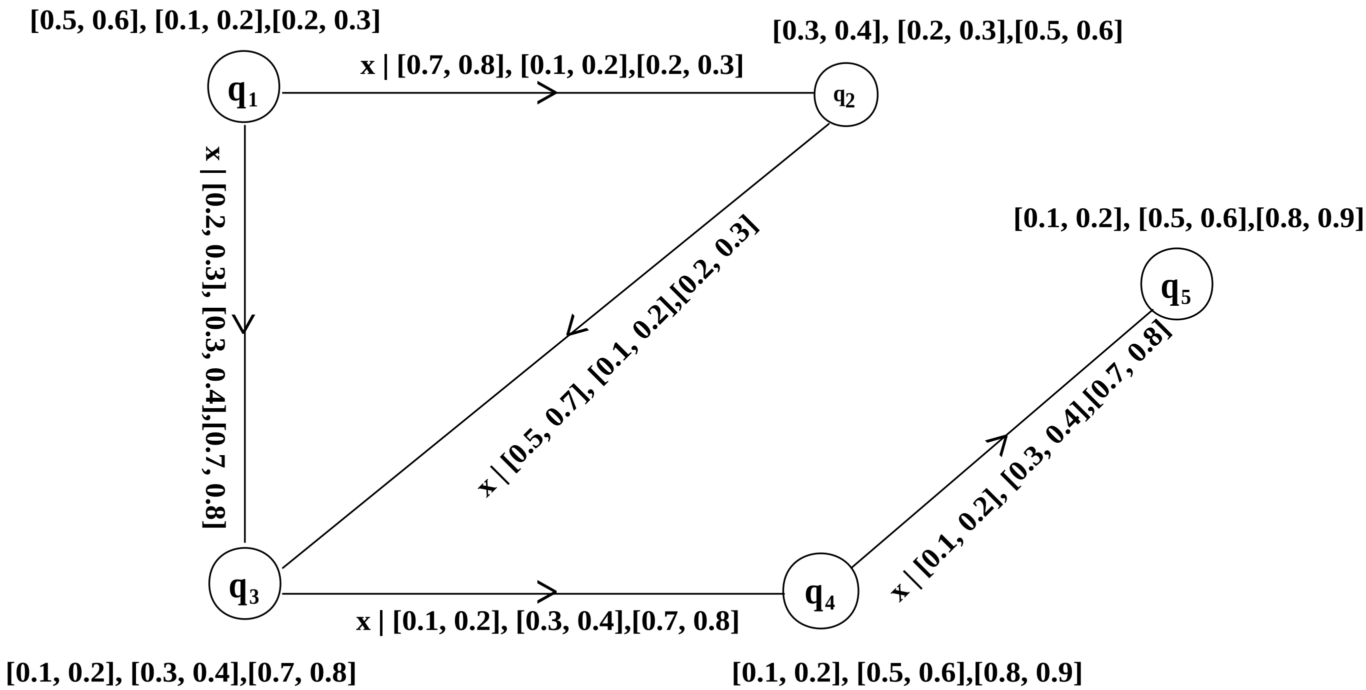

Fig- 3.1

In this case the above interval neutrosophic automaton M is said to be reverse subsystem.

Theorem 3.3. Let $M = (Q,\ \Sigma,\ N)$ be an interval neutrosophic automaton and $R_{N_Q} = \left\{\left\langle \alpha_{R_{N_Q}}, \beta_{R_{N_Q}}, \gamma_{R_{N_Q}} \right\rangle\right\}$ be an interval neutrosophic subset in Q. Then R_{N_Q} is an reverse subsystem of M if and only if $\forall q_i, q_j \in Q, \forall x \in \Sigma^*$,

$\alpha_{R_{N_Q}}(q_j) \leq \vee_{q_i \in Q}\{\alpha_{R_{N_Q}}(q_i) \wedge \alpha_N(q_i, x, q_j)\}$,

$\beta_{R_{N_Q}}(q_j) \geq \wedge_{q_i \in Q}\{\beta_{R_{N_Q}}(q_i) \vee \beta_N(q_i, x, q_j)\}$ and

$\gamma_{R_{N_Q}}(q_j) \leq \wedge_{q_i \in Q}\{\gamma_{R_{N_Q}}(q_i) \vee \gamma_N(q_i, x, q_j)\}$.

