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Some Fundamental Operations on Interval Valued Neutrosophic Hypersoft Set with Their Properties

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Abstract: Multi-criteria decision-making (MCDM) focuses on coordination, choice and planning issues, including multi-criteria. the neutrosophic soft set cannot handle environments involving multiple attributes. In order to overcome these obstacles, the neutrosophic hypersoft set (NHSS) and Interval Value neutrosophic hypersoft set (IVNHSS) are defined. In this paper, we extend the concept of IVNHSS with basic properties. We also developed some basic operations on IVNHSS such as union, intersection, addition, difference, Truth-favorite, and False-favorite, etc. with their desirable properties. Finally, the necessity and possibility operations on IVNHSS with properties are presented in the following research.

Keywords: Soft set; Neutrosophic Set; Interval-valued neutrosophic set; Hypersoft set; Interval-valued neutrosophic hypersoft set.

1. Introduction

Anxiety performs a dynamic part in lots of areas of life such as modeling, medicine, and engineering. However, people have raised a general question, that is, how can we verbalize anxiety in mathematical modeling. Several investigators all over the world have recommended and advised different methodologies to minimize uncertainty. First of all, Zadeh planned the idea of fuzzy sets [1] to resolve these complications which contain anxiety as well as ambiguity. It is seen that sometimes; fuzzy sets can't deal with scenarios. To overcome such scenarios, Turksen [2] suggested the concept of interval-valued fuzzy sets (IVFS). In some cases, we need to debate the suitable representation of the object under the circumstances of anxiety and uncertainty, and regard its unbiased membership value and non-membership value of the suitable representation of the object, that cannot be processed by these fuzzy sets or IVFS. To overcome such concerns, Atanassov projected the theory of IFS in [3]. The theory proposed by Atanassov only considers membership and non-membership values to deal with insufficient data, but the IFS theory cannot deal with incompatible and imprecise information. To deal with this incompatible and imprecise data, Smarandache proposed the idea of NS [4]. Molodtsov [5] proposed a general mathematical tool to deal with uncertain, ambiguous, and undefined substances, called soft sets (SS). Maji et al. [6] extended the work of SS and defined some operations and their attributes. In [7], they also use SS theory to make decisions. Ali et al. [8] Modified the Maji method of SS and developed some new operations with its properties. In [9], they proved De Morgan's SS theory and law by using different operators. Cagman and Enginoglu [10] proposed the concept of soft matrices with operations and discussed their properties. They also introduced a decision-making method to solve problems that contain uncertainty. In [11], they modified the actions proposed by Molodtsov's SS. In [12], the author proposed some new operations for soft matrices, such as soft difference product, soft restricted difference product, soft extended difference product, and weak extended difference product.

Maji [13] put forward the idea of NSS with necessary operations and characteristics. The idea of Possibility NSS was proposed by Karaaslan [14] and introduced a neutrosophic soft decision method to solve those uncertain problems based on And-product. Broumi [15] developed a generalized NSS with certain operations and properties and used the proposed concept for decision-making. To solve the MCDM problem with single-valued neutrosophic numbers proposed by Deli and Subas in [16], they constructed the concept of the cut set of single-valued neutrosophic numbers. Based on the correlation of IFS, the term correlation coefficient of SVNS is introduced [17]. In [18], the idea of simplifying NS introduced some algorithms and aggregation operators, such as weighted arithmetic operators and weighted geometric average operators. They constructed the MCDM method based on the proposed aggregation operator. Zulqarnain et al. [19] extended the fuzzy TOPSIS technique to the Neutrosophic TOPSIS technique and used the developed approach to solve the MCDM problem. Abdel-basset et al [20] presented the integration of TOPSIS methodology decision-making test as well as evaluation laboratory (DEMATEL) solution (TOPSIS) CIIC environment delivers a new method to pick out the proper project. Abdel-basset Mohamed [21] developed an MCDM model to discover along with display screen cancer addressing obscure, anxiety, the incompleteness of reported signs as well as handicapping apparently within cancer or replaceable ailments in the signs and symptoms. Abdel-Basset et al. [22] raised the issue of assessment of the smart emergency response techniques is interpreted as MCDM problem. they suggested a framework by combining three common MCDM strategies which are AHP, TOPSIS, and VIKOR.

All the above-mentioned studies cannot deal with the problems in which attributes of the alternates have their corresponding sub-attributes. To handle such compilations Smarandache [23] generalized the SS to HSS by converting the function to a multi-attribute function to deal with uncertainty. Saqlain et al. [24] developed the generalization of TOPSIS for the NHSS, by using accuracy function they transformed the fuzzy neutrosophic numbers to crisp form. Zulqarnain et al. [25] extended the notion of NHSSs and presented the generalized operations for NHSSs, they also developed the necessity and possibility operations and discussed their desirable features. In [26], the author's proposed the fuzzy Plithogenic hypersoft set in matrix form with some basic operations and properties. Saqlain et al. [27] proposed the aggregate operators on NHSS. In [28], the author extended the NHSS approach and introduced IVNHSS, m-polar, and m-polar IVNHSS. Zulqarnain et al. [29] presented the intuitionistic fuzzy hypersoft set, they developed the TOPSIS technique by developing a correlation coefficient to solve multi-attribute decision making problems. Many other novel researchers are done under neutrosophic environment and their applications in everyday life [30-34].

The following research is organized as follows: Some basic definitions recalled in section 2, which are used in the following research such as SS, NS, NSS, HSS, NHSS, and IVNHSS. We present different operators on IVNHSS such as union, intersection, addition, difference, extended union, extended intersection, truth-favorite, and false-favorite operations in section 3 with properties and prove the De Morgan laws by using union and intersection operators. We also proposed the necessity and possibility operators, OR, and operations with some properties in section 4.

2. Preliminaries

In this section, we recollect some basic definitions such as SS, NSS, NHSS, and IVNHSS which use in the following sequel.

Definition 2.1 [5]

The soft set is a pair (F, Λ) over $\mathbb{U}$ if and only if F: $\Lambda \to P(\mathbb{U})$ is a mapping. That is the parameterized family of subsets of $\mathbb{U}$ known as a SS.

Definition 2.2 [4]

Let $\mathbb{U}$ be a universe and Λ be an NS on $\mathbb{U}$ is defined as $\Lambda = \{< u, u_A(u), v_A(u), w_A(u) > : u \in \mathbb{U}\}$, where $u,\ v,\ w$: $\mathbb{U} \rightarrow\ ]0^-,\ 1^+[$ and $0^- \leq u_\Lambda(u) + v_\Lambda(u) + w_\Lambda(u) \leq 3^+$.

Definition 2.3 [13]

Let $\mathbb{U}$ and Ę̆ are universal set and set of attributes respectively. Let P($\mathbb{U}$) be the set of Neutrosophic values of $\mathbb{U}$ and $\Lambda \subseteq$ Ę̆. A pair (F, Λ) is called an NSS over $\mathbb{U}$ and its mapping is given as

F: $\Lambda \rightarrow (\mathbb{U})$

Definition 2.4 [35]

Let $\mathbb{U}$ be a universal set, then interval valued neutrosophic set can be expressed by the set $\boldsymbol{A} = \{< \boldsymbol{u}, \boldsymbol{u_A(u)}, \boldsymbol{v_A(u)}, \boldsymbol{w_A(u)} > : \boldsymbol{u} \in \mathbb{U}\}$, where $\boldsymbol{u_A}$, $\boldsymbol{v_A}$, and $\boldsymbol{w_A}$ are truth, indeterminacy and falsity membership functions for $\boldsymbol{A}$ respectively, $\boldsymbol{u_A}$, $\boldsymbol{v_A}$, and $\boldsymbol{w_A} \subseteq [0, 1]$ for each $\boldsymbol{u} \in \mathbb{U}$. Where

$\boldsymbol{u_A(u)} = [\boldsymbol{u_A^L(u)}, \boldsymbol{u_A^U(u)}]$

$\boldsymbol{v_A(u)} = [\boldsymbol{v_A^L(u)}, \boldsymbol{v_A^U(u)}]$

$\boldsymbol{w_A(u)} = [\boldsymbol{w_A^L(u)}, \boldsymbol{w_A^U(u)}]$

For each point $\boldsymbol{u} \in \mathbb{U}$, $0 \leq \boldsymbol{u_A(u)} + \boldsymbol{v_A(u)} + \boldsymbol{w_A(u)} \leq 3$ and IVN($\mathbb{U}$) represents the family of all interval valued neutrosophic sets.

