Neutrosophic Sets and Systems, Vol. 41, 2021
University of New Mexico

Simplified Neutrosophic Multiplicative Refined Sets and Their Correlation Coefficients with Application in Medical Pattern Recognition

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Abstract. In this paper, the notion of simplified neutrosophic multiplicative refined set (aka, simplified neutrosophic multiplicative multi-set) is introduced and some basic operational relations are investigated. The correlation coefficient is one of the most frequently used tools to provide the strength of relationship between two fuzzy/neutrosophic (refined) sets. Two different methods are proposed to calculate the correlation coefficients between two simplified neutrosophic multiplicative refined sets. Further, the effectiveness of these methods is demonstrated by dealing with the medical pattern recognition problem under the simplified neutrosophic multiplicative refined set environment.

Keywords: Neutrosophic sets; Simplified neutrosophic multiplicative sets; Simplified neutrosophic multiplicative refined sets; Simplified neutrosophic multiplicative multi-sets; Correlation coefficient; Pattern recognition

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1. Introduction

Most of the problems of real-life involve uncertainty or unknown data, and traditional mathematical tools cannot deal with such problems. The fuzzy sets (FSs), originated by Zadeh [48], is a useful tool to cope with vagueness and ambiguity. In 1986, Atanassov [6] initiated the theory of intuitionistic fuzzy sets (IFSs) extending the FSs. In the following years, many authors studied the the fuzzy set extensions [9, 19, 20, 30, 40] and their matrix representations [23, 25, 27–29]. However, the FSs and their extensions failed to cope with indeterminate and inconsistent information which exist in beliefs system, therefore, Smarandache [36] proposed new concept named neutrosophic set (NS) which generalizes the FSs and IFSs. Later, Wang et al. [42] and Ye [46] introduced specific descriptions of NSs known as single-valued neutrosophic set (SVNS) and simplified neutrosophic set (SNS), motivated from a practical point of view and can be used in real scientific and engineering applications. These theories of NSs have proven useful in the different fields such as medical diagnosis [3,16], decision making [1–5,15,21,26] and so on. In 1995, Smarandache put forward that in some cases the degrees of truth-membership, indeterminacy-membership and falsity-membership in the structure of (single-valued/simplified) NS can be not only in the interval [0,1] but also less than 0 or greater than 1, and presented some real world arguments supporting this assertion. Based on this idea, he introduced the concepts of neutrosophic oversets (when some neutrosophic components are > 1), neutrosophic undersets (when some neutrosophic component is < 0), and neutrosophic offsets (when some neutrosophic components are off the interval [0,1], i.e. some neutrosophic components > 1 and some neutrosophic components < 0) and studied their fundamentals [35, 37, 38].

The multi-set theory was introduced by Yager [45] as generalization of the set theory and then the multi-set was improved by Calude et al. [8]. Occasionally, several authors made a number of generalizations of the multi-set theory. Sebastian and Ramakrishnan [33] described a multi fuzzy set (mFS) which is a generalization of the multi-set. In [11, 34], the authors presented an extension of the notion of mFS to an intuitionstic fuzzy set which was termed to be an intuitionstic fuzzy multi-sets (IFmS). As the concepts of mFS and IFmS failed to deal with indeterminacy, Smarandache [39] extended the classical neutrosophic logic to n-valued refined neutrosophic logic, by refining each neutrosophic component. Meanwhile, Ye and Ye [47] proposed the concept of neutrosophic multi-set (NmS) (aka, neutrosophic refined set (NRS)) and investigated their characteristic properties. Deli et al. [10] studied some aspects of NRSs such as intersection, union, convex and strongly convex in a new way. In recent years, many seminal articles on the NmSs/NRSs have been published [7, 22, 41].

In spite of the fact that the FSs, IFSs and NSs are effective mathematical tools for dealing with uncertainties, these sets use the 0-1 scale, which is distributed symmetrically and uniformly. But, there are real-life issues that need to be scaled as unsymmetrically and non-uniformly. The grading system of universities is the most obvious example of such situations [17]. In dealing with such problems that need to be scaled unsymmetrically and non-uniformly while assigning the variable grades, Saaty [31] proposed the 1-9 scale (or $\frac{1}{9} - 9$ scale) as a useful tool. These different scales lead to the modelling of multiplicative preference relation [32]. In 2013, Xia et al. [44] proposed the idea of intuitionistic multiplicative sets (IMSs) and the intuitionistic multiplicative preference relations (IMPRs). Further, they gave a comparison between 0.1-0.9 and $\frac{1}{9} - 9$ scales as in Table 1.

Table 1. The comparison between 0.1-0.9 and $\frac{1}{9} - 9$ scales [44]

$\frac{1}{9} - 9$ scale	0.1-0.9 scale	Meaning
$\frac{1}{9}$	0.1	Extremely not preferrred
$\frac{1}{7}$	0.2	Very strongly not preferrred
$\frac{1}{5}$	0.3	Strongly not preferrred
$\frac{1}{3}$	0.4	Moderately not preferrred
1	0.5	Equally preferrred
3	0.6	Moderately preferrred
5	0.7	Strongly preferrred
7	0.8	Very strongly preferrred
9	0.9	Extremely preferrred
Other values between $\frac{1}{9}$ and 9	Other values between 0 and 1	Intermediate values used to present compromise

Recently, the theoretical aspects of IMSs and IMPRs have been studied in detail [12–14, 18, 43]. In 2019, Köseoğlu et al. [24] put forward that the IMSs cannot handle real-life problems, which include the indeterminate information in addition to the truth-membership information and falsity-membership information of IMS. To eradicate this restriction, they introduced the concepts of simplified neutrosophic multiplicative set (SNMS) and simplified neutrosophic multiplicative preference relations (SNMPRs).

Moreover, they gave several formulas for measuring the distance between two SNMSs.
There are two main objectives underlying this study. The first is to initiate the theory of simplified neutrosophic multiplicative refined set (SNMRS) (aka, simplified neutrosophic multiplicative multiset). Obviously, the concept of SNMRS is a generalization of IMSs and SNMSs. The second is to propose novel correlation coefficients to numerically determine the relationship between two SNMRSs. By using the proposed correlation coefficients, the ranking of all alternatives (objects) can be achieved. The layout of rest of this paper is presented as follows: In Section 2, the concepts of NSs, SNSs and SNMSs are given. In Section 3, the SNMRSs are conceptualized and their fundamentals such as subset, complement, intersection, union and aggregation operators are studied. In Section 4, the conceptual approaches of correlation coefficients between two SNMRSs are proposed and their characteristic properties are discussed. In Section 5, an example are given to validate the proposed correlation measures and the comparative analysis is presented to demonstrate their effectiveness. In Section 6, the conclusion of this study is summarized.

2. Preliminaries

In this section, some basic concepts of neutrosophic sets, simplified neutrosophic sets and simplified neutrosophic multiplicative sets are recalled.

Let $\mathcal{E}$ be a space of points (object) with a generic element denoted by ε.

Definition 2.1. ([36]) A neutrosophic set (NS) $\mathcal{N}$ in $\mathcal{E}$ is characterized by a truth-membership function $t_{\mathcal{N}} : \mathcal{E} \to]0^-, 1^+[$, an indeterminacy-membership function $\imath_{\mathcal{N}} : \mathcal{E} \to]0^-, 1^+[$, and a falsity-membership function $f_{\mathcal{N}} : \mathcal{E} \to]0^-, 1^+[$. $t_{\mathcal{N}}(\varepsilon)$, $\imath_{\mathcal{N}}(\varepsilon)$ and $f_{\mathcal{N}}(\varepsilon)$ are real standard or non-standard subsets of $]0^-, 1^+[$. There is no restriction on the sum of $t_{\mathcal{N}}(\varepsilon)$, $\imath_{\mathcal{N}}(\varepsilon)$ and $f_{\mathcal{N}}(\varepsilon)$, so $0^- \leq \sup t_{\mathcal{N}}(\varepsilon) + \sup \imath_{\mathcal{N}}(\varepsilon) + \sup f_{\mathcal{N}}(\varepsilon) \leq 3^+$ for $\varepsilon \in \mathcal{E}$.

However, Wang et al. [42] and Ye [46] stated the difficulty of using the NSs of non-standard intervals in practice, and introduced the simplified neutrosophic sets as follows.

Definition 2.2. ([46]) An NS $\mathcal{N}$ is characterized by a truth-membership function $t_{\mathcal{N}} : \mathcal{E} \to [0, 1]$, an indeterminacy-membership function $\imath_{\mathcal{N}} : \mathcal{E} \to [0, 1]$, and a falsity-membership function $f_{\mathcal{N}} : \mathcal{E} \to [0, 1]$. $t_{\mathcal{N}}(\varepsilon)$, $\imath_{\mathcal{N}}(\varepsilon)$ and $f_{\mathcal{N}}(\varepsilon)$ are singleton subintervals/subsets in the standard interval $[0, 1]$, then it is termed to be a simplified neutrosophic set (SNS) and described as

$$\mathcal{N} = \{(\varepsilon, \langle t_{\mathcal{N}}(\varepsilon), \imath_{\mathcal{N}}(\varepsilon), f_{\mathcal{N}}(\varepsilon)\rangle) : \ \varepsilon \in \mathcal{E}\} \tag{1}$$

This kind of NS is is named a single-valued neutrosophic set (SVNS) by Wang et al. [42]. Throughout this paper, we will use the term "simplified neutrosophic set (SNS)".

