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Simplified intuitionistic neutrosophic hypersoft TOPSIS method based on correlation coefficient

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Abstract. In psychology and sociology fields, the information is either dependent or independent and sometimes requires combinations of both. The existing structures like the fuzzy set, intuitionistic fuzzy set, and Pythagorean set have limitations when the information requires combinations of dependent and independent components. To overcome these limitations, we introduce the concept of a simplified intuitionistic neutrosophic hypersoft set. We present some properties of the correlation coefficient, weighted correlation coefficient, and aggregation operators on a simplified intuitionistic hypersoft set. Finally, we develop an algorithm and illustrate with a case study for identifying the leader; who can bring changes to society in the socio-political context.

Keywords: neutrosophic set; intuitionistic set; soft set; hypersoft set.

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1. **Introduction**

Zadeh [32] defined the concept of fuzzy set (FS). The membership value of each element in FS is specified by a real number from the closed interval of [0,1]. Atanassov [5] proposed the notion of an intuitionistic fuzzy set (IFS), an extension of FS. In IFS, the elements possess both membership and non-membership values such that their sum does not exceed unity. Smarandache [22] presented the concept of neutrosophic set (NS), characterized by the values of truth, indeterminacy, and falsity grades for each element of the set. Later, Wang et al. [27] proposed the notion of single-valued NS (SVNS) with a restricted condition for the membership values to overcome the constraints faced in NS. Molodtsov [15] introduced the concept of a soft set to deal with uncertainties. Smarandache [24] presented the concept of hypersoft set (HSS) to overcome the restriction faced in the soft set. Smarandache [23] proposed the concept of degree of dependence and the degree of independence between the components of the FS and NS. Also, for the first time, Smarandache [25] presented the concept of neutrosociology. Chinnadurai and Bobin [8], [9] introduced the concepts of the simplified intuitionistic neutrosophic soft set (SINSS) and interval-valued intuitionistic neutrosophic soft set (IVINSS) and studied some of their properties. In SINSS and IVINSS, the membership grades of truth and falsity are dependent on each other such that their sum cannot exceed one and the membership grade of indeterminacy is independent with a value less than or equal to one. Hence, in SINSS and IVINSS the sum of the membership grades cannot exceed two.

Khan et al. [14] introduced a programming language to solve multi-objective multi-product production planning problems. Smarandache [26] extended for the second time the nonstandard analysis by adding the left monad closed to the right, and right monad closed to the left. New theorems, better notations for monads and binads, and examples of nonstandard neutrosophic operations were discussed. Akram et al. [3] introduced the notion of hesitant fuzzy N-soft sets and used it in decision-making problems. Abdel-Basset et al. [1] presented the concept of type -2 neutrosophic numbers and presented a real case study using the technique of order of preference by similarity to ideal solution (TOPSIS). Abdel-Basset et al. [2] combined the neutrosophic analytical network process (ANP) method and the ViseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method for solving supplier chain management problems. Arora and Harish [4] studied the properties of aggregation operators on IFS. Ayele et al. [6] proposed a method for traffic signal control using an interval-valued neutrosophic soft set. Ejegwa et al. [10] used intuitionistic fuzzy correlation measure and programming language in the medical diagnosis field. Harish and Rishu [12] proposed TOPSIS method based on correlation measures on IFS to solve multi-criteria decision-making (MCDM) problems. Jana and Pal [13] presented the concept of aggregation operators on SVNS for solving MCDM problems. Naeem and Riaz [17] introduced Pythagorean fuzzy soft sets and established some of their algebraic properties. Naeem et al. [18] compared TOPSIS, VIKOR, and generalized aggregation operators models and showed that all the three techniques rendered the same optimal choice. Riaz et al. [20] presented an investment strategic decision making problem to illustrate the application of the Pythagorean m -Polar Fuzzy Weighted Aggregation operators and demonstrated its effectiveness. Naeem et al. [19] discussed an application of Pythagorean m-polar fuzzy sets in the decision-making problem for selecting an appropriate mode of advertisement by using the TOPSIS method. Zulqarnain et al. [33] introduced the concept of intuitionistic fuzzy HSS and used the TOPSIS method based on correlation coefficient (CC). Zulqarnain et al. [34] studied the fundamental operations of interval-valued neutrosophic HSS. Muhammad et al. [16] defined aggregation operators on neutrosophic HSS and studied some properties. Saqlain et al. [21] presented the concepts of single neutrosophic HSS and multi-valued neutrosophic HSS. They used tangent similarity measures to solve MCDM problems.

The main aim of the present study is to rank the alternatives of simplified intuitionistic neutrosophic hypersoft sets (SINHSS) by using aggregation operators and also by using the TOPSIS method based on CC. To the best of our knowledge, research on SINHSS is confined to its theory and related development and applications. Therefore, we examine and provide a suitable solution to the decision-makers in ranking the alternatives. We present a MCDM approach based on TOPSIS, and the effectiveness of this method is demonstrated through the selection of a leader who influences society in a socio-political context. To prove the efficacy of the proposed method, a comparative analysis between the proposed and existing method is illustrated with examples. Thus, the SINHSS is a robust tool to predict uncertainties when the membership grades of truth and falsity are dependent on each other.

The manuscript consists of the following sections. Section 2 briefs on existing definitions. Section 3, 4 and 5 introduces the concept of SINHSS and discusses some properties of CC and weighted CC of SINHSS. Section 6 deals with the simplified intuitionistic neutrosophic hypersoft weighted average operator(SINHSWAO) and simplified intuitionistic neutrosophic hypersoft weighted geometric operator (SINHSWGO). Section 7 highlights the combination of CC with the TOPSIS method. Section 8 shows the significance of the proposed method with comparative analysis. Section 9 ends with a conclusion.

2. **Preliminaries**

We present some of the basic definitions required for this study. Let us consider the following notations throughout this study unless otherwise specified. Let $\mathcal{V}$ be the universe and $v \in \mathcal{V}$, $P(\mathcal{V})$ be the power set of $\mathcal{V}$, $\mathbb{N}$ represents natural numbers, and $\mathcal{S}^U$ represent the collection of simplified intuitionistic neutrosophic sets (SINS) over $\mathcal{V}$.

Definition 2.1. [8] A SINS in $\mathcal{V}$ is of the form $\Omega = \{\langle v, \mathcal{T}_\Omega(v), \mathcal{I}_\Omega(v), \mathcal{F}_\Omega(v)\rangle\}$, where $\mathcal{T}_\Omega(v), \mathcal{I}_\Omega(v), \mathcal{F}_\Omega(v) : \mathcal{V} \to [0,1]$, are the membership values of truth, indeterminacy and falsity of the element $v \in \mathcal{V}$ respectively, such that $0 \leq \mathcal{T}_\Omega(v) + \mathcal{F}_\Omega(v) \leq 1$ and $0 \leq \mathcal{T}_\Omega(v) + \mathcal{I}_\Omega(v) + \mathcal{F}_\Omega(v) \leq 2$.

Definition 2.2. [24] Let $\Delta_1, \Delta_2, ..., \Delta_k$, be distinct attribute sets, whose corresponding sub-attributes are $\Delta_1 = \{\lambda_{11}, \lambda_{12}, ..., \lambda_{1f}\}, \Delta_2 = \{\lambda_{21}, \lambda_{22}, ..., \lambda_{2g}\}, ..., \Delta_k = \{\lambda_{k1}, \lambda_{k2}, ..., \lambda_{kh}\}$, where $1 \leq f \leq p$, $1 \leq g \leq q$, $1 \leq h \leq r$ and $p, q, r \in \mathbb{N}$, such that $\Delta_i \cap \Delta_j = \emptyset$, for each $i, j \in \{1, 2, ..., k\}$ and $i \neq j$. Then the Cartesian product of the distinct attribute sets $\Delta_1 \times \Delta_2 \times ... \times \Delta_k = \tilde{\Delta} = \{\lambda_{1f} \times \lambda_{2g} \times ... \times \lambda_{kh}\}$, represent a collection of multi- attributes. A pair $(\Omega, \tilde{\Delta})$ is called a hypersoft set (HSS) over $\mathcal{V}$, where $\Omega : \tilde{\Delta} \to P(\mathcal{V})$. HSS can be represented as $(\Omega, \tilde{\Delta}) = \left\{(\tilde{\lambda}, \Omega(\tilde{\lambda})) | \tilde{\lambda} \in \tilde{\Delta}, \Omega(\tilde{\lambda}) \in P(\mathcal{V})\right\}$.

3. Simplified intuitionistic neutrosophic hypersoft set

We present the notion of simplified intuitionistic neutrosophic hypersoft set (SINHSS). Also, we discuss some basic properties of correlation coefficient (CC) and weighted CC (WCC) on SINHSS.

Definition 3.1. A pair $(\Omega, \tilde{\Delta})$ is called a SINHSS over $\mathcal{V}$, where $\Omega : \tilde{\Delta} \to \mathcal{S}^U$. SINHSS can be represented as $(\Omega, \tilde{\Delta}) = \left\{(\tilde{\lambda}, \Omega(\tilde{\lambda}))|\tilde{\lambda} \in \tilde{\Delta}, \Omega(\tilde{\lambda}) \in \mathcal{S}^U \in [0,1]\right\}$, where $\Omega(\tilde{\lambda}) = \left\{\left\langle v, \mathcal{T}_{\Omega(\tilde{\lambda})}(v), \mathcal{I}_{\Omega(\tilde{\lambda})}(v), \mathcal{F}_{\Omega(\tilde{\lambda})}(v)\right\rangle |v \in \mathcal{V}\right\}$, $\mathcal{T}_{\Omega(\tilde{\lambda})}(v)$, $\mathcal{I}_{\Omega(\tilde{\lambda})}(v)$ and $\mathcal{F}_{\Omega(\tilde{\lambda})}(v)$ represent the membership values of truth, indeterminacy and falsity, such that $0 \leq \mathcal{T}_{\Omega(\tilde{\lambda})}(v) + \mathcal{F}_{\Omega(\tilde{\lambda})}(v) \leq 1$ and $0 \leq \mathcal{T}_{\Omega(\tilde{\lambda})}(v) + \mathcal{I}_{\Omega(\tilde{\lambda})}(v) + \mathcal{F}_{\Omega(\tilde{\lambda})}(v) \leq 2$.

