



# SVPNS-MADM strategy based on GRA in SVPNS

## Environment

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**Abstract:** In the present study, we present a multi-attribute-decision-making (MADM) strategy in Single Valued Pentapartitioned Neutrosophic Set (SVPNS) environment based on Grey Relational Analysis (GRA) which we call SVPNS- MADM strategy. We define Hamming distance between two single valued pentapartitioned neutrosophic sets and prove its basic properties. The notion of pentapartitioned neutrosophic set is a powerful mathematical tool to deal with incomplete, indeterminate, ignorance, and inconsistent information. In this paper, we extend the neutrosophic GRA strategy to pentapartitioned neutrosophic GRA strategy. Then we employ it to an MADM strategy. Further, we demonstrate the developed MADM strategy by solving an illustrative numerical example that reflects the efficiency and applicability of the proposed strategy.

**Keywords:** Neutrosophic set, Single valued neutrosophic set, Pentapartitioned neutrosophic set; Multi attribute decision making, Grey relational analysis.

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### 1. Introduction

The idea of neutrosophic set (NS) was presented by Smarandache [38], which was a powerful mathematical tool to deal with incomplete, indeterminate, and inconsistent information. The notion of NS and its various extensions have been successfully applied in the many fields such as decision making [1-13, 16-19, 21, 25-31, 36-38, 43, 45], medical diagnosis [32-33, 44], data mining [20], conflict resolution [35], etc. In the recent past, the NSs [6, 22, 23-24, 39-42] have drawn a great attention during the last two decades. Different models of Multi-Attribute Decision Making (MADM) for crisp set,

fuzzy set, intuitionistic fuzzy set and NS environments have been already proposed by so many researchers. Biswas et al. [5] proposed an entropy based grey relational analysis strategy for MADM model under The Single Valued Neutrosophic Set (SVNS) environment. Mondal and Pramanik [16] also proposed a neutrosophic decision making model for clay-brick selection in construction field based on Grey Relational Analysis (GRA) in SVNS environment. The notion of Pentapartitioned Neutrosophic Set (PNS) was grounded by Mallick and Pramanik [15] by in the year 2020, which was also very powerful mathematical tool to deal with the data includes incomplete, indeterminate, ignorance, and unknown information. Since PNS has been restricted in the  $[0,1]$ , we call it Single Valued PNS (SVPNS).

There is no study in the literature relating to MADM in SVPNS environment. To explore the unexplored MADM in SVPNS environment, we present an MADM strategy under in SVPNS environment based on GRA.

The rest of the paper is organized in the following way:

Section 2 recalls some relevant results on SVPNSs. Section 3 presents some new definitions relating to SVPNS that are useful to develop the present paper. Section 4 devotes to develop the GRA based SVPNS-MADM strategy. In section 5, we present a numerical example to demonstrate the proposed SVPNS-MADM strategy. In section 6, we present the concluding remarks and future scope of research.

## 2. Some Relevant Results:

In this section, we give some existing definitions, properties of PNS. We also provide some illustrative examples on PNS.

**Definition 2.1.**[15] Suppose that  $\Psi$  be a non-empty set. Then a PNS  $V$  over  $\Psi$  is defined by:

$V = \{(s, T_V(s), C_V(s), G_V(s), U_V(s), F_V(s)) : s \in \Psi\}$ , where  $T_V(s), C_V(s), G_V(s), U_V(s), F_V(s) \in [0,1]$  are the degree of truth, contradiction, ignorance, unknown, and falsity membership of  $s \in \Psi$ . Therefore  $0 \leq T_V(s) + C_V(s) + G_V(s) + U_V(s) + F_V(s) \leq 5$ .

**Example 2.1.** Let  $\Psi = \{r, s\}$ . Then  $W = \{(r, 0.8, 0.3, 0.4, 0.8, 0.9), (s, 0.9, 0.2, 0.4, 0.5)\}$  is a PNS over  $\Psi$ .

**Definition 2.2.**[15] Suppose that  $V = \{(s, T_V(s), C_V(s), G_V(s), U_V(s), F_V(s)) : s \in \Psi\}$  and  $Y = \{(s, T_Y(s), C_Y(s), G_Y(s), U_Y(s), F_Y(s)) : s \in \Psi\}$  be two PNSs over  $\Psi$ . Then

(i)  $V \cup Y = \{(s, \max\{T_V(s), T_Y(s)\}, \max\{C_V(s), C_Y(s)\}, \min\{G_V(s), G_Y(s)\}, \min\{U_V(s), U_Y(s)\}, \min\{F_V(s), F_Y(s)\}) : s \in \Psi\}$ ;

(ii)  $V \cap Y = \{(s, \min\{T_V(s), T_Y(s)\}, \min\{C_V(s), C_Y(s)\}, \max\{G_V(s), G_Y(s)\}, \max\{U_V(s), U_Y(s)\}, \max\{F_V(s), F_Y(s)\}) : s \in \Psi\}$ ;

(iii)  $V^c = \{(s, F_V(s), U_V(s), 1 - G_V(s), C_V(s), T_V(s)) : s \in \Psi\}$ ;

(iv)  $V \subseteq Y$  iff  $T_V(s) \leq T_Y(s), C_V(s) \leq C_Y(s), G_V(s) \geq G_Y(s), U_V(s) \geq U_Y(s), F_V(s) \geq F_Y(s)$ , for all  $s \in \Psi$ .

**Example 2.2.** Suppose that  $\Psi = \{s, r\}$ . Let  $V = \{(s, 0.3, 0.8, 0.8, 0.5, 0.2), (r, 0.5, 0.9, 0.5, 0.2, 0.3)\}$  and  $Y = \{(s, 0.9, 0.6, 0.6, 0.7, 0.2), (r, 0.8, 0.6, 0.3, 0.1, 0.9)\}$  be two PNSs over  $\Psi$ . Then

- (i)  $V \cup Y = \{(s, 0.9, 0.8, 0.6, 0.5, 0.2), (r, 0.8, 0.9, 0.3, 0.1, 0.3)\}$ ;
- (ii)  $V \cap Y = \{(s, 0.3, 0.6, 0.8, 0.7, 0.2), (r, 0.5, 0.6, 0.5, 0.2, 0.9)\}$ ;
- (iii)  $V^c = \{(s, 0.2, 0.5, 0.2, 0.8, 0.3), (r, 0.3, 0.2, 0.5, 0.9, 0.5)\}$ ,  $Y^c = \{(s, 0.2, 0.7, 0.4, 0.6, 0.9), (r, 0.9, 0.1, 0.7, 0.6, 0.8)\}$

**Example 2.3.** Suppose that  $\Psi = \{s, r\}$ . Let  $V = \{(s, 0.3, 0.3, 0.8, 0.7, 0.2), (r, 0.8, 0.9, 0.5, 0.2, 0.3)\}$  and  $Y = \{(s, 0.9, 0.9, 0.6, 0.7, 0.2), (r, 0.8, 1.0, 0.3, 0.1, 0.1)\}$  be two PNSs over  $\Psi$ . Then  $V \subseteq Y$ .