Definition 2.3. ([39, 47]) A simplified neutrosophic refined set (SNRS) $\widetilde{\mathcal{N}}$ can be defined as follows:

$$\widetilde{\mathcal{N}} = \{(\varepsilon, \langle (t^1_{\widetilde{\mathcal{N}}}(\varepsilon), t^2_{\widetilde{\mathcal{N}}}(\varepsilon), ..., t^q_{\widetilde{\mathcal{N}}}(\varepsilon)), (\imath^1_{\widetilde{\mathcal{N}}}(\varepsilon), \imath^2_{\widetilde{\mathcal{N}}}(\varepsilon), ..., \imath^q_{\widetilde{\mathcal{N}}}(\varepsilon)), (f^1_{\widetilde{\mathcal{N}}}(\varepsilon), f^2_{\widetilde{\mathcal{N}}}(\varepsilon), ..., f^q_{\widetilde{\mathcal{N}}}(\varepsilon))\rangle) : \ \varepsilon \in \mathcal{E}\} \tag{2}$$

where

$$t^1_{\widetilde{\mathcal{N}}}, t^2_{\widetilde{\mathcal{N}}}, ..., t^q_{\widetilde{\mathcal{N}}} : \mathcal{E} \to [0,1], \ \ \imath^1_{\widetilde{\mathcal{N}}}, \imath^2_{\widetilde{\mathcal{N}}}, ..., \imath^q_{\widetilde{\mathcal{N}}} : \mathcal{E} \to [0,1], \text{ and } f^1_{\widetilde{\mathcal{N}}}, f^2_{\widetilde{\mathcal{N}}}, ..., f^q_{\widetilde{\mathcal{N}}} : \mathcal{E} \to [0,1]$$

such that

$$0^- \le \sup t^i_{\widetilde{\mathcal{N}}}(\varepsilon) + \sup \imath^i_{\widetilde{\mathcal{N}}}(\varepsilon) + \sup f^i_{\widetilde{\mathcal{N}}}(\varepsilon) \le 3^+ \ \forall i \in I_q = \{1, 2, ..., q\}.$$

for each $\varepsilon \in \mathcal{E}$. Further, the truth-membership sequence $(t^1_{\widetilde{\mathcal{N}}}(\varepsilon), t^2_{\widetilde{\mathcal{N}}}(\varepsilon), ..., t^q_{\widetilde{\mathcal{N}}}(\varepsilon))$ may be in decreasing/increasing order, and the corresponding indeterminacy-membership sequence $(\imath^1_{\widetilde{\mathcal{N}}}(\varepsilon), \imath^2_{\widetilde{\mathcal{N}}}(\varepsilon), ..., \imath^q_{\widetilde{\mathcal{N}}}(\varepsilon))$ and falsity-membership sequence $(f^1_{\widetilde{\mathcal{N}}}(\varepsilon), f^2_{\widetilde{\mathcal{N}}}(\varepsilon), ..., f^q_{\widetilde{\mathcal{N}}}(\varepsilon))$. Also, q is termed to be the dimension of SNMS $\widetilde{\mathcal{N}}$.

Note 1. In the literature, SNRSs are also referred to as simplified neutrosophic multisets (SNmSs).

Definition 2.4. ([24]) A simplified neutrosophic multiplicative set (SNMS) $\mathcal{M}$ in $\mathcal{E}$ is defined as

$$\mathcal{M} = \{(\varepsilon, \langle \zeta_{\mathcal{M}}(\varepsilon), \eta_{\mathcal{M}}(\varepsilon), \vartheta_{\mathcal{M}}(\varepsilon)\rangle) : \ \varepsilon \in \mathcal{E}\}, \tag{3}$$

which assigns to each element ε a truth-membership information $\zeta_{\mathcal{M}}(\varepsilon)$, an indeterminacy-membership information $\eta_{\mathcal{M}}(\varepsilon)$, and a falsity-membership information $\vartheta_{\mathcal{M}}(\varepsilon)$ with conditions

$$\frac{1}{9} \le \zeta_{\mathcal{M}}(\varepsilon), \eta_{\mathcal{M}}(\varepsilon), \vartheta_{\mathcal{M}}(\varepsilon) \le 9 \text{ and } 0 < \zeta_{\mathcal{M}}(\varepsilon)\vartheta_{\mathcal{M}}(\varepsilon) \le 1. \tag{4}$$

for each $\varepsilon \in \mathcal{E}$.

Note 1. In 1995, Smarandache put forward that in some cases the degrees of truth-membership, indeterminacy-membership and falsity-membership in the structure of (single-valued/simplified) NS can be not only in the interval [0,1] but also greater than 1. Thus, he described the truth-membership function, indeterminacy-membership function and falsity-membership function as $t_{\mathcal{N}}, \imath_{\mathcal{N}}, f_{\mathcal{N}} : \mathcal{E} \to [0, \Omega]$ where $0 < 1 < \Omega$ and Ω is named overlimit. He called this extended type of (single-valued/simplified) NSs as neutrosophic overset [37, 38]. It is noted that the SNMSs are particular case of the neutrosophic oversets.

3. Simplified Neutrosophic Multiplicative Refined Sets

In this section, we initiate the theory of simplified neutrosophic multiplicative refined sets. Also, we derive some basic operations on simplified neutrosophic multiplicative refined sets and study the related properties.

Definition 3.1. Let $\mathcal{E}$ be a space of points (object) with a generic element denoted by ε. A simplified neutrosophic multiplicative refined set (SNMRS) $\widetilde{\mathcal{M}}$ in $\mathcal{E}$ is defined as

$$\begin{aligned} \widetilde{\mathcal{M}} &= \{(\varepsilon, \langle(\zeta^1_{\widetilde{\mathcal{M}}}(\varepsilon), \zeta^2_{\widetilde{\mathcal{M}}}(\varepsilon), ..., \zeta^q_{\widetilde{\mathcal{M}}}(\varepsilon)), (\eta^1_{\widetilde{\mathcal{M}}}(\varepsilon), \eta^2_{\widetilde{\mathcal{M}}}(\varepsilon), ..., \eta^q_{\widetilde{\mathcal{M}}}(\varepsilon)), (\vartheta^1_{\widetilde{\mathcal{M}}}(\varepsilon), \vartheta^2_{\widetilde{\mathcal{M}}}(\varepsilon), ..., \vartheta^q_{\widetilde{\mathcal{M}}}(\varepsilon))\rangle) : \ \varepsilon \in \mathcal{E}\} \\ &= \{\langle \varepsilon, (\zeta^i_{\widetilde{\mathcal{M}}}(\varepsilon))_{i \in I_q}, (\eta^i_{\widetilde{\mathcal{M}}}(\varepsilon))_{i \in I_q}, (\vartheta^i_{\widetilde{\mathcal{M}}}(\varepsilon))_{i \in I_q}\rangle : \ \varepsilon \in \mathcal{E}\} \end{aligned} \tag{5}$$

which assigns to each element ε a sequence of truth-membership information $\zeta^i_{\widetilde{\mathcal{M}}}(\varepsilon)$ $(i = 1, 2, ..., q)$, a sequence of indeterminacy-membership information $\eta^i_{\widetilde{\mathcal{M}}}(\varepsilon)$ $(i = 1, 2, ..., q)$, and a sequence of falsity-membership information $\vartheta^i_{\widetilde{\mathcal{M}}}(\varepsilon)$ $(i = 1, 2, ..., q)$ with conditions

$$\frac{1}{9} \le \zeta^i_{\widetilde{\mathcal{M}}}(\varepsilon), \eta^i_{\widetilde{\mathcal{M}}}(\varepsilon), \vartheta^i_{\widetilde{\mathcal{M}}}(\varepsilon) \le 9 \text{ and } 0 < \zeta^i_{\widetilde{\mathcal{M}}}(\varepsilon)\vartheta^i_{\widetilde{\mathcal{M}}}(\varepsilon) \le 1 \ \ \forall i \in I_q \tag{6}$$

for each $\varepsilon \in \mathcal{E}$. Further, the truth-membership sequence $(\zeta^i_{\widetilde{\mathcal{M}}}(\varepsilon))_{i\in I_q} = (\zeta^1_{\widetilde{\mathcal{M}}}(\varepsilon), \zeta^2_{\widetilde{\mathcal{M}}}(\varepsilon), ..., \zeta^q_{\widetilde{\mathcal{M}}}(\varepsilon))$ may be in decreasing/increasing order, and the corresponding indeterminacy-membership sequence $(\eta^i_{\widetilde{\mathcal{M}}}(\varepsilon))_{i\in I_q} = (\eta^1_{\widetilde{\mathcal{M}}}(\varepsilon), \zeta^2_{\widetilde{\mathcal{M}}}(\varepsilon), ..., \zeta^q_{\widetilde{\mathcal{M}}}(\varepsilon))$ and falsity-membership sequence $(\vartheta^i_{\widetilde{\mathcal{M}}}(\varepsilon))_{i\in I_q} = (\vartheta^1_{\widetilde{\mathcal{M}}}(\varepsilon), \vartheta^2_{\widetilde{\mathcal{M}}}(\varepsilon), ..., \vartheta^q_{\widetilde{\mathcal{M}}}(\varepsilon))$. Also, q is termed to be the dimension of SNMRS $\widetilde{\mathcal{M}}$. For convenience, any element of $\widetilde{\mathcal{M}}$ can be represented as $\psi = \langle (\zeta^i_{\widetilde{\mathcal{M}}})_{i\in I_q}, (\eta^i_{\widetilde{\mathcal{M}}})_{i\in I_q}, (\vartheta^i_{\widetilde{\mathcal{M}}})_{i\in I_q}\rangle$ and it is said to be a simplified neutrosophic multiplicative refined number (SNMRN).

From now on, $SNMRS(\mathcal{E}, q)$ denotes the collection of all q-dimension SNMRSs in $\mathcal{E}$.