Definition 2.5 [23]

Let $\mathbb{U}$ be a universal set and $P(\mathbb{U})$ be a power set of $\mathbb{U}$ and for $n \geq 1$, there are n distinct attributes such as $k_1,\ k_2,\ k_3,\ \ldots,\ k_n$ and $K_1,\ K_2,\ K_3,\ \ldots,\ K_n$ are sets for corresponding values attributes respectively with following conditions such as $K_i \cap K_j = \emptyset\ (i \neq j)$ and $i, j \in \{1,2,3 \ldots n\}$. Then the pair (F, $K_1 \times K_2 \times K_3 \times \ldots \times K_n$) is said to be HSS over $\mathbb{U}$ where F is a mapping from $K_1 \times K_2 \times K_3 \times \ldots \times K_n$ to $P(\mathbb{U})$.

Definition 2.6 [23]

Let $\mathbb{U}$ be a universal set and $P(\mathbb{U})$ be a power set of $\mathbb{U}$ and for $n \geq 1$, there are n distinct attributes such as $k_1,\ k_2,\ k_3,\ \ldots,\ k_n$ and $K_1,\ K_2,\ K_3,\ \ldots,\ K_n$ are sets for corresponding values attributes respectively with following conditions such as $K_i \cap K_j = \emptyset\ (i \neq j)$ and $i, j \in \{1,2,3 \ldots n\}$. Then the pair (F, Λ) is said to be NHSS over $\mathbb{U}$ if there exists a relation $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \Lambda$. F is a mapping from $K_1 \times K_2 \times K_3 \times \ldots \times K_n$ to $P(\mathbb{U})$ and $\mathrm{F}(K_1 \times K_2 \times K_3 \times \ldots \times K_n) = \{< u, u_A(u), v_A(u), w_A(u) > : u \in \mathbb{U}\}$ where u, v, w are membership values for truthness, indeterminacy and falsity respectively such that $u,\ v,\ w$: $\mathbb{U} \rightarrow\ ]0^-,\ 1^+[$ and $0^- \leq u_\Lambda(u) + v_\Lambda(u) + w_\Lambda(u) \leq 3^+$.

Definition 2.7 [28]

Let $\mathbb{U}$ be a universal set and $P(\mathbb{U})$ be a power set of $\mathbb{U}$ and for $n \geq 1$, there are n distinct attributes such as $k_1,\ k_2,\ k_3,\ \ldots,\ k_n$ and $K_1,\ K_2,\ K_3,\ \ldots,\ K_n$ are sets for corresponding values attributes respectively with following conditions such as $K_i \cap K_j = \emptyset\ (i \neq j)$ and $i, j \in \{1,2,3 \ldots n\}$. Then the pair $(F,\ A)$ is said to be IVNHSS over $\mathbb{U}$ if there exists a relation $K_1 \times K_2 \times K_3 \times \ldots \times K_n = A$. Where

F: $K_1 \times K_2 \times K_3 \times \ldots \times K_n \rightarrow (\mathbb{U})$ and

$F(K_1 \times K_2 \times K_3 \times \ldots \times K_n) = \{< u, [u_A^L(u), u_A^U(u)],\ [v_A^L(u), v_A^U(u)],\ [w_A^L(u), w_A^U(u)] > : u \in \mathbb{U}\}$, where u_A^L, v_A^L, and w_A^L are lower and u_A^U, v_A^U, and w_A^U are upper membership values for truthiness, indeterminacy, and falsity respectively for A and $[u_A^L(u), u_A^U(u)]$, $[v_A^L(u), v_A^U(u)]$, $[w_A^L(u), w_A^U(u)] \subseteq [0, 1]$ and $0 \leq sup\, u_A(u) + sup\, v_A(u) + sup\, w_A(u) \leq 3$ for each $u \in \mathbb{U}$.

Example 1 Assume $\mathbb{U} = \{u_1,\ u_2\}$ be a universe of discourse and $E = \{x_1,\ x_2,\ x_3,\ x_4\}$ be a set of attributes. Consider F_A be an IVNHSS over $\mathbb{U}$ can be expressed as follows

$$F_A = \{(x_1, \{\langle u_1, [.6, .8], [.5, 0.9], [.1, .4]\rangle, \langle u_2, [.4, .7], [.3, .9], [.2, .6]\rangle\}),$$
$$(x_2, \{\langle u_1, [.4, .7], [.3, .9], [.3, .5]\rangle, \langle u_2, [0, .3], [.6, .8], [.3, .7]\rangle\}),$$
$$(x_3, \{\langle u_1, [.2, .9], [.1, .5], [.7, .8]\rangle, \langle u_2, [.4, .9], [.1, .6], [.5, .7]\rangle\}),$$
$$(x_4, \{\langle u_1, [.6, .9], [.6, .9], [1, 1]\rangle, \langle u_2, [.5, .9], [.6, .8], [.1, .8]\rangle\})\}.$$

Tablur representation of IVNHSS F_A over $\mathbb{U}$ given as follows

Table 1: Tablur representation of IVNHSS F_A

$\mathbb{U}$	u_1	u_1
x_1	$\langle[.6, .8], [.5, .9], [.1, .4]\rangle$	$\langle[.4, .7], [.3, .9], [.2, .6]\rangle$
x_2	$\langle[.4, .7], [.3, .9], [.3, .5]\rangle$	$\langle[0, .3], [.6, .8], [.3, .7]\rangle$
x_3	$\langle[.2, .9], [.1, .5], [.7, .8]\rangle$	$\langle[.4, .9], [.1, .6], [.5, .7]\rangle$
x_4	$\langle[.6, .9], [.6, .9], [1, 1]\rangle$	$\langle[.5, .9], [.6, .8], [.1, .8]\rangle$

3. Operations on Interval Valued Neutrosophic Hypersoft Set with Properties

In this section, we extend the concept of IVNHSS and introduce some fundamental operations on IVNHSS with their properties.

Definition 3.1

Let F_A and G_B $\in$ IVNHSS over $\mathbb{U}$, then $F_A \subseteq G_B$ if

$inf\,u_A(u) \leq inf\,u_B(u),\ sup\,u_A(u) \leq sup\,u_B(u)$

$inf\,v_A(u) \geq inf\,v_B(u),\ sup\,v_A(u) \geq sup\,v_B(u)$

$inf\,w_A(u) \geq inf\,w_B(u),\ sup\,w_A(u) \geq sup\,w_B(u)$

Example 2 Assume $\mathbb{U} = \{u_1,\ u_2\}$ be a universe of discourse and $E = \{x_1,\ x_2,\ x_3,\ x_4\}$ be a set of attributes. Consider G_B be an IVNHSS over $\mathbb{U}$ can be expressed as follows and F_A given in example 1

$$G_B = \{(x_1, \{\langle u_1, [.6, .9], [.3, .7], [.1, .3]\rangle, \langle u_2, [.6, .9], [.3, .5], [.1, .4]\rangle\}),$$
$$(x_2, \{\langle u_1, [.6, .8], [.2, .5], [.2, .3]\rangle, \langle u_2, [.3, .5], [.4, .7], [.1, .4]\rangle\}),$$
$$(x_3, \{\langle u_1, [.4, .9], [.1, .3], [.4, .6]\rangle, \langle u_2, [.6, 1], [.1, .4], [.3, .4]\rangle\}),$$
$$(x_4, \{\langle u_1, [.7, .9], [.4, .6], [.6, 1]\rangle, \langle u_2, [.5, .7], [.4, .7], [.1, .4]\rangle\})\}.$$

Thus

$F_A \subseteq G_B$.

Definition 3.2

Let $F_A \in$ IVNHSS over $\mathbb{U}$, then

i. Empty IVNHSS can be represented as $F_{\breve{0}}$, and defined as follows $F_{\breve{0}} = \{< u, [0, 0],\ [1, 1],\ [1, 1] >: u \in \mathbb{U}\}$.
ii. Universal IVNHSS can be represented as $F_{\breve{E}}$, and defined as follows $F_{\breve{E}} = \{< u, [0, 0],\ [1, 1],\ [1, 1] >: u \in \mathbb{U}\}$.

iii. The complement of IVNHSS can be defined as follows $F_A^c = \{< u, [w_A^L(u), w_A^U(u)], [1 - v_A^U(u), 1 - v_A^L(u)], [u_A^L(u), u_A^U(u)] > : u \in \mathbb{U}\}$.

Example 3 Assume $\mathbb{U} = \{u_1, u_2\}$ be a universe of discourse and $E = \{x_1, x_2, x_3, x_4\}$ be a set of attributes. The tabular representation of $F_{\breve{0}}$ and $F_{\breve{E}}$ given as follows in table 2 and table 3 respectively.