### 3. Single Valued Pentapartitioned Neutrosophic Set (SVPNS):

**Definition 3.1.** An SVPNS [15]  $Y$  over a fixed set  $\Psi$  are characterized by a truth-membership function ( $T_Y$ ), a contradiction-membership function ( $C_Y$ ), an ignorance-membership function ( $G_Y$ ), an unknown-membership function ( $U_Y$ ), a falsity-membership function ( $F_Y$ ). Here  $T_Y(s), C_Y(s), G_Y(s), U_Y(s), F_Y(s) \in [0, 1], \forall s \in \Psi$ . The SVPNS  $Y$  is denoted as follows:

$$Y = \{(s, T_Y(s), C_Y(s), G_Y(s), U_Y(s), F_Y(s)) : s \in \Psi\}.$$

**Definition 3.2.** Assume that  $B = \{(s, T_B(s), C_B(s), G_B(s), U_B(s), F_B(s)) : s \in \Psi\}$  and  $D = \{(s, T_D(s), C_D(s), G_D(s), U_D(s), F_D(s)) : s \in \Psi\}$  be two SVPNSs [15] over  $\Psi$ . Then,

- (i)  $B \subseteq D$  iff  $T_B(s) \leq T_D(s), C_B(s) \leq C_D(s), G_B(s) \geq G_D(s), U_B(s) \geq U_D(s), F_B(s) \geq F_D(s), \forall s \in \Psi$ ;
- (ii)  $B = D$  iff  $D \subseteq B$  and  $B \subseteq D$ .

**Definition 3.3.** Assume that  $B = \{(s, T_B(s), C_B(s), G_B(s), U_B(s), F_B(s)) : s \in \Psi\}$  and  $D = \{(s, T_D(s), C_D(s), G_D(s), U_D(s), F_D(s)) : s \in \Psi\}$  be two SVPNSs over  $\Psi$ . Let the cardinality of  $\Psi$  be  $n$ . The Hamming distance ( $H_d$ ) between  $B$  and  $D$  is defined by

$$H_d(B, D) = \sum_{s \in \Psi} (|T_B(s) - T_D(s)| + |C_B(s) - C_D(s)| + |G_B(s) - G_D(s)| + |U_B(s) - U_D(s)| + |F_B(s) - F_D(s)|) \quad (1)$$

where  $0 \leq H_d(B, D) \leq 5n$ .

**Example 3.1.** Suppose that  $V = \{(s, 0.3, 0.3, 0.8, 0.7, 0.2), (r, 0.8, 0.9, 0.5, 0.2, 0.3)\}$  and  $Y = \{(s, 0.9, 0.9, 0.6, 0.7, 0.2), (r, 0.8, 1.0, 0.3, 0.1, 0.1)\}$  be two SVPN sets over  $\Psi = \{s, r\}$ . Then, the Hamming distance between  $V$  and  $Y$  is  $H_d(V, Y) = 2$ .

**Theorem 3.1.** The Hamming distance between two SVPNSs is bounded.

**Proof.** Suppose that  $B = \{(s, T_B(s), C_B(s), G_B(s), U_B(s), F_B(s)) : s \in \Psi\}$  and  $D = \{(s, T_D(s), C_D(s), G_D(s), U_D(s), F_D(s)) : s \in \Psi\}$  be two SVPNSs over  $\Psi$ , where cardinality of  $\Psi$  is  $n$ . Therefore,  $0 \leq T_B(s) \leq 1, 0 \leq C_B(s) \leq 1, 0 \leq G_B(s) \leq 1, 0 \leq U_B(s) \leq 1, 0 \leq F_B(s) \leq 1, 0 \leq T_D(s) \leq 1, 0 \leq C_D(s) \leq 1, 0 \leq G_D(s) \leq 1, 0 \leq U_D(s) \leq 1,$  and  $0 \leq F_D(s) \leq 1$ , for each  $s \in \Psi$ . This implies  $0 \leq |T_B(s) - T_D(s)| \leq 1, 0 \leq |C_B(s) - C_D(s)| \leq 1, 0 \leq |G_B(s) - G_D(s)| \leq 1, 0 \leq |U_B(s) - U_D(s)| \leq 1, 0 \leq |F_B(s) - F_D(s)| \leq 1$ , for each  $s \in \Psi$ .

Therefore we have,

$$\begin{aligned} & 0 \leq |T_B(s) - T_D(s)| + |C_B(s) - C_D(s)| + |G_B(s) - G_D(s)| + |U_B(s) - U_D(s)| + |F_B(s) - F_D(s)| \leq 5 \\ \Rightarrow & 0 \leq \sum_{s \in \Psi} (|T_B(s) - T_D(s)| + |C_B(s) - C_D(s)| + |G_B(s) - G_D(s)| + |U_B(s) - U_D(s)| + |F_B(s) - F_D(s)|) \\ & \leq 5n \\ \Rightarrow & 0 \leq H_d(B, D) \leq 5n \\ \Rightarrow & H_d(B, D) \in [0, 5n]. \end{aligned}$$

Therefore, the Hamming distance between two SVPNSs is bounded.

**Theorem 3.2.** Suppose that  $D = \{(s, T_D(s), C_D(s), G_D(s), U_D(s), F_D(s)): s \in \Psi\}$ ,  $B = \{(s, T_B(s), C_B(s), G_B(s), U_B(s), F_B(s)): s \in \Psi\}$  and  $A = \{(s, T_A(s), C_A(s), G_A(s), U_A(s), F_A(s)): s \in \Psi\}$  be three SVPNSs over  $\Psi$ , where cardinality of  $\Psi$  is  $n$ . If  $D \subseteq B \subseteq A$ , then

- (i)  $H_d(D, B) \leq H_d(D, A)$ ;
- (ii)  $H_d(B, A) \leq H_d(D, A)$ .

**Proof.** Let  $D = \{(s, T_D(s), C_D(s), G_D(s), U_D(s), F_D(s)): s \in \Psi\}$ ,  $B = \{(s, T_B(s), C_B(s), G_B(s), U_B(s), F_B(s)): s \in \Psi\}$  and  $A = \{(s, T_A(s), C_A(s), G_A(s), U_A(s), F_A(s)): s \in \Psi\}$  be three SVPNSs over  $\Psi$ , where cardinality of  $\Psi$  is  $n$ .

(i) Suppose that  $D \subseteq B \subseteq A$ . So  $|T_D(s) - T_B(s)| \leq |T_D(s) - T_A(s)|$ ,  $|C_D(s) - C_B(s)| \leq |C_D(s) - C_A(s)|$ ,  $|G_D(s) - G_B(s)| \leq |G_D(s) - G_A(s)|$ ,  $|U_D(s) - U_B(s)| \leq |U_D(s) - U_A(s)|$ ,  $|F_D(s) - F_B(s)| \leq |F_D(s) - F_A(s)|$ , for each  $s \in \Psi$ .

