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Types of System of the neutrosophic linear equations and Cramer's rule

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Abstract: The purpose of this article is studying the types of system of neutrosophic linear equations, where the neutrosophic linear equation and the solution to the neutrosophic linear equation are defined. Also, finding a solution to a linear equation with two variables, the general situation of the solution. In addition to studying the n-variable neutrosophic linear equation and the system of neutrosophic homogeneous linear equations. The most important is the introduction of the concept of Cramer's rule to solve the system of neutrosophic linear equations. Provide enough examples for each case to enhance understanding.

Keywords: neutrosophic liner equation; Cramer's rule; solution of the neutrosophic linear equation.

1. Introduction

As an alternative to the existing logics, Smarandache proposed the Neutrosophic Logic to represent a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, contradiction, where the concept of neutrosophy is a new branch of philosophy introduced by Smarandache [3-13]. He presented the definition of the standard form of neutrosophic real number and conditions for the division of two neutrosophic real numbers to exist, he defined the standard form of neutrosophic complex number, and found root index $n \geq 2$ of a neutrosophic real and complex number [2-4], studying the concept of the Neutrosophic probability [3-5], the Neutrosophic statistics [4][6], and professor Smarandache entered the concept of preliminary calculus of the differential and integral calculus, where he introduced for the first time the notions of neutrosophic mereo-limit, mereo-continuity, mereoderivative, and mereo-integral [1-8]. Madeleine Al- Taha presented results on single valued neutrosophic (weak) polygroups [9].Edalatpanah proposed a new direct algorithm to solve the neutrosophic linear programming where the variables and right-hand side represented with triangular neutrosophic numbers [10]. Chakraborty used pentagonal neutrosophic number in networking problem, and Shortest Path Problem [11-12]. Y.Alhasan studied the concepts of neutrosophic complex numbers, the general exponential form of a neutrosophic complex, and the neutrosophic integrals and integration methods [7-14-17]. On the other hand, M.Abdel-Basset presented study in the science of neutrosophic about an approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number [15]. Also, neutrosophic sets played an important role in applied science such as health care, industry, and optimization [16]. Recently, Alhasan,Y., and Alfahal, A presented study in the neutrosophic differential equations that translate into linear[18].

Mathematical equations are used to solve real problems in our daily life, for example, mathematical equations are used in electronic chips used in all modern machines and devices, such as washing machines and dryers, cars, airplanes, ships, cell phones, computers, space programs and so on. One may be surprised when he learns that there are about 2 million algorithms and mathematical equations in mobile and computer devices.

The paper consists of 5 sections. In 1th section, provides an introduction, in which neutrosophic science review has given. In 2th section, some definitions and examples of neutrosophic real number. The 3th section frames studying types of system of the neutrosophic linear equations. The 4th section introduces the concept of Cramer's rule to solve the system of neutrosophic linear equations. In 5th section, a conclusion to the paper is given.

2. Preliminaries

2.1. Neutrosophic Real Number [4]

Suppose that w is a neutrosophic number, then it takes the following standard form: $w = a + bI$ where a , b are real coefficients, and I represent indeterminacy, such $0.\mathrm{I} = 0$ and $\mathrm{I}^n = \mathrm{I}$, for all positive integers n.

2.2. Division of neutrosophic real numbers [4]

Suppose that w_1, w_2 are two neutrosophic numbers, where

$$w_1 = a_1 + b_1 I \ , \qquad w_2 = a_2 + b_2 I$$

To find $(a_1 + b_1 I) \div (a_2 + b_2 I)$, we can write:

$$\frac{a_1 + b_1 I}{a_2 + b_2 I} \equiv x + yI$$

where x and y are real unknowns.

$$a_1 + b_1 I \equiv (a_2 + b_2 I)(x + yI)$$

$$a_1 + b_1 I \equiv a_2 x + (b_2 x + a_2 y + b_2 y)I$$

by identifying the coefficients, we get

$$a_1 = a_2 x$$

$$b_1 = b_2 x + (a_2 + b_{2)} y$$

We obtain unique one solution only, provided that:

$$\begin{vmatrix} a_2 & 0 \\ b_2 & a_2 + b_2 \end{vmatrix} \neq 0 \quad \Rightarrow \quad a_2(a_2 + b_2) \neq 0$$

Hence: $a_2 \neq 0$ *and* $a_2 \neq -b_2$ are the conditions for the division of two neutrosophic real numbers to exist.