Table 2:Tablur representation of IVNHSS $F_{\breve{0}}$

$\mathbb{U}$	u_1	u_1
x_1	$\langle[0,0],[1,1],[1,1]\rangle$	$\langle[0,0],[1,1],[1,1]\rangle$
x_2	$\langle[0,0],[1,1],[1,1]\rangle$	$\langle[0,0],[1,1],[1,1]\rangle$
x_3	$\langle[0,0],[1,1],[1,1]\rangle$	$\langle[0,0],[1,1],[1,1]\rangle$
x_4	$\langle[0,0],[1,1],[1,1]\rangle$	$\langle[0,0],[1,1],[1,1]\rangle$

Table 3:Tablur representation of IVNHSS $F_{\breve{E}}$

$\mathbb{U}$	u_1	u_1
x_1	$\langle[1,1],[0,0],[0,0]\rangle$	$\langle[1,1],[0,0],[0,0]\rangle$
x_2	$\langle[1,1],[0,0],[0,0]\rangle$	$\langle[1,1],[0,0],[0,0]\rangle$
x_3	$\langle[1,1],[0,0],[0,0]\rangle$	$\langle[1,1],[0,0],[0,0]\rangle$
x_4	$\langle[1,1],[0,0],[0,0]\rangle$	$\langle[1,1],[0,0],[0,0]\rangle$

Proposition 3.3

If $F_A \in$ IVNHSS, then

1. $(F_A^c)^c = F_A$
2. $(F_{\breve{0}})^c = F_{\breve{E}}$
3. $(F_{\breve{E}})^c = F_{\breve{0}}$

Proof 1 Let $F_A = \{< u, [u_A^L(u), u_A^U(u)], [v_A^L(u), v_A^U(u)], [w_A^L(u), w_A^U(u)] > : u \in \mathbb{U}\}$ be an IVNHSS. Then by using definition 3.3(iii), we have

$F_A^c = \{< u, [w_A^L(u), w_A^U(u)], [1 - v_A^U(u), 1 - v_A^L(u)], [u_A^L(u), u_A^U(u)] > : u \in \mathbb{U}\}$

Thus

$(F_A^c)^c = \{< u, [u_A^L(u), u_A^U(u)], [1 - (1 - v_A^L(u)), 1 - (1 - v_A^U(u))], [w_A^L(u), w_A^U(u)] > : u \in \mathbb{U}\}$

$(F_A^c)^c = \{< u, [u_A^L(u), u_A^U(u)], [v_A^L(u), v_A^U(u)], [w_A^L(u), w_A^U(u)] > : u \in \mathbb{U}\}$

$(F_A^c)^c = F_A$

Proof 2

As we know that $F_{\breve{0}} = \{< u, [0,0], [1,1], [1,1] > : u \in \mathbb{U}\}$

By using definition 3.3(iii), we get

$(F_{\breve{0}})^c = \{< u, [1,1], [0,0], [0,0] > : u \in \mathbb{U}\} = F_{\breve{E}}$.

Similarly, we can prove 3.

Definition 3.4

Let F_A and G_B $\in$ IVNHSS over $\mathbb{U}$, then

$$F_A \cup G_B = \left\{ \begin{matrix} (< u, [max\{inf\,u_A(u),\ inf\,u_B(u)\}, max\{sup\,u_A(u),\ sup\,u_B(u)\}], \\ [min\{inf\,v_A(u),\ inf\,v_B(u)\}, min\{sup\,v_A(u),\ sup\,v_B(u)\}], \\ [min\{inf\,w_A(u),\ inf\,w_B(u)\}, min\{sup\,w_A(u),\ sup\,w_B(u)\}] >/ u \in \mathbb{U}) \end{matrix} \right\}. \quad (1)$$

Example 4 Assume $\mathbb{U} = \{u_1,\ u_2\}$ be a universe of discourse and $E = \{x_1,\ x_2,\ x_3,\ x_4\}$ be a set of attributes. Consider F_A and G_B are IVNHSS over $\mathbb{U}$ can be given as follows

$$\begin{aligned} F_A = \{&(x_1, \{\langle u_1, [.6,.8], [.5,.9], [.1,.4]\rangle, \langle u_2, [.4,.7], [.3,.9], [.2,.6]\rangle\}), \\ &(x_2, \{\langle u_1, [.4,.7], [.3,.9], [.3,.5]\rangle, \langle u_2, [.2,.8], [.6,.8], [.3,.7]\rangle\}), \\ &(x_3, \{\langle u_1, [.2,.9], [.1,.5], [.4,.7]\rangle, \langle u_2, [.4,.9], [.1,.6], [.5,.7]\rangle\}), \\ &(x_4, \{\langle u_1, [.6,.9], [.6,.9], [1,1]\rangle, \langle u_2, [.5,.9], [.6,.8], [.1,.8]\rangle\})\} \end{aligned}$$

$$\begin{aligned} G_B = \{&(x_1, \{\langle u_1, [.5,.7], [.5,.7], [.4,.6]\rangle, \langle u_2, [.3,.9], [.3,.6], [.4,.7]\rangle\}), \\ &(x_2, \{\langle u_1, [.3,.8], [.4,.5], [.4,.9]\rangle, \langle u_2, [.4,.7], [.5,.9], [.4,.6]\rangle\}), \\ &(x_3, \{\langle u_1, [.3,.5], [.2,.6], [.3,.8]\rangle, \langle u_2, [.3,1], [.2,.7], [.3,.8]\rangle\}), \\ &(x_4, \{\langle u_1, [.4,.6], [.7,.8], [.4,1]\rangle, \langle u_2, [.4,.8], [.3,.6], [.2,.6]\rangle\})\} \end{aligned}$$

Then

$$\begin{aligned} F_A \cup G_B = \{&(x_1, \{\langle u_1, [.6,.8], [.5,.7], [.1,.4]\rangle, \langle u_2, [.4,.9], [.3,.6], [.2,.6]\rangle\}), \\ &(x_2, \{\langle u_1, [.4,.8], [.3,.5], [.3,.5]\rangle, \langle u_2, [.4,.8], [.5,.8], [.3,.6]\rangle\}), \\ &(x_3, \{\langle u_1, [.3,.9], [.1,.5], [.3,.7]\rangle, \langle u_2, [.4,1], [.1,.6], [.3,.7]\rangle\}), \\ &(x_4, \{\langle u_1, [.6,.9], [.6,.8], [.4\ 1]\rangle, \langle u_2, [.5,.9], [.3,.6], [.1,.6]\rangle\})\} \end{aligned}$$

Proposition 3.5

Let $\mathcal{F}_{\breve{A}}$, $\mathcal{G}_{\breve{B}}$, $\mathcal{H}_{\breve{C}}$ $\in$ IVNHSS over $\mathbb{U}$. Then

1. $\mathcal{F}_{\breve{A}} \cup \mathcal{F}_{\breve{A}} = \mathcal{F}_{\breve{A}}$
2. $\mathcal{F}_{\breve{A}} \cup \mathcal{F}_{\breve{0}} = \mathcal{F}_{\breve{0}}$
3. $\mathcal{F}_{\breve{A}} \cup \mathcal{F}_{\breve{E}} = \mathcal{F}_{\breve{A}}$
4. $\mathcal{F}_{\breve{A}} \cup \mathcal{G}_{\breve{B}} = \mathcal{G}_{\breve{B}} \cup \mathcal{F}_{\breve{A}}$
5. $(\mathcal{F}_{\breve{A}} \cup \mathcal{G}_{\breve{B}}) \cup \mathcal{H}_{\breve{C}} = \mathcal{F}_{\breve{A}} \cup (\mathcal{G}_{\breve{B}} \cup \mathcal{H}_{\breve{C}})$

Proof By using definition 3.4 we can prove easily.