Therefore,

$$\sum_{s \in \Psi} (|T_D(s) - T_B(s)| + |C_D(s) - C_B(s)| + |G_D(s) - G_B(s)| + |U_D(s) - U_B(s)| + |F_D(s) - F_B(s)|) \leq \sum_{s \in \Psi} (|T_D(s) - T_A(s)| + |C_D(s) - C_A(s)| + |G_D(s) - G_A(s)| + |U_D(s) - U_A(s)| + |F_D(s) - F_A(s)|)$$

Now, we have

$$\begin{aligned} H_d(D, B) &= \sum_{s \in \Psi} (|T_D(s) - T_B(s)| + |C_D(s) - C_B(s)| + |G_D(s) - G_B(s)| + |U_D(s) - U_B(s)| + |F_D(s) - F_B(s)|) \\ &\leq \sum_{s \in \Psi} (|T_D(s) - T_A(s)| + |C_D(s) - C_A(s)| + |G_D(s) - G_A(s)| + |U_D(s) - U_A(s)| + |F_D(s) - F_A(s)|) \\ &= H_d(D, A). \end{aligned}$$

Hence,  $H_d(D, B) \leq H_d(D, A)$ .

(ii) Assume that  $D \subseteq B \subseteq A$ . So  $|T_B(s) - T_A(s)| \leq |T_D(s) - T_A(s)|$ ,  $|C_B(s) - C_A(s)| \leq |C_D(s) - C_A(s)|$ ,  $|G_B(s) - G_A(s)| \leq |G_D(s) - G_A(s)|$ ,  $|U_B(s) - U_A(s)| \leq |U_D(s) - U_A(s)|$ ,  $|F_B(s) - F_A(s)| \leq |F_D(s) - F_A(s)|$ , for each  $s \in \Psi$ .

Therefore,

$$\sum_{s \in \Psi} (|T_B(s) - T_A(s)| + |C_B(s) - C_A(s)| + |G_B(s) - G_A(s)| + |U_B(s) - U_A(s)| + |F_B(s) - F_A(s)|) \leq \sum_{s \in \Psi} (|T_D(s) - T_A(s)| + |C_D(s) - C_A(s)| + |G_D(s) - G_A(s)| + |U_D(s) - U_A(s)| + |F_D(s) - F_A(s)|)$$

Now, we have

$$\begin{aligned} H_d(B, A) &= \sum_{s \in \Psi} (|T_B(s) - T_A(s)| + |C_B(s) - C_A(s)| + |G_B(s) - G_A(s)| + |U_B(s) - U_A(s)| + |F_B(s) - F_A(s)|) \\ &\leq \sum_{s \in \Psi} (|T_D(s) - T_A(s)| + |C_D(s) - C_A(s)| + |G_D(s) - G_A(s)| + |U_D(s) - U_A(s)| + |F_D(s) - F_A(s)|) \\ &= H_d(D, A). \end{aligned}$$

Hence,  $H_d(B, A) \leq H_d(D, A)$ .

**Definition 3.4.** Assume that  $B = \{(s, T_B(s), C_B(s), G_B(s), U_B(s), F_B(s)): s \in \Psi\}$  and  $D = \{(s, T_D(s), C_D(s), G_D(s), U_D(s), F_D(s)): s \in \Psi\}$  be two SVPNS sets over  $\Psi$ . Let the cardinality of  $\Psi$  be  $n$ . The normalized Hamming distance (N-H<sub>d</sub>) between  $B$  and  $D$  is defined by

$$N-H_d(B, D) = \frac{1}{5n} \sum_{s \in \Psi} (|T_B(s) - T_D(s)| + |C_B(s) - C_D(s)| + |G_B(s) - G_D(s)| + |U_B(s) - U_D(s)| + |F_B(s) - F_D(s)|) \quad (2)$$

where  $0 \leq N-H_d(B, D) \leq 1$ .

**Example 3.2.** Suppose that  $V$  and  $Y$  are two SVPNSs over  $\Psi = \{s, r\}$  as shown in Example 3.1. Then  $N-H_d(V, Y) = 0.2$ .

**Theorem 3.3.** The Normalized Hamming distance between two SVPNSs is bounded.

**Proof.** Suppose that  $B = \{(s, T_B(s), C_B(s), G_B(s), U_B(s), F_B(s)): s \in \Psi\}$  and  $D = \{(s, T_D(s), C_D(s), G_D(s), U_D(s), F_D(s)): s \in \Psi\}$  be two SVPNSs over  $\Psi$ , where cardinality of  $\Psi$  is  $n$ . Therefore,  $0 \leq T_B(s) \leq 1, 0 \leq C_B(s) \leq 1, 0 \leq G_B(s) \leq 1, 0 \leq U_B(s) \leq 1, 0 \leq F_B(s) \leq 1, 0 \leq T_D(s) \leq 1, 0 \leq C_D(s) \leq 1, 0 \leq G_D(s) \leq 1, 0 \leq U_D(s) \leq 1$ , and  $0 \leq F_D(s) \leq 1$ . This implies  $0 \leq |T_B(s) - T_D(s)| \leq 1, 0 \leq |C_B(s) - C_D(s)| \leq 1, 0 \leq |G_B(s) - G_D(s)| \leq 1, 0 \leq |U_B(s) - U_D(s)| \leq 1, 0 \leq |F_B(s) - F_D(s)| \leq 1$ .

Therefore we have,

$$\begin{aligned} & 0 \leq |T_B(s) - T_D(s)| + |C_B(s) - C_D(s)| + |G_B(s) - G_D(s)| + |U_B(s) - U_D(s)| + |F_B(s) - F_D(s)| \leq 5 \\ \Rightarrow & 0 \leq \sum_{s \in \Psi} (|T_B(s) - T_D(s)| + |C_B(s) - C_D(s)| + |G_B(s) - G_D(s)| + |U_B(s) - U_D(s)| + |F_B(s) - F_D(s)|) \\ & \leq 5n \\ \Rightarrow & 0 \leq \frac{1}{5n} \sum_{s \in \Psi} (|T_B(s) - T_D(s)| + |C_B(s) - C_D(s)| + |G_B(s) - G_D(s)| + |U_B(s) - U_D(s)| + |F_B(s) - F_D(s)|) \leq 1 \\ \Rightarrow & 0 \leq N-Hd(B, D) \leq 1 \\ \Rightarrow & N-Hd(B, D) \in [0, 1]. \end{aligned}$$

**Theorem 3.4.** Suppose that  $D = \{(s, T_D(s), C_D(s), G_D(s), U_D(s), F_D(s)): s \in \Psi\}$ ,  $B = \{(s, T_B(s), C_B(s), G_B(s), U_B(s), F_B(s)): s \in \Psi\}$  and  $A = \{(s, T_A(s), C_A(s), G_A(s), U_A(s), F_A(s)): s \in \Psi\}$  be three SVPNSs over  $\Psi$ , where cardinality of  $\Psi$  is  $n$ . If  $B \subseteq D \subseteq A$ , then

- (i)  $N-Hd(B, D) \leq N-Hd(B, A)$ ;
- (ii)  $N-Hd(D, A) \leq N-Hd(B, A)$ .