Then:

$$\frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} . I$$

3. The neutrosophic linear equation

Defination3.1:
The neutrosophic linear equation of n variables $x_1, x_2, x_3, \dots\dots x_n$, is each equation that takes the form:
$$(a_1 + b_1I)x_1 + (a_2 + b_2I)x_2 + (a_3 + b_3I)x_3 + \cdots + (a_n + b_nI)x_n = c + dI$$
Where:
$a_1, a_2, a_3, \dots, a_n, b_1, b_2, b_3, \dots, b_n, c, d$ are real coefficients, and I represent indeterminacy.
We call $(a_1 + b_1I), (a_2 + b_2I), (a_3 + b_3I), \dots, (a_n + b_nI)$ neutrosophic coefficients of the borders of the equation, and $c + dI$ constant neutrosophic border of the equation.

Remarks3.1:
We call each equation of the form:
$$(a_1 + b_1I)x + (a_2 + b_2I)y = c + dI$$
the two-variable neutrosophic linear equation, where:
$a_1, a_2, a_3, \dots, a_n, b_1, b_2, b_3, \dots, b_n, c, d$ are real coefficients, and I represent indeterminacy.

Remarks3.2:
We call each equation of the form:
$$(a_1 + b_1I)x + (a_2 + b_2I)y + (a_3 + b_3I)z = c + dI$$
the three-variable neutrosophic linear equation, where:
$a_1, a_2, a_3, \dots, a_n, b_1, b_2, b_3, \dots, b_n, c, d$ are real coefficients, and I represent indeterminacy.

Example3.1:
- ✓ $(2 + 3I)x + (4 - 5I)y + (-1 + I)z = 5 + 4I$
- ✓ $(3 - I)x + (9 - 5I)y + (-1 + I)z + (3 - 2I)w = 7 - I$
- ✓ $(1 + I)x + (2 - 5I)y = 6 - 2I$

Defination3.2:
Solution of the neutrosophic linear equation:
$$(a_1 + b_1I)x_1 + (a_2 + b_2I)x_2 + (a_3 + b_3I)x_3 + \cdots + (a_n + b_nI)x_n = c + dI$$
is finding the values of the variables $x_1, x_2, x_3, \dots, x_n$ that make its first term equal to its second term. where a_1, a_2, b_1, b_2, c, d are real coefficients.

Example3.2:
The equation: $(1 + I)x + (2 - 5I)y = 6 - 2I$ accepts the solution
$$x = 1 - I,\ \ y = \frac{5}{2} - \frac{23}{6}I$$
because fulfills the equation:
$$L_1 = (1 + I)(1 - I) + (2 - 5I)\left(\frac{5}{2} - \frac{23}{6}I\right) = 1 - I + 5 - \frac{46}{6}I - \frac{25}{2}I + \frac{115}{6}I = 6 - 2I = L_2$$

3.1 Solution of the two-variables neutrosophic linear equation

For the neutrosophic linear equation:
$$(a_1 + b_1I)x + (a_2 + b_2I)y = c + dI$$
unlimited number of solutions defined by form:
$$S = \left\{(x, y) \in R^2 \cup \{I\}: y = \left(\frac{a_1}{a_2} + \frac{a_2b_1 - a_1b_2}{a_2(a_2 + b_2)}.I\right)x + \frac{\mathrm{c}}{a_2} + \frac{a_2\mathrm{d} - \mathrm{c}b_2}{a_2(a_2 + b_2)}.I\right\}$$
where a_1, a_2, b_1, b_2, c, d are real coefficients, $a_2 \neq 0$ and $a_2 \neq -b_2$, by given a value for one of the two variables, we obtain a value for the other variable.