Definition 3.6

Let F_A and G_B $\in$ IVNHSS over $\mathbb{U}$, then

$$F_A \cap G_B = \left\{ \begin{matrix} < u, [min\{inf\,u_A(u),\ inf\,u_B(u)\}, min\{sup\,u_A(u),\ sup\,u_B(u)\}], \\ [max\{inf\,v_A(u),\ inf\,v_B(u)\}, max\{sup\,v_A(u),\ sup\,v_B(u)\}], \\ [max\{inf\,w_A(u),\ inf\,w_B(u)\}, max\{sup\,w_A(u), sup\,w_B(u)\}] >/ u \in \mathbb{U}) \end{matrix} \right\}. \quad (2)$$

Example 5 Reconsider example 4

$$\begin{aligned} F_A = \{&(x_1, \{\langle u_1, [.6,.8], [.5,.9], [.1,.4]\rangle, \langle u_2, [.4,.7], [.3,.9], [.2,.6]\rangle\}), \\ &(x_2, \{\langle u_1, [.4,.7], [.3,.9], [.3,.5]\rangle, \langle u_2, [.2,.8], [.6,.8], [.3,.7]\rangle\}), \end{aligned}$$

$$(x_3, \{\langle u_1, [.2, .9], [.1, .5], [.4, .7]\rangle, \langle u_2, [.4, .9], [.1, .6], [.5, .7]\rangle\}),$$
$$(x_4, \{\langle u_1, [.6, .9], [.6, .9], [1, 1]\rangle, \langle u_2, [.5, .9], [.6, .8], [.1, .8]\rangle\})\}$$
$$G_B = \{(x_1, \{\langle u_1, [.5, .7], [.5, .7], [.4, .6]\rangle, \langle u_2, [.3, .9], [.3, .6], [.4, .7]\rangle\}),$$
$$(x_2, \{\langle u_1, [.3, .8], [.4, .5], [.4, .9]\rangle, \langle u_2, [.4, .7], [.5, .9], [.4, .6]\rangle\}),$$
$$(x_3, \{\langle u_1, [.3, .5], [.2, .6], [.3, .8]\rangle, \langle u_2, [.3, 1], [.2, .7], [.3, .8]\rangle\}),$$
$$(x_4, \{\langle u_1, [.4, .6], [.7, .8], [.4, 1]\rangle, \langle u_2, [.4, .8], [.3, .6], [.2, .6]\rangle\})\}$$

Then

$$F_A \cap G_B = \{(x_1, \{\langle u_1, [.5, .7], [.5, .9], [.4, .6]\rangle, \langle u_2, [.3, .7], [.3, .9], [.4, .7]\rangle\}),$$
$$(x_2, \{\langle u_1, [.3, .7], [.4, .9], [.4, .9]\rangle, \langle u_2, [.2, .7], [.6, .9], [.4, .7]\rangle\}),$$
$$(x_3, \{\langle u_1, [.2, .5], [.2, .6], [.4, .8]\rangle, \langle u_2, [.3, .9], [.2, .7], [.5, .8]\rangle\}),$$
$$(x_4, \{\langle u_1, [.4, .6], [.7, .9], [1, 1]\rangle, \langle u_2, [.4, .8], [.6, .8], [.2, .8]\rangle\})\}$$

Proposition 3.7

Let $\mathcal{F}_{\breve{A}}, \mathcal{G}_{\breve{B}}, \mathcal{H}_{\breve{C}} \in$ IVNHSS over $\mathbb{U}$. Then

1. $\mathcal{F}_{\breve{A}} \cap \mathcal{F}_{\breve{A}} = \mathcal{F}_{\breve{A}}$
2. $\mathcal{F}_{\breve{A}} \cap \mathcal{F}_{\breve{0}} = \mathcal{F}_{\breve{A}}$
3. $\mathcal{F}_{\breve{A}} \cap \mathcal{F}_{\breve{E}} = \mathcal{F}_{\breve{E}}$
4. $\mathcal{F}_{\breve{A}} \cap \mathcal{G}_{\breve{B}} = \mathcal{G}_{\breve{B}} \cap \mathcal{F}_{\breve{A}}$
5. $(\mathcal{F}_{\breve{A}} \cap \mathcal{G}_{\breve{B}}) \cap \mathcal{H}_{\breve{C}} = \mathcal{F}_{\breve{A}} \cap (\mathcal{G}_{\breve{B}} \cap \mathcal{H}_{\breve{C}})$

Proof By using definition 3.6 we can prove easily.

Proposition 3.8

Let F_A and $G_B \in$ IVNHSS over $\mathbb{U}$, then

1. $(F_A \cup G_B)^C = {F_A}^C \cap {G_B}^C$
2. $(F_A \cap G_B)^C = {F_A}^C \cup {G_B}^C$

Proof 1 As we know that

$F_A = \{< u, u_A(u), v_A(u), w_A(u) > : u \in \mathbb{U}\}$ and $G_B = \{< u, u_B(u), v_B(u), w_B(u) > : u \in \mathbb{U}\}$. Where

$u_A(u) = [inf u_A(u),\ sup u_A(u)]$ or $[u_A^L(u), u_A^U(u)]$, $u_A^L(u) = inf u_A(u)$ and $u_A^U(u) = sup u_A(u)$

$v_A(u) = [inf v_A(u),\ sup v_A(u)]$ or $[v_A^L(u), v_A^U(u)]$, $v_A^L(u) = inf v_A(u)$ and $v_A^U(u) = sup v_A(u)$

$w_A(u) = [\ inf w_A(u)\ ,\ sup w_A(u)\ ]$ or $[w_A^L(u), w_A^U(u)]$, $w_A^L(u) = inf w_A(u)$ and $w_A^U(u) = sup w_A(u)$

$u_B(u) = [inf u_B(u),\ sup u_B(u)]$ or $[u_B^L(u), u_B^U(u)]$, $u_B^L(u) = inf u_B(u)$ and $u_B^U(u) = sup u_B(u)$

$v_B(u) = [inf v_B(u),\ sup v_B(u)]$ or $[v_B^L(u), v_B^U(u)]$, $v_B^L(u) = inf v_B(u)$ and $v_B^U(u) = sup v_B(u)$

$w_B(u) = [\ inf w_B(u)\ ,\ sup w_B(u)\ ]$ or $[w_B^L(u), w_B^U(u)]$, $w_B^L(u) = inf w_B(u)$ and $w_B^U(u) = sup w_B(u)$

Then by using Equation 1

$$F_A \cup G_B = \left\{ \begin{array}{c} (< u, [max\{inf u_A(u),\ inf u_B(u)\}, max\{sup u_A(u),\ sup u_B(u)\}], \\ [min\{inf v_A(u),\ inf v_B(u)\}, min\{sup v_A(u),\ sup v_B(u)\}], \\ [min\{inf w_A(u),\ inf w_B(u)\}, min\{sup w_A(u),\ sup w_B(u)\}] > / u \in \mathbb{U}) \end{array} \right\}$$

By using definition 3.3(iii), we get

$$(F_A \cup G_B)^C = \left\{\begin{matrix} (< u, [min\{inf\,w_A(u),\ inf\,w_B(u)\}, min\{sup\,w_A(u),\ sup\,w_B(u)\}], \\ [1 - min\{sup\,v_A(u),\ sup\,v_B(u)\}, 1 - min\{inf\,v_A(u),\ inf\,v_B(u)\}], \\ [max\{inf\,u_A(u),\ inf\,u_B(u)\}, max\{sup\,u_A(u),\ sup\,u_B(u)\}] >/ u \in \mathbb{U}) \end{matrix}\right\}$$

Now

$${F_A}^C = \{< u, [inf\,w_A(u), sup\,w_A(u)], [1 - sup\,v_A(u), 1 - inf\,v_A(u)], [inf\,u_A(u), sup\,u_A(u)] > : u \in \mathbb{U}\}$$

$${G_B}^C = \{< u, [inf\,w_B(u), sup\,w_B(u)], [1 - sup\,v_B(u), 1 - inf\,v_B(u)], [inf\,u_B(u), sup\,u_B(u)] > : u \in \mathbb{U}\}$$

$${F_A}^C \cap {G_B}^C = \left\{\begin{matrix} (< u, [min\{inf\,w_A(u),\ inf\,w_B(u)\}, min\{sup\,w_A(u),\ sup\,w_B(u)\}], \\ [max\{1 - sup\,v_A(u),\ 1 - sup\,v_B(u)\}, max\{1 - inf\,v_A(u),\ 1 - inf\,v_B(u)\}], \\ [max\{inf\,u_A(u),\ inf\,u_B(u)\}, max\{sup\,u_A(u),\ sup\,u_B(u)\}] >/ u \in \mathbb{U}) \end{matrix}\right\}$$

$${F_A}^C \cap {G_B}^C = \left\{\begin{matrix} (< u, [min\{inf\,w_A(u),\ inf\,w_B(u)\}, min\{sup\,w_A(u),\ sup\,w_B(u)\}], \\ [1 - min\{sup\,v_A(u), sup\,v_B(u)\}, 1 - min\{inf\,v_A(u),\ inf\,v_B(u)\}], \\ [max\{inf\,u_A(u),\ inf\,u_B(u)\}, max\{sup\,u_A(u),\ sup\,u_B(u)\}] >/ u \in \mathbb{U}) \end{matrix}\right\}$$

Hence

$(F_A \cup G_B)^C = {F_A}^C \cap {G_B}^C$

Proof 2

Similar to assertion 1.