Example 3.2. Assume that $\mathcal{E} = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4\}$ is the universal set where all the elements represent some drugs suitable for different infections such as bronchitis, sinusitis, skin infections and ear infections. We can easily categorize these drugs according to their side effects. Thus, the (3-dimension) SNMRS is given as follows:

$$\widetilde{\mathcal{M}} = \left\{ \begin{array}{l} (\varepsilon_1, \langle (\frac{1}{4}, 1, 4), (1, \frac{1}{5}, \frac{1}{2}), (3, \frac{2}{5}, \frac{1}{4})\rangle), (\varepsilon_2, \langle (1, 3, 5), (\frac{1}{2}, \frac{1}{4}, 2), (1, \frac{1}{4}, \frac{1}{5})\rangle), \\ (\varepsilon_3, \langle (\frac{1}{9}, \frac{1}{6}, \frac{1}{2}), (9, 1, \frac{1}{5}), (2, 1, \frac{1}{2})\rangle), (\varepsilon_4, \langle (\frac{5}{4}, 4, 5), (\frac{1}{4}, 4, 5), (\frac{1}{5}, \frac{1}{5}, \frac{1}{9})\rangle) \end{array} \right\}.$$

Consider $(\varepsilon_1, \langle (\frac{1}{4}, 1, 4), (1, \frac{1}{5}, \frac{1}{2}), (3, \frac{2}{5}, \frac{1}{4})\rangle) \in \widetilde{\mathcal{M}}$. Then, $(\frac{1}{4}, 1, 4)$ means the sequence of truth-membership information (scaled between $\frac{1}{9}$ and 9) of side effects of drug ε_1. The sequences of indeterminacy-membership information and falsity-membership information of ε_1 can be interpreted similarly.

Definition 3.3. Let $\widetilde{\mathcal{M}}, \widetilde{\mathcal{M}}_1, \widetilde{\mathcal{M}}_2 \in SNMRS(\mathcal{E}, q)$.

(a): If for each $\varepsilon \in \mathcal{E}$,

$$\zeta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon) \leq \zeta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon),\ \eta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon) \geq \eta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon),\ \vartheta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon) \geq \vartheta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon)\ \ \forall i \in I_q$$

then $\widetilde{\mathcal{M}}_1$ is an SNMR subset of $\widetilde{\mathcal{M}}_2$, denoted by $\widetilde{\mathcal{M}}_1 \subseteq \widetilde{\mathcal{M}}_2$.

(b): If for each $\varepsilon \in \mathcal{E}$,

$$\zeta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon) = \zeta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon),\ \eta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon) = \eta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon),\ \vartheta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon) = \vartheta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon)\ \ \forall i \in I_q$$

then the SNMRSs $\widetilde{\mathcal{M}}_1$ and $\widetilde{\mathcal{M}}_2$ are equal, denoted by $\widetilde{\mathcal{M}}_1 = \widetilde{\mathcal{M}}_2$. That is, $\widetilde{\mathcal{M}}_1 = \widetilde{\mathcal{M}}_2$ iff $\widetilde{\mathcal{M}}_1 \subseteq \widetilde{\mathcal{M}}_2$ and $\widetilde{\mathcal{M}}_2 \subseteq \widetilde{\mathcal{M}}_1$.

(c): The complement of $\widetilde{\mathcal{M}}$, is denoted and defined as

$$\widetilde{\mathcal{M}}^c = \{(\varepsilon, \langle (\vartheta^i_{\widetilde{\mathcal{M}}}(\varepsilon))_{i\in I_q}, (\frac{1}{\eta^i_{\widetilde{\mathcal{M}}}(\varepsilon)})_{i\in I_q}, (\zeta^i_{\widetilde{\mathcal{M}}}(\varepsilon))_{i\in I_q}\rangle) :\ \varepsilon \in \mathcal{E}\}.$$

where $(\frac{1}{\eta^i_{\widetilde{\mathcal{M}}}(\varepsilon)})_{i\in I_q}$ represents the sequence $(\frac{1}{\eta^1_{\widetilde{\mathcal{M}}}(\varepsilon)}, \frac{1}{\eta^2_{\widetilde{\mathcal{M}}}(\varepsilon)}, ..., \frac{1}{\eta^q_{\widetilde{\mathcal{M}}}(\varepsilon)})$

(d): The intersection of $\widetilde{\mathcal{M}}_1$ and $\widetilde{\mathcal{M}}_2$, denoted by $\widetilde{\mathcal{M}}_1 \cap \widetilde{\mathcal{M}}_2$, is described as

$$\widetilde{\mathcal{M}}_1 \cap \widetilde{\mathcal{M}}_2 = \left\{ \left(\varepsilon, \left\langle \begin{array}{l} \left(\min\{\zeta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon), \zeta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon)\}\right)_{i\in I_q}, \\ \left(\max\{\eta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon), \eta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon)\}\right)_{i\in I_q}, \\ \left(\max\{\vartheta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon), \vartheta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon)\}\right)_{i\in I_q} \end{array} \right\rangle \right) :\ \varepsilon \in \mathcal{E} \right\}.$$

(e): The union of $\widetilde{\mathcal{M}}_1$ and $\widetilde{\mathcal{M}}_2$, denoted by $\widetilde{\mathcal{M}}_1 \cup \widetilde{\mathcal{M}}_2$, is described as

$$\widetilde{\mathcal{M}}_1 \cup \widetilde{\mathcal{M}}_2 = \left\{ \left(\varepsilon, \left\langle \begin{array}{c} \left(\max\{\zeta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon), \zeta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon)\}\right)_{i\in I_q}, \\ \left(\min\{\eta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon), \eta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon)\}\right)_{i\in I_q}, \\ \left(\min\{\vartheta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon), \vartheta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon)\}\right)_{i\in I_q} \end{array} \right\rangle \right) : \ \varepsilon \in \mathcal{E} \right\}.$$

For the sequences of truth-membership information in definitions of intersection and union of SNMRSs, $\left(\min\{\zeta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon), \zeta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon)\}\right)_{i\in I_q} = (\min\{\zeta^1_{\widetilde{\mathcal{M}}_1}(\varepsilon), \zeta^1_{\widetilde{\mathcal{M}}_2}(\varepsilon)\}, \min\{\zeta^2_{\widetilde{\mathcal{M}}_1}(\varepsilon), \zeta^2_{\widetilde{\mathcal{M}}_2}(\varepsilon)\}, ..., \min\{\zeta^q_{\widetilde{\mathcal{M}}_1}(\varepsilon), \zeta^q_{\widetilde{\mathcal{M}}_2}(\varepsilon)\})$ and $\left(\max\{\zeta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon), \zeta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon)\}\right)_{i\in I_q} = (\max\{\zeta^1_{\widetilde{\mathcal{M}}_1}(\varepsilon), \zeta^1_{\widetilde{\mathcal{M}}_2}(\varepsilon)\}, \max\{\zeta^2_{\widetilde{\mathcal{M}}_1}(\varepsilon), \zeta^2_{\widetilde{\mathcal{M}}_2}(\varepsilon)\}, ..., \max\{\zeta^q_{\widetilde{\mathcal{M}}_1}(\varepsilon), \zeta^q_{\widetilde{\mathcal{M}}_2}(\varepsilon)\})$. It can be considered similar matches for the indeterminacy-membership information and falsity-membership information in Definition 3.3 (d) and (e).

Theorem 3.4. *Let* $\widetilde{\mathcal{M}}_1, \widetilde{\mathcal{M}}_2, \widetilde{\mathcal{M}}_3 \in SNMRS(\mathcal{E}, q)$.

(i): *If* $\widetilde{\mathcal{M}}_1 \star \widetilde{\mathcal{M}}_2$ *and* $\widetilde{\mathcal{M}}_2 \star \widetilde{\mathcal{M}}_3$ *then* $\widetilde{\mathcal{M}}_1 \star \widetilde{\mathcal{M}}_3$ *for each* $\star \in \{\subseteq, =\}$.

(ii): *If* $\widetilde{\mathcal{M}}_1 \star \widetilde{\mathcal{M}}_2$ *then* $(\widetilde{\mathcal{M}}_1 \bullet \widetilde{\mathcal{M}}_3) \star (\widetilde{\mathcal{M}}_2 \bullet \widetilde{\mathcal{M}}_3)$ *for each* $\star \in \{\subseteq, =\}$ *and* $\bullet \in \{\cap, \cup\}$.

(iii): $\widetilde{\mathcal{M}}_1 \bullet \widetilde{\mathcal{M}}_2 = \widetilde{\mathcal{M}}_2 \bullet \widetilde{\mathcal{M}}_1$ *for each* $\bullet \in \{\cap, \cup\}$.

(iv): $\widetilde{\mathcal{M}}_1 \bullet (\widetilde{\mathcal{M}}_2 \bullet \widetilde{\mathcal{M}}_3) = (\widetilde{\mathcal{M}}_1 \bullet \widetilde{\mathcal{M}}_2) \bullet \widetilde{\mathcal{M}}_3$ *for each* $\bullet \in \{\cap, \cup\}$.

(v): $\widetilde{\mathcal{M}}_1 \bullet (\widetilde{\mathcal{M}}_2 \blacklozenge \widetilde{\mathcal{M}}_3) = (\widetilde{\mathcal{M}}_1 \bullet \widetilde{\mathcal{M}}_2) \blacklozenge (\widetilde{\mathcal{M}}_1 \bullet \widetilde{\mathcal{M}}_3)$ *for each* $\bullet, \blacklozenge \in \{\cap, \cup\}$.

(vi): $(\widetilde{\mathcal{M}}_1 \bullet \widetilde{\mathcal{M}}_2)^c = \widetilde{\mathcal{M}}^c_1 \blacklozenge \widetilde{\mathcal{M}}^c_2$ *for each* $\bullet, \blacklozenge \in \{\cap, \cup\}$ *and* $\bullet \neq \blacklozenge$.

Proof. Let us prove the properties (vi) for $\bullet = \cap$ and $\blacklozenge = \cup$.