Example3.1.1:
Find solution of the equation:
$$(1 + I)x + (2 - 5I)y = 6 - 2I$$

Solution:

$$y=\frac{-(1+I)}{(2-5I)}x+\frac{6-2I}{(2-5I)}$$

$$y=\left(\frac{-1}{2}+\frac{7}{6}I\right)x+3+\frac{13}{3}I$$

Then the set of solutions is:

$$S=\left\{(x,y)\in R^2\cup\{I\}: y=\left(\frac{-1}{2}+\frac{7}{6}I\right)x+3+\frac{13}{3}I\right\}$$

By given any value for the variables x, we obtain a value of the variable y.

3.2 General situation: Solution of the n-variable neutrosophic linear equation

For the neutrosophic linear equation:

$$(a_1+b_1I)x_1+(a_2+b_2I)x_2+(a_3+b_3I)x_3+\cdots+(a_n+b_nI)x_n=c+dI$$

unlimited number of solutions, where $a_1,a_2,a_3,\dots,a_n,b_1,b_2,b_3,\dots,b_n,c,d$ are real coefficients.

4. System of the neutrosophic linear equations

It is a system of neutrosophic linear equations that given by the form:

$$\begin{cases}(a_{11}+b_{11}I)x_1+(a_{12}+b_{12}I)x_2+(a_{13}+b_{13}I)x_3+\cdots+(a_{1n}+b_{1n}I)x_n=c_1+d_1I\\(a_{21}+b_{21}I)x_1+(a_{22}+b_{22}I)x_2+(a_{23}+b_{23}I)x_3+\cdots+(a_{2n}+b_{2n}I)x_n=c_2+d_2I\\ .\dots \quad .... \quad \dots \quad .... \quad \dots\ \dots=\cdots\\ .\dots \quad .... \quad \dots \quad .... \quad \dots=\cdots\\(a_{m1}+b_{m1}I)x_1+(a_{m2}+b_{m2}I)x_2+(a_{m3}+b_{m3}I)x_3+\cdots+(a_{mn}+b_{mn}I)x_n=c_m+d_mI\end{cases}$$

Where:

a_{ij},b_{ij},c_j,d_j are real coefficients, $i=1,\dots,n, j=1,\dots,m,$ and I represent indeterminacy.

4.1 Solution of system of the neutrosophic linear equations

Solution of system of the neutrosophic linear equation:

$$\begin{cases}(a_{11}+b_{11}I)x_1+(a_{12}+b_{12}I)x_2+(a_{13}+b_{13}I)x_3+\cdots+(a_{1n}+b_{1n}I)x_n=c_1+d_1I\\(a_{21}+b_{21}I)x_1+(a_{22}+b_{22}I)x_2+(a_{23}+b_{23}I)x_3+\cdots+(a_{2n}+b_{2n}I)x_n=c_2+d_2I\\ .\dots \quad .... \quad \dots \quad .... \quad \dots\ \dots=\cdots\\ .\dots \quad .... \quad \dots \quad .... \quad \dots=\cdots\\(a_{m1}+b_{m1}I)x_1+(a_{m2}+b_{m2}I)x_2+(a_{m3}+b_{m3}I)x_3+\cdots+(a_{mn}+b_{mn}I)x_n=c_m+d_mI\end{cases} \qquad (*)$$

Where:

a_{ij},b_{ij},c_j,d_j are real coefficients, $i=1,\dots,n, j=1,\dots,m,$ and I represent indeterminacy.

is finding the values of the variables $x_1,x_2,x_3,\dots,x_n$ that fulfill each of the system equations. We call this solution the system solution.