Example 3.2. Let $\mathcal{V} = \{v_1, v_2, v_3\}$ be a set of sociologists responsible to evaluate a leader, the role of the leader is to bring socio-political changes to society. Let Δ_1, Δ_2 and Δ_3 be distinct attribute sets whose corresponding sub-attributes are represented as Δ_1 = leader attributes = $\{\lambda_{11} = \text{personality variables}, \lambda_{12} = \text{cognitive ability and skills}, \lambda_{13} = \text{sense making}\}$, Δ_2 = leader behavior = $\{\lambda_{21} = \text{setting sub culture}, \lambda_{22} = \text{conflict management}\}$, Δ_3 = group behaviors = $\{\lambda_{31} = \text{living the sub culture}\}$. Then $\tilde{\Delta} = \Delta_1 \times \Delta_2 \times \Delta_3$ be distinct attribute sets, such as

$$\tilde{\Delta} = \Delta_1 \times \Delta_2 \times \Delta_3 = \{\lambda_{11}, \lambda_{12}, \lambda_{13}\} \times \{\lambda_{21}, \lambda_{22}\} \times \{\lambda_{31}\}.$$
$$= \left\{(\lambda_{11}, \lambda_{21}, \lambda_{31}), (\lambda_{11}, \lambda_{22}, \lambda_{31}), (\lambda_{12}, \lambda_{21}, \lambda_{31}), (\lambda_{12}, \lambda_{22}, \lambda_{31}), (\lambda_{13}, \lambda_{21}, \lambda_{31}), (\lambda_{13}, \lambda_{22}, \lambda_{31})\right\}.$$
$$= \left\{\tilde{\lambda_1}, \tilde{\lambda_2}, \tilde{\lambda_3}, \tilde{\lambda_4}, \tilde{\lambda_5}, \tilde{\lambda_6}\right\}.$$

A SINHSS $(\Omega, \tilde{\Delta})$ is a collection of subsets of $\mathcal{V}$, given by the sociologists for a leader based on the description in Table 1.

TABLE 1. Shows leadership skills of a leader in SINHSS $(\Omega, \tilde{\Delta})$ form.

$\mathcal{V}$	$\tilde{\lambda_1}$	$\tilde{\lambda_2}$	$\tilde{\lambda_3}$	$\tilde{\lambda_4}$	$\tilde{\lambda_5}$	$\tilde{\lambda_6}$
v_1	$\langle 0.4, 0.9, 0.5\rangle$	$\langle 0.2, 0.5, 0.7\rangle$	$\langle 0.8, 0.9, 0.1\rangle$	$\langle 0.7, 0.9, 0.2\rangle$	$\langle 0.1, 0.4, 0.3\rangle$	$\langle 0.9, 0.9, 0.1\rangle$
v_2	$\langle 0.2, 0.8, 0.5\rangle$	$\langle 0.6, 0.2, 0.4\rangle$	$\langle 0.7, 0.4, 0.2\rangle$	$\langle 0.4, 0.5, 0.4\rangle$	$\langle 0.3, 0.4, 0.2\rangle$	$\langle 0.5, 0.2, 0.1\rangle$
v_3	$\langle 0.4, 0.4, 0.4\rangle$	$\langle 0.3, 0.3, 0.3\rangle$	$\langle 0.2, 0.1, 0.7\rangle$	$\langle 0.5, 0.5, 0.5\rangle$	$\langle 0.4, 1.0, 0.6\rangle$	$\langle 0.4, 0.8, 0.4\rangle$

4. Correlation coefficient for SINHSS

Let $(\Omega_1, \tilde{\Delta}_1) = \left\{(v_i, \mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_i), \mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_i), \mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_i))|v_i \in \mathcal{V}\right\}$ and $(\Omega_2, \tilde{\Delta}_2) = \left\{(v_i, \mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_i), \mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_i), \mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_i))|v_i \in \mathcal{V}\right\}$ be two SINHSS over $\mathcal{V}$.

Definition 4.1. Let $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ be two SINHSS. Then the simplified intuitionistic neutrosophic informational energies of $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ are represented as

$$\Phi(\Omega_1, \tilde{\Delta}_1) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left[(\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\right] \quad (1)$$

$$\Phi(\Omega_2, \tilde{\Delta}_2) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left[(\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\right]. \quad (2)$$

Definition 4.2. Let $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ be two SINHSS. Then the correlation measure between $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ is defined as

$$\mathcal{C}_{\mathcal{M}}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = \sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) + (\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) + (\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_i))\Big]. \quad (3)$$

Proposition 4.3. *Let* $(\Omega_1, \tilde{\Delta}_1)$ *and* $(\Omega_2, \tilde{\Delta}_2)$ *be two SINHSS. Then,*
(i) $\mathcal{C}_{\mathcal{M}}((\Omega_1, \tilde{\Delta}_1), (\Omega_1, \tilde{\Delta}_1)) = \Phi(\Omega_1, \tilde{\Delta}_1)$
(ii) $\mathcal{C}_{\mathcal{M}}((\Omega_2, \tilde{\Delta}_2), (\Omega_2, \tilde{\Delta}_2)) = \Phi(\Omega_2, \tilde{\Delta}_2)$.

Proof. Straight forward □

Definition 4.4. Let $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ be two SINHSS. Then, the CC between $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ is given as

$$\mathcal{C}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = \frac{\mathcal{C}_{\mathcal{M}}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2))}{\sqrt{\Phi(\Omega_1, \tilde{\Delta}_1)}\sqrt{\Phi(\Omega_2, \tilde{\Delta}_2)}} \quad (4)$$

Example 4.5. Let the values of $(\Omega_1, \tilde{\Delta}_1)$ be as in Table 1 and the values of $(\Omega_2, \tilde{\Delta}_2)$ be as in Table 2.

TABLE 2. Shows leadership skills of a leader in SINHSS $(\Omega_2, \tilde{\Delta}_2)$ form.

$\mathcal{V}$	$\tilde{\lambda_1}$	$\tilde{\lambda_2}$	$\tilde{\lambda_3}$	$\tilde{\lambda_4}$	$\tilde{\lambda_5}$	$\tilde{\lambda_6}$
v_1	$\langle 0.2, 0.8, 0.5\rangle$	$\langle 0.6, 0.2, 0.4\rangle$	$\langle 0.7, 0.4, 0.2\rangle$	$\langle 0.4, 0.5, 0.4\rangle$	$\langle 0.3, 0.4, 0.2\rangle$	$\langle 0.5, 0.2, 0.1\rangle$
v_2	$\langle 0.4, 0.4, 0.4\rangle$	$\langle 0.3, 0.3, 0.3\rangle$	$\langle 0.2, 0.1, 0.7\rangle$	$\langle 0.5, 0.5, 0.5\rangle$	$\langle 0.4, 1.0, 0.6\rangle$	$\langle 0.4, 0.8, 0.4\rangle$
v_3	$\langle 0.4, 0.9, 0.5\rangle$	$\langle 0.2, 0.5, 0.7\rangle$	$\langle 0.8, 0.9, 0.1\rangle$	$\langle 0.7, 0.9, 0.2\rangle$	$\langle 0.1, 0.4, 0.3\rangle$	$\langle 0.9, 0.9, 0.1\rangle$

Then, $\mathcal{C}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = 0.7738$.

Proposition 4.6. *Let* $(\Omega_1, \tilde{\Delta}_1)$ *and* $(\Omega_2, \tilde{\Delta}_2)$ *be two SINHSS. Then, the following CC properties hold:*
(i) $0 \leq \mathcal{C}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) \leq 1$*;*

(ii) $\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = \mathcal{C}_C((\Omega_2,\tilde{\Delta}_2),(\Omega_1,\tilde{\Delta}_1))$;
(iii) If $(\Omega_1,\tilde{\Delta}_1) = (\Omega_2,\tilde{\Delta}_2)$, *then* $\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = 1$.

Proof. (i) Obviously, $\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \geq 0$. Now, we present the proof of $\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \leq 1$.

$$\mathcal{C}_{\mathcal{M}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))$$
$$= \sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) + (\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) + (\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_i))\Big].$$
$$= \sum_{k=1}^{m}\Big[\Big((\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_1)) + (\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_1)) + (\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_1))\Big)$$
$$+\Big((\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_2)) + (\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_2)) + (\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_2))\Big) + \ldots$$
$$+\Big((\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_n)) + (\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_n)) + (\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_n))\Big)\Big].$$

By applying Cauchy-Schwarz inequality, we get

$$\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))^2$$
$$\leq \sum_{k=1}^{m}\Big[\Big\{(\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 + (\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2 + \ldots + (\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2\Big\} + \Big\{(\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 + (\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2 +$$
$$\ldots + (\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2\Big\}\Big\{(\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 + (\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2 + \ldots + (\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2\Big\}\Big] \times$$
$$\sum_{k=1}^{m}\Big[\Big\{(\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2 + (\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 + \ldots + (\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2\Big\} + \Big\{(\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2 + (\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 +$$
$$\ldots + (\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2\Big\}\Big\{(\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2 + (\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 + \ldots + (\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2\Big\}\Big].$$

$$\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))^2$$
$$\leq \sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\Big] \times \sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 +$$
$$(\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big].$$

$\Rightarrow \mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))^2 \leq \Phi(\Omega_1,\tilde{\Delta}_1) \times \Phi(\Omega_2,\tilde{\Delta}_2)$.
$\Rightarrow \mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \leq \sqrt{\Phi(\Omega_1,\tilde{\Delta}_1)} \times \sqrt{\Phi(\Omega_2,\tilde{\Delta}_2)}$.
$\Rightarrow \frac{\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))}{\sqrt{\Phi(\Omega_1,\tilde{\Delta}_1)} \times \sqrt{\Phi(\Omega_2,\tilde{\Delta}_2)}} \leq 1$.
By using Definition 4.4, we get $\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \leq 1$.
Hence, $0 \leq \mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \leq 1$. □

Proof. (ii) Straight forward. □

Proof. (iii) $\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = \frac{\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))}{\sqrt{\Phi(\Omega_1,\tilde{\Delta}_1)}\times\sqrt{\Phi(\Omega_2,\tilde{\Delta}_2)}}$.
Since, $(\Omega_1,\tilde{\Delta}_1) = (\Omega_2,\tilde{\Delta}_2)$.
$\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))$

$$= \frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\left[(\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\right]}{\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left[(\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\right]} \times \sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left[(\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\right]}}.$$

$\Rightarrow \mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = 1.$ □

Definition 4.7. Let $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ be two SINHSS. Then, the CC between $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2\tilde{\Delta}_2)$ is defined as

$$\tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = \frac{\mathcal{C}_{\mathcal{M}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))}{\max\left\{\Phi(\Omega_1,\tilde{\Delta}_1),\Phi(\Omega_2,\tilde{\Delta}_2)\right\}}. \tag{5}$$

$\tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))$

$$= \frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\left[(\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_i))+(\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_i))+(\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_i))\right]}{\max\left\{\sum_{k=1}^{m}\sum_{i=1}^{n}\left[(\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\right], \sum_{k=1}^{m}\sum_{i=1}^{n}\left[(\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\right]\right\}}.$$

Proposition 4.8. *Let $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ be two SINHSS. Then, the following CC properties hold:*
(i) $0 \le \tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \le 1$;
(ii) $\tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = \tilde{\mathcal{C}_C}((\Omega_2,\tilde{\Delta}_2),(\Omega_1,\tilde{\Delta}_1))$;
(iii) If $(\Omega_1,\tilde{\Delta}_1) = (\Omega_2,\tilde{\Delta}_2)$, *then* $\tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = 1$.