Proposition 3.9

Let $\mathcal{F}_{\breve{A}}$, $\mathcal{G}_{\breve{B}}$, $\mathcal{H}_{\breve{C}} \in$ IVNHSS over $\mathbb{U}$. Then

1. $\mathcal{F}_{\breve{A}} \cup (\mathcal{G}_{\breve{B}} \cap \mathcal{H}_{\breve{C}}) = (\mathcal{F}_{\breve{A}} \cup \mathcal{G}_{\breve{B}}) \cap (\mathcal{F}_{\breve{A}} \cup \mathcal{H}_{\breve{C}})$
2. $\mathcal{F}_{\breve{A}} \cap (\mathcal{G}_{\breve{B}} \cup \mathcal{H}_{\breve{C}}) = (\mathcal{F}_{\breve{A}} \cap \mathcal{G}_{\breve{B}}) \cup (\mathcal{F}_{\breve{A}} \cap \mathcal{H}_{\breve{C}})$
3. $\mathcal{F}_{\breve{A}} \cup (\mathcal{F}_{\breve{A}} \cap \mathcal{G}_{\breve{B}}) = \mathcal{F}_{\breve{A}}$
4. $\mathcal{F}_{\breve{A}} \cap (\mathcal{F}_{\breve{A}} \cup \mathcal{G}_{\breve{B}}) = \mathcal{F}_{\breve{A}}$

Proof 1 From Equation 2, we have

$$\mathcal{G}_{\breve{B}} \cap \mathcal{H}_{\breve{C}} = \left\{\begin{matrix} (< u, [min\{inf\,u_B(u),\ inf\,u_C(u)\}, min\{sup\,u_B(u),\ sup\,u_C(u)\}], \\ [max\{inf\,v_B(u),\ inf\,v_C(u)\}, max\{sup\,v_B(u),\ sup\,v_C(u)\}], \\ [max\{inf\,w_B(u),\ inf\,w_C(u)\}, max\{sup\,w_B(u),\ sup\,w_C(u)\}] >/ u \in \mathbb{U}) \end{matrix}\right\}$$

$\mathcal{F}_{\breve{A}} \cup (\mathcal{G}_{\breve{B}} \cap \mathcal{H}_{\breve{C}}) =$

$$\left\{\begin{matrix} (< u, [\max\{inf\,u_A(u), min\{inf\,u_B(u),\ inf\,u_C(u)\}\}, \max\{sup\,u_A(u), min\{sup\,u_B(u), sup\,u_C(u)\}\}], \\ [\min\{inf\,v_A(u), max\{inf\,v_B(u),\ inf\,v_C(u)\}\}, \min\{sup\,v_A(u), max\{sup\,v_B(u), sup\,v_C(u)\}\}], \\ [min\{inf\,w_A(u), max\{inf\,w_B(u),\ inf\,w_C(u)\}\}, min\{sup\,w_A(u), max\{sup\,w_B(u),\ sup\,w_C(u)\}\}] >/ u \in \mathbb{U}) \end{matrix}\right\}$$

$$\mathcal{F}_{\breve{A}} \cup \mathcal{G}_{\breve{B}} = \left\{\begin{matrix} (< u, [max\{inf\,u_A(u),\ inf\,u_B(u)\}, max\{sup\,u_A(u),\ sup\,u_B(u)\}], \\ [min\{inf\,v_A(u),\ inf\,v_B(u)\}, min\{sup\,v_A(u),\ sup\,v_B(u)\}], \\ [min\{inf\,w_A(u),\ inf\,w_B(u)\}, min\{sup\,w_A(u),\ sup\,w_B(u)\}] >/ u \in \mathbb{U}) \end{matrix}\right\}.$$

$$\mathcal{F}_{\breve{A}} \cup \mathcal{H}_{\breve{C}} = \left\{\begin{matrix} (< u, [max\{inf\,u_A(u),\ inf\,u_C(u)\}, max\{sup\,u_A(u),\ sup\,u_C(u)\}], \\ [min\{inf\,v_A(u),\ inf\,v_C(u)\}, min\{sup\,v_A(u),\ sup\,v_C(u)\}], \\ [min\{inf\,w_A(u),\ inf\,w_C(u)\}, min\{sup\,w_A(u),\ sup\,w_C(u)\}] >/ u \in \mathbb{U}) \end{matrix}\right\}.$$

$(\mathcal{F}_{\breve{A}} \cup \mathcal{G}_{\breve{B}}) \cap (\mathcal{F}_{\breve{A}} \cup \mathcal{H}_{\breve{C}}) =$

$$\left\{\begin{array}{c}(< u, [\min\{max\{inf\mathcal{u}_A(u),\ inf\mathcal{u}_B(u)\}, max\{inf\mathcal{u}_A(u),\ inf\mathcal{u}_C(u)\}\}, \min\{max\{sup\mathcal{u}_A(u),\ sup\mathcal{u}_B(u)\}, max\{sup\mathcal{u}_A(u),\ sup\mathcal{u}_C(u)\}\}],\\ [\max\{min\{inf\mathcal{v}_A(u),\ inf\mathcal{v}_B(u)\}, min\{inf\mathcal{v}_A(u),\ inf\mathcal{v}_C(u)\}\}, \max\{min\{sup\mathcal{v}_A(u),\ sup\mathcal{v}_B(u)\}, min\{sup\mathcal{v}_A(u),\ sup\mathcal{v}_C(u)\}\}],\\ [max\{min\{inf\mathcal{w}_A(u),\ inf\mathcal{w}_B(u)\}, min\{inf\mathcal{w}_A(u),\ inf\mathcal{w}_C(u)\}\}, max\{min\ \{sup\mathcal{w}_A(u),\ sup\mathcal{w}_B(u)\}, sup\mathcal{w}_A(u),\ sup\mathcal{w}_C(u)\}\}] >/ u \in \mathbb{U})\end{array}\right\}$$

$(\mathcal{F}_{\breve{A}}\ \cup\ \mathcal{G}_{\breve{B}})\ \cap\ (\mathcal{F}_{\breve{A}}\ \cup\ \mathcal{H}_{\breve{C}}) =$

$$\left\{\begin{array}{c}(< u, [\max\{inf\mathcal{u}_A(u), min\{inf\mathcal{u}_B(u),\ inf\mathcal{u}_C(u)\}\}, \max\{sup\mathcal{u}_A(u), min\{sup\mathcal{u}_B(u), sup\mathcal{u}_C(u)\}\}],\\ [\min\{inf\mathcal{v}_A(u), max\{inf\mathcal{v}_B(u),\ inf\mathcal{v}_C(u)\}\}, \min\{sup\mathcal{v}_A(u), max\{sup\mathcal{v}_B(u), sup\mathcal{v}_C(u)\}\}],\\ [min\{inf\mathcal{w}_A(u), max\{inf\mathcal{w}_B(u),\ inf\mathcal{w}_C(u)\}\}, min\{sup\mathcal{w}_A(u), max\{sup\mathcal{w}_B(u),\ sup\mathcal{w}_C(u)\}\}] >/ u \in \mathbb{U})\end{array}\right\}$$

Hence

$\mathcal{F}_{\breve{A}}\ \cup\ (\mathcal{G}_{\breve{B}} \cap\ \mathcal{H}_{\breve{C}}) = (\mathcal{F}_{\breve{A}}\ \cup\ \mathcal{G}_{\breve{B}})\ \cap\ (\mathcal{F}_{\breve{A}}\ \cup\ \mathcal{H}_{\breve{C}})$.

Similarly, we can prove other results.

Definition 3.10

Let F_A, G_B $\in$ IVNHSS, then their extended union is

$$\mathcal{u}\ (F_A\ \cup\ G_B) = \begin{cases}[inf\mathcal{u}_A(u), sup\mathcal{u}_A(u)] & if\ u\ \in\ A-B\\ [inf\mathcal{u}_B(u), sup\mathcal{u}_B(u)] & if\ u\ \in\ B-A\\ [max\{inf\mathcal{u}_A(u),\ inf\mathcal{u}_B(u)\}, max\{sup\mathcal{u}_A(u),\ sup\mathcal{u}_B(u)\}] & if\ u\ \in A\cap B\end{cases}$$

$$\mathcal{v}\ (F_A\ \cup\ G_B) = \begin{cases}[inf\mathcal{v}_A(u), sup\mathcal{v}_A(u)] & if\ u\ \in\ A-B\\ [inf\mathcal{v}_B(u), sup\mathcal{v}_B(u)] & if\ u\ \in\ B-A\\ [min\{inf\mathcal{v}_A(u),\ inf\mathcal{v}_B(u)\}, min\{sup\mathcal{v}_A(u),\ sup\mathcal{v}_B(u)\}] & if\ u\ \in A\cap B\end{cases}$$

$$\mathcal{w}\ (F_A\ \cup\ G_B) = \begin{cases}[inf\mathcal{w}_A(u), sup\mathcal{w}_A(u)] & if\ u\ \in\ A-B\\ [inf\mathcal{w}_B(u), sup\mathcal{w}(u)] & if\ u\ \in\ B-A\\ [min\{inf\mathcal{w}_A(u),\ inf\mathcal{w}_B(u)\}, min\{sup\mathcal{w}_A(u),\ sup\mathcal{w}_B(u)\}] & if\ u\ \in A\cap B\end{cases}$$

