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Systems of Neutrosophic Linear Equations

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Abstract: In the present paper, for first time, a System of Neutrosophic Linear Equations (SNLE) is investigated based on the embedding approach. To this end, the (α,β,γ)-cut is used for transformation of SNLE into a crisp linear system. Furthermore, the existence of a neutrosophic solution to n x n linear system is proved in details and a computational procedure for solving the SNLE is designed. Finally, numerical experiments are presented to show the reliability and efficiency of the method.

Keywords: Neutrosophic set; Neutrosophic number; Neutrosophic linear equation; Neutrosophic linear system; Embedding method.

1. Introduction

A system of linear equations can be defined as:

$$Ax=b, \tag{1}$$

Various equations in the field of scientific modeling that describe the realistic issues like engineering problems and natural phenomena such as differential equations, computational fluid, circuit simulation, cryptography, quantum and structural mechanics, MRI reconstructions, vibroacoustics, linear and non-linear optimization, portfolios, economic modeling, astrophysics, Google page rank, image processing, nano-technology, natural language processing, deep learning, etc., must be solved mathematically. These issues can regularly be diminished to solving of linear systems. There are a huge amount of models to solve this problem, for more details, see [1-15] and the references therein.

Nevertheless, if the assessment of the coefficients of systems is uncertain and imprecise and just some ambiguous understanding regarding the real values of the parameters is accessible, it might be advantageous to characterize them with special numbers related to soft computing. Fuzzy set was introduced by Zadeh [16, 17], as a suitable instrument to express uncertainty in real life situation. After the introduction of fuzzy set, numerous scholars deliberate on this topic (information of some studies can be observed in [18-23]).

Numerous researchers also suggested several strategies to solve linear systems under fuzzy situation. Fuzzy linear systems emerged at least until 1980 [24]; however Friedman et al. [25] launched a particular model to solve a fuzzy linear system where, the matrix coefficient is crisp and the right-hand hand vector is a fuzzy number. Their model later modified by some other scholars; see [26-46].

However, when there is not clarity in information then the measure of non-membership is not the complement of the measure of membership. In these cases, individual measure of membership and non-membership are needed. Keeping this type of situation in consideration, intuitionistic fuzzy set (IFS) was established by Atanassov [47]. Nevertheless, in different branches of sciences and engineering, it was found that two mentioned components are not sufficient to represent some special types of information. In such cases, a component namely 'neutrality' is needed to represent the information completely. Thus, to remove the limitation of IFS and to handle with more possible types of uncertainty in practical situation, Smarandache [48-51] initiated neutrosophic set (NS) as an extension of the classical and all types of fuzzy sets.

This concept divided into two category of the neutrosophic numbers (NNs) and the neutrosophic sets (NSs). The neutrosophic number (NN) introduce a concept of indeterminacy, denoted by $A = m + nI$ ($m, n \in R$), consists of its determinate part m and its indeterminate part nI. In the worst scenario, A can be unknown, *i.e.*, $A = nI$. However, when there is no indeterminacy related to A , in the best scenario, there is only its determinate part *i.e.*, $A = m$ [50, 51]. But, the neutrosophic sets (NSs) represented by a truth-membership degree, an indeterminacy-membership degree, a falsity-membership degree and have some subclasses such as interval neutrosophic set [52-54] , bipolar neutrosophic set [55-57], single-valued neutrosophic set [58-66], multi-valued neutrosophic set [67-98], and neutrosophic linguistic set [69-70] and applied to solve various problems; see [71-78]. It is worth mentioning that NSs and NNs are two different branches in neutrosophic theory and indicate different forms and concepts of information.

