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Semi Homomorphisms and Algebraic Relations Between Strong Refined Neutrosophic Modules and Strong Neutrosophic Modules

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Abstract: The objective of this paper is to define and study the concept of semi homomorphism between two modules defined over different rings. Semi homomorphisms can help in the study of algebraic relations between two modules defined over two different rings. An application of this functions is presented in the case of refined strong neutrosophic modules and neutrosophic strong modules, where strong neutrosophic modules are shown to be semi homomorphic images of their corresponding strong refined neutrosophic modules. Also, this work presents a discussion of the algebraic structure of some semi homomorphisms and isomorphisms.

Keywords: Neutrosophic module, $f-$ semi homomorphism, $f-$ semi isomomorphism, neutrosophic ring, refined neutrosophic ring, classical homomorphism..

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1. Introduction

Neutrosophy as a new kind of logic founded by Smarandache plays an important role in many fields of knowledge as well as algebra. We find many applications of neutrosophic sets in decision making [20,23], medical studies [14], and computer science [21]. Also, many algebraic neutrosophic structures were defined and handled such as neutrosophic rings and neutrosophic modules/spaces [1,6,7,8,18].

Recently, F. Smarandache came with an interesting idea suggests that the indeterminacy I can be split into two different degrees of indeterminacy I_1, I_2. Agboola et. al used this idea to introduce the concept of refined neutrosophic ring [2,4]. Substructures of these rings were studied widely in [1,2,3,5,6,7,10,11] such as neutrosophic isomorphisms, AH-ideals, AHS-homomorphisms, idempotents, and semi idempotents.

Neutrosophic modules were firstly defined over neutrosophic rings in [8] and studied widely in [4], and then they were generalized into refined neutrosophic modules over refined neutrosophic rings in [9]. Recently, they were generalized into n-refined neutrosophic modules [16] over n-refined neutrosophic rings [19].

It is well known that the relationships between two modules can be represented with module homomorphisms if these modules were defined over the same ring. In this work we introduce the concept of semi homomorphism between two modules to help us in the study of the relationships between two different modules defined over two different rings. Our goal is to study the relationships between strong refined neutrosophic modules and strong neutrosophic modules by using the notion of semi homomorphisms.

All rings through this paper are considered commutative with unity 1. Also, all neutrosophic modules and refined neutrosophic modules are considered strong.

2. Preliminaries

Definition 2.1: [6]

Let (R,+,×) be a ring, $R(I) = \{a + bI\ ;\ a, b \in R\}$ is called the neutrosophic ring where I is a neutrosophic element with condition $I^2 = I$.

Remark 2.2 : [5]

The element I can be split into two indeterminacies I_1, I_2 with conditions :

$$\mathrm{I}_1{}^2 = \mathrm{I}_1\ , I_2{}^2 = I_2\ , I_1 I_2 = I_2 I_1 = I_1\ .$$

Definition 2.3 : [5]

If X is a set, then $X(I_1, I_2) = \{(a, bI_1, cI_2); a\ , b\ , c\ \in X\}$ is called the refined neutrosophic set generated by X , I_1, I_2.

Definition 2.4 : [6]

Let (R,+,×) be a ring, $(R(I_1, I_2)\ , +\ , \times)$ is called a refined neutrosophic ring generated by R , I_1, I_2.

Theorem 2.5: [6]

Let (R(I_1, I_2) , + ,×) be a refined neutrosophic ring then it is a ring .

Definition 2.6:[8]

Let (M,+,.) be a module over the ring R, then (M(I),+,.) is called a weak neutrosophic module over the ring R, and it is called a strong neutrosophic module if it is a module over the neutrosophic ring R(I). Elements of M(I) have the form $x + yI; x , y \in M$, i.e M(I) can be written as $M(I) = M + MI$.

Definition 2.7: [8]

Let M(I) be a strong neutrosophic module over the neutrosophic ring R(I) and W(I) be a non empty subset of M(I), then W(I) is called a strong neutrosophic submodule if W(I) itself is a strong neutrosophic module.