Proof. Suppose $R_{N_Q}$is an reverse subsystem of M. Let $q_i, q_j \in Q$ and $x \in \Sigma^*$. We prove this by induction on $|x| = n$. If $n = 0$, then $x = \epsilon$. Now if $q_i = q_j$, then

$\alpha_{R_{N_Q}}(q_j) \wedge \alpha_{N*}(q_i, \epsilon, q_j) = \alpha_{R_{N_Q}}(q_j)$, $\beta_{R_{N_Q}}(q_j) \vee \beta_{N*}(q_i, \epsilon, q_j) = \beta_{R_{N_Q}}(q_j)$, and

$\gamma_{R_{N_Q}}(q_j) \vee \gamma_{N*}(q_i, \epsilon, q_j) = \gamma_{R_{N_Q}}(q_j)$.

Now if $q_i \neq q_j$, then

$\alpha_{R_{N_Q}}(q_i) \wedge \alpha_{N^*}(q_i, \epsilon, q_j) \geq \alpha_{R_{N_Q}}(q_j)$, $\beta_{R_{N_Q}}(q_i) \vee \beta_{N^*}(q_i, \epsilon, q_j) \leq \beta_{R_{N_Q}}(q_j)$, and

$\gamma_{R_{N_Q}}(q_i) \vee \gamma_{N^*}(q_i, \epsilon, q_j) \leq \gamma_{R_{N_Q}}(q_j)$.

Therefore, the statement is true for $n = 0$.

Assume the statement is true for all $y \in \Sigma^*$ such that $|y| = n - 1, n > 0$.

Let $x = ya, |y| = n - 1, y \in \Sigma^*, a \in \Sigma$. Then

$\vee_{q_i \in Q}\{\alpha_{R_{N_Q}}(q_i) \wedge \alpha_N(q_i, x, q_j)\} = \vee_{q_i \in Q}\{\alpha_{R_{N_Q}}(q_i) \wedge \alpha_{N^*}(q_i, ya, q_j)\}$

$= \vee_{q_i \in Q}\{\alpha_{R_{N_Q}}(q_i) \wedge \{\vee_{q_k \in Q}\ \{\alpha_{N^*}(q_i, y, q_k) \wedge \alpha_N(q_k, a, q_j)\}\}\}$

$= \vee_{q_i \in Q}\{\vee_{q_k \in Q}\left\{\alpha_{R_{N_Q}}(q_i) \wedge \alpha_{N^*}(q_i, y, q_k) \wedge \alpha_N(q_k, a, q_j)\right\}\}$
$\geq \vee_{q_k \in Q}\left\{\alpha_{R_{N_Q}}(q_k) \wedge \alpha_N(q_k, a, q_j)\right\}$
$\geq \alpha_{R_{N_Q}}(q_j)$.
$\vee_{q_i \in Q}\{\alpha_{R_{N_Q}}(q_i) \wedge \alpha_N(q_i, x, q_j)\} \geq \alpha_{R_{N_Q}}(q_j)$.
Thus, $\alpha_{R_{N_Q}}(q_j) \leq \vee_{q_i \in Q}\{\alpha_{R_{N_Q}}(q_i) \wedge \alpha_N(q_i, x, q_j)\}$
$\wedge_{q_i \in Q}\{\beta_{R_{N_Q}}(q_i) \vee \beta_N(q_i, x, q_j)\} = \wedge_{q_i \in Q}\{\beta_{R_{N_Q}}(q_i) \vee \beta_N(q_i, ya, q_j)\}$
$= \wedge_{q_i \in Q}\{\beta_{R_{N_Q}}(q_i) \vee \{\wedge_{q_k \in Q}\{\beta_{N^*}(q_i, y, q_k) \vee \beta_N(q_k, a, q_j)\}\}\}$
$= \wedge_{q_i \in Q}\{\wedge_{q_k \in Q}\left\{\beta_{R_{N_Q}}(q_i) \vee \beta_{N^*}(q_i, y, q_k) \vee \beta_N(q_k, a, q_j)\right\}\}$
$\leq \wedge_{q_k \in Q}\left\{\beta_{R_{N_Q}}(q_k) \vee \beta_N(q_k, a, q_j)\right\}$
$\leq \beta_{R_{N_Q}}(q_j)$.
$\wedge_{q_i \in Q}\{\beta_{R_{N_Q}}(q_i) \vee \beta_N(q_i, x, q_j)\} \leq \beta_{R_{N_Q}}(q_j)$
Thus,$\beta_{R_{N_Q}}(q_j) \geq \wedge_{q_i \in Q}\{\beta_{R_{N_Q}}(q_i) \vee \beta_N(q_i, x, q_j)\}$ and
$\wedge_{q_i \in Q}\{\gamma_{R_{N_Q}}(q_i) \vee \gamma_N(q_i, x, q_j)\} = \wedge_{q_i \in Q}\{\gamma_{R_{N_Q}}(q_i) \vee \gamma_N(q_i, ya, q_j)\}$
$= \wedge_{q_i \in Q}\{\gamma_{R_{N_Q}}(q_i) \vee \{\wedge_{q_k \in Q}\{\gamma_{N^*}(q_i, y, q_k) \vee \gamma_N(q_k, a, q_j)\}\}\}$
$= \wedge_{q_i \in Q}\{\wedge_{q_k \in Q}\left\{\gamma_{R_{N_Q}}(q_i) \vee \gamma_{N^*}(q_i, y, q_k) \vee \gamma_N(q_k, a, q_j)\right\}\}$
$\leq \wedge_{q_k \in Q}\left\{\gamma_{R_{N_Q}}(q_k) \vee \gamma_N(q_k, a, q_j)\right\}$
$\leq \gamma_{R_{N_Q}}(q_j)$.
$\wedge_{q_i \in Q}\{\gamma_{R_{N_Q}}(q_i) \vee \gamma_N(q_i, x, q_j)\} \leq \gamma_{R_{N_Q}}(q_j)$.
Thus, $\gamma_{R_{N_Q}}(q_j) \geq \wedge_{q_i \in Q}\{\gamma_{R_{N_Q}}(q_i) \vee \gamma_N(q_i, x, q_j)\}$.
The converse part is obvious.