Like any other framework, system of linear equations has also been the topic of evolution. One of the important developments in this field related to situations that coefficients are defined under conditions of uncertainty and indeterminacy. In fact, one of the expectations of classic linear systems is their crispness of data. However, in circumstances where uncertainty and indeterminacy is an inevitable feature of a real life environment, the assumption of crispness of data seems questionable. Also, there is a lot of ambiguity, indeterminacy, and uncertainty in these problems. The system of linear equations under neutrosophic environment are more useful than crisp and other fuzzy linear systems because user in his/her formulation of the problem is not forced to make a delicate formulation. The use of system of Neutrosophic linear equations (SNLE) is recommended to avert unrealistic modeling. Though there are numerous methodologies to solve various issues under NSs and also some models presented to solve linear systems with NNs [79-80], but to the best of our knowledge, the SNLE has not been discussed sets until now. Therefore, the contributions of this study are as follow:

(i) We present for first time, the system of Neutrosophic Linear Equations (SNLE) problem.
(ii) Based on the (α,β,γ)-cut, we design a strategy for solving SNLE with the single valued neutrosophic numbers (SVNNs).
(iii) Some theorems about SNLE are investigated and the conditions of a strong neutrosophic solution to n x n system of linear equations is proved in details.

This study prearranged as follows: some fundamental information, notions and operations on SVNNs are announced in Section 2. In Section 3, we introduce the SNLE and propose a general model to solve it. To show the efficiency and reliability of the method, numerical tests are provided in Section 4. Lastly, conclusions are offered in Section 5.

2. Some Basic Definitions and Arithmetic Operations

Here, we have deliberated some fundamental definitions regarding the neutrosophic sets and single-valued neutrosophic numbers.

Definition 1 [48-49]. A neutrosophic set A in objects X is described by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$ where, $T_A(x)$: $X \to]0^-,1^+[$, $I_A(x)$: $X \to]0^-,1^+[$, and $F_A(x)$: $X \to]0^-,1^+[$, and

$$0^- \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+.$$

Definition 2 [58].When three membership functions of neutrosophic set A be singleton subsets in the real standard [0, 1],we have a single-valued neutrosophic set (SVNS) A that is denoted by

$$A=\{(x,T_A(x),I_A(x),F_A(x))\,|\,x\in X\}.$$

Definition 3 [59]. A single valued triangular neutrosophic number (SVTrN-number) is denoted by $A^{\aleph}=<(a,b,c),(\mu,\nu,\omega)>$ whose its three membership functions are given as follows:

$$T_{A^{\aleph}}(x)=\begin{cases}\frac{(x-a)}{(b-a)}\mu & a\le x<b,\\ \mu & x=b,\\ \frac{(c-x)}{(c-b)}\mu & b\le x<c,\\ 0 & otherwise.\end{cases}\quad I_{A^{\aleph}}(x)=\begin{cases}\frac{(b-x)}{(b-a)}\nu & a\le x<b,\\ \nu & x=b\\ \frac{(x-c)}{(c-b)}\nu & b\le x<c,\\ 1 & otherwise.\end{cases}\quad F_{A^{\aleph}}(x)=\begin{cases}\frac{(b-x)}{(b-a)}\omega & a\le x<b,\\ \omega & x=b\\ \frac{(x-c)}{(c-b)}\omega & b\le x<c,\\ 1 & otherwise.\end{cases}$$

Definition 3 [59]. Let $A_1^{\aleph}=<(a_1,b_1,c_1),(\mu_1,\nu_1,\omega_1)>$ and $A_2^{\aleph}=<(a_2,b_2,c_2),(\mu_2,\nu_2,\omega_2)>$ be two SVTrN-numbers. Then the arithmetic relations are defined as:

$$(i)A_1^{\aleph}\oplus A_2^{\aleph}=<(a_1+a_2,b_1+b_2,c_1+c_2),(\mu_1\wedge\mu_2,\nu_1\vee\nu_2,\omega_1\vee\omega_2)> \tag{2}$$

$$(ii)\lambda A_1^{\aleph}=\begin{cases}<(\lambda a_1,\lambda b_1,\lambda c_1),(\mu_1,\nu_1,\omega_1)>, & if\quad \lambda>0\\ <(\lambda c_1,\lambda b_1,\lambda a_1),(\mu_1,\nu_1,\omega_1)>, & if\quad \lambda<0\end{cases} \tag{3}$$

Definition 4 [59]. The (α,β,γ)-cut Neutrosophic set F is denoted by $F_{(\alpha,\beta,\gamma)}$, where $\alpha,\beta,\gamma\in[0,1]$ and are fixed numbers such that $\alpha+\beta+\gamma\le 3$ is defined as by $F(\alpha,\beta,\gamma)=\{<T_A(x), I_A(x), F_A(x)> : x\in X, T_A(x)\ge\alpha, I_A(x)\le\beta, F_A(x)\le\gamma\}$.