Definition 2.8:[8]

Let U(I) and W(I) be two strong neutrosophic submodules of M(I) and let $f: U(I) \to W(I)$, we say that f is a neutrosophic vector space homomorphism if

(a) $f(I) = I$.

(b) f is a module homomorphism.

We define the kernel of f by Ker f = { x∈ $M(I)$; $f(x)$ = 0 }.

Definition 2.9: [9]

Let $(X\ (I_1, I_2), +, \cdot)$ be any refined neutrosophic algebraic structure where + and $\cdot$ are ordinary addition and multiplication respectively. I_1 *and* I_2 are the split components of the indeterminacy factor I that is $I = \alpha I_1 + \beta I_2$ with $\alpha, \beta \in R$ *or* C . Also, I_1 *and* I_2 are taken to have the properties $I_1^2 = I_1, I_2^2 = I_2$ and $I_1 I_2 = I_2 I_1 = I_1$.

For any two elements, we define

1) $x + y = (a, bI_1, cI_2) + (d, eI_1, fI_2) = (a + d, (b + e)I_1, (c + f)I_2)$

2) $x \cdot y = (a, bI_1, cI_2) \cdot (d, eI_1, fI_2) = \begin{pmatrix} ad, (ae + bd + be + bf + ce)I_1, \\ (af + cd + cf)I_2 \end{pmatrix}$

Definition 2.10: [9]

Let $(M,+,.)$ be any R-module over a refined neutrosophic ring $R(I_1,I_2)$, The triple $(M(I_1,I_2),+,\cdot)$ is called a strong refined neutrosophic R-module over a refined neutrosophic ring $R(I_1,I_2)$, generated by M, I_1 *and* I_2.

Theorem 2.11: [3]

Let (R ,+,×) be a ring and R(I), R(I_1, I_2) the related neutrosophic ring and refined neutrosophic ring respectively, we have:

(a) There is a ring homomorphism $f: R(I_1, I_2) \to R(I)$.

(b) The additive group (Ker f , +) is isomorphic to the additive group (R,+).

3. Main discussion

Definition 3.1:

Let M be a module over a ring R, N be a module over a ring T, $\varphi: M \to N$ be a well defined map, we say that φ is an f-semi module homomorphism if and only if the following conditions are true:

(a) $\varphi(x+y) = \varphi(x) + \varphi(y)$ for all $x, y \in M$.

(b) There is a ring homomorphism $f: R \to T$ such $\varphi(r.x) = f(r).\varphi(x)$ for all $r \in R, x \in$M.

Remark 3.2 :

(a) The concept of semi homomorphism can be used to study relationships between two modules defined over different rings.

(b) It is easy to see that every homomorphism between two modules defined over the same ring is a semi homomorphism. (Semi homomorphisms generalize classical module homomorphisms).

In the following theorem, we show that every neutrosophic module M(I) defined over a neutrosophic ring R(I) is a semi homomorphic image to the corresponding refined neutrosophic module $M(I_1, I_2)$ over the corresponding refined neutrosophic ring $R(I_1, I_2)$.

Theorem 3.3 :

Let M be a module over a ring R, M(I) be the corresponding strong neutrosophic module over R(I), $M(I_1, I_2)$ be the corresponding strong refined neutrosophic module over $R(I_1, I_2)$. Then $\varphi: M(I_1, I_2) \to M(I)$; $\varphi(a, bI_1, cI_2) = a + (b+c)I$ is a semi homomorphism.

Proof:

Clearly, φ is a well defined map. Let $x = (a, bI_1, cI_2), y = (u, vI_1, wI_2)$ be two arbitrary elements in $M(I_1, I_2)$,

$\varphi(x + y) = (a + u) + (b + v + c + w)I = [a + (b + c)I] + [u + (v + w)I] = \varphi(x) + \varphi(y)$.

According to [3], the map $f: R(I_1, I_2) \to R(I); f(m, nI_1, tI_2) = m + (n + t)I; m, n, t \in R$ is a ring homomorphism.