(iv): From Definition 3.3 (c) and (d), we can write

$$(\widetilde{\mathcal{M}}_1 \cap \widetilde{\mathcal{M}}_2)^c = \left\{ \left(\varepsilon, \left\langle \begin{array}{c} \left(\max\{\vartheta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon), \vartheta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon)\}\right)_{i\in I_q}, \\ \left(\frac{1}{\max\{\eta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon), \eta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon)\}}\right)_{i\in I_q}, \\ \left(\min\{\zeta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon), \zeta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon)\}\right)_{i\in I_q} \end{array} \right\rangle \right) : \ \varepsilon \in \mathcal{E} \right\} \tag{7}$$

For the right side of the equality, we can write

$$\widetilde{\mathcal{M}}^c_k = \{\langle \varepsilon, (\vartheta^i_{\widetilde{\mathcal{M}}_k}(\varepsilon))_{i\in I_q}, (\frac{1}{\eta^i_{\widetilde{\mathcal{M}}_k}(\varepsilon)})_{i\in I_q}, (\zeta^i_{\widetilde{\mathcal{M}}_k}(\varepsilon))_{i\in I_q} \rangle : \ \varepsilon \in \mathcal{E}\}.$$

for $k = 1, 2$ and so

$$\widetilde{\mathcal{M}}^c_1 \cup \widetilde{\mathcal{M}}^c_2 = \left\{ \left(\varepsilon, \left\langle \begin{array}{c} \left(\max\{\vartheta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon), \vartheta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon)\}\right)_{i\in I_q}, \\ \left(\min\{\frac{1}{\eta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon)}, \frac{1}{\eta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon)}\}\right)_{i\in I_q}, \\ \left(\max\{\zeta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon), \zeta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon)\}\right)_{i\in I_q} \end{array} \right\rangle \right) : \ \varepsilon \in \mathcal{E} \right\} \tag{8}$$

Since $\eta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon), \eta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon) \in [\frac{1}{9}, 9]$ for all $i \in I_q$, the equality $\frac{1}{\max\{\eta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon), \eta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon)\}} = \min\{\frac{1}{\eta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon)}, \frac{1}{\eta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon)}\}$ is valid. So, from the Eqs. (7) and (8), we deduce that $(\widetilde{\mathcal{M}}_1 \cap \widetilde{\mathcal{M}}_2)^c = \widetilde{\mathcal{M}}^c_1 \cup \widetilde{\mathcal{M}}^c_2$. Proceeding with similar calculations, it can be demonstrated that $(\widetilde{\mathcal{M}}_1 \cup \widetilde{\mathcal{M}}_2)^c = \widetilde{\mathcal{M}}^c_1 \cap \widetilde{\mathcal{M}}^c_2$.

By using similar techniques, the properties (i)-(v) can be proved, therefore they are omitted. □

Definition 3.5. For $\psi = \langle (\zeta^i)_{i\in I_q}, (\eta^i)_{i\in I_q}, (\vartheta^i)_{i\in I_q} \rangle$, the score, accuracy and certainty functions of ψ are described respectively as follows:

$$f_S(\psi) = \frac{1}{q} \sum_{i\in I_q} \frac{\zeta^i}{\eta^i \vartheta^i} \tag{9}$$

$$f_A(\psi) = \frac{1}{q} \sum_{i\in I_q} \zeta^i \vartheta^i \tag{10}$$

and

$$f_C(\psi) = \frac{1}{q} \sum_{i\in I_q} \zeta^i \tag{11}$$

To compare two SNMRNs ψ_1 and ψ_2, the steps detailed below can be followed:

(1): if $f_S(\psi_1) > f_S(\psi_2)$ then $\psi_1 \succ \psi_2$,
(2): if $f_S(\psi_1) < f_S(\psi_2)$ then $\psi_1 \prec \psi_2$,
(3): if $f_S(\psi_1) = f_S(\psi_2)$ then
- **(i):** if $f_A(\psi_1) > f_A(\psi_2)$ then $\psi_1 \succ \psi_2$,
- **(ii):** if $f_A(\psi_1) < f_A(\psi_2)$ then $\psi_1 \prec \psi_2$,
- **(iii):** if $f_A(\psi_1) = f_A(\psi_2)$ then
  - **(a):** if $f_C(\psi_1) > f_C(\psi_2)$ then $\psi_1 \succ \psi_2$,
  - **(b):** if $f_C(\psi_1) < f_C(\psi_2)$ then $\psi_1 \prec \psi_2$,
  - **(c):** if $f_C(\psi_1) = f_C(\psi_2)$ then $\psi_1 = \psi_2$.

Example 3.6. If we take $\psi_1 = \langle (\frac{1}{4}, \frac{1}{2}, 1), (3, \frac{3}{4}, 3), (1, 1, \frac{1}{2}) \rangle$ and $\psi_2 = \langle (\frac{1}{2}, 1, 1), (\frac{3}{4}, 3, 3), (1, \frac{1}{2}, \frac{1}{4}) \rangle$ then we get $f_S(\psi_1) = f_S(\psi_2) = \frac{17}{36}$. Since the score values of ψ_1 and ψ_2 are equal, by using the accuracy function, we obtain $f_A(\psi_1) = f_A(\psi_2) = \frac{5}{12}$. By considering the certainty function, we calculate as $f_C(\psi_1) = \frac{17}{12}$ and $f_C(\psi_2) = \frac{3}{4}$. Thus, we have $\psi_1 \succ \psi_2$ since $f_C(\psi_1) > f_C(\psi_2)$.

Definition 3.7. Let $\psi = \langle (\zeta^i)_{i\in I_q}, (\eta^i)_{i\in I_q}, (\vartheta^i)_{i\in I_q} \rangle$, $\psi_1 = \langle (\zeta_1^i)_{i\in I_q}, (\eta_1^i)_{i\in I_q}, (\vartheta_1^i)_{i\in I_q} \rangle$ and $\psi_2 = \langle (\zeta_2^i)_{i\in I_q}, (\eta_2^i)_{i\in I_q}, (\vartheta_2^i)_{i\in I_q} \rangle$ be three SNMRNs and $\omega > 0$ be a real number. Then, some operational laws of SNMRNs are described as follows.

(a):

$$\psi_1 \oplus \psi_2 = \Big\langle \ \big(\tfrac{(1+2\zeta_1^i)(1+2\zeta_2^i)-1}{2}\big)_{i\in I_q}, \big(\tfrac{2\eta_1^i\eta_2^i}{(2+\eta_1^i)(2+\eta_2^i)-\eta_1^i\eta_2^i}\big)_{i\in I_q}, \big(\tfrac{2\vartheta_1^i\vartheta_2^i}{(2+\vartheta_1^i)(2+\vartheta_2^i)-\vartheta_1^i\vartheta_2^i}\big)_{i\in I_q} \ \Big\rangle.$$

(b):

$$\psi_1 \otimes \psi_2 = \Big\langle \ \big(\tfrac{2\zeta_1^i\zeta_2^i}{(2+\zeta_1^i)(2+\zeta_2^i)-\zeta_1^i\zeta_2^i}\big)_{i\in I_q}, \big(\tfrac{(1+2\eta_1^i)(1+2\eta_2^i)-1}{2}\big)_{i\in I_q}, \big(\tfrac{(1+2\vartheta_1^i)(1+2\vartheta_2^i)-1}{2}\big)_{i\in I_q} \ \Big\rangle.$$

(c):

$$\omega\psi = \Big\langle \ \big(\tfrac{(1+2\zeta^i)^\omega-1}{2}\big)_{i\in I_q}, \big(\tfrac{2(\eta^i)^\omega}{(2+\eta^i)^\omega-(\eta^i)^\omega}\big)_{i\in I_q}, \big(\tfrac{2(\vartheta^i)^\omega}{(2+\vartheta^i)^\omega-(\vartheta^i)^\omega}\big)_{i\in I_q} \ \Big\rangle.$$

(d):

$$\psi^\omega = \Big\langle \ \big(\tfrac{2(\zeta^i)^\omega}{(2+\zeta^i)^\omega-(\zeta^i)^\omega}\big)_{i\in I_q}, \big(\tfrac{(1+2\eta^i)^\omega-1}{2}\big)_{i\in I_q}, \big(\tfrac{(1+2\vartheta^i)^\omega-1}{2}\big)_{i\in I_q} \ \Big\rangle.$$

(e):

$$\psi^c = \Big\langle \ (\vartheta^i)_{i\in I_q}, (\tfrac{1}{\eta^i})_{i\in I_q}, (\zeta^i)_{i\in I_q} \ \Big\rangle.$$

Example 3.8. We consider ψ_1 and ψ_2 given in Example 3.6. Then, we obtain

$$\begin{aligned}\psi_1 \oplus \psi_2 &= \left\langle \begin{array}{l} \left(\frac{(1+2\times\frac{1}{4})(1+2\times\frac{1}{2})-1}{2}, \frac{(1+2\times\frac{1}{2})(1+2\times1)-1}{2}, \frac{(1+2\times1)(1+2\times1)-1}{2}\right), \\ \left(\frac{2\times3\times\frac{3}{4}}{(2+3)(2+\frac{3}{4})-3\times\frac{3}{4}}, \frac{2\times\frac{3}{4}\times3}{(2+\frac{3}{4})(2+3)-\frac{3}{4}\times3}, \frac{2\times3\times3}{(2+3)(2+3)-3\times3}\right), \\ \left(\frac{2\times1\times1}{(2+1)(2+1)-1\times1}, \frac{2\times1\times\frac{1}{2}}{(2+1)(2+\frac{1}{1})-1\times\frac{1}{2}}, \frac{2\times\frac{1}{2}\times\frac{1}{4}}{(2+\frac{1}{2})(2+\frac{1}{4})-\frac{1}{2}\times\frac{1}{4}}\right) \end{array} \right\rangle \\ &= \langle(1,\frac{5}{2},4),(\frac{9}{23},\frac{9}{23},\frac{9}{8}),(\frac{1}{4},\frac{1}{7},\frac{1}{22})\rangle\end{aligned}$$

and

$$\psi_1^c = \langle(1,1,\frac{1}{2}),(\frac{1}{3},\frac{4}{3},\frac{1}{3}),(\frac{1}{4},\frac{1}{2},1)\rangle.$$

Theorem 3.9. *Let* $\psi = \langle(\zeta^i)_{i\in I_q},(\eta^i)_{i\in I_q},(\vartheta^i)_{i\in I_q})\rangle$, $\psi_1 = \langle(\zeta_1^i)_{i\in I_q},(\eta_1^i)_{i\in I_q},(\vartheta_1^i)_{i\in I_q})\rangle$ *and* $\psi_2 = \langle(\zeta_2^i)_{i\in I_q},(\eta_2^i)_{i\in I_q},(\vartheta_2^i)_{i\in I_q})\rangle$ *be three SNMRNs and* $\omega,\omega_1,\omega_2 > 0$ *be real numbers, then the following properties are valid.*

- **(i):** $\psi_1 \bullet \psi_2 = \psi_2 \bullet \psi_1$ *for each* $\bullet \in \{\oplus,\otimes\}$.
- **(ii):** $\omega(\psi_1 \oplus \psi_2) = \omega\psi_1 \oplus \omega\psi_2$.
- **(iii):** $(\psi_1 \otimes \psi_2)^\omega = \psi_1^\omega \otimes \psi_2^\omega$.
- **(iv):** $\omega_1\psi \oplus \omega_2\psi = (\omega_1+\omega_2)\psi$.
- **(v):** $\psi^{\omega_1} \otimes \psi^{\omega_2} = \psi^{\omega_1+\omega_2}$.
- **(vi):** $(\psi_1 \bullet \psi_2)^c = \psi_1^c \blacklozenge \psi_2^c$ *for each* $\bullet, \blacklozenge \in \{\oplus,\otimes\}$ *and* $\bullet \neq \blacklozenge$.