Proof. (i) Obviously, $\tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \geq 0$. Now, we present the proof of $\tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \leq 1$. $\mathcal{C}_{\mathcal{M}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))$

$$= \sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_i))+(\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) + (\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_i))\Big].$$

$$=\sum_{k=1}^{m}\Bigg[\Big((\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_1))*(\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_1))+(\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_1))*(\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_1))+(\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_1))*(\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_1))\Big)$$
$$+\Big((\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_2))*(\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_2))+(\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_2))*(\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_2))+(\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_2))*(\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_2))\Big)+...$$
$$+\Big((\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_n))*(\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_n))+(\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_n))*(\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_n))+(\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_n))*(\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_n))\Big)\Bigg].$$

By applying Cauchy-Schwarz inequality, we get
$\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))$

$$\leq \Bigg\{\sum_{k=1}^{m}\Bigg[\Big\{(\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2+(\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2+...+(\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2\Big\}+\Big\{(\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2+(\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2+$$
$$...+(\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2\Big\}\Big\{(\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2+(\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2+...+(\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2\Big\}\Bigg]\times$$
$$\sum_{k=1}^{m}\Bigg[\Big\{(\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2+(\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2+...+(\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2\Big\}+\Big\{(\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2+(\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2+$$
$$...+(\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2\Big\}\Big\{(\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2+(\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2+...+(\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2\Big\}\Bigg]\Bigg\}^{\frac{1}{2}}.$$

$\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))$

$$\leq \Bigg\{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\Big]\times\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+ (\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]\Bigg\}^{\frac{1}{2}}.$$

$$\leq \Bigg\{\Bigg(\max\Bigg\{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\Big]\times\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+ (\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]\Bigg\}\Bigg)^2\Bigg\}^{\frac{1}{2}}.$$

$$= \max\Bigg\{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\Big]\times\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+ (\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]\Bigg\}.$$

$\Rightarrow \mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \leq \max\Big\{\Phi(\Omega_1,\tilde{\Delta}_1)\times\Phi(\Omega_2,\tilde{\Delta}_2)\Big\}.$

$\Rightarrow \frac{\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))}{\max\Big\{\Phi(\Omega_1,\tilde{\Delta}_1)\times\Phi(\Omega_2,\tilde{\Delta}_2)\Big\}} \leq 1.$

By using Definition 4.7, we get $\tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \leq 1.$

Hence, $0 \leq \tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \leq 1.$

Proofs of (ii) and (iii) are same as in Proposition 4.6. □

5. **Weighted correlation coefficient for SINHSS**

We present the concept of weighted correlation coefficient (WCC) for SINHSS. WCC facilitates decision-makers (DMs) to provide different weights for each alternative. Consider $\mathcal{D} = \{\mathcal{D}_1,\mathcal{D}_2,...,\mathcal{D}_m\}$ and $\mathcal{W} = \{\mathcal{W}_1,\mathcal{W}_2,...,\mathcal{W}_n\}$ as weight vectors for alternatives and experts, respectively, such that $\mathcal{D}_k,\mathcal{W}_i > 0$ and $\sum\limits_{k=1}^{m}\mathcal{D}_k = 1, \sum\limits_{i=1}^{n}\mathcal{W}_i = 1.$

Definition 5.1. Let $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ be two SINHSS. Then, the WCC between $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ is defined as

$$\mathcal{C}_{C_{\mathcal{W}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = \frac{\mathcal{C}_{\mathcal{M}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))}{\sqrt{\Phi(\Omega_1,\tilde{\Delta}_1)}\sqrt{\Phi(\Omega_2,\tilde{\Delta}_2)}} \tag{6}$$

$$\mathcal{C}_{C_{\mathcal{W}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = \frac{\sum\limits_{k=1}^{m}\mathcal{D}_k\Bigg(\sum\limits_{i=1}^{n}\mathcal{W}_i\Big[\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_i)*\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_i)+\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_i)*\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_i)+\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_i)*\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_i)\Big]\Bigg)}{\sqrt{\sum\limits_{k=1}^{m}\mathcal{D}_k\Bigg(\sum\limits_{i=1}^{n}\mathcal{W}_i\Big[(\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\Big]\Bigg)} \times \sqrt{\sum\limits_{k=1}^{m}\mathcal{D}_k\Bigg(\sum\limits_{i=1}^{n}\mathcal{W}_i\Big[(\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]\Bigg)}}.$$

If $\mathcal{D} = \left\{\frac{1}{m},\frac{1}{m},...,\frac{1}{m},\right\}$ and $\mathcal{W} = \left\{\frac{1}{n},\frac{1}{n},...,\frac{1}{n},\right\}$, then WCC given in Eq.(6) reduces to CC as in Eq.(4).

Example 5.2. Let the values of $(\Omega_1,\tilde{\Delta}_1)$ be as in Table 1 and the values of $(\Omega_2,\tilde{\Delta}_2)$ be as in Table 2.

Then, $\mathcal{C}_{C_{\mathcal{W}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = 0.7903.$

Proposition 5.3. *Let $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ be two SINHSS. Then, the following WCC properties hold:*

(i) $0 \leq \mathcal{C}_{C_{\mathcal{W}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \leq 1$*;*

(ii) $\mathcal{C}_{C_{\mathcal{W}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = \mathcal{C}_{C_{\mathcal{W}}}((\Omega_2,\tilde{\Delta}_2),(\Omega_1,\tilde{\Delta}_1))$*;*

(iii) If $(\Omega_1,\tilde{\Delta}_1) = (\Omega_2,\tilde{\Delta}_2)$*, then* $\mathcal{C}_{C_{\mathcal{W}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = 1.$

Proof. Similar to Proposition 4.6. □

Definition 5.4. Let $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ be two SINHSS. Then, the WCC between $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2 \tilde{\Delta}_2)$ is defined as

$$\mathcal{C}_{\tilde{C_{\mathcal{W}}}}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = \frac{\mathcal{C}_{\mathcal{M}}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2))}{\max\left\{\Phi(\Omega_1, \tilde{\Delta}_1), \Phi(\Omega_2, \tilde{\Delta}_2)\right\}}. \tag{7}$$

$$\mathcal{C}_{\tilde{C_{\mathcal{W}}}}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = \frac{\sum_{k=1}^{m} \mathcal{D}_k \left(\sum_{i=1}^{n} \mathcal{W}_i \left[(\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) + (\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) + (\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) \right] \right)}{\max\left\{ \sum_{k=1}^{m} \mathcal{D}_k \left(\sum_{i=1}^{n} \mathcal{W}_i \left[(\mathcal{T}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{I}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{F}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 \right] \right), \sum_{k=1}^{m} \mathcal{D}_k \left(\sum_{i=1}^{n} \mathcal{W}_i \left[(\mathcal{T}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{I}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{F}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 \right] \right) \right\}}.$$

If $\mathcal{D} = \left\{\frac{1}{m}, \frac{1}{m}, ..., \frac{1}{m}, \right\}$ and $\mathcal{W} = \left\{\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}, \right\}$, then WCC given in Eq.(7) reduces to CC as in Eq.(5).

Proposition 5.5. *Let $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ be two SINHSS. Then, the following WCC properties hold:*
(i) $0 \leq \mathcal{C}_{\tilde{C_{\mathcal{W}}}}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) \leq 1$;
(ii) $\mathcal{C}_{\tilde{C_{\mathcal{W}}}}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = \mathcal{C}_{\tilde{C_{\mathcal{W}}}}((\Omega_2, \tilde{\Delta}_2), (\Omega_1, \tilde{\Delta}_1))$;
(iii) If $(\Omega_1, \tilde{\Delta}_1) = (\Omega_2, \tilde{\Delta}_2)$, *then* $\mathcal{C}_{\tilde{C_{\mathcal{W}}}}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = 1$.

Proof. Similiar to Proposition 4.6. □

6. Aggregation operators for SINHSS

We now present the concept of simplified intuitionistic neutrosophic hypersoft weighted average operator(SINHSWAO) and simplified intuitionistic neutrosophic hypersoft weighted geometric operator (SINHSWGO) by using operational laws. Let κ represent the collection of simplified intuitionistic neutrosophic hypersoft numbers (SINHSNs).

6.1. *Operational laws for SINHSS*

Definition 6.1. Let $\Omega_{e_{11}} = (\mathcal{T}_{11}, \mathcal{I}_{11}, \mathcal{F}_{11})$ and $\Omega_{e_{12}} = (\mathcal{T}_{12}, \mathcal{I}_{12}, \mathcal{F}_{12})$ be two SINHSS and β a positive integer. Then,
(i) $\Omega_{e_{11}} \oplus \Omega_{e_{12}} = \langle \mathcal{T}_{11} + \mathcal{T}_{12} - \mathcal{T}_{11}\mathcal{T}_{12}, \mathcal{I}_{11} + \mathcal{I}_{12} - \mathcal{I}_{11}\mathcal{I}_{12}, \mathcal{F}_{11}\mathcal{F}_{12} \rangle$;
(ii) $\Omega_{e_{11}} \otimes \Omega_{e_{12}} = \langle \mathcal{T}_{11}\mathcal{T}_{12}, \mathcal{I}_{11}\mathcal{I}_{12}, \mathcal{F}_{11} + \mathcal{F}_{12} - \mathcal{F}_{11}\mathcal{F}_{12} \rangle$;
(iii) $\beta\Omega_{e_{11}} = \left\langle \left[(1 - (1 - \mathcal{T}_{11})^{\beta}, (1 - (1 - \mathcal{I}_{11})^{\beta}, (\mathcal{F}_{11})^{\beta} \right] \right\rangle$;
(iv) $(\Omega_{e_{11}})^{\beta} = \left\langle \left[(\mathcal{T}_{11})^{\beta}, (\mathcal{I}_{11})^{\beta}, (1 - (1 - \mathcal{F}_{11})^{\beta} \right] \right\rangle$.