Consider $r = (m, nI_1, tI_2) \in R(I_1, I_2)$, we can write

$\varphi(r.x) = \varphi(m.a, (m.b + n.a + n.b + n.c + t.b)I_1, (t.a + t.c + m.c)I_2) =$

$m.a + (m.b + n.a + n.b + n.c + t.b + t.a + t.c + m.c)I = [m + (n + t)I].[a + (b + c)I] =$

$f(r).\varphi(x)$. Thus the proof is complete.

Theorem 3.4 :

Let M be a module over a ring R, M(I) be the corresponding strong neutrosophic module over R(I), $M(I_1, I_2)$ be the corresponding strong refined neutrosophic module over $R(I_1, I_2), \varphi$ be the semi homomorphism defined in Theorem 3.3. Then

The semi homomorphic image of any strong submodule of $M(I_1, I_2)$ is a strong submodule of M(I).

Proof:

Let N be any strong AH-submodule of $M(I_1, I_2)$, since $(N, +)$ is a subgroup of $(M(I_1, I_2), +)$, we have $(\varphi(N), +)$ is a subgroup of (M(I),+).

Let $y = a + bI$ be an arbitrary element in $\varphi(N)$, $r = u + vI$ be any element in R(I), since φ, f are surjective maps, there are

$x = (a, mI_1, nI_2) \in M(I_1, I_2)$; $\varphi(x) = y$, i.e $m + n = b$ and $g = (u, zI_1, qI_2) \in R(I_1, I_2); f(g) = r$, i.e $z + q = v$,

$r.y = u.a + (v.a + v.b + u.b)I = u.a + (z.a + q.a + z.m + z.n + q.m + q.n + u.m + u.n)I=$

$f(g).\varphi(x) = \varphi(g.x) \in \varphi(N)$.

Now we study the kernel of semi homomorphism.

Definition 3.5:

Let M be a module over a ring R, M(I) be the corresponding strong neutrosophic module over R(I), $M(I_1, I_2)$ be the corresponding strong refined neutrosophic module over $R(I_1, I_2), \varphi$ be the semi homomorphism defined in Theorem 3.3. We define $Ker(\varphi) = \{x \in M(I_1, I_2);\ \varphi(x) = 0\}$.

Theorem 3.6:

Let M be a module over a ring R with unity 1, M(I) be the corresponding strong neutrosophic module over R(I), $M(I_1, I_2)$ be the corresponding strong refined neutrosophic module over $R(I_1, I_2), \varphi$ be the semi homomorphism defined in Theorem 3.3, we have

(a) $Ker(\varphi) = \{(0, xI_1, -xI_2); x \in M\}$.

(b) For each $m \in Ker(\varphi)$ there is $r \in Ker(f)$ and $n \in M(I_1, I_2)$ such $m = r.n$.

(c) $Ker(\varphi)$ is a strong submodule of $M(I_1, I_2)$.

Proof:

(a) Let $z = (y, xI_1, tI_2) \in Ker(\varphi)$, $\varphi(z) = y + (x + t)I = 0$, thus $y = 0, t = -x$.

(b) Consider $m = (0, xI_1, -xI_2) \in Ker(\varphi)$, there is $r = (0, I_1, -I_2) \in Ker(f) and\ n = (2x, xI_1, -xI_2) \in M(I_1, I_2); x \in M$, where

$r.n = (0, xI_1, -xI_2) = m$.

(c) It is clear that $Ker(\varphi)$ is closed under addition. Now suppose that $m = (0, xI_1, -xI_2) \in Ker(\varphi)$ and $r = (a, bI_1, cI_2) \in R(I_1, I_2)$, $r.m = (0, [a.x + c.x]I_1, [-a.x - c.x]I_2) \in Ker(\varphi)$.

Thus our proof is complete.

Example 3.7:

Let $M = Z_3$ be the module of integers modulo 3 over the ring Z, $M(I), M(I_1, I_2)$ be its corresponding neutrosophic and refined neutrosophic modules over $Z(I) and\ Z(I_1, I_2)$ respectively. We have

(a) $f: Z(I_1, I_2) \to Z(I); f(a, bI_1, cI_2) = a + (b + c)I$ is a ring homomorphism.