Proof. Considering Definition 3.7, they can be achieved with simple calculations and so omitted. □

4. Correlation Coefficients for SNMRSs

In this section, we propose some types of correlation coefficients for the SNMRSs, which can be applied to real-life problems.

Suppose that $\widetilde{\mathcal{M}}_1 = \{\langle\varepsilon_j,(\zeta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon_j))_{i\in I_q},(\eta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon_j))_{i\in I_q},(\vartheta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon_j))_{i\in I_q}\rangle : \varepsilon_j \in \mathcal{E}\}$ and $\widetilde{\mathcal{M}}_2 = \{\langle\varepsilon_j,(\zeta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon_j))_{i\in I_q},(\eta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon_j))_{i\in I_q},(\vartheta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon_j))_{i\in I_q}\rangle : \varepsilon_j \in \mathcal{E}\}$ be any two q-dimension SNMRSs in the universal set $\mathcal{E} = \{\varepsilon_j : j = 1,2,...,m\}$ where $\zeta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon_j),\eta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon_j),\vartheta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon_j),\zeta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon_j),\eta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon_j),\vartheta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon_j) \in [\frac{1}{9},9]$, $0 < \zeta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon_j)\vartheta^i_{\widetilde{\mathcal{M}}_1}(\varepsilon_j) \leq 1$ and $0 < \zeta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon_j)\vartheta^i_{\widetilde{\mathcal{M}}_2}(\varepsilon_j) \leq 1$ $(i = 1,2,...,q)$ for each $\varepsilon_j \in \mathcal{E}$. The informational energies of SNMRSs $\widetilde{\mathcal{M}}_1$ and $\widetilde{\mathcal{M}}_2$ are defined as

$$\mathfrak{T}(\widetilde{\mathcal{M}}_1) = \frac{1}{q}\sum_{i\in I_q}\sum_{j=1}^{m}\Big(\frac{\zeta^i_{\mathcal{M}_1}(\varepsilon_j)}{2(1+\zeta^i_{\mathcal{M}_1}(\varepsilon_j))} + \frac{\eta^i_{\mathcal{M}_1}(\varepsilon_j)}{2(1+\eta^i_{\mathcal{M}_1}(\varepsilon_j))} + \frac{\vartheta^i_{\mathcal{M}_1}(\varepsilon_j)}{2(1+\vartheta^i_{\mathcal{M}_1}(\varepsilon_j))}\Big) \tag{12}$$

and

$$\mathfrak{T}(\widetilde{\mathcal{M}}_2) = \frac{1}{q}\sum_{i\in I_q}\sum_{j=1}^{m}\Big(\frac{\zeta^i_{\mathcal{M}_2}(\varepsilon_j)}{2(1+\zeta^i_{\mathcal{M}_2}(\varepsilon_j))} + \frac{\eta^i_{\mathcal{M}_2}(\varepsilon_j)}{2(1+\eta^i_{\mathcal{M}_2}(\varepsilon_j))} + \frac{\vartheta^i_{\mathcal{M}_2}(\varepsilon_j)}{2(1+\vartheta^i_{\mathcal{M}_2}(\varepsilon_j))}\Big) \tag{13}$$

The correlation of the SNMRSs $\widetilde{\mathcal{M}}_1$ and $\widetilde{\mathcal{M}}_2$ is described as

$$\mathfrak{C}(\widetilde{\mathcal{M}}_1, \widetilde{\mathcal{M}}_2) = \frac{1}{q} \sum_{i \in I_q} \sum_{j=1}^{m} \left(\begin{array}{c} \frac{2\zeta^i_{\mathcal{M}_1}(\varepsilon_j)\zeta^i_{\mathcal{M}_2}(\varepsilon_j)}{(2+\zeta^i_{\mathcal{M}_1}(\varepsilon_j))(2+\zeta^i_{\mathcal{M}_2}(\varepsilon_j))-\zeta^i_{\mathcal{M}_1}(\varepsilon_j)\zeta^i_{\mathcal{M}_2}(\varepsilon_j)} \\ +\frac{2\eta^i_{\mathcal{M}_1}(\varepsilon_j)\eta^i_{\mathcal{M}_2}(\varepsilon_j)}{(2+\eta^i_{\mathcal{M}_1}(\varepsilon_j))(2+\eta^i_{\mathcal{M}_2}(\varepsilon_j))-\eta^i_{\mathcal{M}_1}(\varepsilon_j)\eta^i_{\mathcal{M}_2}(\varepsilon_j)} \\ +\frac{2\vartheta^i_{\mathcal{M}_1}(\varepsilon_j)\vartheta^i_{\mathcal{M}_2}(\varepsilon_j)}{(2+\vartheta^i_{\mathcal{M}_1}(\varepsilon_j))(2+\vartheta^i_{\mathcal{M}_2}(\varepsilon_j))-\vartheta^i_{\mathcal{M}_1}(\varepsilon_j)\vartheta^i_{\mathcal{M}_2}(\varepsilon_j)} \end{array} \right) \tag{14}$$

It is clear that the Eq. (14) has the following axioms.

(1) $\mathfrak{C}(\widetilde{\mathcal{M}}_1, \widetilde{\mathcal{M}}_1) = \mathfrak{T}(\widetilde{\mathcal{M}}_1)$.
(2) $\mathfrak{C}(\widetilde{\mathcal{M}}_1, \widetilde{\mathcal{M}}_2) = \mathfrak{C}(\widetilde{\mathcal{M}}_2, \widetilde{\mathcal{M}}_1)$.

The correlation coefficients between two SNMRs $\widetilde{\mathcal{M}}_1$ and $\widetilde{\mathcal{M}}_2$ are defined as follows.

Definition 4.1. Let $\widetilde{\mathcal{M}}_1, \widetilde{\mathcal{M}}_2 \in SNMRS(\mathcal{E}, q)$. Then, the (type-1) correlation coefficient between $\widetilde{\mathcal{M}}_1$ and $\widetilde{\mathcal{M}}_2$ is denoted and defined as

$$\begin{aligned} \kappa_1(\mathcal{M}_1, \mathcal{M}_2) &= \frac{\mathfrak{C}(\widetilde{\mathcal{M}}_1, \widetilde{\mathcal{M}}_2)}{\sqrt{\mathfrak{T}(\widetilde{\mathcal{M}}_1) \cdot \mathfrak{T}(\widetilde{\mathcal{M}}_2)}} \\ &= \frac{\frac{1}{q} \sum_{i \in I_q} \sum_{j=1}^{m} \left(\begin{array}{c} \frac{2\zeta^i_{\mathcal{M}_1}(\varepsilon_j)\zeta^i_{\mathcal{M}_2}(\varepsilon_j)}{(2+\zeta^i_{\mathcal{M}_1}(\varepsilon_j))(2+\zeta^i_{\mathcal{M}_2}(\varepsilon_j))-\zeta^i_{\mathcal{M}_1}(\varepsilon_j)\zeta^i_{\mathcal{M}_2}(\varepsilon_j)} \\ +\frac{2\eta^i_{\mathcal{M}_1}(\varepsilon_j)\eta^i_{\mathcal{M}_2}(\varepsilon_j)}{(2+\eta^i_{\mathcal{M}_1}(\varepsilon_j))(2+\eta^i_{\mathcal{M}_2}(\varepsilon_j))-\eta^i_{\mathcal{M}_1}(\varepsilon_j)\eta^i_{\mathcal{M}_2}(\varepsilon_j)} \\ +\frac{2\vartheta^i_{\mathcal{M}_1}(\varepsilon_j)\vartheta^i_{\mathcal{M}_2}(\varepsilon_j)}{(2+\vartheta^i_{\mathcal{M}_1}(\varepsilon_j))(2+\vartheta^i_{\mathcal{M}_2}(\varepsilon_j))-\vartheta^i_{\mathcal{M}_1}(\varepsilon_j)\vartheta^i_{\mathcal{M}_2}(\varepsilon_j)} \end{array} \right)}{\sqrt{\frac{1}{q} \sum_{i \in I_q} \sum_{j=1}^{m} \left(\begin{array}{c} \frac{\zeta^i_{\mathcal{M}_1}(\varepsilon_j)}{2(1+\zeta^i_{\mathcal{M}_1}(\varepsilon_j))} \\ +\frac{\eta^i_{\mathcal{M}_1}(\varepsilon_j)}{2(1+\eta^i_{\mathcal{M}_1}(\varepsilon_j))} \\ +\frac{\vartheta^i_{\mathcal{M}_1}(\varepsilon_j)}{2(1+\vartheta^i_{\mathcal{M}_1}(\varepsilon_j))} \end{array} \right)} \times \sqrt{\frac{1}{q} \sum_{i \in I_q} \sum_{j=1}^{m} \left(\begin{array}{c} \frac{\zeta^i_{\mathcal{M}_2}(\varepsilon_j)}{2(1+\zeta^i_{\mathcal{M}_2}(\varepsilon_j))} \\ +\frac{\eta^i_{\mathcal{M}_2}(\varepsilon_j)}{2(1+\eta^i_{\mathcal{M}_2}(\varepsilon_j))} \\ +\frac{\vartheta^i_{\mathcal{M}_2}(\varepsilon_j)}{2(1+\vartheta^i_{\mathcal{M}_2}(\varepsilon_j))} \end{array} \right)}} \end{aligned} \tag{15}$$

Theorem 4.2. *Let $\widetilde{\mathcal{M}}_1, \widetilde{\mathcal{M}}_2 \in SNMRS(\mathcal{E}, q)$. For the (type-1) correlation coefficient between $\widetilde{\mathcal{M}}_1$ and $\widetilde{\mathcal{M}}_2$, the following properties are satisfied.*

(i): $\widetilde{\mathcal{M}}_1 = \widetilde{\mathcal{M}}_2 \Rightarrow \kappa_1(\mathcal{M}_1, \mathcal{M}_2) = 1$.
(ii): $\kappa_1(\mathcal{M}_1, \mathcal{M}_2) = \kappa_1(\mathcal{M}_2, \mathcal{M}_1)$.
(iii): $\frac{1}{9} \leq \kappa_1(\mathcal{M}_1, \mathcal{M}_2) \leq 9$.