Theorem 3.4. Let$M = (Q,\ \Sigma,\ N)$ be an interval neutrosophic automaton. Let $R_{N_{Q_1}}$, and $R_{N_{Q_2}}$ be reverse subsystems of M. Then $R_{N_{Q_1}} \vee R_{N_{Q_2}}$ is an reverse subsystem of M.

Proof. Since $R_{N_{Q_1}}$ and $R_{N_{Q_2}}$ are reverse subsystem of an interval neutrosophic automaton M. Then $\forall\ q_i, q_j\ \in\ Q$ and $x\ \in\ \Sigma$ such that
$\alpha_{R_{N_{Q_1}}}(q_j) \leq \vee_{q_i \in Q}\{\alpha_{R_{N_{Q_1}}}(q_i) \wedge \alpha_N(q_i, x, q_j)\}$,
$\beta_{R_{N_{Q_1}}}(q_j) \geq \wedge_{q_i \in Q}\{\beta_{R_{N_{Q_1}}}(q_i) \vee \beta_N(q_i, x, q_j)\}$,
$\gamma_{R_{N_{Q_1}}}(q_j) \geq \wedge_{q_i \in Q}\{\gamma_{R_{N_{Q_1}}}(q_i) \vee \gamma_N(q_i, x, q_j)\}$ and
$\alpha_{R_{N_{Q_2}}}(q_j) \leq \vee_{q_i \in Q}\{\alpha_{R_{N_{Q_2}}}(q_i) \wedge \alpha_N(q_i, x, q_j)\}$,
$\beta_{R_{N_{Q_2}}}(q_j) \leq \wedge_{q_i \in Q}\{\beta_{R_{N_{Q_2}}}(q_i) \vee \beta_N(q_i, x, q_j)\}$,
$\gamma_{R_{N_{Q_2}}}(q_j) \leq \wedge_{q_i \in Q}\{\gamma_{R_{N_{Q_2}}}(q_i) \vee \gamma_N(q_i, x, q_j)\}$.
Now to prove $R_{N_{Q_1}} \vee R_{N_{Q_2}}$ is reverse subsystem of interval neutrosophic automaton M, it is enough to prove that
$(\alpha_{R_{N_{Q_1}}} \vee \alpha_{R_{N_{Q_2}}})(q_j) \leq \vee_{q_i \in Q}\{(\alpha_{R_{N_{Q_1}}} \vee \alpha_{R_{N_{Q_2}}})(q_i) \wedge \alpha_N(q_i, x, q_j)\}$,
$(\beta_{R_{N_{Q_1}}} \vee \beta_{R_{N_{Q_2}}})(q_j) \geq \wedge_{q_i \in Q}\{(\beta_{R_{N_{Q_1}}} \vee \beta_{R_{N_{Q_2}}})(q_i) \vee \beta_N(q_i, x, q_j)\}$, and
$(\gamma_{R_{N_{Q_1}}} \vee \gamma_{R_{N_{Q_2}}})(q_j) \geq \wedge_{q_i \in Q}\{(\gamma_{R_{N_{Q_1}}} \vee \gamma_{R_{N_{Q_2}}})(q_i) \vee \gamma_N(q_i, x, q_j)\}$.