(b) $\varphi: M(I_1, I_2) \to M(I);\ \varphi(x, yI_1, zI_2) = x + (y + z)I$ is an f −semi module homomorphism.

(c) $Ker(\varphi) = \{(0, xI_1, -xI_2); x \in M\} = \{(0,0,0), (0, I_1, 2I_2), (0,2I_1, I_2)\}$.

Definition 3.8:

Let M be a module over a ring R with unity 1, M(I) be the corresponding strong neutrosophic module over R(I), $M(I_1, I_2)$ be the corresponding strong refined neutrosophic module over $R(I_1, I_2), \varphi$ be the semi homomorphism defined in Theorem 3.3. We define

$M(I_1,I_2)/Ker(\varphi) = \{a + Ker(\varphi); a \in M(I_1,I_2)\} = \{(x, yI_1, zI_2) + Ker(\varphi); x, y, z \in M\}$.

$M(I_1,I_2)/Ker(\varphi)$ is called neutrosophic semi factor.

Definition 3.9:

We define operations on the semi factor $M(I_1,I_2)/Ker(\varphi)$ as follows:

(a) Addition: for each $a + Ker(\varphi), b + Ker(\varphi) \in M(I_1,I_2)/Ker(\varphi)$ *, we have*

$\big(a + Ker(\varphi)\big) + \big(b + Ker(\varphi)\big) = (a + b) + Ker(\varphi)$.

(b) Multiplication by a scalar: for each $r \in R(I_1,I_2), a + Ker(\varphi) \in M(I_1,I_2)/Ker(\varphi)$, we have

$r.\big(a + Ker(\varphi)\big) = r.a + Ker(\varphi)$.

Theorem 3.10:

Addition and Multiplication by a scalar are well defined operations on $M(I_1,I_2)/Ker(\varphi)$.

Proof:

Suppose that $a + Ker(\varphi) = b + Ker(\varphi)$, and $c + Ker(\varphi) = d + Ker(\varphi)$ then $a - b, c - d \in Ker(\varphi)$, thus $(a + c) - (b + d) \in Ker(\varphi)$, hence $a + c + Ker(\varphi) = b + d + Ker(\varphi)$.

Now assume that $a + Ker(\varphi) = b + Ker(\varphi)$ and $r = s \in R(I_1,I_2)$, then

$r.\big(a + Ker(\varphi)\big) = s.(b + Ker(\varphi))$. Thus the proof is complete.

Theorem 3.11:

$(M(I_1,I_2)/Ker(\varphi), +, .)$ is a module over the refined neutrosophic ring $R(I_1,I_2)$.

Proof:

Firstly, we remark that $M(I_1,I_2)/Ker(\varphi)$ is closed under addition and multiplication.

Since R is a ring with unity, we find that $1 \in R(I_1,I_2)$. Let $a + Ker(\varphi), b + Ker(\varphi)$ be two arbitrary elements in $M(I_1,I_2)/Ker(\varphi)$, r, s be two arbitrary elements in $R(I_1,I_2)$. Now we have

$1.\big(a + Ker(\varphi)\big) = a + Ker(\varphi)$, $(r + s)\big(a + Ker(\varphi)\big) = r.\big(a + Ker(\varphi)\big) + s.(b + Ker(\varphi))$,

$r.\big[\big(a + Ker(\varphi)\big) + \big(b + Ker(\varphi)\big)\big] = r.\big(a + b + Ker(\varphi)\big) = r.(a + b) + Ker(\varphi) = [r.a + Ker(\varphi)] + [r.b + Ker(\varphi)] = r.\big(a + Ker(\varphi)\big) + r.(b + Ker(\varphi))$.

Also, $M(I_1,I_2)/Ker(\varphi)$ is an abelian group with respect to addition. Thus it is a module over the ring $R(I_1,I_2)$.

Definition 3.12:

Let M be a module over a ring R, N be a module over a ring T, $\varphi: M \to N$ be a semi module homomorphism, we say that φ is a semi isomorphism if and only if it is a bijective map.

M, N are called semi isomorphic modules.