6.2. *Simplified intuitionistic neutrosophic hypersoft weighted average operator*

Definition 6.2. Let $\mathcal{D}_k$ and $\mathcal{W}_i$ be weight vectors for alternatives and experts, respectively, such that $\mathcal{D}_k, \mathcal{W}_i > 0$ and $\sum_{k=1}^{m} \mathcal{D}_k = 1, \sum_{i=1}^{n} \mathcal{W}_i = 1$ and $\Omega_{e_{ik}} = (\mathcal{T}_{ik}, \mathcal{I}_{ik}, \mathcal{F}_{ik})$ be a SINHSN, where $i = \{1, 2, ...n\}$, $k = \{1, 2, ...m\}$. Then, $\mathcal{A} : \kappa^n \to \kappa$, SINHSWAO is represented as

$$\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{nm}}) = \bigoplus_{k=1}^{m} \mathcal{D}_k \bigg(\bigoplus_{i=1}^{n} \mathcal{W}_i \Omega_{e_{ik}} \bigg).$$

Theorem 6.3. *Let $\Omega_{e_{ik}} = (\mathcal{T}_{ik}, \mathcal{I}_{ik}, \mathcal{F}_{ik})$ be a SINHSN, where $i = \{1, 2, ...n\}$, $k = \{1, 2, ...m\}$. Then, the aggregated value of SINHSWAO is also a SINHSN, which is given by*

$$\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{nm}}) = \Bigg\langle 1 - \prod_{k=1}^{m} \bigg(\prod_{i=1}^{n} \Big(1 - \mathcal{T}_{ik} \Big)^{\mathcal{W}_i} \bigg)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{m} \bigg(\prod_{i=1}^{n} \Big(1 - \mathcal{I}_{ik} \Big)^{\mathcal{W}_i} \bigg)^{\mathcal{D}_k}, \prod_{k=1}^{m} \bigg(\prod_{i=1}^{n} \Big(\mathcal{F}_{ik} \Big)^{\mathcal{W}_i} \bigg)^{\mathcal{D}_k} \Bigg\rangle.$$

Proof. If $n = 1$, then $\mathcal{W}_1 = 1$. By using Definition 6.1, we get

$$\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{1m}}) = \bigoplus_{k=1}^{m} \mathcal{D}_k \Omega_{e_{1k}}.$$

$$= \Bigg\langle 1 - \prod_{k=1}^{m} \bigg(\prod_{i=1}^{1} \Big(1 - \mathcal{T}_{ik} \Big)^{\mathcal{W}_i} \bigg)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{m} \bigg(\prod_{i=1}^{1} \Big(1 - \mathcal{I}_{ik} \Big)^{\mathcal{W}_i} \bigg)^{\mathcal{D}_k}, \prod_{k=1}^{m} \bigg(\prod_{i=1}^{1} \Big(\mathcal{F}_{ik} \Big)^{\mathcal{W}_i} \bigg)^{\mathcal{D}_k} \Bigg\rangle.$$

If $m = 1$, then $\mathcal{D}_1 = 1$. By using Definition 6.2, we get

$$\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{21}}, ..., \Omega_{e_{n1}}) = \bigoplus_{i=1}^{n} \mathcal{W}_i \Omega_{e_{i1}}.$$

$$= \Bigg\langle 1 - \prod_{k=1}^{1} \bigg(\prod_{i=1}^{n} \Big(1 - \mathcal{T}_{ik} \Big)^{\mathcal{W}_i} \bigg)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{1} \bigg(\prod_{i=1}^{n} \Big(1 - \mathcal{I}_{ik} \Big)^{\mathcal{W}_i} \bigg)^{\mathcal{D}_k}, \prod_{k=1}^{1} \bigg(\prod_{i=1}^{n} \Big(\mathcal{F}_{ik} \Big)^{\mathcal{W}_i} \bigg)^{\mathcal{D}_k} \Bigg\rangle.$$

Hence, the results hold for $n = 1$ and $m = 1$.

Now, if $m = l_1 + 1$ and $n = l_2$, then,

$$\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{l_2(l_1+1)}}) = \bigoplus_{k=1}^{l_1+1} \mathcal{D}_k \bigg(\bigoplus_{i=1}^{l_2} \mathcal{W}_i \Omega_{e_{ik}} \bigg).$$

$$= \Bigg\langle 1 - \prod_{k=1}^{l_1+1} \bigg(\prod_{i=1}^{l_2} \Big(1 - \mathcal{T}_{ik} \Big)^{\mathcal{W}_i} \bigg)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{l_1+1} \bigg(\prod_{i=1}^{l_2} \Big(1 - \mathcal{I}_{ik} \Big)^{\mathcal{W}_i} \bigg)^{\mathcal{D}_k}, \prod_{k=1}^{l_1+1} \bigg(\prod_{i=1}^{l_2} \Big(\mathcal{F}_{ik} \Big)^{\mathcal{W}_i} \bigg)^{\mathcal{D}_k} \Bigg\rangle.$$

Similarly, if $m = l_1$, $n = l_2 + 1$, then,

$$\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{(l_2+1)l_1}}) = \bigoplus_{k=1}^{l_1} \mathcal{D}_k \bigg(\bigoplus_{i=1}^{l_2+1} \mathcal{W}_i \Omega_{e_{ik}} \bigg).$$

$$= \Bigg\langle 1 - \prod_{k=1}^{l_1} \bigg(\prod_{i=1}^{l_2+1} \Big(1 - \mathcal{T}_{ik} \Big)^{\mathcal{W}_i} \bigg)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{l_1} \bigg(\prod_{i=1}^{l_2+1} \Big(1 - \mathcal{I}_{ik} \Big)^{\mathcal{W}_i} \bigg)^{\mathcal{D}_k}, \prod_{k=1}^{l_1} \bigg(\prod_{i=1}^{l_2+1} \Big(\mathcal{F}_{ik} \Big)^{\mathcal{W}_i} \bigg)^{\mathcal{D}_k} \Bigg\rangle.$$

Now, if $m = l_1 + 1$, $n = l_2 + 1$, then,

$\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{(l_2+1)(l_1+1)}})$

$$= \bigoplus_{k=1}^{l_1+1} \mathcal{D}_k \Big(\bigoplus_{i=1}^{l_2+1} \mathcal{W}_i \Omega_{e_{ik}} \Big).$$

$$= \bigoplus_{k=1}^{l_1+1} \mathcal{D}_k \Big(\bigoplus_{i=1}^{l_2} \mathcal{W}_i \Omega_{e_{ik}} \Big) \bigoplus_{k=1}^{l_1+1} \mathcal{D}_k \Big(\mathcal{W}_{l_2+1} \Omega_{e_{(l_2+1)k}} \Big).$$

$\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{(l_2+1)(l_1+1)}})$

$$= \Bigg\langle 1 - \prod_{k=1}^{l_1+1} \Bigg(\prod_{i=1}^{l_2} \Big(1 - \mathcal{T}_{ik}\Big)^{\mathcal{W}_i} \Bigg)^{\mathcal{D}_k} \oplus 1 - \prod_{k=1}^{l_1+1} \Bigg(\Big(1 - \mathcal{T}_{(l_2+1)k}\Big)^{\mathcal{W}_{(l_2+1)}} \Bigg)^{\mathcal{D}_k},$$
$$1 - \prod_{k=1}^{l_1+1} \Bigg(\prod_{i=1}^{l_2} \Big(1 - \mathcal{I}_{ik}\Big)^{\mathcal{W}_i} \Bigg)^{\mathcal{D}_k} \oplus 1 - \prod_{k=1}^{l_1+1} \Bigg(\Big(1 - \mathcal{I}_{(l_2+1)k}\Big)^{\mathcal{W}_{(l_2+1)}} \Bigg)^{\mathcal{D}_k},$$
$$\prod_{k=1}^{l_1+1} \Bigg(\prod_{i=1}^{l_2} \Big(\mathcal{F}_{ik}\Big)^{\mathcal{W}_i} \Bigg)^{\mathcal{D}_k} \oplus \prod_{k=1}^{l_1+1} \Bigg(\Big(\mathcal{F}_{(l_2+1)k}\Big)^{\mathcal{W}_{(l_2+1)}} \Bigg)^{\mathcal{D}_k} \Bigg\rangle.$$

$$= \Bigg\langle 1 - \prod_{k=1}^{l_1+1} \Bigg(\prod_{i=1}^{l_2+1} \Big(1 - \mathcal{T}_{ik}\Big)^{\mathcal{W}_i} \Bigg)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{l_1+1} \Bigg(\prod_{i=1}^{l_2+1} \Big(1 - \mathcal{I}_{ik}\Big)^{\mathcal{W}_i} \Bigg)^{\mathcal{D}_k}, \prod_{k=1}^{l_1+1} \Bigg(\prod_{i=1}^{l_2+1} \Big(\mathcal{F}_{ik}\Big)^{\mathcal{W}_i} \Bigg)^{\mathcal{D}_k} \Bigg\rangle.$$

Hence, the results hold for $n = l_2 + 1$ and $m = l_1 + 1$.

Therefore, by induction method, the result is true $\forall\ m, n \geq 1$.

Since

$0 \leq \mathcal{T}_{ik} + \mathcal{F}_{ik} \leq 1$ and $0 \leq \mathcal{I}_{ik} \leq 1$.

$$\Leftrightarrow 1 - \prod_{k=1}^{m} \Bigg(\prod_{i=1}^{n} \Big(1 - \mathcal{T}_{ik}\Big)^{\mathcal{W}_i} \Bigg)^{\mathcal{D}_k} + \prod_{k=1}^{m} \Bigg(\prod_{i=1}^{n} \Big(\mathcal{F}_{ik}\Big)^{\mathcal{W}_i} \Bigg)^{\mathcal{D}_k} \leq 1$$
$$\text{and} 1 - \prod_{k=1}^{m} \Bigg(\prod_{i=1}^{n} \Big(1 - \mathcal{I}_{ik}\Big)^{\mathcal{W}_i} \Bigg)^{\mathcal{D}_k} \leq 1.$$

$$\Leftrightarrow 1 - \prod_{k=1}^{m} \Bigg(\prod_{i=1}^{n} \Big(1 - \mathcal{T}_{ik}\Big)^{\mathcal{W}_i} \Bigg)^{\mathcal{D}_k} + \prod_{k=1}^{m} \Bigg(\prod_{i=1}^{n} \Big(1 - \mathcal{T}_{ik}\Big)^{\mathcal{W}_i} \Bigg)^{\mathcal{D}_k} \leq 1$$
$$\text{and } 1 - \prod_{k=1}^{m} \Bigg(\prod_{i=1}^{n} \Big(1 - \mathcal{I}_{ik}\Big)^{\mathcal{W}_i} \Bigg)^{\mathcal{D}_k} \leq 1.$$

$$\Leftrightarrow 1 - \prod_{k=1}^{m} \Bigg(\prod_{i=1}^{n} \Big(1 - \mathcal{T}_{ik}\Big)^{\mathcal{W}_i} \Bigg)^{\mathcal{D}_k} + \prod_{k=1}^{m} \Bigg(\prod_{i=1}^{n} \Big(1 - \mathcal{T}_{ik}\Big)^{\mathcal{W}_i} \Bigg)^{\mathcal{D}_k} + 1 - \prod_{k=1}^{m} \Bigg(\prod_{i=1}^{n} \Big(1 - \mathcal{I}_{ik}\Big)^{\mathcal{W}_i} \Bigg)^{\mathcal{D}_k} \leq 2.$$