**Proof.** Let  $D = \{(s, T_D(s), C_D(s), G_D(s), U_D(s), F_D(s)): s \in \Psi\}$ ,  $B = \{(s, T_B(s), C_B(s), G_B(s), U_B(s), F_B(s)): s \in \Psi\}$  and  $A = \{(s, T_A(s), C_A(s), G_A(s), U_A(s), F_A(s)): s \in \Psi\}$  be three SVPNSs over  $\Psi$ , where cardinality of  $\Psi$  is  $n$ .

(i) Suppose that  $B \subseteq D \subseteq A$ . So  $|T_B(s) - T_D(s)| \leq |T_B(s) - T_A(s)|, |C_B(s) - C_D(s)| \leq |C_B(s) - C_A(s)|, |G_B(s) - G_D(s)| \leq |G_B(s) - G_A(s)|, |U_B(s) - U_D(s)| \leq |U_B(s) - U_A(s)|, |F_B(s) - F_D(s)| \leq |F_B(s) - F_A(s)|$ , for each  $s \in \Psi$ .

Therefore,

$$\begin{aligned} & \frac{1}{5n} \sum_{s \in \Psi} (|T_B(s) - T_D(s)| + |C_B(s) - C_D(s)| + |G_B(s) - G_D(s)| + |U_B(s) - U_D(s)| + |F_B(s) - F_D(s)|) \\ & \leq \frac{1}{5n} \sum_{s \in \Psi} (|T_B(s) - T_A(s)| + |C_B(s) - C_A(s)| + |G_B(s) - G_A(s)| + |U_B(s) - U_A(s)| + |F_B(s) - F_A(s)|) \end{aligned}$$

Now, we have

$$\begin{aligned} & N-Hd(B, D) \\ & = \frac{1}{5n} \sum_{s \in \Psi} (|T_B(s) - T_D(s)| + |C_B(s) - C_D(s)| + |G_B(s) - G_D(s)| + |U_B(s) - U_D(s)| + |F_B(s) - F_D(s)|) \\ & \leq \frac{1}{5n} \sum_{s \in \Psi} (|T_B(s) - T_A(s)| + |C_B(s) - C_A(s)| + |G_B(s) - G_A(s)| + |U_B(s) - U_A(s)| + |F_B(s) - F_A(s)|) \\ & = N-Hd(B, A). \end{aligned}$$

Hence,  $N-Hd(B, D) \leq N-Hd(B, A)$ .

(ii) Assume that  $B \subseteq D \subseteq A$ . So  $|T_D(s) - T_A(s)| \leq |T_B(s) - T_A(s)|, |C_D(s) - C_A(s)| \leq |C_B(s) - C_A(s)|, |G_D(s) - G_A(s)| \leq |G_B(s) - G_A(s)|, |U_D(s) - U_A(s)| \leq |U_B(s) - U_A(s)|, |F_D(s) - F_A(s)| \leq |F_B(s) - F_A(s)|$ , for each  $s \in \Psi$ .

Therefore,

$$\frac{1}{5n} \sum_{s \in \Psi} (|T_D(s) - T_A(s)| + |C_D(s) - C_A(s)| + |G_D(s) - G_A(s)| + |U_D(s) - U_A(s)| + |F_D(s) - F_A(s)|)$$

$$\leq \frac{1}{5n} \sum_{s \in \Psi} (|T_B(s) - T_A(s)| + |C_B(s) - C_A(s)| + |G_B(s) - G_A(s)| + |U_B(s) - U_A(s)| + |F_B(s) - F_A(s)|)$$

Now, we have

$$N-H_d(D, A)$$

$$= \frac{1}{5n} \sum_{s \in \Psi} (|T_D(s) - T_A(s)| + |C_D(s) - C_A(s)| + |G_D(s) - G_A(s)| + |U_D(s) - U_A(s)| + |F_D(s) - F_A(s)|)$$

$$\leq \frac{1}{5n} \sum_{s \in \Psi} (|T_B(s) - T_A(s)| + |C_B(s) - C_A(s)| + |G_B(s) - G_A(s)| + |U_B(s) - U_A(s)| + |F_B(s) - F_A(s)|)$$

$$= N-H_d(B, A).$$

Hence,  $N-H_d(D, A) \leq N-H_d(B, A)$ .

#### 4. SVPNS-MADM strategy based on GRA:

Choosing an alternative from a set of possible alternatives based on some attributes is a challenging task for a Decision Maker (DM). For that the DM should have to plan an MADM strategy to take the decision. Assume that  $L = \{L_1, L_2, \dots, L_p\}$  is the collection of some possible alternatives and  $S = \{S_1, S_2, \dots, S_q\}$  is the family of attributes. The DM provides their evaluation information for every alternative  $L_i$  ( $i=1, 2, \dots, p$ ) based on the attribute  $S_j$  ( $j=1, 2, \dots, q$ ) in terms of Single Valued Pentapartitioned Neutrosophic Numbers (SVPNNs). So the whole evaluation information of all alternatives can be expressed by a decision matrix.

The steps of the proposed SVPNS-MADM strategy are presented as follows:

**Step-1:** Construct the decision matrix using SVPNS

The whole evaluation assessment of every alternative  $L_i$  ( $i = 1, 2, \dots, p$ ) over the attributes  $S_j$  ( $j = 1, 2, \dots, q$ ) is presented in terms of SVPNNs  $E_{L_i} = \{(S_j, T_{ij}(L_i, S_j), C_{ij}(L_i, S_j), G_{ij}(L_i, S_j), U_{ij}(L_i, S_j), F_{ij}(L_i, S_j)) : S_j \in S\}$ , where  $(T_{ij}(L_i, S_j), C_{ij}(L_i, S_j), G_{ij}(L_i, S_j), U_{ij}(L_i, S_j), F_{ij}(L_i, S_j)) = (T_{ij}, C_{ij}, G_{ij}, U_{ij}, F_{ij})$  (in short) is the evaluation assessment of alternative  $L_i$  ( $i = 1, 2, \dots, n$ ) over the attribute  $S_j$  ( $j = 1, 2, \dots, m$ ).

Then the decision matrix (D) is given by:

D	$S_1$	$S_2$	...	...	$S_m$
$L_1$	$\langle T_{11}(L_1, S_1), C_{11}(L_1, S_1), G_{11}(L_1, S_1), U_{11}(L_1, S_1), F_{11}(L_1, S_1) \rangle$	$\langle T_{12}(L_1, S_2), C_{12}(L_1, S_2), G_{12}(L_1, S_2), U_{12}(L_1, S_2), F_{12}(L_1, S_2) \rangle$	....	....	$\langle T_{1m}(L_1, S_m), C_{1m}(L_1, S_m), G_{1m}(L_1, S_m), U_{1m}(L_1, S_m), F_{1m}(L_1, S_m) \rangle$
$L_2$	$\langle T_{21}(L_2, S_1), C_{21}(L_2, S_1), G_{21}(L_2, S_1), U_{21}(L_2, S_1), F_{21}(L_2, S_1) \rangle$	$\langle T_{22}(L_2, S_2), C_{22}(L_2, S_2), G_{22}(L_2, S_2), U_{22}(L_2, S_2), F_{22}(L_2, S_2) \rangle$	...	...	$\langle T_{2m}(L_2, S_m), C_{2m}(L_2, S_m), G_{2m}(L_2, S_m), U_{2m}(L_2, S_m), F_{2m}(L_2, S_m) \rangle$