Also, If $A^{\aleph}=<(a,b,c),(\mu,\nu,\omega)>$ then (α,β,γ)-cut is given by:

$$A^{\aleph}_{(\alpha,\beta,\gamma)}=\left\langle\begin{array}{l}[(a+\alpha(b-a))\mu,(c-\alpha(c-b))\mu],\\ [(b-\beta(b-a))\nu,(b+\beta(c-b))\nu],\\ [(b-\gamma(b-a))\omega,(b+\gamma(c-b))\omega]\end{array}\right\rangle \tag{4}$$

3. System of Neutrosophic Linear Equations (SNLE)

Consider the $n\times n$ linear system with the following equations:

$$\begin{cases}a_{11}x_1^{\aleph}+a_{12}x_2^{\aleph}+.............+a_{1n}x_n^{\aleph}=b_1^{\aleph},\\ a_{21}x_1^{\aleph}+a_{22}x_2^{\aleph}+.............+a_{2n}x_n^{\aleph}=b_2^{\aleph},\\ .\\ .\\ .\\ .\\ .\\ a_{n1}x_1^{\aleph}+a_{n2}x_2^{\aleph}+.............+a_{nn}x_n^{\aleph}=b_n^{\aleph}.\end{cases} \tag{5}$$

The matrix form of the Eq.(5) is as follows:

$$Ax^{\aleph} = b^{\aleph}, \tag{6}$$

where, the coefficient matrix $A = (a_{ij})$ is a crisp $n \times n$ matrix and $b_i^{\aleph}, i = 1, 2, ..., n$ is a neutrosophic number. The Eq.(6) is called a system of neutrosophic linear equations (SNLE).

Let the solution of the SNLE of Eq.(6) be $x^{\aleph}$ and its (α,β,γ)-cut be $x^{\aleph}_{(\alpha,\beta,\gamma)} = ([\underline{x}^T(\alpha), \bar{x}^T(\alpha)],$ $[\underline{x}^I(\beta), \bar{x}^I(\beta)], [\underline{x}^F(\gamma), \bar{x}^F(\gamma)])$ .If the (α,β,γ)-cut of $b^{\aleph}$ be $x^{\aleph}_{(\alpha,\beta,\gamma)} = ([\underline{b}^T(\alpha), \bar{b}^T(\alpha)],$ $[\underline{b}^I(\beta), \bar{b}^I(\beta)], [\underline{b}^F(\gamma), \bar{b}^F(\gamma)])$, then The SNLE of (6) can be written as:

$$\begin{cases} \underline{\sum_{j=1}^{n} a_{ij} x_j^T(\alpha)} = \sum_{j=1}^{n} a_{ij} \underline{x_j^T(\alpha)} = \underline{b_i^T(\alpha)}, \\ \overline{\sum_{j=1}^{n} a_{ij} x_j^T(\alpha)} = \sum_{j=1}^{n} \overline{a_{ij} x_j^T(\alpha)} = \overline{b_i^T(\alpha)}, \\ \underline{\sum_{j=1}^{n} a_{ij} x_j^I(\beta)} = \sum_{j=1}^{n} \underline{a_{ij} x_j^I(\beta)} = \underline{b_i^I(\beta)}, \\ \overline{\sum_{j=1}^{n} a_{ij} x_j^I(\beta)} = \sum_{j=1}^{n} \overline{a_{ij} x_j^I(\beta)} = \overline{b_i^I(\beta)}, \\ \underline{\sum_{j=1}^{n} a_{ij} x_j^F(\gamma)} = \sum_{j=1}^{n} \underline{a_{ij} x_j^F(\gamma)} = \underline{b_i^F(\gamma)}, \\ \overline{\sum_{j=1}^{n} a_{ij} x_j^F(\gamma)} = \sum_{j=1}^{n} \overline{a_{ij} x_j^F(\gamma)} = \overline{b_i^F(\gamma)}. \end{cases} \tag{7}$$