Definition 3.11

Let F_A, G_B $\in$ IVNHSS, then their extended intersection is

$$\mathcal{u}\ (F_A\ \cap\ G_B) = \begin{cases}[inf\mathcal{u}_A(u), sup\mathcal{u}_A(u)] & if\ u\ \in\ A-B\\ [inf\mathcal{u}_B(u), sup\mathcal{u}_B(u)] & if\ u\ \in\ B-A\\ [min\{inf\mathcal{u}_A(u),\ inf\mathcal{u}_B(u)\}, min\{sup\mathcal{u}_A(u),\ sup\mathcal{u}_B(u)\}] & if\ u\ \in A\cap B\end{cases}$$

$$\mathcal{v}\ (F_A\ \cap\ G_B) = \begin{cases}[inf\mathcal{v}_A(u), sup\mathcal{v}_A(u)] & if\ u\ \in\ A-B\\ [inf\mathcal{v}_B(u), sup\mathcal{v}_B(u)] & if\ u\ \in\ B-A\\ [max\{inf\mathcal{v}_A(u),\ inf\mathcal{v}_B(u)\}, max\{sup\mathcal{v}_A(u),\ sup\mathcal{v}_B(u)\}] & if\ u\ \in A\cap B\end{cases}$$

$$\mathcal{w}\ (F_A\ \cap\ G_B) = \begin{cases}[inf\mathcal{w}_A(u), sup\mathcal{w}_A(u)] & if\ u\ \in\ A-B\\ [inf\mathcal{w}_B(u), sup\mathcal{w}(u)] & if\ u\ \in\ B-A\\ [max\{inf\mathcal{w}_A(u),\ inf\mathcal{w}_B(u)\}, max\{sup\mathcal{w}_A(u),\ sup\mathcal{w}_B(u)\}] & if\ u\ \in A\cap B\end{cases}$$

Definition 3.12

Let F_A and G_B $\in$ IVNHSS over $\mathbb{U}$, then their difference defined as follows

$$F_A\ \setminus\ G_B\ = \left\{\begin{array}{c}(< u, [min\{inf\mathcal{u}_A(u),\ inf\mathcal{u}_B(u)\}, min\{sup\mathcal{u}_A(u),\ sup\mathcal{u}_B(u)\}],\\ [max\{inf\mathcal{v}_A(u),\ 1-\sup\mathcal{v}_B(u)\}, max\{sup\mathcal{v}_A(u),\ 1-\inf\mathcal{v}_B(u)\}],\\ [max\{inf\mathcal{w}_A(u),\ inf\mathcal{w}_B(u)\}, max\{sup\mathcal{w}_A(u),\ sup\mathcal{w}_B(u)\}] >/ u \in \mathbb{U})\end{array}\right\}. \quad (3)$$

Example 6 Reconsider example 4

$$F_A\ \setminus\ G_B\ = \{(x_1, \{\langle u_1, [.5, .7], [.5, .9], [.4, .6]\rangle, \langle u_2, [.3, .7], [.4, .9], [.4, .7]\rangle\}),$$
$$(x_2, \{\langle u_1, [.3, .7], [.5, .9], [.4, .9]\rangle, \langle u_2, [.2, .7], [.6, .8], [.4, .7]\rangle\}),$$

$$(x_3, \{\langle u_1, [.2, .5], [.4, .8], [.4, .8]\rangle, \langle u_2, [.3, .9], [.3, .8], [.5, .8]\rangle\}),$$
$$(x_4, \{\langle u_1, [.4, .6], [.6, .9], [1, 1]\rangle, \langle u_2, [.4, .8], [.6, .8], [.2, .8]\rangle\})\}$$

Definition 3.13

Let F_A and $G_B \in$ IVNHSS over $\mathbb{U}$, then their addition defined as follows

$$F_A + G_B = \left\{\begin{matrix} (< u, [min\{inf\,u_A(u) + inf\,u_B(u), 1\}, min\{sup\,u_A(u) + sup\,u_B(u), 1\}], \\ [min\{inf\,v_A(u) + inf\,v_B(u), 1\}, min\{sup\,v_A(u) + sup\,v_B(u), 1\}], \\ [min\{inf\,w_A(u) + inf\,w_B(u), 1\}, min\{sup\,w_A(u) + sup\,w_B(u), 1\}] >/ u \in \mathbb{U}) \end{matrix}\right\}. \quad (4)$$

Example 7 Reconsider example 4

$$F_A + G_B = \{(x_1, \{\langle u_1, [1.0, 1.0], [1.0, 1.0], [0.5, 1.0]\rangle, \langle u_2, [0.7, 1.0], [0.6, 1.0], [0.6, 1.0]\rangle\}),$$
$$(x_2, \{\langle u_1, [0.7, 1.0], [0.7, 1.0], [0.7, 1.0]\rangle, \langle u_2, [0.6, 1.0], [1.0, 1.0], [0.7, 1.0]\rangle\}),$$
$$(x_3, \{\langle u_1, [0.5, 1.0], [0.3, 1.0], [0.7, 1.0]\rangle, \langle u_2, [0.7, 1.0], [0.3, 1.0], [0.8, 1.0]\rangle\}),$$
$$(x_4, \{\langle u_1, [1.0, 1.0], [1.0, 1.0], [1.0, 1.0]\rangle, \langle u_2, [0.9, 1.0], [0.9, 1.0], [0.3, 1.0]\rangle\})\}.$$

Definition 3.14

Let $F_A \in$ IVNHSS over $\mathbb{U}$, then its scalar multiplication is represented as $F_A.\breve{a}$, where $\breve{a} \in [0, 1]$ and defined as follows

$$F_A.\breve{a} = \left\{\begin{matrix} (< u, [min\{inf\,u_A(u).\breve{a}, 1\}, min\{sup\,u_A(u).\breve{a}, 1\}], \\ [min\{inf\,v_A(u).\breve{a}, 1\}, min\{sup\,v_A(u).\breve{a}, 1\}], \\ [min\{inf\,w_A(u).\breve{a}, 1\}, min\{sup\,w_A(u).\breve{a}, 1\}] >/ u \in \mathbb{U}) \end{matrix}\right\}. \quad (5)$$

Definition 3.15

Let $F_A \in$ IVNHSS over $\mathbb{U}$, then its scalar division is represented as $F_A/\breve{a}$, where $\breve{a} \in [0, 1]$ and defined as follows

$$F_A/\breve{a} = \left\{\begin{matrix} (< u, [min\{inf\,u_A(u)/\breve{a}, 1\}, min\{sup\,u_A(u)/\breve{a}, 1\}], \\ [min\{inf\,v_A(u)/\breve{a}, 1\}, min\{sup\,v_A(u)/\breve{a}, 1\}], \\ [min\{inf\,w_A(u)/\breve{a}, 1\}, min\{sup\,w_A(u)/\breve{a}, 1\}] >/ u \in \mathbb{U}) \end{matrix}\right\}. \quad (6)$$

Definition 3.16

Let $F_A \in$ IVNHSS over $\mathbb{U}$, then Truth-Favorite operator on F_A is denoted by $\widetilde{\Delta}F_A$ and defined as follows

$$\widetilde{\Delta}F_A = \left\{\begin{matrix} (< u, [min\{inf\,u_A(u) + inf\,v_A(u), 1\}, min\{sup\,u_A(u) + sup\,v_A(u), 1\}], [0, 0], \\ [inf\,w_A(u), sup\,w_A(u)] >/ u \in \mathbb{U}) \end{matrix}\right\}. \quad (7)$$

Example 8 Reconsider example 1

$$\widetilde{\Delta}F_A = \{(x_1, \{\langle u_1, [1, 1], [0, 0], [.1, .4]\rangle, \langle u_2, [.7, 1], [0, 0], [.2, .6]\rangle\}),$$
$$(x_2, \{\langle u_1, [.7, 1], [0, 0], [.3, .5]\rangle, \langle u_2, [.6, 1], [0, 0], [.3, .7]\rangle\}),$$
$$(x_3, \{\langle u_1, [.3, 1], [0, 0], [.7, .8]\rangle, \langle u_2, [.5, 1], [0, 0], [.5, .7]\rangle\}),$$
$$(x_4, \{\langle u_1, [1, 1], [0, 0], [1, 1]\rangle, \langle u_2, [1, 1], [0, 0], [.1, .8]\rangle\})\}$$

Proposition 3.17

Let $\mathcal{F}_{\breve{A}}, \mathcal{G}_{\breve{B}} \in$ IVNHSS over $\mathbb{U}$, then

1. $\widetilde{\Delta}\widetilde{\Delta}\mathcal{F}_{\breve{A}} = \widetilde{\Delta}\mathcal{F}_{\breve{A}}$
2. $\widetilde{\Delta}(\mathcal{F}_{\breve{A}} \cup \mathcal{G}_{\breve{B}}) \subseteq \widetilde{\Delta}\mathcal{F}_{\breve{A}} \cup \widetilde{\Delta}\mathcal{G}_{\breve{B}}$

3. $\tilde{\Delta}(\mathcal{F}_{\breve{A}} \cap \mathcal{G}_{\breve{B}}) \subseteq \tilde{\Delta}\mathcal{F}_{\breve{A}} \cap \tilde{\Delta}\mathcal{G}_{\breve{B}}$
4. $\tilde{\Delta}(\mathcal{F}_{\breve{A}} + \mathcal{G}_{\breve{B}}) = \tilde{\Delta}\mathcal{F}_{\breve{A}} + \tilde{\Delta}\mathcal{G}_{\breve{B}}$

Proof of the above proposition is easily obtained by using definitions 3.4, 3.6, 3.13, and 3.16.