Remarks4.1.1:

We distinguish three cases to solve the system$(*)$:

1. It may have unique solution
2. It may be impossible to solve
3. It may have Unlimited number of solutions

4.2 Cramer's rule to solve system of the neutrosophic linear equations

Let the system of the neutrosophic linear equations:

$$\begin{cases} (a_{11}+b_{11}I)x_1+(a_{12}+b_{12}I)x_2+(a_{13}+b_{13}I)x_3+\dots+(a_{1n}+b_{1n}I)x_n=c_1+d_1I \\ (a_{21}+b_{21}I)x_1+(a_{22}+b_{22}I)x_2+(a_{23}+b_{23}I)x_3+\dots+(a_{2n}+b_{2n}I)x_n=c_2+d_2I \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots = \dots \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots = \dots \\ (a_{n1}+b_{n1}I)x_1+(a_{n2}+b_{n2}I)x_2+(a_{n3}+b_{n3}I)x_3+\dots+(a_{nn}+b_{nn}I)x_n=c_n+d_nI \end{cases} \qquad (*)$$

with the n variables and the n equations, A the neutrosophic coefficient matrix of the system, and suppose $\det(A)=a+bI$, We distinguish the following cases:

1. If $\det(A)\neq 0+0I \; or \; a\neq 0 \; or \; a\neq -b$, then the system has unique solution given by the formula:

$$x_i=\frac{\det(x_i)}{\det(A)} \; ; \; i=1,2,\dots,n$$

Where $\det(x_i)$ is the determinant produced by the determinant $\det(A)$ by replacing the column of constants with the i -order column.

2. If $a=0 \; or \; a=-b$, then the system is impossible to solve.
3. If $\det(A)=0+0I$, then we distinguish tow cases:
   - If one of the determinants $\det(x_1)$, $\det(x_2),\dots,\det(x_n)$, is not equal to zero, then the system is impossible to solve.
   - If $\det(x_i)=0+0I; \; i=1,2,\dots,n$, then the system is impossible to solve or it have unlimited number of solutions.

4.2.1 Solve the system of two linear equations by two variables

Let:

$$(a_{11}+b_{11}I)x+(a_{12}+b_{12}I)\mathrm{y}=c_1+d_1I$$
$$(a_{21}+b_{21}I)x+(a_{22}+b_{22}I)\mathrm{y}=c_2+d_2I$$

$$A=\begin{bmatrix} a_{11}+b_{11}I & a_{12}+b_{12}I \\ a_{21}+b_{21}I & a_{22}+b_{22}I \end{bmatrix}$$

$$\det(A)=\begin{vmatrix} a_{11}+b_{11}I & a_{12}+b_{12}I \\ a_{21}+b_{21}I & a_{22}+b_{22}I \end{vmatrix}=$$

$$\det(x)=\begin{vmatrix} c_1+d_1I & a_{12}+b_{12}I \\ c_2+d_2I & a_{22}+b_{22}I \end{vmatrix} \quad , \quad \det(y)=\begin{vmatrix} a_{11}+b_{11}I & c_1+d_1I \\ a_{21}+b_{21}I & c_2+d_2I \end{vmatrix}$$

Suppose $\det(A)=a+bI$, We distinguish the following cases:

1. If $\det(A)\neq 0+0I \; or \; a\neq 0 \; or \; a\neq -b$, then the system has unique solution given by the formulas:

$$x=\frac{\det(x)}{\det(A)} \quad and \quad y=\frac{\det(y)}{\det(A)}$$

2. If $a=0 \; or \; a=-b$, then the system is impossible to solve.
3. If $\det(A)=0+0I$, then we distinguish tow cases:
   - If one of the determinants $\det(x)$, $\det(y)$, is not equal to zero, then the system is impossible to solve.

- If $\det(x_i) = 0 + 0I;\ i = 1,2,\dots,n$, then the system has unlimited number of solutions.

Note: To verify the solution, we substitute the value of x and y into one of the equations.

Example4.2.1:

Let:

$$(2+I)x + 3y = 5 + I$$
$$(3-2I)x + 2y = I$$

Find solution of the system.