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.	.	.	.	.	.
.	.	.	.	.	.
$L_n$	$\langle T_{n1}(L_n, S_1), C_{n1}(L_n, S_1), G_{n1}(L_n, S_1), U_{n1}(L_n, S_1), F_{n1}(L_n, S_1) \rangle$	$\langle T_{n2}(L_n, S_2), C_{n2}(L_n, S_2), G_{n2}(L_n, S_2), U_{n2}(L_n, S_2), F_{n2}(L_n, S_2) \rangle$	...	...	$\langle T_{nm}(L_n, S_m), C_{nm}(L_n, S_m), G_{nm}(L_n, S_m), U_{nm}(L_n, S_m), F_{nm}(L_n, S_m) \rangle$

**Step-2:** Determine the weights for the attributes.

In every MADM strategy, the weights of the attributes play an important role in making decision. If the weights of the information of all attributes are completely unknown to the decision makers, then by using the following compromise function, the decision maker can find the weights of the attributes.

**Compromise Function:** The compromise function of  $L$  is defined as follows:

$$\xi_j = \sum_{i=1}^n (3 + T_{ij}(L_i, S_j) + C_{ij}(L_i, S_j) - G_{ij}(L_i, S_j) - U_{ij}(L_i, S_j) - F_{ij}(L_i, S_j)) / 5 \tag{3}$$

Then the weights of the  $j$ th attribute is defined by  $w_j = \frac{\xi_j}{\sum_{j=1}^m \xi_j}$  (4)

Here  $\sum_{j=1}^m w_j = 1$ .

**Step-3:** Construct the Ideal Pentapartitioned Neutrosophic Estimates Reliability Solution (IPNERS) and Ideal Pentapartitioned Neutrosophic Estimates Un-Reliability Solution (IPNEURS) for the decision matrix:

The IPNERS for the decision matrix is presented as:

$$R^+ = [\langle T_1^+, C_1^+, G_1^+, U_1^+, F_1^+ \rangle, \langle T_2^+, C_2^+, G_2^+, U_2^+, F_2^+ \rangle, \dots, \langle T_m^+, C_m^+, G_m^+, U_m^+, F_m^+ \rangle], \tag{5}$$

where  $T_j^+ = \max \{T_{ij}(L_i, S_j): i=1, 2, 3, \dots, n\}$ ,  $C_j^+ = \max \{C_{ij}(L_i, S_j): i=1, 2, 3, \dots, n\}$ ,  $G_j^+ = \min \{G_{ij}(L_i, S_j): i=1, 2, 3, \dots, n\}$ ,  $U_j^+ = \min \{U_{ij}(L_i, S_j): i=1, 2, 3, \dots, n\}$ , and  $F_j^+ = \min \{F_{ij}(L_i, S_j): i=1, 2, 3, \dots, n\}$ .

The IPNEURS for the decision matrix is presented as:

$$R^- = [\langle T_1^-, C_1^-, G_1^-, U_1^-, F_1^- \rangle, \langle T_2^-, C_2^-, G_2^-, U_2^-, F_2^- \rangle, \dots, \langle T_m^-, C_m^-, G_m^-, U_m^-, F_m^- \rangle], \tag{6}$$

where  $T_j^- = \min \{T_{ij}(L_i, S_j): i=1, 2, 3, \dots, n\}$ ,  $C_j^- = \min \{C_{ij}(L_i, S_j): i=1, 2, 3, \dots, n\}$ ,  $G_j^- = \max \{G_{ij}(L_i, S_j): i=1, 2, 3, \dots, n\}$ ,  $U_j^- = \max \{U_{ij}(L_i, S_j): i=1, 2, 3, \dots, n\}$ , and  $F_j^- = \max \{F_{ij}(L_i, S_j): i=1, 2, 3, \dots, n\}$ .

**Step-4:** Determination of Pentapartitioned Neutrosophic Grey Relational Coefficient (PNGRC) of each alternative from IPNERS & IPNEURS.

The PNGRC of each alternative from IPNERS is presented as:

$$G_{ij}^+ = \frac{\min_i \min_j \Delta_{ij}^+ + k \max_i \max_j \Delta_{ij}^+}{\Delta_{ij}^+ + k \max_i \max_j \Delta_{ij}^+}, \text{ where } \Delta_{ij}^+ = Hd (\langle T_j^+, C_j^+, G_j^+, U_j^+, F_j^+ \rangle, \langle T_{ij}, C_{ij}, G_{ij}, U_{ij}, F_{ij} \rangle), i=1,2,\dots,n$$

and  $j=1,2,\dots,m$ , and  $k \in [0,1]$ .

The PNGRC of each alternative from IPNEURS is given below:

$$G_{ij}^- = \frac{\min_i \min_j \Delta_{ij}^- + k \max_i \max_j \Delta_{ij}^-}{\Delta_{ij}^- + k \max_i \max_j \Delta_{ij}^-}, \text{ where } \Delta_{ij}^- = Hd (< T_{ij}, C_{ij}, G_{ij}, U_{ij}, F_{ij} >, < T_j^-, C_j^-, G_j^-, U_j^-, F_j^- >), i=1, 2, \dots,$$

$n$ , and  $j=1, 2, \dots, m$ , and  $k \in [0,1]$ .

Here  $G_{ij}^+$  and  $G_{ij}^-$  are the identification coefficient used to adjust the range of the comparison environment, and to control level of differences of the relation coefficients. The comparison environment remains unchanged when  $k = 1$  and the comparison environment disappears when  $k = 0$ . If the identification coefficient is smaller, then the range of grey relational coefficient will become so large. Generally,  $k = 0.5$  is considered for decision making situation.

**Step-5:** Determine the PNGRC

The PNGRC of each alternative from IPNERS and IPNEURS are defined as follows:

$$G_i^+ = \sum_{j=1}^q w_j G_{ij}^+ \tag{7}$$

where  $i = 1, 2, \dots, n$ ,

$$\text{and } G_i^- = \sum_{j=1}^n w_j G_{ij}^- \tag{8}$$

where  $i = 1, 2, \dots, m$ .

**Step-6:** Determine the pentapartitioned neutrosophic relative relational degree.

The pentapartitioned neutrosophic relative relational degree of each alternatives is can be defined as follows:

$$\mathfrak{R}_i = \frac{G_i^+}{G_i^+ + G_i^-} \tag{9}$$

where  $i = 1, 2, \dots, n$ .

**Step-7:** Rank the alternatives.

The ranking order of all alternatives can be determined according to the ascending order of the pentapartitioned relative relational degree. The alternative with highest value of  $\mathfrak{R}_i$  indicates the best alternative.