Now, $(\alpha_{R_{N_{Q_1}}} \vee \alpha_{R_{N_{Q_2}}})(q_j) = (\alpha_{R_{N_{Q_1}}}(q_j) \vee \alpha_{R_{N_{Q_2}}}(q_j))$
$\leq \{\vee_{q_i \in Q}\{\alpha_{R_{N_{Q_1}}}(q_i) \wedge \alpha_N(q_i, x, q_j)\}\} \vee \{\vee_{q_i \in Q}\{\alpha_{R_{N_{Q_2}}}(q_i) \wedge \alpha_N(q_i, x, q_j)\}\}$
$= \{\vee_{q_i \in Q}\{\alpha_{R_{N_{Q_1}}}(q_i) \vee \alpha_{R_{N_{Q_2}}}(q_i) \wedge \alpha_N(q_i, x, q_j)\}\}$
$= \vee_{q_i \in Q}\{(\alpha_{R_{N_{Q_1}}} \vee \alpha_{R_{N_{Q_2}}})(q_i) \wedge \alpha_N(q_i, x, q_j)\}$
Thus, $(\alpha_{R_{N_{Q_1}}} \vee \alpha_{R_{N_{Q_2}}})(q_j) \leq \vee_{q_i \in Q}\{(\alpha_{R_{N_{Q_1}}} \vee \alpha_{R_{N_{Q_2}}})(q_i) \wedge \alpha_N(q_i, x, q_j)\}$,—(1)
$(\beta_{R_{N_{Q_1}}} \vee \beta_{R_{N_{Q_2}}})(q_j) = (\beta_{R_{N_{Q_1}}}(q_j) \vee \beta_{R_{N_{Q_2}}}(q_j))$
$\geq \{\wedge_{q_i \in Q}\{\beta_{R_{N_{Q_1}}}(q_i) \vee \beta_N(q_i, x, q_j)\}\} \vee \{\wedge_{q_i \in Q}\{\beta_{R_{N_{Q_2}}}(q_i) \vee \beta_N(q_i, x, q_j)\}\}$
$= \{\wedge_{q_i \in Q}\{\beta_{R_{N_{Q_1}}}(q_i) \vee \beta_{R_{N_{Q_2}}}(q_i) \vee \beta_N(q_i, x, q_j)\}\}$
$= \wedge_{q_i \in Q}\{(\beta_{R_{N_{Q_1}}} \vee \beta_{R_{N_{Q_2}}})(q_i) \vee \beta_N(q_i, x, q_j)\}$,
Thus, $(\beta_{R_{N_{Q_1}}} \vee \beta_{R_{N_{Q_2}}})(q_j) \geq \wedge_{q_i \in Q}\{(\beta_{R_{N_{Q_1}}} \vee \beta_{R_{N_{Q_2}}})(q_i) \vee \beta_N(q_i, x, q_j)\}$,—(2) and
$(\gamma_{R_{N_{Q_1}}} \vee \gamma_{R_{N_{Q_2}}})(q_j) = (\gamma_{R_{N_{Q_1}}}(q_j) \vee \gamma_{R_{N_{Q_2}}}(q_j))$
$\geq \{\wedge_{q_i \in Q}\{\gamma_{R_{N_{Q_1}}}(q_i) \vee \gamma_N(q_i, x, q_j)\}\} \vee \{\wedge_{q_i \in Q}\{\gamma_{R_{N_{Q_2}}}(q_i) \vee \gamma_N(q_i, x, q_j)\}\}$
$= \{\wedge_{q_i \in Q}\{\gamma_{R_{N_{Q_1}}}(q_i) \vee \gamma_{R_{N_{Q_2}}}(q_i) \vee \gamma_N(q_i, x, q_j)\}\}$
$= \wedge_{q_i \in Q}\{(\gamma_{R_{N_{Q_1}}} \vee \gamma_{R_{N_{Q_2}}})(q_i) \vee \gamma_N(q_i, x, q_j)\}$.
Thus, $(\gamma_{R_{N_{Q_1}}} \vee \gamma_{R_{N_{Q_2}}})(q_j) \geq \wedge_{q_i \in Q}\{(\gamma_{R_{N_{Q_1}}} \vee \gamma_{R_{N_{Q_2}}})(q_i) \vee \gamma_N(q_i, x, q_j)\}$.—(3)
Hence from (1), (2), and (3),$R_{N_{Q_1}} \vee R_{N_{Q_2}}$ is reverse subsystem of interval neutrosophic automaton M.

