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On Separation Axioms with New Constructions in Fuzzy Neutrosophic Topological Spaces

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Abstract: The purpose of this .research is, to define a fuzzy neutrosophic points in fuzzy neutrosophic. topological space namely [FNPs]. Also, we have study some new types of points in separation axioms T_i, where i= 0,1,2 with some new construction based of fuzzy neutrosophic topological spaces as extension of Fatimah et al. work [*1*]. Then, we investigate many theorems and examples to present and discuss.

Keywords: fuzzy neutrosophic point; fuzzy neutrosophic topology; separation axioms.

1. Introduction

The concept of neutrosophic set theory introduced by Smarandache [2] and get the introduction of the term neutrosophic. components,$(T,I,F,)$ which refers to the membership, non-membership and between them the indeterminacy values . Then, Salama et al..[3,4] study some .basic concepts and theire operations, of the neutrosophic.crisp set for building. new branches. of neutrosophic. mathematic. Then, many authors studied and presented the term of neutrosophic set theory and some of its applications in their works, (see[5,6,7,8,9,10]).

Recently, many concepts of neutrosophic topological spaces have been extended in fuzzy neutrosophic topological spaces by the authors (see [11-20]). In this work, we put some basic concepts of the neutrosophic. set, with their operations, and because, of their .useful and wide applications to solve many problems, we used these concepts of fuzzy neutrosophic sets as generalized of Ahmed et al. study [21] to define new types of neutrosophic points based of our space, also our interest is to study separation axioms T_0, T_1, T_2 with new construction as extension of Fatimah et al. work [1] in fuzzy neutrosophic topological spaces by definitions, propositions and counter examples so in this paper, several types of fuzzy neutrosophic points in separation axioms via fuzzy neutrosophic topological spaces are going to be studied. Finally, we used the new concepts and definitions to examine the relationship between them in details.

2. Preliminaries:

In this part of our research, we will refer to some definitions and operations which are useful in our study.

Definition 2.1 [9]: "Let X_N be a non-empty fixed set. The fuzzy neutrosophic set (FNS), S_N is an object having the form $S_N = \{< x, \mu_{S_N}(x), \sigma_{S_N}(x), v_{S_N}(x) >: x \in X_N\}$ where the functions $\mu_{S_N}, \sigma_{S_N}, v_{S_N}: X_N \to [0, 1]$ denote the degree of membership function (namely $\mu_{S_N}(x)$), the degree of indeterminacy function (namely $\sigma_{S_N}(x)$) and the degree of non-membership function (namely $v_{S_N}(x)$) respectively of each element $x \in X_N$ to the set S_N and $0 \leq \mu_{S_N}(x) + \sigma_{S_N}(x) + v_{S_N}(x) \leq 3$, for each $x \in X_N$."

Remark 2.2 [15]: "FNS $\mu_N = \{< u, \lambda_{\mu N}(u), \sigma_{\mu N}(u), V_{\mu N}(u) >: u \in U\}$ can be identified to an ordered triple $< u, \lambda_{\mu N}, \sigma_{\mu N}, V_{\mu N} >$ in [0,1] on U."

Definition 2.3 [9]: "Fuzzy neutrosophic topology (FNT) on a non-empty set X_N is a family τ of fuzzy neutrosophic subsets in X_N satisfying the following axioms.

i. $0_N, 1_N \in \tau$,

ii. $S_{N_1} \wedge S_{N_2} \in \tau$ for any $S_{N_1}, S_{N_2} \in \tau$,

iii. $\vee S_{N_j} \in \tau$, $\forall \{S_{N_j}: j \in J\} \subseteq \tau$.

In this case the pair (X_N, τ) is called fuzzy neutrosophic topological space (FNTS). The elements of τ are called fuzzy neutrosophic -open sets (FN-open set). The complement of FN-open set in the FNTS (X_N, τ) is called fuzzy neutrosophic-closed set (FN-closed set)."

Definition 2.4 [9]: "Let $S = \langle \mu_S(x), \sigma_S(x), \gamma_S(x) >$ be a NS on X_N, then the complement of the set S (S^c, for short) maybe defined as three kinds of complements:

(C_1) $S^c = \{< x, 1 - \mu_S(x), 1 - \gamma_S(x) >: x \in X_N\}$,

(C_2) $S^c = \{< x, \gamma_S(x), \sigma_S(x), \mu_S(x) >: x \in X_N\}$,

(C_3) $S^c = \{< x, \gamma_S(x), 1 - \sigma_S(x), \mu_S(x) >: x \in X_N\}$."

Definition 2.5 [10]**:** **"**Let X_N be a non-empty set and two NS*s* S with M in the form $S =< \mu_S(x), \sigma_S(x), \gamma_S(x) >$, $M = < \mu_M(x), \sigma_M(x), \gamma_M(x) >$, then we may consider two possible definitions for subsets ($S \subseteq M$) may be defined as :

(1) $S \subseteq M \Leftrightarrow \mu_S(x) \leq \mu_M(x)$, $\gamma_S(x) \geq \gamma_M(x)$ and, $\sigma_S(x) \leq \sigma_M(x)$ $\forall x \in X_N$,

(2) $S \subseteq M \Leftrightarrow \mu_S(x) \leq \mu_M(x), \gamma_S(x) \geq \gamma_M(x)$ and $\sigma_S(x) \geq \sigma_M(x) \forall x \in X_N$."

Proposition 2.6 [10]**:** **"**For any neutrosophic set S the following are holds:

(1) $0_N \subseteq S$, $0_N \subseteq 0_N$,

(2) $S \subseteq 1_N$, $1_N \subseteq 1_N$."

Also, the intersection can be written as $S \wedge M$ and may be defined by:

(I_1) $S \wedge M = <x, \mu_S(x) \wedge \mu_M(x), \sigma_S(x) \wedge \sigma_M(x), \gamma_S(x) \vee \gamma_M(x) >$,

(I_2) $S \wedge M = <x, \mu_S(x) \wedge \mu_M(x), \sigma_S(x) \vee \sigma_M(x), \gamma_S(x) \vee \gamma_M(x) >$.

Finally, the union can be written as $S \vee M$ may be defined by:

(U_1) $S \vee M = <x, \mu_S(x) \vee \mu_M(x), \sigma_S(x) \vee \sigma_M(x), \gamma_S(x) \wedge \gamma_M(x)>$,

(U_2) $S \vee M = <x, \mu_S(x) \vee \mu_M(x), \sigma_S(x) \wedge \sigma_M(x), \gamma_S(x) \wedge \gamma_M(x)>$.