**Step-8:** End.



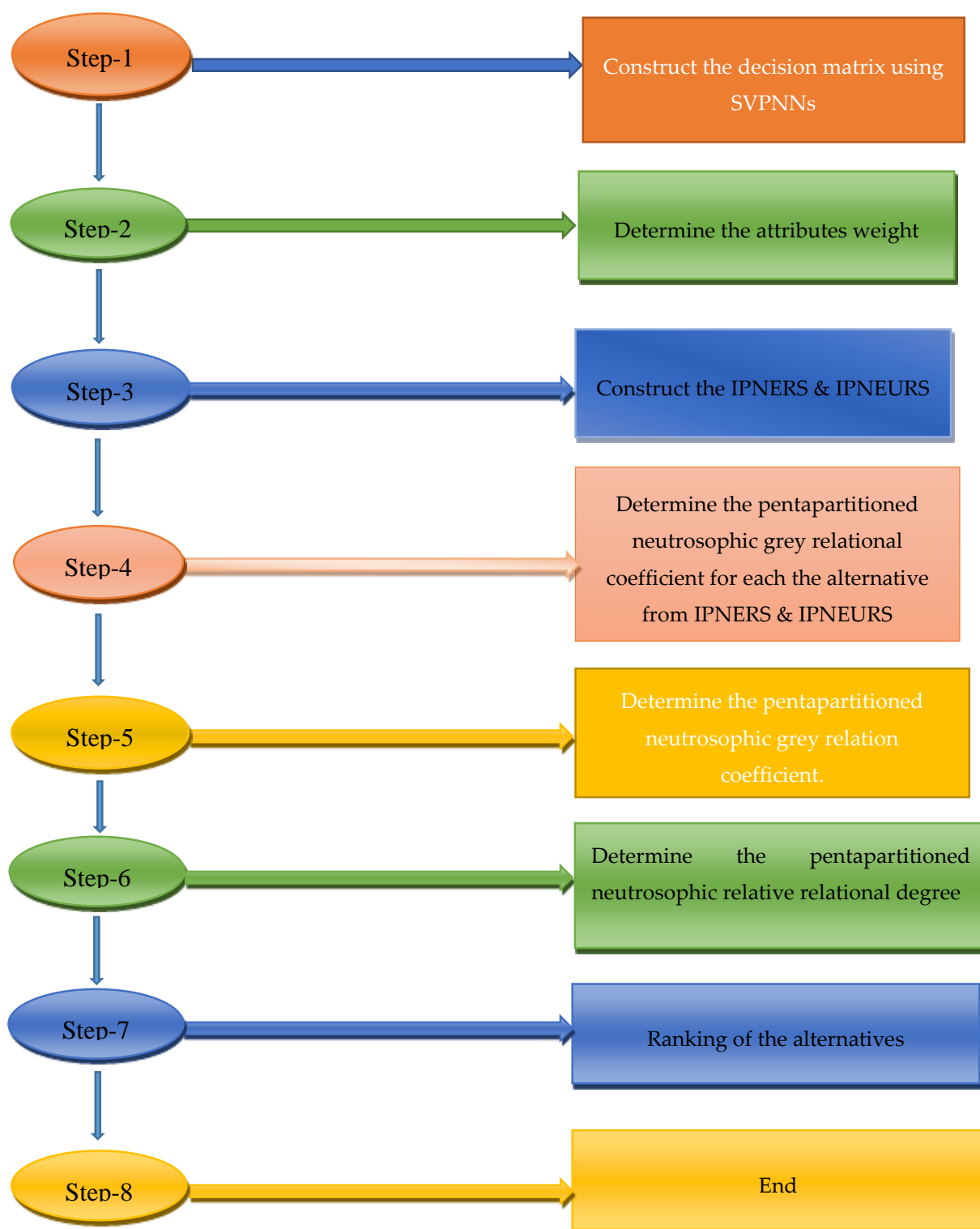


Figure 1: Flow chart of the proposed SVPNS-MADM strategy.

**5. Validation of Proposed Model:**

In this section, we present a numerical example namely “Selection of supplier to buy electronic goods for an institution” to validate the proposed strategy.

**5.1. Selection of supplier to buy electronic goods for an institution:**

In every government/private institutions, lots of electronic goods namely Computer, Printer, Scanner, Projector, AC, etc. are required for the purpose of official uses. To buy a particular or all electronic goods, the institutions must select a suitable private company for giving the tender against some attributes. So, the selection of best private company by the institution for purchasing the necessary electronic goods can be considered as an MADM problem. For the selection of suitable private company, the decision maker selects four major attributes namely  $S_1$ : Cost of the products;  $S_2$ : Quality of the products;  $S_3$ : Service of the Company;  $S_4$ : Reliability.

Then, the developed MADM strategy is presented using the following steps.

**Step-1:** Determine the decision matrix in single valued pentapartitioned neutrosophic environment. The decision maker provides the evaluation information for all the alternatives over the attributes as shown in Table-1

Table-1:

	$S_1$	$S_2$	$S_3$	$S_4$
$L_1$	(0.9,0.3,0.1,0.5,0.2)	(0.8,0.2,0.2,0.1,0.4)	(0.9,0.1,0.3,0.1,0.3)	(0.9,0.1,0.2,0.3,0.4)
$L_2$	(0.8,0.1,0.3,0.3,0.2)	(0.9,0.2,0.3,0.4,0.2)	(0.6,0.1,0.2,0.3,0.2)	(0.9,0.2,0.1,0.2,0.2)
$L_3$	(0.9,0.4,0.2,0.3,0.1)	(0.8,0.3,0.4,0.1,0.1)	(0.5,0.1,0.1,0.2,0.1)	(0.8,0.3,0.1,0.3,0.1)

**Step-2:** Determine the weights of attributes

By using the eq. (3) and (4), we get the weight vector as follows:

$$(w_1, w_2, w_3, w_4) = (0.261728, 0.249383, 0.234568, 0.254321).$$

**Step-3:** Determine the IPNERS & IPNEURS

The IPNERS ( $R^+$ ) and IPNEURS ( $R^-$ ) for the decision matrix are presented in the Table-3.

Table-3:

	$S_1$	$S_2$	$S_3$	$S_4$
$L_1$	(0.9,0.3,0.1,0.5,0.2)	(0.8,0.2,0.2,0.1,0.4)	(0.9,0.1,0.3,0.1,0.3)	(0.9,0.1,0.2,0.3,0.4)
$L_2$	(0.8,0.1,0.3,0.3,0.2)	(0.9,0.2,0.3,0.4,0.2)	(0.6,0.1,0.2,0.3,0.2)	(0.9,0.2,0.1,0.2,0.2)
$L_3$	(0.9,0.4,0.2,0.3,0.1)	(0.8,0.3,0.4,0.1,0.1)	(0.5,0.1,0.1,0.2,0.1)	(0.8,0.3,0.1,0.3,0.1)
$R^+$	(0.9,0.4,0.1,0.3,0.1)	(0.9,0.3,0.2,0.1,0.1)	(0.9,0.1,0.1,0.1,0.1)	(0.9,0.3,0.1,0.2,0.1)

$R^-$	(0.8,0.1,0.3,0.5,0.2)	(0.8,0.2,0.4,0.4,0.4)	(0.5,0.1,0.3,0.3,0.3)	(0.8,0.1,0.2,0.3,0.4)
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**Step-4:** Determine the PNGRC for each of the alternative from IPNERS & IPNEURS.