Therefore, the aggregated value given by SINHSWAO is also a SINHSN. □

Example 6.4. Let us consider the same values mentioned in Example 3.2. Also, let $\mathcal{W}_i = \{0.50, 0.30, 0.20\}$ and $\mathcal{D}_k = \{0.14, 0.13, 0.23, 0.20, 0.18, 0.12\}$ be the weight of sociologists and attributes, respectively. Then,

$\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{36}})$

$$= \left\langle 1 - \prod_{k=1}^{6}\left(\prod_{i=1}^{3}\left(1-\mathcal{T}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{6}\left(\prod_{i=1}^{3}\left(1-\mathcal{I}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}, \prod_{k=1}^{6}\left(\prod_{i=1}^{3}\left(\mathcal{F}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}\right\rangle.$$

$$= \langle 0.55, 1.00, 0.27\rangle.$$

6.3. *Simplified intuitionistic neutrosophic hypersoft weighted geometric operator*

Definition 6.5. Let $\mathcal{D}_k$ and $\mathcal{W}_i$ be weight vectors for alternatives and experts, respectively, such that $\mathcal{D}_k, \mathcal{W}_i > 0$ and $\sum_{k=1}^{m} \mathcal{D}_k = 1, \sum_{i=1}^{n} \mathcal{W}_i = 1$ and $\Omega_{e_{ik}} = (\mathcal{T}_{ik}, \mathcal{I}_{ik}, \mathcal{F}_{ik})$ be a SINHSN, where $i = \{1, 2, ...n\}$, $k = \{1, 2, ...m\}$. Then, $\mathcal{G} : \kappa^n \to \kappa$, SINHSWGO is defined as

$$\mathcal{G}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{nm}}) = \bigotimes_{k=1}^{m}\left(\bigotimes_{i=1}^{n}\left(\Omega_{e_{ik}}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}.$$

Theorem 6.6. *Let $\Omega_{e_{ik}} = (\mathcal{T}_{ik}, \mathcal{I}_{ik}, \mathcal{F}_{ik})$ be a SINHSN, where $i = \{1, 2, ...n\}$, $k = \{1, 2, ...m\}$. Then, the aggregated value of SINHSWGO is also a SINHSN, which is given by*

$$\mathcal{G}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{nm}}) = \left\langle \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(\mathcal{T}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}, \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(\mathcal{I}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\mathcal{F}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}\right\rangle.$$

Proof. Similar to Theorem 6.3. □

Example 6.7. Let us consider the same values mentioned in Example 3.2 and the weight of sociologists and attributes be as in Example 6.4. Then,

$\mathcal{G}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{36}})$

$$= \left\langle \prod_{k=1}^{6}\left(\prod_{i=1}^{3}\left(\mathcal{T}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}, \prod_{k=1}^{6}\left(\prod_{i=1}^{3}\left(\mathcal{I}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{6}\left(\prod_{i=1}^{3}\left(1-\mathcal{F}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}\right\rangle.$$

$$= \langle 0.40, 0.52, 0.37\rangle.$$

7. MCDM problems based on TOPSIS and CC method

TOPSIS method helps to find the best alternative based on minimum and maximum distance from the neutrosophic positive ideal solution (NPIS) and neutrosophic negative ideal solution (NNIS). Also, when TOPSIS method is combined with CC instead of similarity measures, it provides reliable results for predicting the closeness coefficients. We present an algorithm and a case study to illustrate the SINHSS TOPSIS method based on CC.

7.1. *Algorithm to solve MCDM problems with SINHSS data based on TOPSIS and CC method*

Let $\mathcal{A} = \left\{\mathcal{A}^1, \mathcal{A}^2, ..., \mathcal{A}^x\right\}$ be a set of selected leaders aspiring to bring in socio-political changes to society and $\mathcal{V} = \{v_1, v_2, ..., v_n\}$ be a set of sociologists responsible to evaluate the leaders with weights $\mathcal{W}_i = (\mathcal{W}_1, \mathcal{W}_2, ..., \mathcal{W}_n)$, such that $\mathcal{W}_i > 0$ and $\sum_{i=1}^{n} \mathcal{W}_i = 1$. Let $\tilde{\Delta} = \left\{\tilde{\lambda}_1, \tilde{\lambda}_2, ..., \tilde{\lambda}_m\right\}$ be a set of multi-valued sub-attributes with weights $\mathcal{D}_k = (\mathcal{D}_1, \mathcal{D}_2, ..., \mathcal{D}_m)$, such that $\mathcal{D}_k > 0$ and $\sum_{k=1}^{m} \mathcal{D}_k = 1$. The evaluation of leaders $\mathcal{A}^t$, $(t = 1, 2, ..., x)$ performed by the sociologists v_i, $(i = 1, 2, ..., n)$ based on the multi-valued sub-attributes $\tilde{\lambda}_k$, $(k = 1, 2, ..., m)$ are given in SINHSS form and represented as $\Omega^t_{ik} = \left\langle \mathcal{T}^t_{ik}, \mathcal{I}^t_{ik}, \mathcal{F}^t_{ik} \right\rangle$, such that $0 \leq \mathcal{T}^t_{ik} + \mathcal{F}^t_{ik} \leq 1$ and $0 \leq \mathcal{T}^t_{ik} + \mathcal{I}^t_{ik} + \mathcal{F}^t_{ik} \leq 2 \ \forall \ i, k$.

Step 1. Construct the matrix for each multi-valued sub-attributes in SINHSS form as below:

$$[\mathcal{A}^t, \tilde{\Delta}]_{n\times m} = [\mathcal{A}^t]_{n\times m} = \begin{array}{c} \\ v_1 \\ v_2 \\ \vdots \\ v_n \end{array} \begin{array}{c} \begin{array}{cccc} \tilde{\lambda}_1 & \tilde{\lambda}_2 & \cdots & \tilde{\lambda}_m \end{array} \\ \begin{bmatrix} \langle \mathcal{T}^t_{11}, \mathcal{I}^t_{11}, \mathcal{F}^t_{11} \rangle & \langle \mathcal{T}^t_{12}, \mathcal{I}^t_{12}, \mathcal{F}^t_{12} \rangle & \cdots & \langle \mathcal{T}^t_{1m}, \mathcal{I}^t_{1m}, \mathcal{F}^t_{1m} \rangle \rangle \\ \langle \mathcal{T}^t_{21}, \mathcal{I}^t_{21}, \mathcal{F}^t_{21} \rangle & \langle \mathcal{T}^t_{22}, \mathcal{I}^t_{22}, \mathcal{F}^t_{22} \rangle & \cdots & \langle \mathcal{T}^t_{2m}, \mathcal{I}^t_{2m}, \mathcal{F}^t_{2m} \rangle \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \mathcal{T}^t_{n1}, \mathcal{I}^t_{n1}, \mathcal{F}^t_{n1} \rangle & \langle \mathcal{T}^t_{n2}, \mathcal{I}^t_{n2}, \mathcal{F}^t_{n2} \rangle & \cdots & \langle \mathcal{T}^t_{nm}, \mathcal{I}^t_{nm}, \mathcal{F}^t_{nm} \rangle \rangle \end{bmatrix} \end{array}$$

Step 2. Obtain the weighted decision matrix for each multi-valued sub-attributes,

$$[\tilde{A}^t_{ik}]_{n\times m}$$
$$= \left\langle 1 - \prod_{k=1}^{m} \left(\prod_{i=1}^{n} \left(1 - \mathcal{T}_{ik}\right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{m} \left(\prod_{i=1}^{n} \left(1 - \mathcal{I}_{ik}\right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k}, \prod_{k=1}^{m} \left(\prod_{i=1}^{n} \left(\mathcal{F}_{ik}\right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k} \right\rangle$$
$$= \left\langle \tilde{\mathcal{T}}_{ik}, \tilde{\mathcal{I}}_{ik}, \tilde{\mathcal{F}}_{ik} \right\rangle.$$

Step 3. Determine the NPIS and NNIS for weighted SINHSS as below:

$$\tilde{\mathcal{A}}^{+} = \left\langle \tilde{\mathcal{T}}^{+}, \tilde{\mathcal{I}}^{+}, \tilde{\mathcal{F}}^{+} \right\rangle_{n\times m} = \left\langle \tilde{\mathcal{T}}^{(\vee_{ij})}, \tilde{\mathcal{I}}^{(\wedge_{ij})}, \tilde{\mathcal{F}}^{(\wedge_{ij})} \right\rangle \text{ and}$$
$$\tilde{\mathcal{A}}^{-} = \left\langle \tilde{\mathcal{T}}^{-}, \tilde{\mathcal{I}}^{-}, \tilde{\mathcal{F}}^{-} \right\rangle_{n\times m} = \left\langle \tilde{\mathcal{T}}^{(\wedge_{ij})}, \tilde{\mathcal{I}}^{(\wedge_{ij})}, \tilde{\mathcal{F}}^{(\vee_{ij})} \right\rangle,$$

where $\vee_{ij} = \arg\max_t \left\{\varphi^t_{ij}\right\}$ and $\wedge_{ij} = \arg\min_t \left\{\varphi^t_{ij}\right\}$.

Step 4. Determine the CC for each alternative from NPIS and NNIS.

$$\chi^t = \mathcal{C}_C(\tilde{A}^t, \tilde{A}^{+}) = \frac{\mathcal{C}_{\mathcal{M}}(\tilde{A}^t, \tilde{A}^{+})}{\sqrt{\Phi(\tilde{A}^t)} * \sqrt{(\Phi \tilde{A}^{+})}} \text{ and}$$

$$\lambda^t = \mathcal{C}_C(\tilde{A}^t, \tilde{A}^{-}) = \frac{\mathcal{C}_{\mathcal{M}}(\tilde{A}^t, \tilde{A}^{-})}{\sqrt{\Phi(\tilde{A}^t)} * \sqrt{\Phi(\tilde{A}^{-})}}$$

Step 5. Compute the closeness coefficient of neutrosophic ideal solution as below:

$$\epsilon^t = \frac{1 - \lambda^t}{2 - \chi^t - \lambda^t}$$

Step 6. Arrange the ϵ^t values in descending order and determine the rank of the alternatives $\mathcal{A}^t$, $(t = 1, 2, ..., x)$. The one with the maximum value is the best alternative.