3. Some New Separation Axioms in Fuzzy Neutrosophic Topological Spaces

In this section, we present T_i- separation axioms where I = 0,1,2 based of fuzzy neutrosophic topological spaces and introduced it after giving some definitions of as follows:

Definition 3.1: The object having the from $S = \langle S_1, S_2, S_3 \rangle$ is called:

1. A fuzzy neutrosophic set of Type 1 [FNS/Type1] if satisfying
   $S_1 \wedge S_2 = 0, S_1 \wedge S_3 = 0$ and $S_2 \wedge S_3 = 0$,
2. A fuzzy neutrosophic set of Type 2 [FNS/Type2] if satisfying
   $S_1 \wedge S_2 = 0, S_1 \wedge S_3 = 0$ and $S_2 \wedge S_3 = 0$, $S_1 \vee S_2 \vee S_3 = 1$,
3. A fuzzy neutrosophic set of Type 3 [FNS/Type3] if satisfying
   $S_1 \wedge S_2 \wedge S_3 = 0$, $S_1 \vee S_2 \vee S_3 = 1$.

Example 3.2: Let $X_N = \{a\}$, then:

1. $S = < 0.5, 0, 0 >$ is a FNS in X_N,

Type 1: $S_1 \wedge S_2 = 0.5 \wedge 0 = 0, S_1 \wedge S_3 = 0.5 \wedge 0 = 0, S_2 \wedge S_3 = 0$

Therefore FNS is FN_1.

2. $S = < 1,0,0 >$ is an (FNS) in X_N,

Type 2 : $S_1 \wedge S_2 = 1 \wedge 0 = 0$, $S_1 \wedge S_3 = 1 \wedge 0 = 0, S_2 \wedge S_3 = 0$ _

$S_1 \vee S_2 \vee S_3 = 1$

Therefore FNS is FN_2.

3. $S = < 0.8, 1, 0 >$ is an (FNS) in X_N.

Type 3 : $S_1 \wedge S_2 \wedge S_3 = 0$, $S_1 \vee S_2 \vee S_3 = 1$.

Therefore FNS is FN_3.

Remark 3.3: For the FNS we have:

1. Every FN_2 is FN_1,
2. Every FN_2 is FN_3.

The proof is directly.

The converse of Remark 3.3 is not true as it shown in the next example.

Example 3.4: Let $X_N = \{a\}$, then:

1. $S =< 0.5,0,0 >$ is an (FNS) in X_N,

Type 1: $S_1 \wedge S_2 = 0.5 \wedge 0 = 0, S_1 \wedge S_3 = 0.5 \wedge 0 = 0, S_2 \wedge S_3 = 0.$

Therefore FNS is FN_1 but is not FN_2.

2. $S =< 0.8, 1, 0 >$ is an (FNS) in X_N.

Type 3 : $S_1 \wedge S_2 \wedge S_3 = 0$, $S_1 \vee S_2 \vee S_3 = 1.$

Therefore FNS is FN_1 but is not FN_2.

Definition 3.5: Types of FNSs 0_N and 1_N in X_N can defined as follows :

1. 0_N may be defined in many ways as a FNS as four ways:
   1. Type 1 : $0_N =< 0,0,1 >$,
   2. Type 2 : $0_N =< 0,1,1 >$,
   3. Type 3 : $0_N =< 0,1,0 >$,
   4. Type 4 : $0_N =< 0,0,0 >$.
2. 1_N may be defined in many ways as a FNS as:
   1. Type 1 : $1_N =< 1,0,0 >$,
   2. Type 2 : $1_N =< 1,1,0 >$,
   3. Type 3 : $1_N =< 1,0,1 >$,
   4. Type 4 : $1_N =< 1,1,1 >$.

Definition 3.6: Let X_N be a non - empty set and the FNSs α and β in form $\alpha =< \alpha_1, \alpha_2, \alpha_3 >$, $\beta =< \beta_1, \beta_2, \beta_3 >$, then we may consider two possible definitions for subsets $\alpha \subseteq \beta$, may be defined in two ways :

1. $\alpha \subseteq \beta \Leftrightarrow \alpha_1 \subseteq \beta_1, \alpha_2 \subseteq \beta_2$ and $\alpha_3 \subseteq \beta_3$,
2. $\alpha \subseteq \beta \Leftrightarrow \alpha_1 \subseteq \beta_1, \beta_2 \subseteq \alpha_2$ and $\beta_3 \subseteq \alpha_3$.

Definition 3.7: For all x ,y, z belonging to a non – empty set X_N, the fuzzy neutrosophic points related to x ,y, z are defined as follows:

1. $x_{N_1} =< \{x\},0,0 >$, is called a fuzzy neutrosophic point (FNP_{N_1}) in X_N,
2. $y_{N_2} =< 0, \{y\},0 >$, is called a fuzzy neutrosophic point (FNP_{N_2}) in X_N,
3. $z_{N_3} =< 0,0, \{z\} >$, is called a fuzzy neutrosophic point (FNP_{N_3}) in X_N.

The set of all fuzzy neutrosophic points (FNP_{N_1}, FNP_{N_2}, FNP_{N_3}) is denoted by FNP_N.

Definition 3.8: Let X_N be anon - empty set and x, y, z $\in X_N$. Then the fuzzy neutrosophic point:

1. x_{N_1} is belonging to the fuzzy neutrosophic set $S = < S_1, S_2, S_3 >$, denoted by $x_{N_1} \in S$, if $x \in S_1$ where in x_{N_1} does not belong to the fuzzy neutrosophic set S denoted by $x_{N_1} \notin S$, if $x_{N_1} \notin S_1$,
2. y_{N_2} is belonging to the fuzzy neutrosophic set $S = < S_1, S_2, S_3 >$, denoted by $y_{N_2} \in S$, if $y \in S_2$ in contrast y_{N_2} does not belong to the fuzzy neutrosophic set S denoted by $y_{N_2} \notin S$, if $y_{N_2} \notin S_2$,
3. z_{N_3} is belonging to the fuzzy neutrosophic set $S = < S_1, S_2, S_3 >$, denoted by $z_{N_3} \in S$, if $z \in S_3$ in contrast z_{N_3} does not belong to the fuzzy neutrosophic set S denoted by $z_{N_3} \notin S$, if $z_{N_3} \notin S_3$.

Definition 3.9: Let (X_N, τ) be a FNTS, Then X_N is called:

1- $FN_1 T_0$ -space for every two different points from X_N are x_{N_1}, y_{N_1} there exists two fuzzy neutrosophic open set S ,M inX_N such that $x_{N_1} \in S$, $y_{N_1} \notin S$ and $x_{N_1} \notin M$, $y_{N_1} \in M$,

2- $F N_2 T_0$ -space for every two different points from X_N are x_{N_2}, y_{N_2} there exists two fuzzy neutrosophic open set S ,M inX_N such that $x_{N_2} \in S$, $y_{N_2} \notin S$ and $x_{N_2} \notin M$, $y_{N_2} \in M$,

3- $FN_3 T_0$ -space for every two different points from X_N are x_{N_3}, y_{N_3} there exists two fuzzy neutrosophic open set S, M inX_N such that $x_{N_3} \in S$, $y_{N_3} \notin S$ and $x_{N_3} \notin M$, $y_{N_3} \in M$.