Theorem 3.5. Let$M = (Q,\ \Sigma,\ N)$ be an interval neutrosophic automaton. and $R_{N_{Q_1}}$, and $R_{N_{Q_2}}$ be reverse subsystems of M. Then $R_{N_{Q_1}} \wedge R_{N_{Q_2}}$ is reverse subsystem of M.

Proof:

Since $R_{N_{Q_1}}$ and $R_{N_{Q_2}}$ are reverse subsystem of interval neutrosophic automaton M. Then $\forall\ q_i, q_j\ \in\ Q$ and $x\ \in\ \Sigma$ such that
$\alpha_{R_{N_{Q_1}}}(q_j) \leq \vee_{q_i \in Q}\{\alpha_{R_{N_{Q_1}}}(q_i) \wedge \alpha_N(q_i, x, q_j)\}$,
$\beta_{R_{N_{Q_1}}}(q_j) \geq \wedge_{q_i \in Q}\{\beta_{R_{N_{Q_1}}}(q_i) \vee \beta_N(q_i, x, q_j)\}$,
$\gamma_{R_{N_{Q_1}}}(q_j) \geq \wedge_{q_i \in Q}\{\gamma_{R_{N_{Q_1}}}(q_i) \vee \gamma_N(q_i, x, q_j)\}$ and
$\alpha_{R_{N_{Q_2}}}(q_j) \leq \vee_{q_i \in Q}\{\alpha_{R_{N_{Q_2}}}(q_i) \wedge \alpha_N(q_i, x, q_j)\}$,
$\beta_{R_{N_{Q_2}}}(q_j) \leq \wedge_{q_i \in Q}\{\beta_{R_{N_{Q_2}}}(q_i) \vee \beta_N(q_i, x, q_j)\}$,
$\gamma_{R_{N_{Q_2}}}(q_j) \leq \wedge_{q_i \in Q}\{\gamma_{R_{N_{Q_2}}}(q_i) \vee \gamma_N(q_i, x, q_j)\}$.
Now we have to prove $R_{N_{Q_1}} \wedge R_{N_{Q_2}}$ is a reverse subsystem of M.
It is enough to prove that
$(\alpha_{R_{N_{Q_1}}} \wedge \alpha_{R_{N_{Q_2}}})(q_j) \leq \vee_{q_i \in Q}\{(\alpha_{R_{N_{Q_1}}} \wedge \alpha_{R_{N_{Q_2}}})(q_i) \wedge \alpha_N(q_i, x, q_j)\}$,
$(\beta_{R_{N_{Q_1}}} \wedge \beta_{R_{N_{Q_2}}})(q_j) \geq \wedge_{q_i \in Q}\{(\beta_{R_{N_{Q_1}}} \wedge \beta_{R_{N_{Q_2}}})(q_i) \vee \beta_N(q_i, x, q_j)\}$, and
$(\gamma_{R_{N_{Q_1}}} \wedge \gamma_{R_{N_{Q_2}}})(q_j) \geq \wedge_{q_i \in Q}\{(\gamma_{R_{N_{Q_1}}} \wedge \gamma_{R_{N_{Q_2}}})(q_i) \vee \gamma_N(q_i, x, q_j)\}$.
Now, $(\alpha_{R_{N_{Q_1}}} \wedge \alpha_{R_{N_{Q_2}}})(q_j) = (\alpha_{R_{N_{Q_1}}}(q_j) \wedge \alpha_{R_{N_{Q_2}}}(q_j))$
$\leq \{\vee_{q_i \in Q}\{\alpha_{R_{N_{Q_1}}}(q_i) \wedge \alpha_N(q_i, x, q_j)\}\} \wedge \{\vee_{q_i \in Q}\{\alpha_{R_{N_{Q_2}}}(q_i) \wedge \alpha_N(q_i, x, q_j)\}\}$