7.2. *Application based on TOPSIS and CC method*

Let $\mathcal{A} = \{\mathcal{A}^1, \mathcal{A}^2, \mathcal{A}^3, \mathcal{A}^4\}$ be a set of leaders aspiring to bring in socio-political changes with their leadership skills and Δ_1 and Δ_2 be distinct attribute sets whose corresponding sub-attributes are represented as $\Delta_1 = \text{leader attributes} = \{\lambda_{11} = \text{personality variables}, \lambda_{12} = \text{cognitive ability and skills}\}$, $\Delta_2 = \text{leader behaviors} = \{\lambda_{21} = \text{setting sub culture}, \lambda_{22} = \text{conflict management}\}$. Then $\tilde{\Delta} = \Delta_1 \times \Delta_2$ be distinct attribute sets, such as

$$\tilde{\Delta} = \Delta_1 \times \Delta_2 = \{\lambda_{11}, \lambda_{12}\} \times \{\lambda_{21}, \lambda_{22}\}.$$
$$= \Big\{(\lambda_{11}, \lambda_{21}), (\lambda_{11}, \lambda_{22}), (\lambda_{12}, \lambda_{21}), (\lambda_{12}, \lambda_{21})\Big\}.$$
$$= \Big\{\tilde{\lambda_1}, \tilde{\lambda_2}, \tilde{\lambda_3}, \tilde{\lambda_4}\Big\} \text{ with weights } \mathcal{D}_k = (0.20, 0.25, 0.30, 0.25).$$

An expert team selects a set of sociologists and provides the weightage depending on their tenure and knowledge. Let $\mathcal{V} = \{v_1, v_2, v_3, v_4\}$ be a set of sociologists responsible to evaluate the leaders with weights $\mathcal{W}_i = (0.35, 0.15, 0.30, 0.20)$. This study aims to find a leader who can bring major socio-political changes in a larger way to society.

Step 1. Construct $\mathcal{A}^1$, $\mathcal{A}^2$, $\mathcal{A}^3$ and $\mathcal{A}^4$ matrices for each multi-valued sub-attributes in SINHSS form.

Table 3. Representation of values in SINHSS form for $\mathcal{A}^1$.

$\mathcal{A}^1$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
v_1	$\langle 0.65, 0.92, 0.34\rangle$	$\langle 0.55, 0.48, 0.25\rangle$	$\langle 0.78, 0.88, 0.21\rangle$	$\langle 0.23, 0.24, 0.35\rangle$
v_2	$\langle 0.55, 0.72, 0.24\rangle$	$\langle 0.65, 0.56, 0.25\rangle$	$\langle 0.55, 0.77, 0.12\rangle$	$\langle 0.43, 0.45, 0.45\rangle$
v_3	$\langle 0.63, 0.87, 0.35\rangle$	$\langle 0.45, 0.76, 0.35\rangle$	$\langle 0.67, 0.55, 0.32\rangle$	$\langle 0.41, 0.67, 0.55\rangle$
v_4	$\langle 0.53, 0.79, 0.45\rangle$	$\langle 0.67, 0.34, 0.31\rangle$	$\langle 0.57, 0.66, 0.42\rangle$	$\langle 0.32, 0.87, 0.53\rangle$

Table 4. Representation of values in SINHSS form for $\mathcal{A}^2$.

$\mathcal{A}^2$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
v_1	$\langle 0.74, 0.27, 0.24\rangle$	$\langle 0.69, 0.43, 0.25\rangle$	$\langle 0.54, 0.22, 0.12\rangle$	$\langle 0.32, 0.67, 0.24\rangle$
v_2	$\langle 0.44, 0.95, 0.54\rangle$	$\langle 0.79, 0.56, 0.15\rangle$	$\langle 0.66, 0.33, 0.31\rangle$	$\langle 0.42, 0.78, 0.15\rangle$
v_3	$\langle 0.35, 0.85, 0.45\rangle$	$\langle 0.57, 0.32, 0.25\rangle$	$\langle 0.53, 0.44, 0.21\rangle$	$\langle 0.52, 0.89, 0.43\rangle$
v_4	$\langle 0.45, 0.76, 0.35\rangle$	$\langle 0.82, 0.78, 0.16\rangle$	$\langle 0.64, 0.55, 0.24\rangle$	$\langle 0.34, 0.91, 0.61\rangle$

Table 5. Representation of values in SINHSS form for $\mathcal{A}^3$.

$\mathcal{A}^3$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
v_1	$\langle 0.65, 0.78, 0.34\rangle$	$\langle 0.65, 0.78, 0.21\rangle$	$\langle 0.65, 0.65, 0.23\rangle$	$\langle 0.63, 0.34, 0.19\rangle$
v_2	$\langle 0.45, 0.55, 0.42\rangle$	$\langle 0.54, 0.88, 0.19\rangle$	$\langle 0.58, 0.45, 0.33\rangle$	$\langle 0.53, 0.47, 0.25\rangle$
v_3	$\langle 0.55, 0.76, 0.35\rangle$	$\langle 0.75, 0.33, 0.24\rangle$	$\langle 0.46, 0.35, 0.45\rangle$	$\langle 0.23, 0.78, 0.34\rangle$
v_4	$\langle 0.35, 0.45, 0.24\rangle$	$\langle 0.58, 0.44, 0.25\rangle$	$\langle 0.74, 0.25, 0.19\rangle$	$\langle 0.45, 0.81, 0.17\rangle$

Table 6. Representation of values in SINHSS form for $\mathcal{A}^4$.

$\mathcal{A}^4$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
v_1	$\langle 0.81, 0.46, 0.12\rangle$	$\langle 0.35, 0.45, 0.24\rangle$	$\langle 0.23, 0.32, 0.42\rangle$	$\langle 0.54, 0.93, 0.45\rangle$
v_2	$\langle 0.64, 0.56, 0.14\rangle$	$\langle 0.59, 0.65, 0.34\rangle$	$\langle 0.33, 0.43, 0.52\rangle$	$\langle 0.45, 0.48, 0.38\rangle$
v_3	$\langle 0.54, 0.76, 0.23\rangle$	$\langle 0.63, 0.76, 0.26\rangle$	$\langle 0.12, 0.54, 0.72\rangle$	$\langle 0.56, 0.79, 0.41\rangle$
v_4	$\langle 0.76, 0.45, 0.16\rangle$	$\langle 0.67, 0.88, 0.31\rangle$	$\langle 0.18, 0.65, 0.45\rangle$	$\langle 0.66, 0.58, 0.34\rangle$

Step 2. Obtain $\tilde{\mathcal{A}}^1$, $\tilde{\mathcal{A}}^2$, $\tilde{\mathcal{A}}^3$ and $\tilde{\mathcal{A}}^4$, the weighted matrices for each multi-valued sub-attributes.

Table 7. Representation of weighted values in SINHSS form for $\tilde{\mathcal{A}}^1$.

$\tilde{\mathcal{A}}^1$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
v_1	$\langle 0.0709, 0.1621, 0.9273\rangle$	$\langle 0.0675, 0.0557, 0.8858\rangle$	$\langle 0.1470, 0.1996, 0.8489\rangle$	$\langle 0.0227, 0.0238, 0.9123\rangle$
v_2	$\langle 0.0237, 0.0375, 0.9581\rangle$	$\langle 0.0387, 0.0304, 0.9494\rangle$	$\langle 0.0353, 0.0640, 0.9090\rangle$	$\langle 0.0209, 0.0222, 0.9705\rangle$
v_3	$\langle 0.0580, 0.1153, 0.9390\rangle$	$\langle 0.0439, 0.1016, 0.9243\rangle$	$\langle 0.0950, 0.0694, 0.9026\rangle$	$\langle 0.0388, 0.0798, 0.9562\rangle$
v_4	$\langle 0.0298, 0.0606, 0.9686\rangle$	$\langle 0.0540, 0.0206, 0.9432\rangle$	$\langle 0.0494, 0.0627, 0.9493\rangle$	$\langle 0.0191, 0.0970, 0.9688\rangle$

Table 8. Representation of weighted values in SINHSS form $\tilde{\mathcal{A}}^2$.

$\tilde{\mathcal{A}}^2$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_2$	$\tilde{\lambda}_4$
v_1	$\langle 0.0900, 0.0218, 0.9050\rangle$	$\langle 0.0975, 0.0480, 0.8858\rangle$	$\langle 0.0784, 0.0258, 0.8005\rangle$	$\langle 0.0332, 0.0925, 0.8827\rangle$
v_2	$\langle 0.0173, 0.0860, 0.9817\rangle$	$\langle 0.0569, 0.0304, 0.9314\rangle$	$\langle 0.0474, 0.0179, 0.9487\rangle$	$\langle 0.0203, 0.0552, 0.9314\rangle$
v_3	$\langle 0.0256, 0.1076, 0.9533\rangle$	$\langle 0.0614, 0.0286, 0.9013\rangle$	$\langle 0.0657, 0.0509, 0.8690\rangle$	$\langle 0.0536, 0.1526, 0.9387\rangle$
v_4	$\langle 0.0237, 0.0555, 0.9589\rangle$	$\langle 0.0822, 0.0730, 0.9125\rangle$	$\langle 0.0595, 0.0468, 0.9180\rangle$	$\langle 0.0206, 0.1135, 0.9756\rangle$

Step 3. Determine the NPIS and NNIS from the weighted matrices, $\tilde{\mathcal{A}}^1$, $\tilde{\mathcal{A}}^2$, $\tilde{\mathcal{A}}^3$ and $\tilde{\mathcal{A}}^4$.

$$\tilde{\mathcal{A}}^{+} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{cccc} \tilde{\lambda}_1 & \tilde{\lambda}_2 & \tilde{\lambda}_3 & \tilde{\lambda}_4 \end{array} \begin{bmatrix} \langle 0.1098, 0.0218, 0.8621\rangle & \langle 0.0975, 0.0480, 0.8724\rangle & \langle 0.1470, 0.0258, 0.8005\rangle & \langle 0.0834, 0.0238, 0.8648\rangle \\ \langle 0.0302, 0.0237, 0.9428\rangle & \langle 0.0569, 0.0304, 0.9314\rangle & \langle 0.0474, 0.0179, 0.9090\rangle & \langle 0.0280, 0.0222, 0.9314\rangle \\ \langle 0.0580, 0.0821, 0.9156\rangle & \langle 0.0988, 0.0286, 0.8985\rangle & \langle 0.0950, 0.0381, 0.8690\rangle & \langle 0.0598, 0.0798, 0.9223\rangle \\ \langle 0.0555, 0.0237, 0.9294\rangle & \langle 0.0822, 0.0206, 0.9125\rangle & \langle 0.0777, 0.0172, 0.9052\rangle & \langle 0.0526, 0.0425, 0.9153\rangle \end{bmatrix}$$

Table 9. Representation of weighted values in SINHSS form for $\tilde{\mathcal{A}}^3$.