Example 3.10: Let $X_N = \{a,b,c\}$ and $\tau = \{0_N, 1_N, S\}$,

1. If $S = \{< \frac{a}{0.8,0,0} >, < \frac{b}{0,0,0} >, < \frac{c}{0,0,0} >\}$.

So, $x_{N_1} = \{< \frac{a}{0.8,0,0} >, < \frac{b}{0,0,0} >, < \frac{c}{0,0,0} >\} \neq y_{N_1} = \{< \frac{a}{0.7,0,0} >, < \frac{b}{0,0,0} >, < \frac{c}{0,0,0} >\} \in X_N$.

There is a FNOS in (X_N, τ) say $x_{N_1} = \{< \frac{a}{0.8,0,0} >, < \frac{b}{0,0,0} >, < \frac{c}{0,0,0} >\} \in S$ but $y_{N_1} = \{< \frac{a}{0.7,0,0} >, < \frac{b}{0,0,0} >, < \frac{c}{0,0,0} >\} \notin S$.

Therefore, (X_N, τ) is $FN_1 T_0$ -space .

2. If $S = \{< \frac{a}{1,1,0} > , < \frac{b}{0,1,0} >, < \frac{c}{0,1,0} > \}$.So, $x_{N_2} = \{< \frac{a}{1,1,0} >, < \frac{b}{1,0,0} >, < \frac{c}{0,1,0} >\} \neq y_{N_2} = \{< \frac{a}{0,1,0} >, < \frac{b}{0,1,0} >, < \frac{c}{0,1,0} >\} \in X_N$. There is a FNOS in (X_N, τ) say $x_{N_2} = \{< \frac{a}{1,1,0} >, < \frac{b}{1,0,0} >, < \frac{c}{0,1,0} >\} \in S$ but, $y_{N_2} = \{< \frac{a}{0,1,0} >, < \frac{b}{0,1,0} >, < \frac{c}{0,1,0} >\} \notin S$.

Therefore, (X_N, τ) is $FN_2 T_0$ -space

3. If $S = \{<\frac{a}{1,0.9,0}>,<\frac{b}{0,0,1}>,<\frac{c}{0,0,1}>\}$.

So, $x_{N_3} = \{<\frac{a}{0,0,1}>,<\frac{b}{0,0,1}>,<\frac{c}{0,0.9,1},>\} \neq y_{N_3} = \{<\frac{a}{0,0,1}>,<\frac{b}{0,0,1}>,<\frac{c}{0.6,1,0}>\} \in X_N$.

There is a FNOS in (X_N, τ) say, $x_{N_3} = \{<\frac{a}{0,0,1}>,<\frac{b}{0,0,1}>,<\frac{c}{0,0.9,1},>\} \in S$ but, $y_{N_3} = \{<\frac{a}{0,0,1}>,<\frac{b}{0,0,1}>,<\frac{c}{0.6,1,0}>\} \notin S$. Therefore, (X_N, τ) is $FN_3\,T_0$ -space .

Definition 3.11: Suppose that (X_N, τ) is a FNTS, Then X_N is called:

1- $FN_1\,T_1$ -space for every two different points from X_N are x_{N_1}, y_{N_1} there exists two fuzzy neutrosophic open set S ,M inX_N such that $x_{N_1} \in S$, $y_{N_1} \notin S$ and $x_{N_1} \notin M$, $y_{N_1} \in M$,
2- $FN_2\,T_1$ -space for every two different points from X_N are x_{N_2}, y_{N_2} there exists two fuzzy neutrosophic open set S ,M inX_N such that $x_{N_2} \in S$, $y_{N_2} \notin S$ and $x_{N_2} \notin M$, $y_{N_2} \in M$,
3- $FN_3\,T_1$ -space for every two different points from X_N are x_{N_3}, y_{N_3} there exists two fuzzy neutrosophic open set S ,M inX_N such that $x_{N_3} \in S$, $y_{N_3} \notin S$ and $x_{N_3} \notin M$, $y_{N_3} \in M$.

Example 3.12: Let $X_N = \{a,b,c\}$, $\tau = \{0_N, 1_N, S, M, S\wedge M, S \vee M\}$,Then.

1. If $S = \{<\frac{a}{0.5,0,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,0,0}>\}$and

$M = \{<\frac{a}{0.3,0,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,0,0}>\}$.

So, $x_{N_1} = \{<\frac{a}{0.5,0,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,0,0}>\} \neq y_{N_1} = \{<\frac{a}{0,0.3,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,0,0}>\} \in X_N$.

There is a FNOS in (X_N, τ), say $x_{N_1} = \{<\frac{a}{0.5,0,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,0,0}>\} \in S$, $x_{N_1} = \{<\frac{a}{0.5,0,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,0,0}>\} \notin M$ and $y_{N_1} = \{<\frac{a}{0,0.3,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,0,0}>\} \in M$, $y_{N_1} = \{<\frac{a}{0,0.3,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,0,0}>\} \notin S$. Therefore, (X_N, τ) is $FN_1\,T_1$ -space.

2. If $S = \{<\frac{a}{1,1,0}>,<\frac{b}{0,1,0}>,<\frac{c}{0,1,0}>\}$, and

$M = \{<\frac{a}{0,0,0}>,<\frac{b}{0,1,0}>,<\frac{c}{0,1,0}>\}$

So, $x_{N_2} = \{< \frac{a}{0,1,0} >,< \frac{b}{1,1,0} >,< \frac{c}{0,1,0} >\} \neq y_{N_2} = \{< \frac{a}{0,1,0} >,< \frac{b}{0,0,0} >,< \frac{c}{0,1,0} >\} \in X_N$

There is a FNOS in (X_N, τ), say $x_{N_2} = \{< \frac{a}{0,1,0} >,< \frac{b}{1,1,0} >,< \frac{c}{0,1,0} >\} \in S$, $x_{N_2} = \{< \frac{a}{0,1,0} >,< \frac{b}{1,1,0} >,< \frac{c}{0,1,0} >\} \notin M$ .and $y_{N_2} = \{< \frac{a}{0,1,0} >,< \frac{b}{0,0,0} >,< \frac{c}{0,1,0} >\}$

$\in M, y_{N_2} = \{< \frac{a}{0,1,0} >,< \frac{b}{0,0,0} >,< \frac{c}{0,1,0} >\} \notin S$. Therefore, (X_N, τ) is $FN_2\, T_1$ -space .

3. If $S = \{< \frac{a}{1,0.2,0} >,< \frac{b}{0,0,1} >,< \frac{c}{0,0,1} >\}$ and

   $M = \{< \frac{a}{0,0.7,1} >, < \frac{b}{0,0,1} >,< \frac{c}{0,0,1} >\}$

$S \wedge M = \{< \frac{a}{0,0.2,0} >,< \frac{b}{0,0,1} >,< \frac{c}{0,0,1} >\}$

$S \vee M = \{< \frac{a}{1,0.7,1} >,< \frac{b}{0,0,1} >,< \frac{c}{0,0,1} >\}$.