Theorem 3.13:

Let M be a module over a ring R with unity 1, M(I) be the corresponding strong neutrosophic module over R(I), $M(I_1, I_2)$ be the corresponding strong refined neutrosophic module over $R(I_1, I_2), \varphi$ be the semi homomorphism defined in Theorem 3.3. Then $M(I_1, I_2)/Ker(\varphi)$ is semi isomorphic to M(I).

Proof:

Define h: $M(I_1, I_2)/Ker(\varphi) \to M(I); h\big(a + Ker(\varphi)\big) = \varphi(a)$.

(a) h is well defined

Assume that $a + Ker(\varphi) = b + Ker(\varphi)$, then $a - b \in Ker(\varphi)$, hence $\varphi(a - b) = 0$, this means $\varphi(a) = \varphi(b)$.

(b) h is a semi homomorphism

Let $a + Ker(\varphi), b + Ker(\varphi)$ be two arbitrary elements in $M(I_1, I_2)/Ker(\varphi)$, and $r = (r_0, r_1 I_1,\ r_2 I_2)$ be an arbitrary element in $R(I_1, I_2)$, we have

$h([a + Ker(\varphi)] + [b + Ker(\varphi)]) = h\big(a + b + Ker(\varphi)\big) = \varphi(a + b)$=

$\varphi(a) + \varphi(b) = h(a + Ker(\varphi)) + h(b + Ker(\varphi))$.

$h(r.[a + Ker(\varphi)]) = h\big(r.a + Ker(\varphi)\big) = \varphi(r.a) = f(r).\varphi(a) = f(r).h(a + Ker(\varphi))$.

(c) h is a bijective map

It is easy to see that h is surjective. Now suppose that

$h([a + Ker(\varphi)]) = h([b + Ker(\varphi)])$, then $\varphi(a) = \varphi(b), this\ implies\ \varphi(a - b) = 0$, hence

$a - b \in Ker(\varphi), so\ a + Ker(\varphi) = b + Ker(\varphi)$. Thus h is a semi isomorphism.

Thus we get the proof.

Theorem 3.14:

Let $\varphi: M \to N, \tau: N \to L$ be two semi homomorphisms, where M, N, L are three modules over the rings

R, T, S respectively. Then $\tau o\varphi: M \to L$ is a semi homomorphism.

Proof:

Suppose that φ is an $f - semi\ homomorphism$, where $f: R \to T$ is a ring homomorphism, and τ is a $g - semi\ homomorphism$, where $g: T \to S$ is a ring homomorphism. It is clear that $gof: R \to S$ is a ring homomorphism. Now we prove that $\tau o\varphi: M \to L$ is a $gof - semi\ homomorphism$.

Let x, y be two arbitrary elements in M, and r be any element in the ring R, we have

$\tau o\varphi(x + y) = \tau\big(\varphi(x) + \varphi(y)\big) = \tau o\varphi(x) + \tau o\varphi(y)$.

$\tau o\varphi(r.x) = \tau\big(\varphi(r.x)\big) = \tau\big(f(r).\varphi(x)\big) = g\big(f(r)\big)\tau\big(\varphi(x)\big) = gof(r).\tau o\varphi(x)$. Hence $\tau o\varphi$ is $gof - semi\ homomorphism$.

Remark 3.15:

The previous theorem shows that the set of all semi homomorphisms between a module M and itself is closed under multiplication.

The following theorem shows that any module M will be a semi homomorphic image to its corresponding neutrosophic module M(I).

Theorem 3.16:

Let M be a module over R, M(I) be its corresponding neutrosophic module over R(I).

Then M is a semi homomorphic image to M(I).

Proof:

According to [3], there is a ring homomorphism $f: R(I) \to R; f(r + sI) = r; r, s \in R$,

we define $g: M(I) \to M; g(x + yI) = x;\ x, y \in M$.

It is clear that for every $m, n \in M(I)$, we get $g(m + n) = g(m) + g(n)$. Also, we have $g([r + s.I].[$[illegible]$+ yI]) = g(r.x + [r.y + s.x + s.y]I) = r.x = f(r + s.I).g(x + yI)$, thus g is a semi homomorphism.

The previous result shows that neutrosophic module (which they were defined using logic) have an algebraic origin, since they can be represented by semi homomorphisms.