Proof. The proofs of (i) and (ii) are obvious from the Eq. (15). Let us prove the assertion (iii).
(iii): Let $\widetilde{\mathcal{M}}_1, \widetilde{\mathcal{M}}_2 \in SNMRS(\mathcal{E}, q)$. Since for each $\varepsilon_j \in \mathcal{E}$,

$$\frac{1}{9} \leq \zeta^i_{\mathcal{M}_1}(\varepsilon_j) \leq 9 \quad \forall i \in I_q \tag{16}$$

it implies that

$$\frac{1}{20} \leq \frac{1}{2(1+\zeta^i_{\mathcal{M}_1}(\varepsilon_j))} \leq \frac{9}{20} \quad \forall i \in I_q \tag{17}$$

Hence, we obtain

$$\frac{(\zeta^i_{\mathcal{M}_1}(\varepsilon_j))^2}{20} \leq \frac{(\zeta^i_{\mathcal{M}_1}(\varepsilon_j))^2}{2(1+\zeta^i_{\mathcal{M}_1}(\varepsilon_j))} \leq \frac{9(\zeta^i_{\mathcal{M}_1}(\varepsilon_j))^2}{20} \quad \forall i \in I_q \tag{18}$$

for each $\varepsilon_j \in \mathcal{E}$. Likewise, for the indeterminacy-membership and falsity-membership, we obtain the following inequalities:

$$\frac{(\eta^i_{\mathcal{M}_1}(\varepsilon_j))^2}{20} \leq \frac{(\eta^i_{\mathcal{M}_1}(\varepsilon_j))^2}{2(1+\eta^i_{\mathcal{M}_1}(\varepsilon_j))} \leq \frac{9(\eta^i_{\mathcal{M}_1}(\varepsilon_j))^2}{20} \quad \forall i \in I_q \tag{19}$$

and

$$\frac{(\vartheta^i_{\mathcal{M}_1}(\varepsilon_j))^2}{20} \leq \frac{(\vartheta^i_{\mathcal{M}_1}(\varepsilon_j))^2}{2(1+\vartheta^i_{\mathcal{M}_1}(\varepsilon_j))} \leq \frac{9(\vartheta^i_{\mathcal{M}_1}(\varepsilon_j))^2}{20} \quad \forall i \in I_q \tag{20}$$

for each $\varepsilon_j \in \mathcal{E}$. By adding Eqs. (18), (19) and (20), we have

$$\begin{aligned}\frac{(\zeta^i_{\mathcal{M}_1}(\varepsilon_j))^2+(\eta^i_{\mathcal{M}_1}(\varepsilon_j))^2+(\vartheta^i_{\mathcal{M}_1}(\varepsilon_j))^2}{20} &\leq \frac{(\zeta^i_{\mathcal{M}_1}(\varepsilon_j))^2}{2(1+\zeta^i_{\mathcal{M}_1}(\varepsilon_j))}+\frac{(\eta^i_{\mathcal{M}_1}(\varepsilon_j))^2}{2(1+\eta^i_{\mathcal{M}_1}(\varepsilon_j))}+\frac{(\vartheta^i_{\mathcal{M}_1}(\varepsilon_j))^2}{2(1+\vartheta^i_{\mathcal{M}_1}(\varepsilon_j))}\\ &\leq \frac{9((\zeta^i_{\mathcal{M}_1}(\varepsilon_j))^2+(\eta^i_{\mathcal{M}_1}(\varepsilon_j))^2+(\vartheta^i_{\mathcal{M}_1}(\varepsilon_j))^2)}{20} \quad \forall i \in I_q \end{aligned}\tag{21}$$

for each $\varepsilon_j \in \mathcal{E}$. By using Eq. (12), we have the following inequality for informational energy of SNMRS $\mathcal{M}_1$.

$$\frac{1}{20q}\sum_{i\in I_q}\sum_{j=1}^{m}\begin{pmatrix}(\zeta^i_{\mathcal{M}_1}(\varepsilon_j))^2\\+(\eta^i_{\mathcal{M}_1}(\varepsilon_j))^2\\+(\vartheta^i_{\mathcal{M}_1}(\varepsilon_j))^2\end{pmatrix} \leq \mathfrak{T}(\widetilde{\mathcal{M}}_1) \leq \frac{9}{20q}\sum_{i\in I_q}\sum_{j=1}^{m}\begin{pmatrix}(\zeta^i_{\mathcal{M}_1}(\varepsilon_j))^2\\+(\eta^i_{\mathcal{M}_1}(\varepsilon_j))^2\\+(\vartheta^i_{\mathcal{M}_1}(\varepsilon_j))^2\end{pmatrix} \tag{22}$$

Similarly, we can obtain the following inequality for informational energy of SNMRS $\mathcal{M}_2$.

$$\frac{1}{20q}\sum_{i\in I_q}\sum_{j=1}^{m}\begin{pmatrix}(\zeta^i_{\mathcal{M}_2}(\varepsilon_j))^2\\+(\eta^i_{\mathcal{M}_2}(\varepsilon_j))^2\\+(\vartheta^i_{\mathcal{M}_2}(\varepsilon_j))^2\end{pmatrix} \leq \mathfrak{T}(\widetilde{\mathcal{M}}_2) \leq \frac{9}{20q}\sum_{i\in I_q}\sum_{j=1}^{m}\begin{pmatrix}(\zeta^i_{\mathcal{M}_2}(\varepsilon_j))^2\\+(\eta^i_{\mathcal{M}_2}(\varepsilon_j))^2\\+(\vartheta^i_{\mathcal{M}_2}(\varepsilon_j))^2\end{pmatrix} \tag{23}$$

On the other hand, we can easily deduce that

$$\frac{\zeta^i_{\mathcal{M}_1}(\varepsilon_j)\zeta^i_{\mathcal{M}_2}(\varepsilon_j)}{20} \leq \frac{2\zeta^i_{\mathcal{M}_1}(\varepsilon_j)\zeta^i_{\mathcal{M}_2}(\varepsilon_j)}{(2+\zeta^i_{\mathcal{M}_1}(\varepsilon_j))(2+\zeta^i_{\mathcal{M}_2}(\varepsilon_j))-\zeta^i_{\mathcal{M}_1}(\varepsilon_j)\zeta^i_{\mathcal{M}_2}(\varepsilon_j)} \leq \frac{9\zeta^i_{\mathcal{M}_1}(\varepsilon_j)\zeta^i_{\mathcal{M}_2}(\varepsilon_j)}{20} \quad \forall i \in I_q \tag{24}$$

for each $\varepsilon_j \in \mathcal{E}$, and so

$$\begin{aligned}\frac{1}{20q}\sum_{i\in I_q}\sum_{j=1}^{m}\zeta^i_{\mathcal{M}_1}(\varepsilon_j)\zeta^i_{\mathcal{M}_2}(\varepsilon_j) &\leq \frac{1}{20q}\sum_{i\in I_q}\sum_{j=1}^{m}\frac{2\zeta^i_{\mathcal{M}_1}(\varepsilon_j)\zeta^i_{\mathcal{M}_2}(\varepsilon_j)}{(2+\zeta^i_{\mathcal{M}_1}(\varepsilon_j))(2+\zeta^i_{\mathcal{M}_2}(\varepsilon_j))-\zeta^i_{\mathcal{M}_1}(\varepsilon_j)\zeta^i_{\mathcal{M}_2}(\varepsilon_j)}\\ &\leq \frac{9}{20q}\sum_{i\in I_q}\sum_{j=1}^{m}\zeta^i_{\mathcal{M}_1}(\varepsilon_j)\zeta^i_{\mathcal{M}_2}(\varepsilon_j)\end{aligned}\tag{25}$$

Likewise, for the indeterminacy-membership and falsity-membership, the following results can be obtained:

$$\frac{1}{20q}\sum_{i\in I_q}\sum_{j=1}^{m}\eta^i_{\mathcal{M}_1}(\varepsilon_j)\eta^i_{\mathcal{M}_2}(\varepsilon_j) \leq \frac{1}{20q}\sum_{i\in I_q}\sum_{j=1}^{m}\frac{2\eta^i_{\mathcal{M}_1}(\varepsilon_j)\eta^i_{\mathcal{M}_2}(\varepsilon_j)}{(2+\eta^i_{\mathcal{M}_1}(\varepsilon_j))(2+\eta^i_{\mathcal{M}_2}(\varepsilon_j))-\eta^i_{\mathcal{M}_1}(\varepsilon_j)\eta^i_{\mathcal{M}_2}(\varepsilon_j)} \leq \frac{9}{20q}\sum_{i\in I_q}\sum_{j=1}^{m}\eta^i_{\mathcal{M}_1}(\varepsilon_j)\eta^i_{\mathcal{M}_2}(\varepsilon_j) \quad (26)$$

and

$$\frac{1}{20q}\sum_{i\in I_q}\sum_{j=1}^{m}\vartheta^i_{\mathcal{M}_1}(\varepsilon_j)\vartheta^i_{\mathcal{M}_2}(\varepsilon_j) \leq \frac{1}{20q}\sum_{i\in I_q}\sum_{j=1}^{m}\frac{2\vartheta^i_{\mathcal{M}_1}(\varepsilon_j)\vartheta^i_{\mathcal{M}_2}(\varepsilon_j)}{(2+\vartheta^i_{\mathcal{M}_1}(\varepsilon_j))(2+\vartheta^i_{\mathcal{M}_2}(\varepsilon_j))-\vartheta^i_{\mathcal{M}_1}(\varepsilon_j)\vartheta^i_{\mathcal{M}_2}(\varepsilon_j)} \leq \frac{9}{20q}\sum_{i\in I_q}\sum_{j=1}^{m}\vartheta^i_{\mathcal{M}_1}(\varepsilon_j)\vartheta^i_{\mathcal{M}_2}(\varepsilon_j) \quad (27)$$