So, $x_{N_3} = \{< \frac{a}{0,0,1} >,<, \frac{b}{0,0,1} >,< \frac{c}{0,0.2,1} >\} \neq y_{N_3} = \{< \frac{a}{0,0,1} >,<, \frac{b}{0,0,1} >,< \frac{c}{0,0.7,1} >\} \in X_N$

There is a FNOS in (X_N, τ) say, $x_{N_3} = \{< \frac{a}{0,0,1} >,<, \frac{b}{0,0,1} >,< \frac{c}{0,0.2,1} >\} \in S$, $x_{N_3} = \{< \frac{a}{0,0,1} >,<, \frac{b}{0,0,1} >,< \frac{c}{0,0.2,1} >\} \notin M$ and $y_{N_3} = \{< \frac{a}{0,0,1} >,<, \frac{b}{0,0,1} >,< \frac{c}{0,0.7,1} >\} \in M$, $y_{N_3} = \{< \frac{a}{0,0,1} >,<, \frac{b}{0,0,1} >,< \frac{c}{0,0.7,1} >\} \notin S$. Therefore, (X_N, τ) is $FN_3\, T_1$ -space .

Definition 3.13: Suppose that (X_N, τ) is a FNTS. Then X_N is called:

1- $FN_1\, T_2$ -space for every two different points from X_N are x_{N_1}, y_{N_1} there exists two fuzzy neutrosophic open set S,M inX_N such that $x_{N_1} \in S$, $y_{N_1} \notin S$ and $x_{N_1} \notin M, y_{N_1} \in M$ with $S \wedge M =< 0,0,0 >$,
2- $FN_2\, T_2$ -space for every two different points from X_N are x_{N_2}, y_{N_2} there exists two fuzzy neutrosophic open set S ,M inX_N such that $x_{N_2} \in S$, $y_{N_2} \notin S$ and $x_{N_2} \notin M$, $y_{N_2} \in M$ with $S \wedge M =< 0,1,0 >$,
3- $FN_3\, T_2$ -space for every two different points from X_N are x_{N_3}, y_{N_3} there exists two fuzzy neutrosophic open set S ,M inX_N such that $x_{N_3} \in S$, $y_{N_3} \notin S$ and $x_{N_3} \notin M, y_{N_3} \in M$ with $S \wedge M = < 0,0,1 >$.

Example 3.14: Let $X_N = \{a, b, c\}$, $\tau = \{0_N, 1_N, S, M, S\vee M\}$,Then.

1. If $S = \{<\frac{a}{0.6,0,0}>, <\frac{b}{0,0,0}>, <\frac{c}{0,0,0}>\}$, and

$M = \{<\frac{a}{0,0.8,0}>, <\frac{b}{0,0,0}>, <\frac{c}{0,0,0}>\}$

$S\vee M = \{<\frac{a}{0,6,0.8,0}>, <\frac{b}{0,0,0}>, <\frac{c}{0,0,0}>\}$.

So, $S \wedge M = <0,0,0>$

Let $x_{N_1} = \{<\frac{a}{0.6,0,0}>, <\frac{b}{0,0,0}>, <\frac{c}{0,0,0}>\} \neq y_{N_1} = \{<\frac{a}{0,0.8,0}>, <\frac{b}{0,0,0}>, <\frac{c}{0,0,0}>\} \in X_N$

There is a FNOS in (X_N, τ), say, $x_{N_1} = \{<\frac{a}{0.6,0,0}>, <\frac{b}{0,0,0}>, <\frac{c}{0,0,0}>\} \in S$, $x_{N_1} = \{<\frac{a}{0.6,0,0}>, <\frac{b}{0,0,0}>, <\frac{c}{0,0,0}>\} \notin M$ and$y_{N_1} = \{<\frac{a}{0,0.8,0}>, <\frac{b}{0,0,0}>, <\frac{c}{0,0,0}>\} \in M$, $y_{N_1} = \{<\frac{a}{0,0.8,0}>, <\frac{b}{0,0,0}>, <\frac{c}{0,0,0}>\} \notin S$. Therefore, (X_N, τ) is $N_1 T_2$ -space .

2. If $S = \{<\frac{a}{1,1,0}>, <\frac{b}{0,1,,0}>, <\frac{c}{0,1,,0}>\}$and

$M = \{<\frac{a}{0,1,0}>, <\frac{b}{0,1,,0}>, <\frac{c}{0,1,,0}>\}$

So, $S \wedge M = <0,1,0>$

Then, $x_{N_2} = \{<\frac{a}{0,1,0}>, <\frac{b}{1,1,,0}>, <\frac{c}{0,1,,0}>\} \quad \neq y_{N_2} = \{<\frac{a}{0,1,0}>, <\frac{b}{0,1,,0}>, <\frac{c}{0,1,,0}>\} \in X_N$

There is a FNOS in(X_N, τ), say $x_{N_2} = \{<\frac{a}{0,1,0}>, <\frac{b}{1,1,,0}>, <\frac{c}{0,1,,0}>\} \in S$, $x_{N_2} = \{<\frac{a}{0}, \frac{b}{1}, \frac{c}{0}>, <\frac{a}{1}, \frac{b}{1}, \frac{c}{0}>, <\frac{a}{0}, \frac{b}{1}, \frac{c}{0}>\} \notin M$ and $y_{N_2} = \{<\frac{a}{0,1,0}>, <\frac{b}{0,1,,0}>, <\frac{c}{0,1,,0}>\} \in M$, $y_{N_2} = \{<\frac{a}{0,1,0}>, <\frac{b}{0,1,,0}>, <\frac{c}{0,1,,0}>\} \notin S$. Therefore, (X_N, τ) is $FN_2 T_2$ -space .

3. If $S = \{<\frac{a}{1,0.9,0}>, <\frac{b}{0,0,1}>, <\frac{c}{0,0,1}>\}$, and

$M = \{<\frac{a}{0,0,1}>, <\frac{b}{0,0,1}>, <\frac{c}{0,0,1}>\}$

$S\vee M=\{<\frac{a}{1,0.9,1}>,<\frac{b}{0,0,1}>,<\frac{c}{0,0,1}>\}$

So, $S\wedge M=<0,0,1>$

Then, $x_{N_3}=\{<\frac{a}{0,0,1}>,<\frac{b}{0,0,1}>,<\frac{c}{1,0.9,1}>\}\neq y_{N_3}=\{<\frac{a}{0,0,1}>,<\frac{b}{0,0,1}>,<\frac{c}{0,0,1}>\}\in X_N$

There is a FNOS in (X_N,τ) say, $x_{N_3}=\{<\frac{a}{0,0,1}>,<\frac{b}{0,0,1}>,<\frac{c}{1,0.9,1}>\}\in S$, $x_{N_3}=\{<\frac{a}{0,0,1}>,<\frac{b}{0,0,1}>,<\frac{c}{1,0.9,1}>\}\notin M$ and $y_{N_3}=\{<\frac{a}{0,0,1}>,<\frac{b}{0,0,1}>,<\frac{c}{0,0,1}>\}\in M$, $y_{N_3}=\{<\frac{a}{0,0,1}>,<\frac{b}{0,0,1}>,<\frac{c}{0,0,1}>\}\notin S$. Therefore, (X_N,τ) is $FN_3\ T_2$-space .