Definition 3.18

Let $F_A \in$ IVNHSS over $\mathbb{U}$, then False-Favorite operator on F_A is denoted by $\tilde{\nabla}F_A$ and defined as follows

$$\tilde{\nabla}F_A = \left\{ \begin{matrix} (< u, [inf\,u_A(u), sup\,u_A(u)], [0,0], \\ [min\{inf\,w_A(u) + inf\,v_A(u), 1\}, min\{sup\,w_A(u) + sup\,v_A(u), 1\}] >/ u \in \mathbb{U}) \end{matrix} \right\}. \quad (8)$$

Example 9 Reconsider example 1

$$\begin{aligned} \tilde{\nabla}F_A = \{&(x_1, \{\langle u_1, [.6,.8], [0,0], [.6,1]\rangle, \langle u_2, [.4,.7], [0,0], [.5,1]\rangle\}), \\ &(x_2, \{\langle u_1, [.4,.7], [0,0], [.6,1]\rangle, \langle u_2, [0,.3], [0,0], [.9,1]\rangle\}), \\ &(x_3, \{\langle u_1, [.2,.9], [0,0], [.8,1]\rangle, \langle u_2, [.4,.9], [0,0], [.6,1]\rangle\}), \\ &(x_4, \{\langle u_1, [.6,.9], [0,0], [1,1]\rangle, \langle u_2, [.5,.9], [0,0], [.7,1]\rangle\})\} \end{aligned}$$

Proposition 3.19

Let $\mathcal{F}_{\breve{A}}$ and $\mathcal{G}_{\breve{B}} \in$ IVNHSS over $\mathbb{U}$, then

1. $\tilde{\nabla}\tilde{\nabla}\mathcal{F}_{\breve{A}} = \tilde{\nabla}\mathcal{F}_{\breve{A}}$
2. $\tilde{\nabla}(\mathcal{F}_{\breve{A}} \cup \mathcal{G}_{\breve{B}}) \subseteq \tilde{\nabla}\mathcal{F}_{\breve{A}} \cup \tilde{\nabla}\mathcal{G}_{\breve{B}}$
3. $\tilde{\nabla}(\mathcal{F}_{\breve{A}} \cap \mathcal{G}_{\breve{B}}) \subseteq \tilde{\nabla}\mathcal{F}_{\breve{A}} \cap \tilde{\nabla}\mathcal{G}_{\breve{B}}$
4. $\tilde{\nabla}(\mathcal{F}_{\breve{A}} + \mathcal{G}_{\breve{B}}) = \tilde{\nabla}\mathcal{F}_{\breve{A}} + \tilde{\nabla}\mathcal{G}_{\breve{B}}$

Proof of the above proposition is easily obtained by using definitions 3.4, 3.6, 3.13, and 3.18.

4. Necessity and Possibility Operations on IVNHSS

In this section, some further operations on IVNHSS are developed such as OR-Operation, And-Operation, necessity, and possibility operations with some properties.

Definition 4.1

Let F_A and $G_B \in$ IVNHSS over $\mathbb{U}$, then OR-Operator is represented by $F_A \vee G_B$ and defined as follows

$$\begin{aligned} u\,(F_{A \times B}) &= [max\{inf\,u_A(u), inf\,u_B(u)\}, max\{sup\,u_A(u), sup\,u_B(u)\}], \\ v\,(F_{A \times B}) &= [min\{inf\,v_A(u), inf\,v_B(u)\}, min\{sup\,v_A(u), sup\,v_B(u)\}], \\ w\,(F_{A \times B}) &= [min\{inf\,w_A(u), inf\,w_B(u)\}, min\{sup\,w_A(u), sup\,w_B(u)\}]. \end{aligned}$$

Definition 4.2

Let F_A and $G_B \in$ IVNHSS over $\mathbb{U}$, then And-Operator is represented by $F_A \wedge G_B$ and defined as follows

$$\begin{aligned} u\,(F_{A \times B}) &= [min\{inf\,u_A(u), inf\,u_B(u)\}, min\{sup\,u_A(u), sup\,u_B(u)\}], \\ v\,(F_{A \times B}) &= [max\{inf\,v_A(u), inf\,v_B(u)\}, max\{sup\,v_A(u), sup\,v_B(u)\}], \\ w\,(F_{A \times B}) &= [max\{inf\,w_A(u), inf\,w_B(u)\}, max\{sup\,w_A(u), sup\,w_B(u)\}]. \end{aligned}$$

Proposition 4.3

Let $\mathcal{F}_{\breve{A}}$, $\mathcal{G}_{\breve{B}}$, $\mathcal{H}_{\breve{C}} \in$ IVNHSSs, then

1. $\mathcal{F}_{\check{A}} \vee \mathcal{G}_{\check{B}} = \mathcal{G}_{\check{B}} \vee \mathcal{F}_{\check{A}}$
2. $\mathcal{F}_{\check{A}} \wedge \mathcal{G}_{\check{B}} = \mathcal{G}_{\check{B}} \wedge \mathcal{F}_{\check{A}}$
3. $\mathcal{F}_{\check{A}} \vee (\mathcal{G}_{\check{B}} \vee \mathcal{H}_{\check{C}}) = (\mathcal{F}_{\check{A}} \vee \mathcal{G}_{\check{B}}) \vee \mathcal{H}_{\check{C}}$
4. $\mathcal{F}_{\check{A}} \wedge (\mathcal{G}_{\check{B}} \wedge \mathcal{H}_{\check{C}}) = (\mathcal{F}_{\check{A}} \wedge \mathcal{G}_{\check{B}}) \wedge \mathcal{H}_{\check{C}}$
5. $(\mathcal{F}_{\check{A}} \vee \mathcal{G}_{\check{B}})^c = \mathcal{F}^c(\check{A}) \wedge \mathcal{G}^c(\check{B})$
6. $(\mathcal{F}_{\check{A}} \wedge \mathcal{G}_{\check{B}})^c = \mathcal{F}^c(\check{A}) \vee \mathcal{G}^c(\check{B})$

Proof We can prove easily by using definitions 4.1 and 4.2.

Definition 4.4

Let $F_A \in$ IVNHSS over $\mathbb{U}$, then necessity operator IVNHSS represented as $\oplus\ F_A$ and defined as follows

$$\oplus\ F_A = \{< u, [inf\,u_A(u), sup\,u_A(u)], [inf\,v_A(u), sup\,v_A(u)], [1 - sup\,u_A(u), 1 - inf\,u_A(u)] > : u \in \mathbb{U}\}$$

Example 10 Reconsider example 1

$$\begin{aligned}\oplus F_A = \{&(x_1, \{\langle u_1, [.6, .8], [.5, 0.9], [.2, .4]\rangle, \langle u_2, [.4, .7], [.3, .9], [.3, .6]\rangle\}),\\ &(x_2, \{\langle u_1, [.4, .7], [.3, .9], [.3, .6]\rangle, \langle u_2, [0, .3], [.6, .8], [.7, 1]\rangle\}),\\ &(x_3, \{\langle u_1, [.2, .9], [.1, .5], [.1, .8]\rangle, \langle u_2, [.4, .9], [.1, .6], [.1, .6]\rangle\}),\\ &(x_4, \{\langle u_1, [.6, .9], [.6, .9], [.1, .4]\rangle, \langle u_2, [.5, .9], [.6, .8], [.1, .5]\rangle\})\}\end{aligned}$$

Definition 4.5

Let $F_A \in$ IVNHSS over $\mathbb{U}$, then possibility operator on IVNHSS represented as $\otimes\ F_A$ and defined as follows

$$\otimes\ F_A = \{(< u, [1 - sup\,w_A(u), 1 - inf\,w_A(u)],\ [inf\,v_A(u),\ sup\,v_A(u)],\ [inf\,w_A(u),\ sup\,w_A(u)] > /\ u \in \mathbb{U}\})\}$$