If we define $x_i^{\aleph} = (\underline{x}_1^T, ..., \underline{x}_n^T, \bar{x}_1^T, ..., \bar{x}_n^T, \underline{x}_1^I, ..., \underline{x}_n^I, \bar{x}_1^I, ..., \bar{x}_n^I, \underline{x}_1^F, ..., \underline{x}_n^F, \bar{x}_1^F, ..., \bar{x}_n^F)^T$ and $b_i^{\aleph} = (\underline{b}_1^T, ..., \underline{b}_n^T, \bar{b}_1^T, ..., \bar{b}_n^T, \underline{b}_1^I, ..., \underline{b}_n^I, \bar{b}_1^I, ..., \bar{b}_n^I, \underline{b}_1^F, ..., \underline{b}_n^F, \bar{b}_1^F, ..., \bar{b}_n^F)^T$, then following Friedman et al., (1998) we must solve an 6n×6n crisp linear system as:

$$HX=B \tag{8}$$

Where,

$$H = \begin{bmatrix} D_{2n\times 2n} & [0]_{2n\times 2n} & [0]_{2n\times 2n} \\ [0]_{2n\times 2n} & D_{2n\times 2n} & [0]_{2n\times 2n} \\ [0]_{2n\times 2n} & [0]_{2n\times 2n} & D_{2n\times 2n} \end{bmatrix}, B = \begin{bmatrix} B^T \\ B^I \\ B^F \end{bmatrix}. \tag{9}$$

Also $D = (d_{ij})$,and obtain as follows:

$$\begin{cases} a_{ij} \geq 0 \rightarrow d_{ij} = a_{ij}, & d_{i+n,j+n} = a_{ij}, \\ a_{ij} < 0 \rightarrow d_{i,j+n} = -a_{ij}, & d_{i+n,j} = -a_{ij} \end{cases} \tag{10}$$

and any d_{ij} which is not determined by (10) is zero. Also:

$$D = \begin{bmatrix} D_1 & -D_2 \\ -D_2 & D_1 \end{bmatrix}, B^T = \begin{bmatrix} \underline{b}^T \\ \overline{b}^T \end{bmatrix}, B^I = \begin{bmatrix} \underline{b}^I \\ \overline{b}^I \end{bmatrix}, B^F = \begin{bmatrix} \underline{b}^F \\ \overline{b}^F \end{bmatrix}.$$

Where, $D_1, D_2 \geq 0, D = D_1 - D_2$.

Since H is a block diagonal matrix, to reduce the computational complexity, we need only to solve the following 2n×2n crisp linear systems:

$$Dx^i = B^i, \quad i = T, I, F. \tag{11}$$

Worthy mentioning that the matrix D may be singular even if A is nonsingular; see the following example:

Example 1. The matrix $A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$ of the SNLE is nonsingular, while $D = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ is singular.

In other sense, a SNLE represented by a nonsingular matrix *A* may be have no solution or an infinite number of solutions. Next, following the Friedman et al., (1998), we study some theorems regarding the properties of D.

Theorem 1. D is nonsingular iff $A = D_1 + D_2$ and $D_1 - D_2$ are nonsingular.