Note: Veereswari Y. [11] defined and construct several FNTSs as in the next definition so, we used it to study some new kinds of separation axioms with some relations and examples.

Definition 3.15 [11]: Let (X_N,τ) be a FNTS on X_N Then ,we can also construct several FNTSs on X_N in the following ways:

1- $\tau_{0.1}=\{[\,]S:S\in\tau\}$, where $[\,]S=<x,\mu_S(x),\sigma_S(x),1-\mu_S(x)>=$ $FS=<S_1,S_2,S_1^c>$,
2- $\tau_{0.2}=\{<>S:S\in\tau\}$, where $<>S=<x,1-V_S(x),\sigma_S(x),V_S(x)>=$ $SE=<S_3^c,S_2,S_3>$.

Now, we defined and construct two new FNTSs from the FNTS (X_N,τ) as the next definition.

Definition 3.16: Let (X_N,τ) be a FNTS such that τ is not indiscrete such that $\tau=\{0_N,1_N\}\vee\{S_i,i\in J\}$. Then we can construct two (FNTSs) on X_N as follows:

1- $\tau^1=\{0_N,1_N\}\vee\{S_1\}$,
2- $\tau^2=\{0_N,1_N\}\vee\{S_2\}$.

Example 3.17: Let $X_N=\{a,b,c\}$, $\tau=\{0_N,1_N,S,M,S\vee M\}$,Then.

1. If $S=\{<\frac{a}{0.6,0,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,0,0}>\}$, and

$M=\{<\frac{a}{0,0.8,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,0,0}>\}$

$S\vee M=\{<\frac{a}{0,6,0.8,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,0,0}>\}$.

So, $S \wedge M = < 0,0,0 >$

$\tau^1 = \{0_N, 1_N\} \vee \{0.6,0,0\} \vee \{0,0.8,0\}$

$\tau^1 = \{0_N, 1_N, \{0.6,0,0\}, \{0,0.8,0\}\}$.

$\tau^2 = \{0_N, 1_N\} \vee \{S_2\} \vee \{M_2\}$

$\tau^2 = \{0_N, 1_N\} \vee \{0,0,0\} \vee \{0,0,0\}$

$\tau^2 = \{0_N, 1_N, \{0,0,0\}, \{0,0,0\}\}$.

Let $x_{N_1} = \{< \frac{a}{0.6,0,0} >, < \frac{b}{0,0,0} >, < \frac{c}{0,0,0} >\} \neq y_{N_1} = \{< \frac{a}{0,0.8,0} >, < \frac{b}{0,0,0} >, < \frac{c}{0,0,0} >\} \in X_N$

There is a FNOS in (X_N, τ) say, $x_{N_1} = \{< \frac{a}{0.6,0,0} >, < \frac{b}{0,0,0} >, < \frac{c}{0,0,0} >\} \in S, x_{N_1} = \{< \frac{a}{0.6,0,0} >, < \frac{b}{0,0,0} >, < \frac{c}{0,0,0} >\} \notin M$ and $y_{N_1} = \{< \frac{a}{0,0.8,0} >, < \frac{b}{0,0,0} >, < \frac{c}{0,0,0} >\} \in M, y_{N_1} = \{< \frac{a}{0,0.8,0} >, < \frac{b}{0,0,0} >, < \frac{c}{0,0,0} >\} \notin S$. Therefore, (X_N, τ) is $FN_1\, T_2$ -space .

2. If $S = \{< \frac{a}{1,1,0} >, < \frac{b}{0,1,,0} >, < \frac{c}{0,1,,0} >\}$ and

$$M = \{< \frac{a}{0,1,0} >, < \frac{b}{0,1,,0} >, < \frac{c}{0,1,,0} >\}$$

So, $S \wedge M = < 0,1,0 >$

$\tau^1 = \{0_N, 1_N\} \vee \{S_1\} \vee \{M_1\}$

$\tau^1 = \{0_N, 1_N\} \vee \{1,1,0\} \vee \{0,1,0\}$

$\tau^1 = \{0_N, 1_N, \{1,0,0\}, \{0,1,0\}\}$.

$\tau^2 = \{0_N, 1_N\} \vee \{S_2\} \vee \{M_2\}$

$\tau^2 = \{0_N, 1_N\} \vee \{0,1,0\} \vee \{0,1,0\}$

$\tau^2 = \{0_N, 1_N, \{0,1,0\}, \{0,1,0\}\}$.

If $x_{N_2} = \{< \frac{a}{0,1,0} >, < \frac{b}{1,1,,0} >, < \frac{c}{0,1,,0} >\} \neq y_{N_2} = \{< \frac{a}{0,1,0} >, < \frac{b}{0,1,,0} >, < \frac{c}{0,1,,0} >\} \in X_N$

There is a FNOS in (X_N, τ) say, $x_{N_2} = \{< \frac{a}{0,1,0} >, < \frac{b}{1,1,,0} >, < \frac{c}{0,1,,0} >\} \in S$, $x_{N_2} = \{< \frac{a}{0}, \frac{b}{1}, \frac{c}{0} >, < \frac{a}{1}, \frac{b}{1}, \frac{c}{0} >, < \frac{a}{0}, \frac{b}{1}, \frac{c}{0} >\} \notin M$ and $y_{N_2} = \{< \frac{a}{0,1,0} >, < \frac{b}{0,1,,0} >, < \frac{c}{0,1,,0} >\} \in M$, $y_{N_2} = \{< \frac{a}{0,1,0} >, < \frac{b}{0,1,,0} >, < \frac{c}{0,1,,0} >\} \notin S$. Therefore, (X_N, τ) is $FN_2\, T_2$ -space .

3. If $S = \{< \frac{a}{1,0.9,0} >, < \frac{b}{0,0,1} >, < \frac{c}{0,0,1} >\}$, and

   $M = \{< \frac{a}{0,0,1} >, < \frac{b}{0,0,1} >, < \frac{c}{0,0,1} >\}$

$S \vee M = \{< \frac{a}{1,0.9,1} >, < \frac{b}{0,0,1} >, < \frac{c}{0,0,1} >\}$

So, $S \wedge M = < 0,0,1 >$ and ,$\tau^1 = \{0_N, 1_N\} \vee \{S_1\} \vee \{M_1\}$.