Example 11 Reconsider example 1

$$\begin{aligned}\otimes\ F_A = \{&(x_1, \{\langle u_1, [.6, .9], [.5, 0.9], [.1, .4]\rangle, \langle u_2, [.4, .8], [.3, .9], [.2, .6]\rangle\}),\\ &(x_2, \{\langle u_1, [.5, .7], [.3, .9], [.3, .5]\rangle, \langle u_2, [.3, .7], [.6, .8], [.3, .7]\rangle\}),\\ &(x_3, \{\langle u_1, [.2, .3], [.1, .5], [.7, .8]\rangle, \langle u_2, [.3, .5], [.1, .6], [.5, .7]\rangle\}),\\ &(x_4, \{\langle u_1, [0, 0], [.6, .9], [1, 1]\rangle, \langle u_2, [.2, .9], [.6, .8], [.1, .8]\rangle\})\}\end{aligned}$$

Proposition 4.6

Let $\mathcal{F}_{\check{A}}$ and $\mathcal{G}_{\check{B}} \in$ IVNHSS over $\mathbb{U}$, then

1. $\oplus\ (\mathcal{F}_{\check{A}} \cup \mathcal{G}_{\check{B}}) = \oplus\ \mathcal{F}_{\check{A}} \cup \oplus\ \mathcal{G}_{\check{B}}$
2. $\oplus\ (\mathcal{F}_{\check{A}} \cap \mathcal{G}_{\check{B}}) = \oplus\ \mathcal{F}_{\check{A}} \cap \oplus\ \mathcal{G}_{\check{B}}$

Proof 1. As we know that

$\mathcal{F}_{\check{A}} = \{(< u, [inf\,u_A(u),\ sup\,u_A(u)],\ [inf\,v_A(u),\ sup\,v_A(u)],\ [inf\,w_A(u),\ sup\,w_A(u)] > /\ u \in \mathbb{U}\})\}$ and

$\mathcal{G}_{\check{B}} = \{(< u, [inf\,u_B(u),\ sup\,u_B(u)],\ [inf\,v_B(u),\ sup\,v_B(u)],\ [inf\,w_B(u),\ sup\,w_B(u)] > /\ u \in \mathbb{U}\})\}$

Then by using definition 3.5, we get

$$\mathcal{F}_{\check{A}} \cup \mathcal{G}_{\check{B}} = \left\{\begin{matrix}(< u, [max\{inf\,u_A(u),\ inf\,u_B(u)\}, max\{sup\,u_A(u),\ sup\,u_B(u)\}],\\ [min\{inf\,v_A(u),\ inf\,v_B(u)\}, min\{sup\,v_A(u),\ sup\,v_B(u)\}],\\ [min\{inf\,w_A(u),\ inf\,w_B(u)\}, min\{sup\,w_A(u),\ sup\,w_B(u)\}] > /\ u \in \mathbb{U})\end{matrix}\right\}.$$

By using the necessity operator, we get

$$\oplus\ (\mathcal{F}_{\check{A}} \cup \mathcal{G}_{\check{B}}) = \left\{\begin{matrix} (< u, [max\{inf\mathcal{u}_A(u),\ inf\mathcal{u}_B(u)\}, max\{sup\mathcal{u}_A(u),\ sup\mathcal{u}_B(u)\}], \\ [min\{inf\mathcal{v}_A(u),\ inf\mathcal{v}_B(u)\}, min\{sup\mathcal{v}_A(u),\ sup\mathcal{v}_B(u)\}], \\ [1 - max\{sup\mathcal{u}_A(u),\ sup\mathcal{u}_B(u)\}, 1 - max\{inf\mathcal{u}_A(u),\ inf\mathcal{u}_B(u)\}] >/ u \in \mathbb{U}) \end{matrix}\right\}.$$

$\oplus\ (\mathcal{F}_{\check{A}} \cup \mathcal{G}_{\check{B}}) =$

$$\left\{\begin{matrix} (< u, [max\{inf\mathcal{u}_A(u),\ inf\mathcal{u}_B(u)\}, max\{sup\mathcal{u}_A(u),\ sup\mathcal{u}_B(u)\}], \\ [min\{inf\mathcal{v}_A(u),\ inf\mathcal{v}_B(u)\}, min\{sup\mathcal{v}_A(u),\ sup\mathcal{v}_B(u)\}], \\ [min\{1 - sup\mathcal{u}_A(u),\ 1 - sup\mathcal{u}_B(u)\}, min\{1 - inf\mathcal{u}_A(u),\ 1 - inf\mathcal{u}_B(u)\}] >/ u \in \mathbb{U}) \end{matrix}\right\}.$$

$\oplus\ \mathcal{F}_{\check{A}} = \{(< u, [inf\mathcal{u}_A(u),\ sup\mathcal{u}_A(u)],\ [inf\mathcal{v}_A(u),\ sup\mathcal{v}_A(u)],\ [1 - sup\mathcal{u}_A(u), 1 - inf\mathcal{u}_A(u)] >/ u \in \mathbb{U}\})\}$ and

$\oplus\ \mathcal{G}_{\check{B}} = \{(< u, [inf\mathcal{u}_B(u),\ sup\mathcal{u}_B(u)],\ [inf\mathcal{v}_B(u),\ sup\mathcal{v}_B(u)],\ [1 - sup\mathcal{u}_B(u), 1 - inf\mathcal{u}_B(u)] >/ u \in \mathbb{U}\})\}$

Again, by using definition 3.5 we get

$\oplus\ \mathcal{F}_{\check{A}} \cup \oplus\ \mathcal{G}_{\check{B}} =$

$$\left\{\begin{matrix} (< u, [max\{inf\mathcal{u}_A(u),\ inf\mathcal{u}_B(u)\}, max\{sup\mathcal{u}_A(u),\ sup\mathcal{u}_B(u)\}], \\ [min\{inf\mathcal{v}_A(u),\ inf\mathcal{v}_B(u)\}, min\{sup\mathcal{v}_A(u),\ sup\mathcal{v}_B(u)\}], \\ [min\{1 - sup\mathcal{u}_A(u),\ 1 - sup\mathcal{u}_B(u)\}, min\{1 - inf\mathcal{u}_A(u),\ 1 - inf\mathcal{u}_B(u)\}] >/ u \in \mathbb{U}) \end{matrix}\right\}$$

Hence

$\oplus\ (\mathcal{F}_{\check{A}} \cup \mathcal{G}_{\check{B}}) = \oplus\ \mathcal{F}_{\check{A}} \cup \oplus\ \mathcal{G}_{\check{B}}$

Similarly, we can prove assertion 2.

Proposition 4.7

Let $\mathcal{F}_{\check{A}}$ and $\mathcal{G}_{\check{B}}$ $\in$ IVNHSS, then we have the following

1. $\oplus(\mathcal{F}_{\check{A}} \wedge \mathcal{G}_{\check{B}}) = \oplus\mathcal{F}_{\check{A}} \wedge \oplus\mathcal{G}_{\check{B}}$
2. $\oplus(\mathcal{F}_{\check{A}} \vee \mathcal{G}_{\check{B}}) = \oplus\mathcal{F}_{\check{A}} \vee \oplus\mathcal{G}_{\check{B}}$
3. $\otimes(\mathcal{F}_{\check{A}} \wedge \mathcal{G}_{\check{B}}) = \otimes\mathcal{F}_{\check{A}} \wedge \otimes\mathcal{G}_{\check{B}}$
4. $\otimes(\mathcal{F}_{\check{A}} \vee \mathcal{G}_{\check{B}}) = \otimes\mathcal{F}_{\check{A}} \vee \otimes\mathcal{G}_{\check{B}}$

Proof By using definitions 4.1, 4.2, 4.4, and 4.5 the proof of the above proposition can be obtained easily.

5. Conclusion

In this paper, we study NHSS and IVNHSS with some basic definitions and examples. We extend the work on IVNHSS and proposed some fundamental operations on IVNHSS such as union, intersection, extended union, extended intersection, addition, and difference, etc. are developed with their properties and proved the De Morgan laws by using union, intersection, OR-operation, and And-Operation. We also developed the addition, difference, scalar multiplication, Truth-Favorite, and False-Favorite operators on IVNHSS. Finally, the concept of necessity and possibility operations on IVNHSS with properties are presented. For future trends, we can develop the interval-valued neutrosophic hypersoft matrices by using proposed operations and use them for decision making. Furthermore, several other operators such as weighted average, weighted geometric, interaction weighted average, interaction weighted geometric, etc. can be developed with their decision-making approaches to solve MCDM problems.

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