$\tilde{A}^3$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
v_1	$\langle 0.0709, 0.1006, 0.9273\rangle$	$\langle 0.0878, 0.1241, 0.8724\rangle$	$\langle 0.1044, 0.1044, 0.8571\rangle$	$\langle 0.0834, 0.0358, 0.8648\rangle$
v_2	$\langle 0.0178, 0.0237, 0.9744\rangle$	$\langle 0.0287, 0.0765, 0.9397\rangle$	$\langle 0.0383, 0.0266, 0.9514\rangle$	$\langle 0.0280, 0.0236, 0.9494\rangle$
v_3	$\langle 0.0468, 0.0821, 0.9390\rangle$	$\langle 0.0988, 0.0296, 0.8985\rangle$	$\langle 0.0540, 0.0381, 0.9307\rangle$	$\langle 0.0195, 0.1074, 0.9223\rangle$
v_4	$\langle 0.0171, 0.0237, 0.9446\rangle$	$\langle 0.0425, 0.0286, 0.9331\rangle$	$\langle 0.0777, 0.0172, 0.9052\rangle$	$\langle 0.0295, 0.0797, 0.9153\rangle$

Table 10. Representation of weighted values in SINHSS form for $\tilde{\mathcal{A}}^4$.

$\tilde{A}^4$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
v_1	$\langle 0.1098, 0.0423, 0.8621\rangle$	$\langle 0.0370, 0.0510, 0.8827\rangle$	$\langle 0.0271, 0.0397, 0.9130\rangle$	$\langle 0.0657, 0.2076, 0.9326\rangle$
v_2	$\langle 0.0302, 0.0244, 0.9428\rangle$	$\langle 0.0329, 0.0387, 0.9604\rangle$	$\langle 0.0179, 0.0250, 0.9711\rangle$	$\langle 0.0222, 0.0243, 0.9644\rangle$
v_3	$\langle 0.0456, 0.0821, 0.9156\rangle$	$\langle 0.0719, 0.1016, 0.9040\rangle$	$\langle 0.0115, 0.0676, 0.9709\rangle$	$\langle 0.0598, 0.1105, 0.9354\rangle$
v_4	$\langle 0.0555, 0.0237, 0.9294\rangle$	$\langle 0.0540, 0.1006, 0.9432\rangle$	$\langle 0.0119, 0.0611, 0.9533\rangle$	$\langle 0.0526, 0.0425, 0.9475\rangle$

$$\tilde{\mathcal{A}}^- = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{cccc} \tilde{\lambda}_1 & \tilde{\lambda}_2 & \tilde{\lambda}_3 & \tilde{\lambda}_4 \\ \end{array}$$

$$\begin{bmatrix} \langle 0.0709, 0.0218, 0.9273\rangle & \langle 0.0370, 0.0480, 0.8858\rangle & \langle 0.0271, 0.0258, 0.9130\rangle & \langle 0.0227, 0.0238, 0.9326\rangle \\ \langle 0.0173, 0.0218, 0.9817\rangle & \langle 0.0287, 0.0304, 0.9604\rangle & \langle 0.0179, 0.0179, 0.9711\rangle & \langle 0.0203, 0.0222, 0.9705\rangle \\ \langle 0.0256, 0.0237, 0.9533\rangle & \langle 0.0439, 0.0286, 0.9243\rangle & \langle 0.0115, 0.0381, 0.9709\rangle & \langle 0.0195, 0.0798, 0.9562\rangle \\ \langle 0.0171, 0.0237, 0.9686\rangle & \langle 0.0425, 0.0206, 0.9432\rangle & \langle 0.0119, 0.0172, 0.9533\rangle & \langle 0.0191, 0.0425, 0.9756\rangle \end{bmatrix}$$

Step 4. Determine the CC for the alternatives by using the values of NPIS and NNIS.

$$\chi^1 = 0.9968, \chi^2 = 0.9981, \chi^3 = 0.9983 \text{ and } \chi^4 = 0.9962.$$

$$\lambda^1 = 0.9961, \lambda^2 = 0.9975, \lambda^3 = 0.9979 \text{ and } \lambda^4 = 0.9975.$$

Step 5. Compute the closeness coefficient of neutrosophic ideal solution as below.

$$\epsilon^1 = 0.5493, \epsilon^2 = 0.5682, \epsilon^3 = 0.5526 \text{ and } \epsilon^4 = 0.3968.$$

Step 6. Arrange the values in descending order.

$$\epsilon^2 > \epsilon^3 > \epsilon^1 > \epsilon^4.$$

$$\Rightarrow \mathcal{A}^2 > \mathcal{A}^3 > \mathcal{A}^1 > \mathcal{A}^4.$$

Hence, $\mathcal{A}^2$ is the best leader among the group and can play a significant role in bringing socio-political changes to society.

8. Comparative Analysis

We compare existing TOPSIS methods with the proposed method. Also, we provide examples to show the advantage of the TOPSIS method based on CC instead of distance or similarity measures.

Example 8.1. Consider the SINHSS values mentioned in Table 10. By applying the existing neutrosophic simplified TOPSIS method discussed in Elhassouny and Smarandache [11],

$$\mathcal{A}^1 = \mathcal{A}^2 = \mathcal{A}^3 = \mathcal{A}^4 = 0.47.$$

Hence, it is not possible to identify the best alternative.
By applying the proposed method,

$$\mathcal{A}^4 = 0.58, \mathcal{A}^3 = 0.56, \mathcal{A}^2 = 0.51 \text{ and } \mathcal{A}^1 = 0.48$$

Hence, the best alternative is $\mathcal{A}^4$.

TABLE 11. Representation of values in SINHSS form for $\mathcal{A}^i$.

$\mathcal{A}^i$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$	$\tilde{\lambda}_5$	$\tilde{\lambda}_6$
$\mathcal{A}^1$	$\langle 0.55, 0.89, 0.34\rangle$	$\langle 0.46, 0.87, 0.25\rangle$	$\langle 0.62, 0.54, 0.11\rangle$	$\langle 0.23, 0.91, 0.35\rangle$	$\langle 0.55, 0.77, 0.24\rangle$	$\langle 0.63, 0.44, 0.21\rangle$
$\mathcal{A}^2$	$\langle 0.65, 0.87, 0.24\rangle$	$\langle 0.72, 0.56, 0.12\rangle$	$\langle 0.45, 0.56, 0.12\rangle$	$\langle 0.25, 0.93, 0.45\rangle$	$\langle 0.44, 0.95, 0.54\rangle$	$\langle 0.78, 0.57, 0.15\rangle$
$\mathcal{A}^3$	$\langle 0.76, 0.85, 0.14\rangle$	$\langle 0.57, 0.76, 0.24\rangle$	$\langle 0.67, 0.55, 0.32\rangle$	$\langle 0.58, 0.67, 0.55\rangle$	$\langle 0.42, 0.75, 0.45\rangle$	$\langle 0.57, 0.54, 0.25\rangle$
$\mathcal{A}^4$	$\langle 0.53, 0.65, 0.45\rangle$	$\langle 0.71, 0.69, 0.11\rangle$	$\langle 0.57, 0.66, 0.33\rangle$	$\langle 0.42, 0.87, 0.54\rangle$	$\langle 0.55, 0.89, 0.14\rangle$	$\langle 0.69, 0.56, 0.16\rangle$

Example 8.2. Consider the SINHSS values mentioned in Table 11. By applying the existing TOPSIS method discussed in Biswas et al. [7],

$$\mathcal{A}^1 = \mathcal{A}^2 = \mathcal{A}^3 = \mathcal{A}^4 = 0.48.$$

Hence, it is not possible to identify the best alternative.
By applying the proposed method,

$$\mathcal{A}^3 = 0.57, \mathcal{A}^2 = 0.54, \mathcal{A}^4 = 0.51 \text{ and } \mathcal{A}^1 = 0.06$$

Hence, the best alternative is $\mathcal{A}^3$.

TABLE 12. Representation of values in SINHSS form for $\mathcal{A}^i$.

$\mathcal{A}^i$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$	$\tilde{\lambda}_5$	$\tilde{\lambda}_6$
$\mathcal{A}^1$	$\langle 0.45, 0.79, 0.45\rangle$	$\langle 0.34, 0.34, 0.65\rangle$	$\langle 0.47, 0.54, 0.52\rangle$	$\langle 0.52, 0.23, 0.42\rangle$	$\langle 0.41, 0.77, 0.49\rangle$	$\langle 0.35, 0.44, 0.61\rangle$
$\mathcal{A}^2$	$\langle 0.65, 0.85, 0.24\rangle$	$\langle 0.66, 0.88, 0.12\rangle$	$\langle 0.45, 0.56, 0.12\rangle$	$\langle 0.47, 0.93, 0.24\rangle$	$\langle 0.44, 0.95, 0.54\rangle$	$\langle 0.78, 0.57, 0.15\rangle$
$\mathcal{A}^3$	$\langle 0.76, 0.85, 0.14\rangle$	$\langle 0.57, 0.76, 0.24\rangle$	$\langle 0.67, 0.55, 0.32\rangle$	$\langle 0.71, 0.88, 0.23\rangle$	$\langle 0.52, 0.75, 0.45\rangle$	$\langle 0.59, 0.54, 0.21\rangle$
$\mathcal{A}^4$	$\langle 0.53, 0.65, 0.45\rangle$	$\langle 0.71, 0.69, 0.11\rangle$	$\langle 0.57, 0.66, 0.33\rangle$	$\langle 0.42, 0.83, 0.54\rangle$	$\langle 0.42, 0.79, 0.14\rangle$	$\langle 0.49, 0.56, 0.34\rangle$

Example 8.3. Consider the SINHSS values mentioned in Table 13. By combining the existing neutrosophic simplified TOPSIS method discussed in Elhassouny and Smarandache [11], with the similarity measures given in Table 12, it is not possible to identify the best alternative. However, by using the proposed method, the best alternative is identified for all the cases, as shown in Table 14.

TABLE 13. Framework of existing similarity measures.