Solution:

$$A = \begin{bmatrix} 2+I & 3 \\ 3-2I & 2 \end{bmatrix}$$

$$\det A = \begin{vmatrix} 2+I & 3 \\ 3-2I & 2 \end{vmatrix} = 4 + 2I - 9 + 6I = -5 + 8I \neq 0 + 0I$$

Then the system has one solution, is:

$$\det x = \begin{vmatrix} 5+I & 3 \\ I & 2 \end{vmatrix} = 10 - I \quad , \quad \det y = \begin{vmatrix} 2+I & 5+I \\ 3-2I & I \end{vmatrix} = -15 + 12I$$

$$x = \frac{\det x}{\det A} = \frac{10 - I}{-5 + 8I} = -2 + 5I \quad and \quad y = \frac{\det y}{\det A} = \frac{-15 + 12I}{-5 + 8I} = 3 - 4I$$

$$(x, y) = (-2 + 5I, 3 - 4I)$$

To verify the solution, we substitute the value of x and y into the first equation:

$$(2+I)x + 3y = (2+I)(-2+5I) + 3(3-4I)$$
$$= -4 - 2I + 10I + 5I + 9 - 12I = 5 + I$$

Example4.2.2:

Let:

$$2Ix + 7y = I$$
$$3Ix + y = 2I$$

Find solution of the system.

Solution:

$$A = \begin{bmatrix} 2I & 7 \\ 3I & 1 \end{bmatrix}$$

$$\det A = \begin{vmatrix} 2I & 7 \\ 3I & 1 \end{vmatrix} = 2I - 21I = 0 - 19I$$

This does not fulfill the condition: $\det A = \mathrm{a} + \mathrm{b}I$, $\mathrm{a} \neq 0$ and $\mathrm{a} \neq -\mathrm{b}$

Because:

$$\det x = \begin{vmatrix} I & 7 \\ 2I & 1 \end{vmatrix} = I - 14I = 0 - 13I$$

Then:

$$x = \frac{\det x}{\det A} = \frac{0 - 13I}{0 - 19I} \ (undefined)$$

So, the system is impossible to solve.

Example4.2.3:

Let:

$$(2+I)x + 3y = 5 + I$$
$$(3-2I)x + y = I$$

Find solution of the system.

Solution:

$$A = \begin{bmatrix} 2+I & 3 \\ 3-2I & 1 \end{bmatrix}$$

$$\det A = \begin{vmatrix} 2+I & 3 \\ 3-2I & 1 \end{vmatrix} = 2 + I - 9 + 6I = -7 + 7I$$

W This does not fulfill the condition: $\det A = \mathrm{a} + \mathrm{b}I$, $\mathrm{a} \neq 0$ and $\mathrm{a} \neq -\mathrm{b}$, where: $(a = -7, b = 7 \; and \; a = -b)$.

So, the system is impossible to solve.

Example4.2.4:

Let:

$$(1 + I)x + (3 - I)y = 2 - 3I$$

$$(2 + 2I)x + (6 - 2I)y = 4 - 6I$$

Find solution of the system.

Solution:

$$A = \begin{bmatrix} 1+I & 3-I \\ 2+2I & 6-2I \end{bmatrix}$$

$$\det A = \begin{vmatrix} 1+I & 3-I \\ 2+2I & 6-2I \end{vmatrix} = 0 + 0I$$

$$\det x = \begin{vmatrix} 2-3I & 3-I \\ 4-6I & 6-2I \end{vmatrix} = 0 + 0I \;, \quad \det y = \begin{vmatrix} 1+I & 2-3I \\ 2+2I & 4-6I \end{vmatrix} = 0 + 0I$$

As: $\det A = \det x = \det y = 0 + 0I$, so the system has Unlimited number of solutions defined by form:

$$S = \left\{(x, y) \in R^2 \cup \{I\}: y = \left(\frac{1}{3} + \frac{2}{3}.I\right)x + \frac{2}{3} - \frac{7}{6}.I\right\}$$

By given any value for the variable x, we obtain a value of the variable y.