The PNGRC of each alternative from IPNERS and IPNEURS is presented in the Table-4 ,Table-5, Table-6, and Table-7, respectively.

Table-4:

	$S_1$	$S_2$	$S_3$	$S_4$	$\min_i \Delta_{ij}^+$	$\max_i \Delta_{ij}^+$
$\Delta_{1j}^+$	0.4	0.5	0.4	0.7	0.4	0.7
$\Delta_{2j}^+$	0.7	0.6	0.7	0.2	0.2	0.7
$\Delta_{3j}^+$	0.1	0.3	0.5	0.2	0.1	0.5
$\min_i \min_j \Delta_{ij}^+$	0.1					
$\max_i \max_j \Delta_{ij}^+$	0.7					

Table-5:

	$S_1$	$S_2$	$S_3$	$S_4$	$\min_i \Delta_{ij}^-$	$\max_i \Delta_{ij}^-$
$\Delta_{1j}^-$	0.5	0.5	0.6	0.1	0.1	0.6
$\Delta_{2j}^-$	0.2	0.4	0.3	0.6	0.2	0.6
$\Delta_{3j}^-$	0.8	0.7	0.5	0.6	0.5	0.8
$\min_i \min_j \Delta_{ij}^-$	0.2					
$\max_i \max_j \Delta_{ij}^-$	0.8					

Table-6:

$G_{ij}^+$	$S_1$	$S_2$	$S_3$	$S_4$
$L_1$	0.6	0.5294	0.6	0.4286
$L_2$	0.4286	0.4737	0.4286	0.8182
$L_3$	1	0.6923	0.5294	0.8182

Table-7:

$G_{ij}^-$	$S_1$	$S_2$	$S_3$	$S_4$
$L_1$	0.5556	0.5556	0.5	1
$L_2$	0.8333	0.625	0.7143	0.5
$L_3$	0.4167	0.4546	0.5556	0.5

**Step-5:** Determine the PNGRC.

The PNGRCs  $G_i^+$  and  $G_i^-$  of each alternative ( $S_i, i = 1, 2, 3, 4$ ) from IPNERS and IPNEURS are presented in Table-8.

Table-8:

	$G_i^+$	$G_i^-$
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$L_1$	0.538803	0.655578
$L_2$	0.538931	0.668675
$L_3$	0.766642	0.479918

**Step-6:** Determine the pentapartitioned neutrosophic relative relational degree.

The pentapartitioned neutrosophic relative relational degree ( $\mathfrak{R}_i$ ) of each alternative ( $A_i$ ,  $i = 1, 2, 3, 4$ ) is presented in the following Table 9.

Table-9:

	$\mathfrak{R}_i = \frac{G_i^+}{G_i^+ + G_i^-}$
$L_1$	0.4511148
$L_2$	0.4462805
$L_3$	0.6150061

**Step-7:** Rank the alternatives.

From Table-9, it is clear that  $\mathfrak{R}_2 < \mathfrak{R}_1 < \mathfrak{R}_3$ . Therefore,  $L_3$  is the best suitable alternative to choose.

## 5. Conclusions

In the study, we have proposed Hamming distance and proves its basic properties for PNSs. We have further developed a GRA based SVPNS-MADM strategy in PNS environment. We also validate the proposed SVPNS-MADM strategy by solving an illustrative decision-making problem.

The proposed SVPNS-MADM strategy can also be used to deal with the other decision-making problems such as brick selection [19], stock trending analysis [14], logistic center location selection [29], teacher selection [34], etc.

We further hope that the proposed MADM strategy will open up a new avenue of research in pentapartitioned neutrosophic set environments.

## References:

1. Abdel-Basset, M., Saleh, M., Gamal, A., & Smarandache, F. (2019). An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. *Applied Soft Computing*, 77, 438-452.
2. Banerjee, D., Giri, B. C., Pramanik, S., & Smarandache, F. (2017). GRA for multi attribute decision making in neutrosophic cubic set environment. *Neutrosophic Sets and Systems*, 15, 60-69. [doi.org/10.5281/zenodo.570938](https://doi.org/10.5281/zenodo.570938)
3. Biswas, P., Pramanik, S., & Giri, B. C. (2019). NH-MADM strategy in neutrosophic hesitant fuzzy set environment based on extended GRA. *Informatica*, 30(2), 213-242.
4. Biswas, P., Pramanik, S., & Giri, B. C. (2016). GRA method of multiple attribute decision making with single valued neutrosophic hesitant fuzzy set information. In F. Smarandache,

- & S. Pramanik (Eds.), New trends in neutrosophic theory and applications (pp. 55-63). Brussels: Pons Editions.
5. Biswas P, Pramanik S, Giri, BC (2014) Entropy based grey relational analysis method for multi-attribute decision making under single valued neutrosophic assessments. *Neutrosophic Sets and Systems* 2, 102-110.
  6. Broumi, S., Bakali, A., Talea, M., Smarandache, F., Uluçay, V., Sahin, S., Dey, A., Dhar, M., Tan, R. P., de Oliveira, A., & Pramanik, S. (2018). Neutrosophic sets: An overview. In F. Smarandache, & S. Pramanik (Eds., vol.2), New trends in neutrosophic theory and applications (pp. 403-434). Brussels: Pons Editions.
  7. Dalapati, S., Pramanik, S., Alam, S., Smarandache, S., & Roy, T.K. (2017). IN-cross entropy based MAGDM strategy under interval neutrosophic set environment. *Neutrosophic Sets and Systems*, 18, 43-57.
  8. Dey, P. P., Pramanik, S. & Giri, B. C. (2015). Generalized neutrosophic soft multi-attribute group decision making based on TOPSIS. *Critical Review*, 11, 41-55.
  9. Dey, P.P., Pramanik, S., & Giri, B.C. (2015). An extended grey relational analysis based interval neutrosophic multi attribute decision making for weaver selection. *Journal of New Theory*, 9, 82-93.
  10. Dey, P.P., Pramanik, S., & Giri, B.C. (2016). Extended projection based models for solving multiple attribute decision making problems with interval valued neutrosophic information. In F. Smarandache, & S. Pramanik (Eds.), *New Trends in Neutrosophic Theory and Applications*. Pons Edition, Brussels, 127-140.
  11. Dey, P. P., Pramanik, S. & Giri, B. C. (2016). An extended grey relational analysis based multiple attribute decision making in interval neutrosophic uncertain linguistic setting. *Neutrosophic Sets and Systems*, 11, 21-30. [doi.org/10.5281/zenodo.571228](https://doi.org/10.5281/zenodo.571228)
  12. Dey, P. P., Pramanik, S. & Giri, B. C. (2016). Neutrosophic soft multi-attribute decision making based on grey relational projection method. *Neutrosophic Sets and Systems*, 11, 98-106. [doi.org/10.5281/zenodo.571576](https://doi.org/10.5281/zenodo.571576)
  13. Dey, P.P., S. Pramanik, & Giri, B.C. (2016). TOPSIS for solving multi-attribute decision making problems under bi-polar neutrosophic environment. In F. Smarandache, & S. Pramanik (Eds.), New trends in neutrosophic theory and applications (pp. 65-77). Brussels: Pons Editions.
  14. Jha, S., Kumar, R., Son, L. H., Biswas, J. M., Khari, M., Yadav, N., & Smarandache, F. (2019) Neutrosophic soft set decision making for stock trending analysis. *Evolving Systems* 10(4): 621-627
  15. Mallick, R., & Pramanik, S. (2020). Pentapartitioned neutrosophic set and its properties. *Neutrosophic Sets and Systems*, 36, 184-192.