$= \{\vee_{q_i \in Q}\{\alpha_{R_{N_{Q_1}}}(q_i) \wedge \alpha_{R_{N_{Q_2}}}(q_i) \wedge \alpha_N(q_i, x, q_j)\}\}$
$= \vee_{q_i \in Q}\{(\alpha_{R_{N_{Q_1}}} \wedge \alpha_{R_{N_{Q_2}}})(q_i) \wedge \alpha_N(q_i, x, q_j)\}$,
Thus, $(\alpha_{R_{N_{Q_1}}} \wedge \alpha_{R_{N_{Q_2}}})(q_j) \leq \vee_{q_i \in Q}\{(\alpha_{R_{N_{Q_1}}} \wedge \alpha_{R_{N_{Q_2}}})(q_i) \wedge \alpha_N(q_i, x, q_j)\}$,—(4) $(\beta_{R_{N_{Q_1}}} \wedge \beta_{R_{N_{Q_2}}})(q_j) = (\beta_{R_{N_{Q_1}}}(q_j) \wedge \beta_{R_{N_{Q_2}}}(q_j))$
$\geq \{\wedge_{q_i \in Q}\{\beta_{R_{N_{Q_1}}}(q_i) \vee \beta_N(q_i, x, q_j)\}\} \wedge \{\wedge_{q_i \in Q}\{\beta_{R_{N_{Q_2}}}(q_i) \vee \beta_N(q_i, x, q_j)\}\}$
$= \{\wedge_{q_i \in Q}\{\beta_{R_{N_{Q_1}}}(q_i) \wedge \beta_{R_{N_{Q_2}}}(q_i) \vee \beta_N(q_i, x, q_j)\}\}$
$= \wedge_{q_i \in Q}\{(\beta_{R_{N_{Q_1}}} \wedge \beta_{R_{N_{Q_2}}})(q_i) \vee \beta_N(q_i, x, q_j)\}$, Thus, $(\beta_{R_{N_{Q_1}}} \wedge \beta_{R_{N_{Q_2}}})(q_j) \geq \wedge_{q_i \in Q}\{(\beta_{R_{N_{Q_1}}} \wedge \beta_{R_{N_{Q_2}}})(q_i) \vee \beta_N(q_i, x, q_j)\}$,—(5) and
$(\gamma_{R_{N_{Q_1}}} \wedge \gamma_{R_{N_{Q_2}}})(q_j) = (\gamma_{R_{N_{Q_1}}}(q_j) \wedge \gamma_{R_{N_{Q_2}}}(q_j))$
$\geq \{\wedge_{q_i \in Q}\{\gamma_{R_{N_{Q_1}}}(q_i) \vee \gamma_N(q_i, x, q_j)\}\} \wedge \{\wedge_{q_i \in Q}\{\gamma_{R_{N_{Q_2}}}(q_i) \vee \gamma_N(q_i, x, q_j)\}\}$
$= \{\wedge_{q_i \in Q}\{\gamma_{R_{N_{Q_1}}}(q_i) \wedge \gamma_{R_{N_{Q_2}}}(q_i) \vee \gamma_N(q_i, x, q_j)\}\}$
$= \wedge_{q_i \in Q}\{(\gamma_{R_{N_{Q_1}}} \wedge \gamma_{R_{N_{Q_2}}})(q_i) \vee \gamma_N(q_i, x, q_j)\}$
Thus, $(\gamma_{R_{N_{Q_1}}} \wedge \gamma_{R_{N_{Q_2}}})(q_j) \geq \wedge_{q_i \in Q}\{(\gamma_{R_{N_{Q_1}}} \wedge \gamma_{R_{N_{Q_2}}})(q_i) \vee \gamma_N(q_i, x, q_j)\}$.—(6)
Thus, From (4), (5) and (6) $R_{N_{Q_1}} \wedge R_{N_{Q_2}}$ is reverse subsystem of interval neutrosophic automaton M.

4. Conclusions

In this paper, we introduce reverse subsystem of interval neutrosophic automata with example. Also, we establish necessary and sufficient condition for R_{N_Q} to be a reverse subsystem in interval neutrosophic automaton. Finally, we prove that the union and intersection of reverse subsystems of interval neutrosophic automaton is reverse subsystem of an interval neutrosophic automaton.

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