4.2.2 Solve the system of three linear equations by three variables

Let:

$$(a_{11} + b_{11}I)x + (a_{12} + b_{12}I)\mathrm{y} + (a_{13} + b_{13}I)z = c_1 + d_1I \quad (1)$$
$$(a_{21} + b_{21}I)x + (a_{22} + b_{22}I)\mathrm{y} + (a_{23} + b_{23}I)z = c_2 + d_2I \quad (2)$$
$$(a_{33} + b_{33}I)x + (a_{33} + b_{33}I)\mathrm{y} + (a_{33} + b_{33}I)x = c_3 + d_3I \quad (3)$$

$$A = \begin{bmatrix} a_{11} + b_{11}I & a_{12} + b_{12}I & a_{13} + b_{13}I \\ a_{21} + b_{21}I & a_{22} + b_{22}I & a_{23} + b_{23}I \\ a_{33} + b_{33}I & a_{33} + b_{33}I & a_{33} + b_{33}I \end{bmatrix}$$

$$\det A = \begin{vmatrix} a_{11} + b_{11}I & a_{12} + b_{12}I & a_{13} + b_{13}I \\ a_{21} + b_{21}I & a_{22} + b_{22}I & a_{23} + b_{23}I \\ a_{33} + b_{33}I & a_{33} + b_{33}I & a_{33} + b_{33}I \end{vmatrix}$$

$$\det x = \begin{vmatrix} c_1 + d_1I & a_{12} + b_{12}I & a_{13} + b_{13}I \\ c_2 + d_2I & a_{22} + b_{22}I & a_{23} + b_{23}I \\ c_3 + d_3I & a_{33} + b_{33}I & a_{33} + b_{33}I \end{vmatrix} \;, \quad \det y = \begin{vmatrix} a_{11} + b_{11}I & c_1 + d_1I & a_{13} + b_{13}I \\ a_{21} + b_{21}I & c_2 + d_2I & a_{23} + b_{23}I \\ a_{33} + b_{33}I & c_3 + d_3I & a_{33} + b_{33}I \end{vmatrix}$$

$$\det z = \begin{vmatrix} a_{11} + b_{11}I & a_{12} + b_{12}I & c_1 + d_1I \\ a_{21} + b_{21}I & a_{22} + b_{22}I & c_2 + d_2I \\ a_{33} + b_{33}I & a_{33} + b_{33}I & c_3 + d_3I \end{vmatrix}$$

We will discuss the second case of (3):

If $\det(A) = 0 + 0I$ *and* $\det(x_i) = 0 + 0I$; $i = 1,2, \dots, n$, then we are looking for the solutions of two of the system of equations, such as {(1), (2)}:

- ✓ If the system {(1), (2)} is impossible to solve, then the system {(1), (2), (3)} is impossible to solve.

- ✓ If the system {(1), (2)} has unlimited number of solutions from the form $x = g(z), y = h(z)$, (maybe $g(z)$ *or* $h(z)$ is constant), then we substitution in (3) to obtain on the equation of the form:

$$0.z = \beta;\ \beta \in R \cup \{I\}$$

This equation is impossible to solve or it have unlimited number of solutions (According to β value).

Example4.2.5:

Let:

$$(2 + 2I)x + (3 + 3I)y + (-5 - 5I)z = 1 \qquad (1)$$

$$(1 + I)x + (-1 - I)y + (1 + I)z = 2 \qquad (2)$$

$$(5 + 5I)x + (5 + 5I)y + (-9 - 9I)z = 4 \qquad (3)$$

Find solution of the system.

Solution:

$$A = \begin{bmatrix} 2+2I & 3+3I & -5-5I \\ 1+I & -1-I & 1+I \\ 5+5I & 5+5I & -9-9I \end{bmatrix}$$

$$\det A = \begin{vmatrix} 2+2I & 3+3I & -5-5I \\ 1+I & -1-I & 1+I \\ 5+5I & 5+5I & -9-9I \end{vmatrix} = 0 + 0I$$

$$\det x = \begin{vmatrix} 1 & 3+3I & -5-5I \\ 2 & -1-I & 1+I \\ 4 & 5+5I & -9-9I \end{vmatrix} = 0 + 0I \quad , \quad \det y = \begin{vmatrix} 2+2I & 1 & -5-5I \\ 1+I & 2 & 1+I \\ 5+5I & 4 & -9-9I \end{vmatrix} = 0 + 0I$$

$$\det z = \begin{vmatrix} 2+2I & 3+3I & 1 \\ 1+I & -1-I & 2 \\ 5+5I & 5+5I & 4 \end{vmatrix} = 0 + 0I$$