Remark 3.17:

According to Theorem 3.13, we find that $M(I)/_{Kerg}$ is semi isomorphic to M.

$Ker(g) = \{yI; y \in M\} = MI$. Hence $M(I)/_{MI}$ is semi isomorphic to M.

Example 3. 18*:*

Let $M = Z_6$ be a module over the ring of integers Z, M(I)= $\{x + yI; x, y \in Z_6\}$ be its corresponding neutrosophic module, we have

(a) $\ker(g) = MI = \{yI; y \in Z_6\} = \{0, I, 2I, 3I, 4I, 5I\}$.

(b) $M(I)/_{MI} = \{A + MI; A \in M(I)\} = \{MI, 1 + MI, 2 + MI, 3 + MI, 4 + MI, 5 + MI\}$.

Which is semi isomorphic to M.

Definition 3.19:

Let R be any ring with unity, M be a module over R, we define

(a) The set of all semi homomorphisms from a module M to itself is denoted by

$S_M = \{\varphi: M \to M;\ \varphi$ is a semi homomorphism$\}$.

(b) The set of all semi isomorphisms from a module M to itself is denoted by $SI_M = \{\varphi: M \to M;\ \varphi\ is\ a\ semi\ isomorphism\}$.

(c) The set of all f −semi homomorphisms between a module M and itself is denoted by $f - S_M$.

(d) The set of all f −semi isomorphisms from a module M to itself is denoted by $f - SI_M$.

The following theorem clarifies the algebraic structure of semi homomorphisms.

Theorem 3.20:

Let M be a module over a ring R, $S_M, SI_M,\ f - S_M, f - SI_M$ be the sets defined above, we have:

(a) $(f - S_M, +, .)$ is a module over the ring R.

(b) ([illegible]$I_M, o)$ is a semi group.

(c) If f is an isomorphism with property $fof = I$ (identity map), then for every $\varphi, \tau \in f - SI_M$, we have $\varphi o \tau$ is a module isomorphism.

Proof:

(a) Let g, h be any two $f - semi\ homomorphisms$. They are additive homomorphisms on M, thus $(g + h)(x + y) = (g + h)(x) + (g + h)(y)$ for every $x, y \in M$ clearly. Also, we have for every $r \in R$ and $x \in M$: $(g+h)(r.x) = g(r.x) + h(r.x) = f(r).g(x) + f(r).h(x) = f(r).(g+h)(x)$, hence $g+h$ is an

$f - semi\ homomorphism$.

One the other hand, g has an inverse with respect to addition, which is $-g$, and $t(x) = 0$ is the identity in $f - S_M$, thus $(f - S_M, +)$ is an abelian group.

Now, let $a \in R$ be an arbitrary element, then the map $(a.g): M \to M; (a.g)(x) = a.g(x)$ is an $f - semi\ homomorphism$ clearly. It is easy to check that the rest of module's axioms are true.

(b) SI_M is closed under (o), and (o) is an associative operation, and the identity map $I \in SI_M$. thus the proof holds easily.

(c) It is well known that $\varphi o \tau$ is an additive homomorphism according to Theorem 3.14.

Now we shall prove that $\varphi o \tau\ (r.x) = r.\varphi o \tau$ (x), for every $r \in R$ and $x \in M$.

$\varphi o \tau\ (r.x) = \varphi(\tau(r.x)) = \varphi[f(r).\tau(x)] = f(f(r)).\varphi o \tau\ (x) = I(r).\varphi o \tau\ (x) = r.\varphi o \tau\ (x)$. Thus $\varphi o \tau$ is a classical homomorphism.

4. Conclusions

In this article, we have defined the concept of semi homomorphism and semi isomorphism between two modules defined over different rings. Also, we applied this concept to study the algebraic relation between refined neutrosophic strong module and neutrosophic strong module, and between a module and its corresponding neutrosophic strong module.

The main result of this work is to prove that every strong neutrosophic module is a semi homomorphic image of the corresponding strong refined neutrosophic module.

In particular, we have constructed some examples to clarify the validity of our work.

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