$\tau^1 = \{0_N, 1_N\} \vee \{1,0.9,1\} \vee \{0,0,1\}$

$\tau^1 = \{0_N, 1_N, \{1,0.9,1\}, \{0,0,0\}\}$,

$\tau^2 = \{0_N, 1_N\} \vee \{S_2\} \vee \{M_2\}$

$\tau^2 = \{0_N, 1_N\} \vee \{0,0,1\} \vee \{0,0,1\}$

$\tau^2 = \{0_N, 1_N, \{0,0,1\}, \{0,0,1\}\}$.

If we put, $x_{N_3} = \{< \frac{a}{0,0,1} >, < \frac{b}{0,0,1} >, < \frac{c}{1,0.9,1} >\} \neq y_{N_3} = \{< \frac{a}{0,0,1} >, < \frac{b}{0,0,1} >, < \frac{c}{0,0,1} >\} \in X_N$

There is a FNOS in (X_N, τ) say, $x_{N_3} = \{< \frac{a}{0,0,1} >, < \frac{b}{0,0,1} >, < \frac{c}{1,0.9,1} >\} \in S$,

$x_{N_3} = \{< \frac{a}{0,0,1} >, < \frac{b}{0,0,1} >, < \frac{c}{1,0.9,1} >\} \notin M$ and$y_{N_3} = \{< \frac{a}{0,0,1} >, < \frac{b}{0,0,1} >, < \frac{c}{0,0,1} >\} \in M$, $y_{N_3} = \{< \frac{a}{0,0,1} >, < \frac{b}{0,0,1} >, < \frac{c}{0,0,1} >\} \notin S$. Therefore, (X_N, τ) is $FN_3\, T_2$ -space .

Definition 3.18: A FNTS (X_N, τ) is called:

1. FNT_0-space if (X_N, τ) is FN_1T_0-space, FN_2T_0-space and FN_3T_0-space.
2. FNT_1-space if (X_N, τ) is FN_1T_1-space, FN_2T_1-space and FN_3T_1-space.
3. FNT_2-space if (X_N, τ) is FN_2T_0-space, FN_2T_2-space and FN_3T_2-space.

The next theorem gave the relations between the FNT_1-space and the new defined construction $\tau_{0.1}$ and τ^1.

Theorem 3.19: Let (X_N, τ) be a FNTS, then the following are equivalent:

(i)- (X_N, τ) is a FN T_1 -space,

(ii)- $(X_N, \tau_{0.1})$ is a FN T_1 -space,

(iii)- (X_N, τ^1) is a FN T_1 -space.

Proof. (i) $\Longrightarrow$ (ii) Let $x_N, y_N \in X_N$ such that $x_N \neq y_N$ then there exist.

$U_{x_N} = < S_1, S_2, S_3 >$ and $V_{y_N} = < M_1, M_2, M_3 >$, such that

$x_N \in U_{x_N}$, if $x_N \in S_1$ and $y_N \in V_{y_N}$, if $y_N \in M_1$

Since $FU_{x_N} = < S_1, S_2, S_1^c >$ and $FV_{y_N} = < M_1, M_2, M_1^c >$.

Then, $x_N \in S_1$ and $x_N \notin S_1^c$, So $x_N \in FU_{x_N}$, $x_N \notin FV_{y_N} \Longrightarrow$ $x_N \notin M_1$ or $x_N \in M_1^c$.

Now if $x_N \notin M_1$, then $x_N \in M_1^c$.

Therefore, $x_N \notin FV_{y_N}$. If $x_N \in M_1^c$,then $x_N \notin M_1^c$ and since $M_1 \wedge M_1^c = 0_N$.

So, $x_N \in M_1^c$,Thus $x_N \notin FV_{y_N}$.

Similarly; $y_N \in V_{y_N}$ and $x_N \notin FV_{y_N}$. Therefore, $(X_N, \tau_{0.1})$ is a FN T_1 -space .

(ii) $\Longrightarrow$ (iii): Suppose that $x_N, y_N \in X_N$ such that $x_N \neq y_N$, then there exist.

$FU_{x_N} = < S_1, S_2, S_1^c >$ and $FV_{y_N} = < M_1, M_2, M_1^c >$in $\tau_{0.1}$.

Where: $U_{x_N} = < S_1, S_2, S_3 >$ and $V_{y_N} = < M_1, M_2, M_3 >$ in τ.

such that $x_N \in FU_{x_N}, y_N \in FV_{y_N}$, $x_N \notin FV_{y_N}$ and $y_N \notin FU_{x_N}$.

Thus , $x_N \in S_1$and not in M_1 and $y_N$in M_1not in S_1, there (X_N, τ^1) is a FN T_1 -space.

(iii) $\Longrightarrow$ (i) Let $x_N, y_N \in X_N$ such that $x_N \neq y_N$ then, there exist, $x_N \in S_1$ and $x_N \notin M_1$ with $y_N \in M_1$, $y_N \notin S_1$, where $S_1$and M_1 are in τ^1.

Put, $U_{x_N} = < S_1, S_2, S_3 >$ and $V_{y_N} = < M_1, M_2, M_3 >$.

So, $U_{x_N}$and $V_{y_N}$are in τ and satisfy T_1 . Therefore, (X_N, τ) is a FNT_1 -space.

Example 3.20: Let $X_N = \{a,b,c\}$, $\tau = \{0_N, 1_N, S, M, S\vee M, S\wedge M\}$,Then.

$S = \{< \frac{a}{1,0.4,0} >,< \frac{b}{0,0,1} >,< \frac{c}{0,0,1} >\}$,and

$M = \{< \frac{a}{0,0.5,1} >,< \frac{b}{0,0,1} >,< \frac{c}{0,0,1} >\}$

$S\vee M = \{< \frac{a}{1,0.5,1} >,< \frac{b}{0,0,1} >,< \frac{c}{0,0,1} >\}$

$S\wedge M = \{< \frac{a}{0,0.4,0} >,< \frac{b}{0,0,1} >,< \frac{c}{0,0,1} >\}$

If $x_{N_3} = \{< \frac{a}{0,0,1} >,< \frac{b}{0,0,1} >,< \frac{c}{1,0.4,0} >\} \neq y_{N_3} = \{< \frac{a}{0,0,1} >,< \frac{b}{0,0,1} >,< \frac{c}{0,0.5,1} >\} \in X_N$.

For the set S, $\tau_{0.1} = \{< \frac{a}{1,0.4,0} >,< \frac{b}{0,0,1} >,< \frac{c}{0,0,6,1} >\}$.

For the set M, $\tau_{0.1} = \{< \frac{a}{0,0.5,1} >,< \frac{b}{0,0,1} >,< \frac{c}{1,0.5,0} >\}$.