As: $\det A = 0 + 0I = \det x = \det y = \det z = 0 + 0I$, so we are looking for a system solution {(1), (2)}, then:

$$x = \frac{1}{5}\left(2z + 7 - \frac{7}{2}I\right)$$

$$y = \frac{1}{5}\left(7z - 3 + \frac{3}{2}I\right)$$

By substitution in (3), we get:

$$(0 + 0I)z = 0 + 0I$$

This equation has unlimited number of solutions, so the system {(1), (2), (3)} has unlimited number of solutions given by:

$$S = \left\{(x, y, z) \in R^3 \cup \{I\}: x = \frac{1}{5}\left(2z + 7 - \frac{7}{2}I\right), y = \frac{1}{5}\left(7z - 3 + \frac{3}{2}I\right)\right\}$$

Or:

$$S = \left\{\left(\frac{1}{5}\left(2z + 7 - \frac{7}{2}I\right), \frac{1}{5}\left(7z - 3 + \frac{3}{2}I\right), z\right); z \in R \cup \{I\}\right\}$$

Example4.2.6:

Let:

$$(2 + I)x + (1 + I)y + (3 - I)Z = 2 + I$$

$$(-1+I)x+(3-2I)y+(1+3I)Z=4+2I$$
$$(3+2I)x+(4-I)y+(2-3I)Z=5-I$$

Find solution of the system.

Solution:

$$A=\begin{bmatrix} 2+I & 1+I & 3-I \\ -1+I & 3-2I & 1+3I \\ 3+2I & 4-I & 2-3I \end{bmatrix}$$

$$\det A=\begin{vmatrix} 2+I & 1+I & 3-I \\ -1+I & 3-2I & 1+3I \\ 3+2I & 4-I & 2-3I \end{vmatrix}=-30+21I$$

$$\det x=\begin{vmatrix} 2+I & 1+I & 3-I \\ 4+2I & 3-2I & 1+3I \\ 2-3I & 4-I & 2-3I \end{vmatrix}=4+29I \quad , \quad \det y=\begin{vmatrix} 2+I & 2+I & 3-I \\ -1+I & 4+2I & 1+3I \\ 3+2I & 2-3I & 2-3I \end{vmatrix}=-35-31I$$

$$\det z=\begin{vmatrix} 2+I & 1+I & 2+I \\ -1+I & 3-2I & 4+2I \\ 3+2I & 4-I & 2-3I \end{vmatrix}=-11+14I$$

Then:

$$x=\frac{\det x}{\det A}=\frac{4+29I}{-30+21I}=\frac{-2}{15}-\frac{477}{135}I$$

$$y=\frac{\det y}{\det A}=\frac{-35-31I}{-30+21I}=\frac{7}{6}+\frac{37}{6}I$$

$$z=\frac{\det z}{\det A}=\frac{-11+14I}{-30+21I}=\frac{11}{30}-\frac{7}{10}I$$

$$(x,y,z)=\left(\frac{-2}{15}-\frac{477}{135}I,\frac{7}{6}+\frac{37}{6}I,\frac{11}{30}-\frac{7}{10}I\right)$$

To verify the solution, we substitute the value of x and y into the first equation:

$$(2+I)\left(\frac{-2}{15}-\frac{477}{135}I\right)+(1+I)\left(\frac{7}{6}+\frac{37}{6}I\right)+(3-I)\left(\frac{11}{30}-\frac{7}{10}I\right)=2+I$$

Example4.2.7:

Let:

$$(2+I)x+(1+I)y+(3-I)z=1+I \qquad (1)$$
$$(4+2I)x+(2+2I)y+(6-2I)z=1+2I \qquad (2)$$
$$(6+3I)x+(3+3I)y+(9-3I)z=-1+3I \qquad (3)$$

Find solution of the system.