16. Mondal, K., Pramanik, S. (2015). Neutrosophic decision making model for clay-brick selection in construction field based on grey relational analysis. *Neutrosophic Sets and Systems*, 9, 72-79.
17. Mondal, K., & Pramanik, S. (2015). Neutrosophic tangent similarity measure and its application to multiple attribute decision making. *Neutrosophic sets and systems*, 9, 80–87.
18. Mondal, K., Pramanik, S., & Giri, B.C. (2018). Single valued neutrosophic hyperbolic sine similarity measure based MADM strategy. *Neutrosophic Sets and Systems*, 20, 3-11.
19. Mondal, K., Pramanik, S., & Smarandache, F. (2014). Intuitionistic fuzzy multi-criteria group decision making approach to quality-brick selection problem. *Journal of Applied Quantitative Methods*, 9 (2), 35-50.
20. Mondal, K., Pramanik, S., & Smarandache, F. (2016). Role of neutrosophic logic in data mining. In F. Smarandache, & S. Pramanik (Eds.), *New Trends in Neutrosophic Theory and Application*. Pons Editions, Brussels, 15-23.
21. Mukherjee, A., & Das, R. (2020). Neutrosophic bipolar vague soft set and its application to decision making problems. *Neutrosophic Sets and Systems*, 32, 410-424.
22. Nguyen G. N., Son, L. H., Ashour, A. S., & Dey, N. (2019). A survey of the state-of-the-arts on neutrosophic sets in biomedical diagnoses. *International Journal of Machine Learning and Cybernetics*, 10: 1–13. <https://doi.org/10.1007/s13042-017-0691-7>
23. Peng X, Dai J (2020) A bibliometric analysis of neutrosophic set: Two decades review from 1998 to 2017. *Artificial Intelligence Review* 53(1), 199-255.
24. Pramanik S (2020) Rough neutrosophic set: an overview. In F. Smarandache, & Broumi, S. (Eds.), *Neutrosophic theories in communication, management and information technology*. Nova Science Publishers, pp 275-311.
25. Pramanik, S., & Dalapati, S. (2016). GRA based multi criteria decision making in generalized neutrosophic soft set environment. *Global Journal of Engineering Science and Research Management*, 3(5), 153-169.
26. Pramanik, S., Dalapati, S., & Roy, T. K. (2016). Logistics center location selection approach based on neutrosophic multicriteria decision making, In F. Smarandache, & S. Pramanik (Eds.), *New Trends in Neutrosophic Theory and Application*. Pons Editions, Brussels, 161-174.
27. Pramanik, S., Dalapati, S., Alam, S., Smarandache, F., & Roy, T. K. (2018). NC-cross entropy based MADM strategy in neutrosophic cubic set environment. *Mathematics*, 6(5), 67.
28. Pramanik, S., Dalapati, S., Alam, S., Smarandache, F., & Roy, T. K. (2018). NS-cross entropy-based MAGDM under single-valued neutrosophic set environment. *Information*, 9(2), 37.
29. Pramanik, S., Dalapati, S., & Roy, T. K. (2018). Neutrosophic multi-attribute group decision making strategy for logistic center location selection. In F. Smarandache, M. A. Basset & V. Chang (Eds.), *Neutrosophic Operational Research*, Vol. III. Pons Asbl, Brussels, 13-32.

30. Pramanik, S., Dey, P. P., & Giri, B. C. (2015). TOPSIS for single valued neutrosophic soft expert set based multi-attribute decision making problems. *Neutrosophic Sets and Systems*, 10, 88-95.
31. Pramanik, S., & Mallick, R. (2020). Extended GRA-based MADM strategy with single-valued trapezoidal neutrosophic numbers. In M. Abdel-Basset, & F. Smarandache (Eds.), *Neutrosophic sets in decision analysis and operations research* (pp. 150-179). Hershey, PA: IGI Global. doi:10.4018/978-1-7998-2555-5.ch008
32. Pramanik, S., & Mondal, K. (2015). Cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. *Global Journal of Advanced Research*, 2 (1), 212–220.
33. Pramanik, S., & Mondal, K. (2015). Cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis. *Journal of New Theory*, 4, 464–471.
34. Pramanik, S., & Mukhopadhyaya, D. (2011). Grey relational analysis based intuitionistic fuzzy multi criteria group decision-making approach for teacher selection in higher education. *International Journal of Computer Applications*, 34 (10), 21-29.
35. Pramanik, S., & Roy, T.K. (2014). Neutrosophic game theoretic approach to Indo-Pak conflict over Jammu-Kashmir. *Neutrosophic Sets and Systems*, 2, 82-101.
36. Roy, R., Pramanik, S., & Roy, T. K. (2020). Interval rough neutrosophic TOPSIS strategy for multi-attribute decision making. In M. Abdel-Basset, & F. Smarandache (Eds.), *Neutrosophic Sets in Decision Analysis and Operations Research* (pp. 98-118). Hershey, PA: IGI Global. doi:10.4018/978-1-7998-2555-5.ch005
37. Şahin, R., & Liu, P. (2016). Maximizing deviation method for neutrosophic multiple attribute decision making with incomplete weight information. *Neural Computing and Applications*, 27(7), 2017-2029.
38. Smarandache, F. (1998). A unifying field in logics, neutrosophy: neutrosophic probability, set and logic. Rehoboth, *American Research Press*.
39. Smarandache, F., Khalid, H. E., & Essa, A. K. (2018). *Neutrosophic logic: The revolutionary logic in science and philosophy*. Brussels: EuropaNova.
40. Smarandache, F. & Pramanik, S. (Eds). (2018). *New trends in neutrosophic theory and applications*, Vol.2. Brussels: Pons Editions.
41. Smarandache, F. & Pramanik, S. (Eds). (2016). *New trends in neutrosophic theory and applications*. Brussels: Pons Editions.
42. Wang, H., Smarandache, F., Zhang, Y. Q., & Sunderraman, R. (2010). Single valued neutrosophic sets. *Multispace and Multistructure*, 4, 410–413.
43. Ye, J. (2013). Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. *International Journal of General Systems*, 42(4), 386-394.

44. Ye J (2015) Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses. *Artificial Intelligence in Medicine* 63(3):171–179
45. Zhang, C., Li, D., Kang, X., Liang, Y., Broumi, S., & Sangaiah, A. K. (2020). Multi-Attribute Group Decision Making Based on Multigranulation Probabilistic Models with Interval-Valued Neutrosophic Information. *Mathematics*, 8(2), 223.

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