So, by using Eq. (15), we obtain

$$\frac{\frac{1}{20q}\xi}{\frac{9}{20q}(\sqrt{\mu}\times\sqrt{\nu})} \leq \kappa_1(\mathcal{M}_1,\mathcal{M}_2) \leq \frac{\frac{9}{20q}\xi}{\frac{1}{20q}(\sqrt{\mu}\times\sqrt{\nu})} \quad (28)$$

and so

$$\frac{1}{9}\frac{\xi}{\sqrt{\mu}\times\sqrt{\nu}} \leq \kappa_1(\mathcal{M}_1,\mathcal{M}_2) \leq 9\frac{\xi}{\sqrt{\mu}\times\sqrt{\nu}} \quad (29)$$

where

$$\xi=\sum_{i\in I_q}\sum_{j=1}^{m}\begin{pmatrix}\zeta^i_{\mathcal{M}_1}(\varepsilon_j)\zeta^i_{\mathcal{M}_2}(\varepsilon_j)\\ +\eta^i_{\mathcal{M}_1}(\varepsilon_j)\eta^i_{\mathcal{M}_2}(\varepsilon_j)\\ +\vartheta^i_{\mathcal{M}_1}(\varepsilon_j)\vartheta^i_{\mathcal{M}_2}(\varepsilon_j)\end{pmatrix},\ \mu=\sum_{i\in I_q}\sum_{j=1}^{m}\begin{pmatrix}(\zeta^i_{\mathcal{M}_1}(\varepsilon_j))^2\\ +(\eta^i_{\mathcal{M}_1}(\varepsilon_j))^2\\ +(\vartheta^i_{\mathcal{M}_1}(\varepsilon_j))^2\end{pmatrix},\ \nu=\sum_{i\in I_q}\sum_{j=1}^{m}\begin{pmatrix}(\zeta^i_{\mathcal{M}_2}(\varepsilon_j))^2\\ +(\eta^i_{\mathcal{M}_2}(\varepsilon_j))^2\\ +(\vartheta^i_{\mathcal{M}_2}(\varepsilon_j))^2\end{pmatrix}$$

Thus, we conclude that $\frac{1}{9} \leq \kappa_1(\mathcal{M}_1,\mathcal{M}_2) \leq 9$. □

Definition 4.3. Let $\widetilde{\mathcal{M}}_1,\widetilde{\mathcal{M}}_2 \in SNMRS(\mathcal{E},q)$. Then, the (type-2) correlation coefficient between $\widetilde{\mathcal{M}}_1$ and $\widetilde{\mathcal{M}}_2$ is denoted and defined as

$$\begin{aligned}\kappa_2(\mathcal{M}_1,\mathcal{M}_2) &= \frac{\mathfrak{C}(\widetilde{\mathcal{M}}_1,\widetilde{\mathcal{M}}_2)}{\max\{\mathfrak{T}(\widetilde{\mathcal{M}}_1),\mathfrak{T}(\widetilde{\mathcal{M}}_2)\}}\\ &= \frac{\frac{1}{q}\sum_{i\in I_q}\sum_{j=1}^{m}\begin{pmatrix}\frac{2\zeta^i_{\mathcal{M}_1}(\varepsilon_j)\zeta^i_{\mathcal{M}_2}(\varepsilon_j)}{(2+\zeta^i_{\mathcal{M}_1}(\varepsilon_j))(2+\zeta^i_{\mathcal{M}_2}(\varepsilon_j))-\zeta^i_{\mathcal{M}_1}(\varepsilon_j)\zeta^i_{\mathcal{M}_2}(\varepsilon_j)}\\ +\frac{2\eta^i_{\mathcal{M}_1}(\varepsilon_j)\eta^i_{\mathcal{M}_2}(\varepsilon_j)}{(2+\eta^i_{\mathcal{M}_1}(\varepsilon_j))(2+\eta^i_{\mathcal{M}_2}(\varepsilon_j))-\eta^i_{\mathcal{M}_1}(\varepsilon_j)\eta^i_{\mathcal{M}_2}(\varepsilon_j)}\\ +\frac{2\vartheta^i_{\mathcal{M}_1}(\varepsilon_j)\vartheta^i_{\mathcal{M}_2}(\varepsilon_j)}{(2+\vartheta^i_{\mathcal{M}_1}(\varepsilon_j))(2+\vartheta^i_{\mathcal{M}_2}(\varepsilon_j))-\vartheta^i_{\mathcal{M}_1}(\varepsilon_j)\vartheta^i_{\mathcal{M}_2}(\varepsilon_j)}\end{pmatrix}}{\max\left\{\frac{1}{q}\sum_{i\in I_q}\sum_{j=1}^{m}\begin{pmatrix}\frac{\zeta^i_{\mathcal{M}_1}(\varepsilon_j)}{2(1+\zeta^i_{\mathcal{M}_1}(\varepsilon_j))}\\ +\frac{\eta^i_{\mathcal{M}_1}(\varepsilon_j)}{2(1+\eta^i_{\mathcal{M}_1}(\varepsilon_j))}\\ +\frac{\vartheta^i_{\mathcal{M}_1}(\varepsilon_j)}{2(1+\vartheta^i_{\mathcal{M}_1}(\varepsilon_j))}\end{pmatrix},\frac{1}{q}\sum_{i\in I_q}\sum_{j=1}^{m}\begin{pmatrix}\frac{\zeta^i_{\mathcal{M}_2}(\varepsilon_j)}{2(1+\zeta^i_{\mathcal{M}_2}(\varepsilon_j))}\\ +\frac{\eta^i_{\mathcal{M}_2}(\varepsilon_j)}{2(1+\eta^i_{\mathcal{M}_2}(\varepsilon_j))}\\ +\frac{\vartheta^i_{\mathcal{M}_2}(\varepsilon_j)}{2(1+\vartheta^i_{\mathcal{M}_2}(\varepsilon_j))}\end{pmatrix}\right\}}\end{aligned} \quad (30)$$

Theorem 4.4. *Let $\widetilde{\mathcal{M}}_1, \widetilde{\mathcal{M}}_2 \in SNMRS(\mathcal{E}, q)$. For the (type-2) correlation coefficient between $\widetilde{\mathcal{M}}_1$ and $\widetilde{\mathcal{M}}_2$, the following properties are valid.*

(i): $\widetilde{\mathcal{M}}_1 = \widetilde{\mathcal{M}}_2 \Rightarrow \kappa_2(\mathcal{M}_1, \mathcal{M}_2) = 1$.

(ii): $\kappa_2(\mathcal{M}_1, \mathcal{M}_2) = \kappa_2(\mathcal{M}_2, \mathcal{M}_1)$.

(iii): $\frac{1}{9} \leq \kappa_2(\mathcal{M}_1, \mathcal{M}_2) \leq 9$.

Proof. They can be demonstrated similar to the proof of Theorem 4.2. □

5. An Application of Correlation Coefficients of SNMRSs in Medical Pattern Recognition

In order to demonstrate the application of the proposed correlation coefficients, we consider the following medical pattern recognition problem under the SNMRS environment.

Example 5.1. Scientists divided coronaviruses into four sub-groupings, called alpha, beta, gamma and delta. Five of beta viruses can infect people: OC43, HKU1, MERS-CoV, SARS-CoV and SARS-CoV-2 (COVID-19). Specially, we focus on three dangerous types of beta viruses: (1) MERS-CoV, (2) SARS-CoV and (3) SARS-CoV-2. We consider the patterns of MERS-CoV, SARS-CoV and SARS-CoV-2 based on the symptoms which are specified by experts as sequences of truth-membership information, indeterminacy-membership information and falsity-membership information (they are scaled between $\frac{1}{9}$ and 9) as a result of investigation and experiments. Suppose that the patterns of MERS-CoV, SARS-CoV and SARS-CoV-2 for the symptoms $\varepsilon_1, \varepsilon_2$ and ε_3 (i.e., $\mathcal{E} = \{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$) are given as follows respectively.

$$\mathcal{P}_1 = \left\{ \begin{array}{l} (\varepsilon_1, \langle (2,4,5,7,8), (\frac{3}{5}, 1, 2, \frac{5}{2}, 6), (\frac{1}{3}, \frac{1}{6}, \frac{1}{5}, \frac{1}{8}, \frac{1}{9}) \rangle), \\ (\varepsilon_2, \langle (\frac{2}{9}, \frac{1}{2}, 1, 3, 4), (\frac{1}{4}, 2, \frac{3}{4}, 1, 5), (\frac{1}{8}, 1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}) \rangle), \\ (\varepsilon_3, \langle (\frac{1}{9}, \frac{1}{3}, \frac{1}{2}, 1, 4), (\frac{1}{5}, 7, \frac{2}{3}, \frac{1}{5}, \frac{1}{2}), (9, 3, 2, 1, \frac{1}{6}) \rangle) \end{array} \right\},$$

$$\mathcal{P}_2 = \left\{ \begin{array}{l} (\varepsilon_1, \langle (\frac{1}{8}, \frac{1}{2}, 2, 3, 7), (3, \frac{1}{2}, 1, \frac{1}{5}, \frac{1}{7}), (8, 1, \frac{1}{6}, \frac{1}{7}, \frac{1}{6}) \rangle), \\ (\varepsilon_2, \langle (\frac{1}{3}, \frac{1}{2}, 1, 4, 6), (\frac{1}{3}, \frac{1}{4}, \frac{1}{8}, 2, \frac{1}{3}), (6, 2, \frac{1}{2}, \frac{1}{4}, \frac{1}{6}) \rangle), \\ (\varepsilon_3, \langle (\frac{1}{3}, \frac{1}{2}, 2, 3, 9), (6, \frac{1}{2}, \frac{1}{3}, 4, \frac{1}{7}), (1, 2, \frac{1}{4}, \frac{1}{5}, \frac{1}{9}) \rangle) \end{array} \right\}$$

and

$$\mathcal{P}_3 = \left\{ \begin{array}{l} (\varepsilon_1, \langle (1,4,5,6,9), (\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, 2, 1), (\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}) \rangle), \\ (\varepsilon_2, \langle (\frac{1}{2}, 2, 3, 5, 8), (\frac{1}{4}, \frac{1}{9}, 1, 3, 2), (\frac{1}{6}, \frac{1}{2}, \frac{1}{4}, \frac{1}{7}, \frac{1}{8}) \rangle), \\ (\varepsilon_3, \langle (\frac{1}{3}, \frac{1}{2}, 1, 2, 3), (\frac{1}{4}, \frac{1}{7}, 1, \frac{1}{2}, 1), (2, 2, \frac{1}{6}, \frac{1}{6}, \frac{1}{9}) \rangle) \end{array} \right\}.$$