Solution:

$$A=\begin{bmatrix} 2+I & 1+I & 3-I \\ 4+2I & 2+2I & 6-2I \\ 6+3I & 3+3I & 9-3I \end{bmatrix}$$

$$\det A=\begin{vmatrix} 2+I & 1+I & 3-I \\ 4+2I & 2+2I & 6-2I \\ 6+3I & 3+3I & 9-3I \end{vmatrix}=0+0I$$

$$\det x=\begin{vmatrix}1+I & 1+I & 3-I\\ 1+2I & 2+2I & 6-2I\\ -1+3I & 3+3I & 9-3I\end{vmatrix}=20I$$

As: $\det A=0+0I$ *and* $\det x\neq 0+0I$, so the system is impossible to solve.

4.3 System of the homogeneous neutrosophic linear equations

Defination4.3.1:

It is a system of homogeneous neutrosophic linear equations that given by the form:

$$\begin{cases}(a_{11}+b_{11}I)x_1+(a_{12}+b_{12}I)x_2+(a_{13}+b_{13}I)x_3+\cdots+(a_{1n}+b_{1n}I)x_n=0+0I\\(a_{21}+b_{21}I)x_1+(a_{22}+b_{22}I)x_2+(a_{23}+b_{23}I)x_3+\cdots+(a_{2n}+b_{2n}I)x_n=0+0I\\ \quad .\dots\ \dots.\quad \dots\quad \dots.\quad \dots\quad \dots\quad =\cdots\\ \quad .\dots\ \dots.\quad \dots\quad \dots.\quad \dots\quad \dots\quad =\cdots\\(a_{m1}+b_{m1}I)x_1+(a_{m2}+b_{m2}I)x_2+(a_{m3}+b_{m3}I)x_3+\cdots+(a_{mn}+b_{mn}I)x_n=0+0I\end{cases}$$

Where:

$a_{ij}, b_{ij},\ c_j, d_j$ are real coefficients, $i=1,\dots,n\,, j=1,\dots,m,$ and I represent indeterminacy.

Remarks4.3.1:

We distinguish two cases to solve the system of the homogeneous neutrosophic linear equations:

1. It may have one solution
2. It may have Unlimited number of solutions

Hence it is always a solvable system (It is not impossible to solve).

Example4.3.1:

Let:

$$(2+I)x+(1+I)y+(3-I)z=0$$

$$(-1+I)x+(3-2I)y+(1+3I)z=0$$

$$(3+2I)x+(4-I)y+(2-3I)z=0$$

Find solution of the system.

Solution:

$$A=\begin{bmatrix}2+I & 1+I & 3-I\\ -1+I & 3-2I & 1+3I\\ 3+2I & 4-I & 2-3I\end{bmatrix}$$

$$\det A=\begin{vmatrix}2+I & 1+I & 3-I\\ -1+I & 3-2I & 1+3I\\ 3+2I & 4-I & 2-3I\end{vmatrix}=-30+21I$$

$$\det x=\begin{vmatrix}0 & 1+I & 3-I\\ 0 & 3-2I & 1+3I\\ 0 & 4-I & 2-3I\end{vmatrix}=0\ ,\quad \det y=\begin{vmatrix}2+I & 0 & 3-I\\ -1+I & 0 & 1+3I\\ 3+2I & 0 & 2-3I\end{vmatrix}=0$$

$$\det z=\begin{vmatrix}2+I & 1+I & 0\\ -1+I & 3-2I & 0\\ 3+2I & 4-I & 0\end{vmatrix}=0$$

Then:

$$x=\frac{\det x}{\det A}=0,\quad y=\frac{\det y}{\det A}=0\,,\quad z=\frac{\det z}{\det A}=0$$

$$(x,y,z)=(0,0,0)$$

5. Conclusions

This study was presented based on the importance of linear equations in our lives, where we introduced the concept of neutrosophic linear equation and its types. In addition to the introduction of Cramer's rule to solve the neutrosophic system of equations. This study can be generalized and applied in several fields of application in our current reality, traffic as an example.

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