Then, for τ we have $\tau_{0.1} = \{0_N, 1_N, < \frac{a}{1,0.4,0} >, < \frac{b}{0,0,1} >, < \frac{c}{0,0,6,1} >, < \frac{a}{0,0.5,1} >, < \frac{b}{0,0,1} >, < \frac{c}{1,0.5,0} >\}$and

$\tau^1 = \{0_N,1_N\} \vee \{S_1\} \vee \{M_1\} = \{0_N,1_N\} \vee \{1,0.4,0\} \vee \{0,0.5,1\}$

That is $\tau^1 = \{0_N,1_N, \{1,0.4,0\}, \{0,0.5,1\}\}$.

Then, There is a FNOS in X_N say, $x_{N_3} = \{< \frac{a}{0,0,1} >,< \frac{b}{0,0,1} >,< \frac{c}{1,0.4,0} >\} \in S$, $x_{N_3} = \{< \frac{a}{0,0,1} >,< \frac{b}{0,0,1} >, < \frac{c}{1,0.4,0} >\} \notin M$ and $y_{N_3} = \{< \frac{a}{0,0,1} >, < \frac{b}{0,0,1} >, < \frac{c}{0,0.5,1} >\} \in M$, $y_{N_3} = \{< \frac{a}{0,0,1} >, < \frac{b}{0,0,1} >,< \frac{c}{0,0.5,1} >\} \notin S$. Therefore, (X_N, τ) is $FN_3\,T_2$ -space .

The next theorem gave the relations between the FNT$_1$-space and the new defined construction $\tau_{0.2}$ and τ^2.

Theorem 3.21: Let (X_N, τ) be a FN T_1 -space, then:

(i)- $(X_N, \tau_{0.2})$ is a FN T_1–space, (ii)- (X_N, τ^2) is a FN T_1 –space.

Proof. (i) Let (X_N, τ) be a FNT_1 -space and Let x_N, y_N be any elements in X_N such that $x_N \neq y_N$ then there exists $U_{x_N} =< S_1, S_2, S_3 >$and $V_{y_N} =< M_1, M_2, M_3 >$, such that $x_N \in U_{x_N}, y_N \in V_{y_N}$, $x_N \notin V_{y_N}$ and $y_N \notin U_{x_N}$.

Thus $x_N \in U_{x_N}$if $x_N \in S_1$, $x_N \notin S_3$ and $y_N \in M_1$, $y_N \notin M_3$.

Also, $x_N \notin M_1$, $y_N \notin S_1$.

Since $SU_{x_N} =< S_3^c, S_2, S_3 >$ and $SV_{y_N} =< M_3^c, M_2, M_3 >$. Then $x_N \notin S_3$, so $x_N \in S_3^c$ and $y_N \in M_3^c$.

Thus $x_N \in SU_{x_N}$ and $y_N \in SV_{y_N}$.

Similarly, we can show $x_N \notin SV_{y_N}$ and $y_N \notin SU_{x_N}$.

Therefore, $(X_N, \tau_{0.2})$ is a FN T_1 -space.

(ii) Suppose that $x_N, y_N \in X_N$ such that $x_N \neq y_N$ then, there exists $SU_{x_N} =< S_3^c, S_2, S_3 >$ and $SV_{y_N} =< M_3^c, M_2, M_3 >$ in $\tau_{0.2}$. So, there exist $U_{x_N} =< S_1, S_2, S_3 >$and $V_{y_N} =< M_1, M_2, M_3 >$ in τ such that $x_N \in SU_{x_N}$, $y_N \in SV_{y_N}$, $x_N \notin SV_{y_N}$ and $y_N \notin SU_{x_N}$.

Thus $x_N \in S_3$ and not in M_3 and $y_N \in M_3$ not in S_3.

Therefore, (X_N, τ^2) is a FN T_1-space .

Remark 3.22: The converse of Theorem 3.21 is not true in general .The following examples show these cases .

Example 3.23: Let $X_N = \{a, b, c\}$,and Let $\tau = \{0_N, 1_N, S, M, S \wedge M, S \vee M\}$,where

$S = \{< (\frac{a}{1,0.7,0}),(\frac{b}{1,0.5,0}),(\frac{c}{1,0.8,0}) >\}$, and $M = \{< (\frac{a}{1,0.7,0}),(\frac{b}{1,0,0}),(\frac{c}{1,0.4,0}) >\}$,

$S \wedge M = \{< (\frac{a}{1,0.7,0}), (\frac{b}{1,0,0}),(\frac{c}{1,0.8,0}) >\}$, and $S \vee M = \{< (\frac{a}{1,0.7,0}),(\frac{b}{1,0.5,0}),(\frac{c}{1,0.4,0}) >\}$

1. For the set S, $\tau_{0.2} = \{<> S: S \in \tau\}$, where $<> S =< x, 1 - V_S(x), \sigma_S(x), V_S(x) >$

$$=< (\frac{a}{0,0.2,1}), (\frac{b}{1,0.5,0}), (\frac{c}{1,0.8,0}) >, \text{ and}$$

2. For the set M, $\tau_{0.2} = \{<> M: M \in \tau\}$, where $<> M =< x, 1 - V_M(x), \sigma_M(x), V_M(x) >$

$$=< (\frac{a}{0,0.6,1}),(\frac{b}{1,0,0}),(\frac{c}{1,0.4,0}) >$$

Then, for τ we have $\tau_{0.2} = \{0_N, 1_N, (\frac{a}{0,0.2,1}), (\frac{b}{1,0.5,0}), (\frac{c}{1,0.8,0}), (\frac{a}{0,0.6,1}), (\frac{b}{1,0,0}), (\frac{c}{1,0.4,0})\}$.

Also, $\tau^2 = \{0_N, 1_N\} \vee \{S_2\} \vee \{M_2\}$

$= \{0_N, 1_N\} \vee \{(\frac{b}{1,0.5,0})\} \vee \{(\frac{b}{1,0,0})\}$.

Remark 3.24: For the FNTS (X_N, τ) we have:

1. Every FNT$_2$-space is FNT$_1$-space.
2. Every FNT$_1$-space is FNT$_0$-space.

The proof is directly from definitions 3.9, 3.11 and 3.13.

The converse of Remark 3.24 is not true as it shown in the next example.

Example 3.25: Let $X_N = \{a,b,c\}$, $\tau = \{0_N, 1_N, S\}$,

1. If $S = \{< \frac{a}{0.8,0,0} >, < \frac{b}{0,0,0} >, < \frac{c}{0,0,0} >\}$.

So, $x_{N_1} = \{< \frac{a}{0.8,0,0} >, < \frac{b}{0,0,0} >, < \frac{c}{0,0,0} >\} \neq y_{N_1} = \{< \frac{a}{0.7,0,0} >, < \frac{b}{0,0,0} >, < \frac{c}{0,0,0} >\} \in X_N$.

There is a FNOS in (X_N, τ) say $x_{N_1} = \{< \frac{a}{0.8,0,0} >, < \frac{b}{0,0,0} >, < \frac{c}{0,0,0} >\} \in S$ but $y_{N_1} = \{< \frac{a}{0.7,0,0} >, < \frac{b}{0,0,0} >, < \frac{c}{0,0,0} >\} \notin S$.