Experts (or doctors) often come across slightly different versions (i.e., unknown patterns) of viruses: MERS-CoV, SARS-CoV and SARS-CoV-2. Suppose that an expert come across an unknown pattern $\mathcal{P}$ which will be reorganized as an SNMRS in $\mathcal{E}$, where

$$\mathcal{P} = \left\{ \begin{array}{l} (\varepsilon_1, \langle (2,4,5,7,9), (\frac{1}{3}, \frac{1}{5}, \frac{1}{8}, 2, 2), (\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{7}, \frac{1}{9}) \rangle), \\ (\varepsilon_2, \langle (\frac{1}{4}, \frac{1}{2}, 3, 4, 9), (\frac{1}{2}, \frac{1}{9}, 1, 3, 4), (\frac{1}{4}, \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{9}) \rangle), \\ (\varepsilon_3, \langle (\frac{1}{5}, \frac{1}{3}, 1, 2, 4), (\frac{1}{4}, \frac{1}{8}, 1, \frac{1}{2}, 1), (1, 2, \frac{1}{6}, \frac{1}{6}, \frac{1}{9}) \rangle) \end{array} \right\}.$$

The motivation of this problem is to classify the pattern $\mathcal{P}$ in one of the classes $\mathcal{P}_1, \mathcal{P}_2$ and $\mathcal{P}_3$. For this purpose, the correlation coefficients κ_1 and κ_2 described in Eqs. (4.1) and (4.3) can be used.
By calculating the the (type-1) correlation coefficients between $\mathcal{P}$ and $\mathcal{P}_k$ $(k = 1, 2, 3)$, we can get

$$\kappa_1(\mathcal{P}_1, \mathcal{P}) = 2.507027, \kappa_1(\mathcal{P}_2, \mathcal{P}) = 2.078367 \text{ and } \kappa_1(\mathcal{P}_3, \mathcal{P}) = 2.727818$$

As a result of (type-1) correlation coefficients, the ranking of $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{P}_3$ is obtained as $\mathcal{P}_2 \prec \mathcal{P}_1 \prec \mathcal{P}_3$, and thus it is most convenient to classify the pattern $\mathcal{P}$ with the pattern $\mathcal{P}_3$ (SARS-CoV-2).
Similarly, by using Eq. (4.3), we have the following (type-2) correlation coefficients between $\mathcal{P}$ and $\mathcal{P}_k$ $(k = 1, 2, 3)$

$$\kappa_2(\mathcal{P}_1, \mathcal{P}) = 2.365829, \kappa_2(\mathcal{P}_2, \mathcal{P}) = 2.032161 \text{ and } \kappa_2(\mathcal{P}_3, \mathcal{P}) = 2.718598$$

Consequently, the ranking of these three patterns is $\mathcal{P}_2 \prec \mathcal{P}_1 \prec \mathcal{P}_3$, and therefore it is most convenient to classify the pattern $\mathcal{P}$ with the pattern $\mathcal{P}_3$.

Comparison and Discussion: In 2018, Garg [14] proposed new correlation coefficients for IMSs and presented their applications in handling decision making. For Examples 1, 2 and 3 in Section 4 of [14], if we assume the 1-dimension simplified neutrosophic multiplicative refined value (i.e., simplified neutrosophic multiplicative value) $\langle \rho, 1, \sigma \rangle$ instead of the priority value $\langle \rho, \sigma \rangle$ of alternative under the IMS environment then the proposed (type-1 and type-2) correlation coefficients (in this paper) can be applied to these problems and the comparison results in Table 2 are obtained.

Table 2. Results of comparing the proposed ones with the correlation coefficients of IMSs

Problems	Ranking for correlation coefficients of IMSs	Ranking for correlation coefficients of SNMRSs
Example 1 in [14]	$K_1(X_4, X^*) > K_1(X_1, X^*) > K_1(X_3, X^*) > K_1(X_2, X^*)$	$\kappa_1(X_4, X^*) > \kappa_1(X_1, X^*) > \kappa_1(X_3, X^*) > \kappa_1(X_2, X^*)$
	$K_2(X_4, X^*) > K_2(X_1, X^*) > K_2(X_3, X^*) > K_2(X_2, X^*)$	$\kappa_2(X_4, X^*) > \kappa_2(X_1, X^*) > \kappa_2(X_3, X^*) > \kappa_2(X_2, X^*)$
Example 2 in [14]	$K_1(C_2, P) > K_1(C_1, P) > K_1(C_3, P)$	$\kappa_1(C_2, P) > \kappa_1(C_1, P) > \kappa_1(C_3, P)$
	$K_2(C_2, P) > K_2(C_1, P) > K_2(C_3, P)$	$\kappa_2(C_2, P) > \kappa_2(C_1, P) > \kappa_2(C_3, P)$
Example 3 in [14]	$K_1(P, Q_2) > K_1(P, Q_1) > K_1(P, Q_5) > K_1(P, Q_3) > K_1(P, Q_4)$	$\kappa_1(P, Q_2) > \kappa_1(P, Q_1) > \kappa_1(P, Q_5) > \kappa_1(P, Q_3) > \kappa_1(P, Q_4)$
	$K_2(P, Q_2) > K_2(P, Q_5) > K_2(P, Q_1) > K_2(P, Q_3) > K_2(P, Q_4)$	$\kappa_2(P, Q_2) > \kappa_2(P, Q_5) > \kappa_2(P, Q_1) > \kappa_2(P, Q_3) > \kappa_2(P, Q_4)$

In 2016, Broumi and Deli [7] studied the correlation measure of (simplified) neutrosophic refined sets and applied them to the problems of medical diagnosis and pattern recognition. For Examples 4.1 and 4.2 in Section 4 of [7], considering the matches between $0 - 1$ and $\frac{1}{9} - 9$ scales given in Table 1 in the Introduction for the priority value $\langle (T^1, T^2, ..., T^p), (I^1, I^2, ..., I^p), (F^1, F^2, ..., F^p) \rangle$ of alternative under the (simplified) NRS environment, we can apply the proposed (type-1 and type-2) correlation coefficients to these problems and the comparison results are presented in Table 3.

Table 3. Results of comparing the proposed ones with the correlation coefficients of NRSs

Problems	Ranking for correlation coefficient of NRSs	Ranking for correlation coefficients of SNMRSs
Example 4.1 in [7]	$\rho_{NRS}(P_1, D_2) > \rho_{NRS}(P_1, D_3) > \rho_{NRS}(P_1, D_4) > \rho_{NRS}(P_1, D_1)$	$\kappa_1(P_1, D_2) > \kappa_1(P_1, D_3) > \kappa_1(P_1, D_4) > \kappa_1(P_1, D_1)$
	$\rho_{NRS}(P_2, D_3) > \rho_{NRS}(P_2, D_2) > \rho_{NRS}(P_2, D_1) > \rho_{NRS}(P_2, D_4)$	$\kappa_1(P_2, D_3) > \kappa_1(P_2, D_2) > \kappa_1(P_2, D_1) > \kappa_1(P_2, D_4)$
	$\rho_{NRS}(P_3, D_3) > \rho_{NRS}(P_3, D_2) > \rho_{NRS}(P_3, D_4) > \rho_{NRS}(P_3, D_1)$	$\kappa_1(P_3, D_3) > \kappa_1(P_3, D_2) > \kappa_1(P_3, D_4) > \kappa_1(P_3, D_1)$
Example 4.2 in [7]	$\rho_{NRS}(Pat.I, Pat.III) > \rho_{NRS}(Pat.II, Pat.III)$	$\kappa_1(Pat.I, Pat.III) > \kappa_1(Pat.II, Pat.III)$ $\kappa_2(Pat.I, Pat.III) > \kappa_2(Pat.II, Pat.III)$

Consequently, we can say that the correlation coefficients of SNMRSs are generalized forms of correlation coefficients of both IMSs and NRSs (by considering Table 1 in the Introduction). These support that the range of application areas of the proposed correlation coefficients is quite wide and therefore advantageous in many situations.

6. Conclusions

In this paper, we have established a new extension of SNMS named as SNMRS which is more efficient and flexible structure to deal with ambiguity. The space for SNMRSs is larger than those of IMSs and SNMSs. We have founded some significant results in the framework of SNMRS. We have presented new correlation coefficients under the SNMRS environment and their application in medical pattern recognition. We hope that the findings in this study will be helpful for researchers handling with various real-life problems that involve uncertainties. Further, the proposed approaches may be extended in new directions including information fusion, aggregation and measures. The next research will aim to explore the real-life applications related to the concepts based on the extensions of SNMRSs.

Funding: This research received no external funding.

Conflicts of Interest: The author declares no conflict of interest.

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Received: Jan 15, 2021. Accepted: March 8, 2021.