Existing similarity measures
$\mathcal{S}_J(\psi_1,\psi_2)$ [28]$= \frac{1}{n}\sum_{i=1}^{n} \frac{\tilde{\mathcal{J}}}{(\mathcal{T}^2_{\psi_1}(u_i)+\mathcal{I}^2_{\psi_1}(u_i)+\mathcal{F}^2_{\psi_1}(u_i))+(\mathcal{T}^2_{\psi_2}(u_i)+\mathcal{I}^2_{\psi_2}(u_i)+\mathcal{F}^2_{\psi_2}(u_i))-\tilde{\mathcal{J}}}$,
where $\tilde{\mathcal{J}} = \mathcal{T}_{\psi_1}(u_i)\mathcal{T}_{\psi_2}(u_i) + \mathcal{I}_{\psi_1}(u_i)\mathcal{I}_{\psi_2}(u_i) + \mathcal{F}_{\psi_1}(u_i)\mathcal{F}_{\psi_2}(u_i)$.
$\mathcal{S}_D(\psi_1,\psi_2)$ [28]$= \frac{1}{n}\sum_{i=1}^{n} \frac{2(\mathcal{T}_{\psi_1}(u_i)\mathcal{T}_{\psi_2}(u_i)+\mathcal{I}_{\psi_1}(u_i)\mathcal{I}_{\psi_2}(u_i)+\mathcal{F}_{\psi_1}(u_i)\mathcal{F}_{\psi_2}(u_i))}{(\mathcal{T}^2_{\psi_1}(u_i)+\mathcal{I}^2_{\psi_1}(u_i)+\mathcal{F}^2_{\psi_1}(u_i))+(\mathcal{T}^2_{\psi_2}(u_i)+\mathcal{I}^2_{\psi_2}(u_i)+\mathcal{F}^2_{\psi_2}(u_i))}$.
$\mathcal{S}_1(\psi_1,\psi_2)$ [29]$= \frac{1}{n}\sum_{i=1}^{n} \cos\left[\frac{\pi}{2}\max(\lvert\mathcal{T}_{\psi_1}(u_i)-\mathcal{T}_{\psi_2}(u_i)\rvert, \lvert\mathcal{I}_{\psi_1}(u_i)-\mathcal{I}_{\psi_2}(u_i)\rvert, \lvert\mathcal{F}_{\psi_1}(u_i)-\mathcal{F}_{\psi_2}(u_i)\rvert)\right]$.
$\mathcal{S}_2(\psi_1,\psi_2)$ [29]$= \frac{1}{n}\sum_{i=1}^{n} \cos\left[\frac{\pi}{6}(\lvert\mathcal{T}_{\psi_1}(u_i)-\mathcal{T}_{\psi_2}(u_i)\rvert + \lvert\mathcal{I}_{\psi_1}(u_i)-\mathcal{I}_{\psi_2}(u_i)\rvert + \lvert\mathcal{F}_{\psi_1}(u_i)-\mathcal{F}_{\psi_2}(u_i)\rvert)\right]$.
$\mathcal{S}_3(\psi_1,\psi_2)$ [30]$= \frac{1}{n}\sum_{i=1}^{n} \tan\left[\frac{\pi}{4}\max(\lvert\mathcal{T}_{\psi_1}(u_i)-\mathcal{T}_{\psi_2}(u_i)\rvert, \lvert\mathcal{I}_{\psi_1}(u_i)-\mathcal{I}_{\psi_2}(u_i)\rvert, \lvert\mathcal{F}_{\psi_1}(u_i)-\mathcal{F}_{\psi_2}(u_i)\rvert)\right]$.
$\mathcal{S}_4(\psi_1,\psi_2)$ [30]$= \frac{1}{n}\sum_{i=1}^{n} \tan\left[\frac{\pi}{12}(\lvert\mathcal{T}_{\psi_1}(u_i)-\mathcal{T}_{\psi_2}(u_i)\rvert + \lvert\mathcal{I}_{\psi_1}(u_i)-\mathcal{I}_{\psi_2}(u_i)\rvert + \lvert\mathcal{F}_{\psi_1}(u_i)-\mathcal{F}_{\psi_2}(u_i)\rvert)\right]$.
$\mathcal{S}_5(\psi_1,\psi_2)$ [31]$= \frac{1}{n}\sum_{i=1}^{n} \cot\left[\frac{\pi}{4}+\frac{\pi}{4}\max(\lvert\mathcal{T}_{\psi_1}(u_i)-\mathcal{T}_{\psi_2}(u_i)\rvert, \lvert\mathcal{I}_{\psi_1}(u_i)-\mathcal{I}_{\psi_2}(u_i)\rvert, \lvert\mathcal{F}_{\psi_1}(u_i)-\mathcal{F}_{\psi_2}(u_i)\rvert)\right]$.
$\mathcal{S}_6(\psi_1,\psi_2)$ [31]$= \frac{1}{n}\sum_{i=1}^{n} \cot\left[\frac{\pi}{4}+\frac{\pi}{6}(\lvert\mathcal{T}_{\psi_1}(u_i)-\mathcal{T}_{\psi_2}(u_i)\rvert + \lvert\mathcal{I}_{\psi_1}(u_i)-\mathcal{I}_{\psi_2}(u_i)\rvert + \lvert\mathcal{F}_{\psi_1}(u_i)-\mathcal{F}_{\psi_2}(u_i)\rvert)\right]$.

TABLE 14. Representation of values in SINHSS form for $\mathcal{A}^i$.

$\mathcal{A}^i$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$	$\tilde{\lambda}_5$	$\tilde{\lambda}_6$
$\mathcal{A}^1$	$\langle 0.55, 0.89, 0.34\rangle$	$\langle 0.46, 0.87, 0.25\rangle$	$\langle 0.62, 0.54, 0.11\rangle$	$\langle 0.23, 0.91, 0.35\rangle$	$\langle 0.55, 0.77, 0.24\rangle$	$\langle 0.63, 0.44, 0.21\rangle$
$\mathcal{A}^2$	$\langle 0.65, 0.87, 0.24\rangle$	$\langle 0.72, 0.56, 0.12\rangle$	$\langle 0.45, 0.56, 0.12\rangle$	$\langle 0.25, 0.93, 0.45\rangle$	$\langle 0.44, 0.95, 0.54\rangle$	$\langle 0.78, 0.57, 0.15\rangle$
$\mathcal{A}^3$	$\langle 0.76, 0.85, 0.14\rangle$	$\langle 0.57, 0.76, 0.24\rangle$	$\langle 0.67, 0.55, 0.32\rangle$	$\langle 0.58, 0.67, 0.55\rangle$	$\langle 0.42, 0.75, 0.45\rangle$	$\langle 0.57, 0.38, 0.25\rangle$
$\mathcal{A}^4$	$\langle 0.53, 0.65, 0.45\rangle$	$\langle 0.71, 0.69, 0.11\rangle$	$\langle 0.57, 0.66, 0.33\rangle$	$\langle 0.42, 0.87, 0.54\rangle$	$\langle 0.55, 0.89, 0.14\rangle$	$\langle 0.69, 0.46, 0.16\rangle$

TABLE 15. Comparison of existing similarity measures with proposed method.

Unable to rank using existing similarity measures	Able to rank using Proposed method
$\mathcal{S}_J(\psi_1,\psi_2)$ [28]$\Rightarrow \mathcal{A}^1=\mathcal{A}^2=\mathcal{A}^3=\mathcal{A}^4=0.50$	$\mathcal{A}^4=0.58,>\mathcal{A}^3=0.56,>\mathcal{A}^2=0.51,>\mathcal{A}^1=0.50.$
$\mathcal{S}_D(\psi_1,\psi_2)$ [28]$\Rightarrow \mathcal{A}^1=\mathcal{A}^2=\mathcal{A}^3=\mathcal{A}^4=0.50$	$\mathcal{A}^4=0.58,>\mathcal{A}^3=0.56,>\mathcal{A}^2=0.51,>\mathcal{A}^1=0.50.$
$\mathcal{S}_1(\psi_1,\psi_2)$ [29]$\Rightarrow \mathcal{A}^1=\mathcal{A}^2=\mathcal{A}^3=\mathcal{A}^4=0.50$	$\mathcal{A}^4=0.58,>\mathcal{A}^3=0.56,>\mathcal{A}^2=0.51,>\mathcal{A}^1=0.50.$
$\mathcal{S}_2(\psi_1,\psi_2)$ [29]$\Rightarrow \mathcal{A}^1=\mathcal{A}^2=\mathcal{A}^3=\mathcal{A}^4=0.50$	$\mathcal{A}^4=0.58,>\mathcal{A}^3=0.56,>\mathcal{A}^2=0.51,>\mathcal{A}^1=0.50.$
$\mathcal{S}_3(\psi_1,\psi_2)$ [30]$\Rightarrow \mathcal{A}^1=\mathcal{A}^2=\mathcal{A}^3=\mathcal{A}^4=0.50$	$\mathcal{A}^4=0.58,>\mathcal{A}^3=0.56,>\mathcal{A}^2=0.51,>\mathcal{A}^1=0.50.$
$\mathcal{S}_4(\psi_1,\psi_2)$ [30]$\Rightarrow \mathcal{A}^1=\mathcal{A}^2=\mathcal{A}^3=\mathcal{A}^4=0.50$	$\mathcal{A}^4=0.58,>\mathcal{A}^3=0.56,>\mathcal{A}^2=0.51,>\mathcal{A}^1=0.50.$
$\mathcal{S}_5(\psi_1,\psi_2)$ [31]$\Rightarrow \mathcal{A}^1=\mathcal{A}^2=\mathcal{A}^3=\mathcal{A}^4=0.50$	$\mathcal{A}^4=0.58,>\mathcal{A}^3=0.56,>\mathcal{A}^2=0.51,>\mathcal{A}^1=0.50.$
$\mathcal{S}_6(\psi_1,\psi_2)$ [31]$\Rightarrow \mathcal{A}^1=\mathcal{A}^2=\mathcal{A}^3=\mathcal{A}^4=0.50$	$\mathcal{A}^4=0.58,>\mathcal{A}^3=0.56,>\mathcal{A}^2=0.51,>\mathcal{A}^1=0.50.$

9. Conclusions

Existing theories fail to handle the information when each component is interrelated. To overcome this limitation, we establish the properties of a simplified intuitionistic neutrosophic hypersoft set. We propose an application based on the TOPSIS method to identify a leader in a socio-political context. We apply CC instead of the usual distance or similarity measures in the TOPSIS method to understand the closeness coefficients in a better way. We have presented a comparative study between the proposed method and the existing TOPSIS method to prove the reliability of the proposed model. The proposed concept may be extended to algebraic structures, $\mathcal{N}$ soft set, and other hybrid structures. Apart from the theoretical dimension, the discussed concepts may be implemented to real-world challenges in fields such as psychology, economics, pattern recognition, artificial intelligence, and many more.

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