Therefore, (X_N, τ) is $FN_1\,T_0$ –space but is not $FN_1\,T_1$ –space and not FN_1T_2 –space.

2. If $S = \{< \frac{a}{1,1,0} >, < \frac{b}{0,1,0} >, < \frac{c}{0,1,0} >\}$.

So, $x_{N_2} = \{< \frac{a}{1,1,0} >, < \frac{b}{1,0,0} >, < \frac{c}{0,1,0} >\} \neq y_{N_2} = \{< \frac{a}{0,1,0} >, < \frac{b}{0,1,0} >, < \frac{c}{0,1,0} >\} \in X_N$. There is a FNOS in X_N say $x_{N_2} = \{< \frac{a}{1,1,0} >, < \frac{b}{1,0,0} >, < \frac{c}{0,1,0} >\} \in S$ but, $y_{N_2} = \{< \frac{a}{0,1,0} >, < \frac{b}{0,1,0} >, < \frac{c}{0,1,0} >\} \notin S$.

Therefore, (X_N, τ) is $FN_2\,T_0$ -space but, is not $FN_2\,T_1$ –space and not FN_2T_2 –space.

3. If $S=\{<\frac{a}{1,0.9,0}>,<\frac{b}{0,0,1}>,<\frac{c}{0,0,1}>\}$.

So, $x_{N_3}=\{<\frac{a}{0,0,1}>,<\frac{b}{0,0,1}>,<\frac{c}{0,0.9,1},>\}\neq y_{N_3}=\{<\frac{a}{0,0,1}>,<\frac{b}{0,0,1}>,<\frac{c}{0.6,1,0}>\}\in X_N$.

There is a FNOTS in (X_N,τ) say, $x_{N_3}=\{<\frac{a}{0,0,1}>,<\frac{b}{0,0,1}>,<\frac{c}{0,0.9,1},>\}\in S$ but, $y_{N_3}=\{<\frac{a}{0,0,1}>,<\frac{b}{0,0,1}>,<\frac{c}{0.6,1,0}>\}\notin S$.

Therefore, (X_N,τ) is $FN_3\ T_0$ -space but is not $FN_3\ T_1$ -space and not $FN_3\ T_2$ -space.

Example 3.26: Let $X_N=\{a,b,c\}$, $\tau=\{0_N,1_N,S,M,S\wedge M,S\vee M\}$,Then.

1. If $S=\{<\frac{a}{0.5,0,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,0,0}>\}$and

   $M=\{<\frac{a}{0.3,0,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,0,0}>\}$.

   So, $x_{N_1}=\{<\frac{a}{0.5,0,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,0,0}>\}\neq y_{N_1}=\{<\frac{a}{0,0.3,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,0,0}>\}\in X_N$.

There is a FNOS in (X_N,τ) say $x_{N_1}=\{<\frac{a}{0.5,0,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,0,0}>\}\in S$, $x_{N_1}=\{<\frac{a}{0.5,0,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,0,0}>\}\notin M$ and $y_{N_1}=\{<\frac{a}{0,0.3,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,0,0}>\}\in M$, $y_{N_1}=\{<\frac{a}{0,0.3,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,0,0}>\}\notin S$. Therefore, (X_N,τ) is $FN_1\ T_1$ –space but is not $FN_1\ T_2$ -space.

2. If $S=\{<\frac{a}{1,1,0}>,<\frac{b}{0,1,0}>,<\frac{c}{0,1,0}>\}$, and

   $M=\{<\frac{a}{0,0,0}>,<\frac{b}{0,1,0}>,<\frac{c}{0,1,0}>\}$

So, $x_{N_2}=\{<\frac{a}{0,1,0}>,<\frac{b}{1,1,0}>,<\frac{c}{0,1,0}>\}\neq y_{N_2}=\{<\frac{a}{0,1,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,1,0}>\}\in X_N$

There is a FNOS in (X_N,τ) say $x_{N_2}=\{<\frac{a}{0,1,0}>,<\frac{b}{1,1,0}>,<\frac{c}{0,1,0}>\}\in S$, $x_{N_2}=\{<\frac{a}{0,1,0}>,<\frac{b}{1,1,0}>,<\frac{c}{0,1,0}>\}\notin M$ .and $y_{N_2}=\{<\frac{a}{0,1,0}>,<\frac{b}{0,0,0}>,<\frac{c}{0,1,0}>\}$

$\in$ M,$y_{N_2} = \{< \frac{a}{0,1,0} >,< \frac{b}{0,0,0} >,< \frac{c}{0,1,0} >\} \notin$ S.

Therefore, (X_N, τ) is $FN_2\, T_1$ -space but is not $FN_2\, T_2$ -space.

3. If $S = \{< \frac{a}{1,0.2,0} >,< \frac{b}{0,0,1} >,< \frac{c}{0,0,1} >\}$ and

$M = \{< \frac{a}{0,0.7,1} >, < \frac{b}{0,0,1} >,< \frac{c}{0,0,1} >\}$

$S \wedge M = \{< \frac{a}{0,0.2,0} >,< \frac{b}{0,0,1} >,< \frac{c}{0,0,1} >\}$

$S \vee M = \{< \frac{a}{1,0.7,1} >,< \frac{b}{0,0,1} >,< \frac{c}{0,0,1} >\}$.

So, $x_{N_3} = \{< \frac{a}{0,0,1} >,<, \frac{b}{0,0,1} >,< \frac{c}{0,0.2,1} >\} \neq y_{N_3} = \{< \frac{a}{0,0,1} >,<, \frac{b}{0,0,1} >,< \frac{c}{0,0.7,1} >\} \in X_N$

There is a FNOS in (X_N, τ) say, $x_{N_3} = \{< \frac{a}{0,0,1} >,<, \frac{b}{0,0,1} >,< \frac{c}{0,0.2,1} >\} \in S, x_{N_3} = \{< \frac{a}{0,0,1} >,<, \frac{b}{0,0,1} >,< \frac{c}{0,0.2,1} >\} \notin M$ and $y_{N_3} = \{< \frac{a}{0,0,1} >,<, \frac{b}{0,0,1} >,< \frac{c}{0,0.7,1} >\} \in M$, $y_{N_3} = \{< \frac{a}{0,0,1} >,<, \frac{b}{0,0,1} >,< \frac{c}{0,0.7,1} >\} \notin S$. Therefore, (X_N, τ) is $FN_3\, T_1$ -space but is not $FN_3\, T_2$.

4. Conclusions

In this research, the new type of fuzzy neutrosophic separation axioms has been defined in the fuzzy neutrosophic topological spaces by several new types of points and new constructions was studied. And many useful examples are presented to clear the newconcepts introduced. Also, proof some new theorems and characterizations relations among the new concepts and the other type are going to be found.

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