Theorem 2. If D^{-1} exists it must have the same structure as D, *i.e,*

$$D^{-1} = \begin{pmatrix} E & F \\ F & E \end{pmatrix}$$

Definition 5. Let $x_i^{\aleph} = (\underline{x}_1^T, \ldots, \underline{x}_n^T, \overline{x}_1^T, \ldots, \overline{x}_n^T, \underline{x}_1^I, \ldots, \underline{x}_n^I, \overline{x}_1^I, \ldots, \overline{x}_n^I, \underline{x}_1^F, \ldots, \underline{x}_n^F, \overline{x}_1^F, \ldots, \overline{x}_n^F)^T$

be the unique solution of Eq.(5). If $\forall k \in \{1, 2, \ldots, n\}$: $\underline{x}_k^T \leq \overline{x}_k^T, \underline{x}_k^I \leq \overline{x}_k^I$ and $\underline{x}_k^F \leq \overline{x}_k^F$, then the solution $x_i^{\aleph}$ is called a strong neutrosophic solution. Otherwise, it is a weak neutrosophic solution.

Theorem 3. Assume that $D = \begin{pmatrix} D_1 & D_2 \\ D_2 & D_1 \end{pmatrix}$ be a nonsingular matrix. Then Eq.(5) has a strong solution if and only if:

$$(D_1 - D_2)^{-1}(\underline{b}^i - \overline{b}^i) \leq 0, \quad i = T, I, F. \tag{12}$$

Proof. From the system (11) we obtain:

$$\begin{pmatrix} D_1 & D_2 \\ D_2 & D_1 \end{pmatrix}\begin{pmatrix} \underline{x}^i \\ \overline{x}^i \end{pmatrix} = \begin{pmatrix} \underline{b}^i \\ \overline{b}^i \end{pmatrix}, \qquad i = T, I, F$$

Hence,

$$D_1\underline{x}^i - D_2\overline{x}^i = \underline{b}^i \tag{13}$$

$$-D_2\underline{x}^i + D_1\overline{x}^i = \overline{b}^i \tag{14}$$

From (13) and (14) we have:

$$\begin{cases} (D_1 + D_2)\underline{x}^i - (D_1 + D_2)\overline{x}^i = \underline{b}^i - \overline{b}^i, \\ (D_1 + D_2)(\underline{x} - \overline{x}) = \underline{b}^i - \overline{b}^i. \end{cases}$$

From Theorem 1, $D_1 - D_2$ is nonsingular. So,

$$(\underline{x}^i - \overline{x}^i) = (D_1 - D_2)^{-1}(\underline{b}^i - \overline{b}^i) \tag{15}$$

By the Definition 5, $\underline{x}^i - \overline{x}^i \le 0$ if Eq. (5) has a strong solution. Henceforth (12) holds. Conversely, if (12) holds, by Eq.(15), we have $\underline{x}^i - \overline{x}^i \le 0$.□

From the theorems 1 and 3, we conclude this result:

Theorem 4. The SNLE has a strong solution if and only if the following conditions hold:

1. The matrices $A = D_1 + D_2$ and $D_1 - D_2$ are both nonsingular.
2. $(D_1 - D_2)^{-1}(\underline{b}^i - \overline{b}^i) \le 0.$

4. Numerical Example

Here, we provide an experiment to demonstrate the consequences gained in former sections.

Example 2. Consider the following SNLE:

$$\begin{cases} x_1^{\aleph} - x_2^{\aleph} = <(0,1,2);(0.9,0.4,0.2)>, \\ x_1^{\aleph} + 3x_2^{\aleph} = <(4,5,7);(0.8,0.3,0.3)>. \end{cases} \tag{16}$$

The extended 4 × 4 matrix is

$$D=\begin{bmatrix}1&0&0&-1\\1&3&0&0\\0&-1&1&0\\0&0&1&3\end{bmatrix}.$$

Since the matrices $A=D_1+D_2$ and $D_1-D_2=\begin{bmatrix}1&1\\1&3\end{bmatrix}$ are both nonsingular, then by Theorem1,

it is easy to see that the matrix D is nonsingular. Therefor, D^{-1} exists and based on Theorem 2, it must have the same structure as D. If we obtain this inverse, we can see that the Theorem 2 is true:

$$D^{-1}=\begin{bmatrix}\frac{9}{8}&\frac{-1}{8}&\frac{-3}{8}&\frac{3}{8}\\ \frac{-3}{8}&\frac{3}{8}&\frac{1}{8}&\frac{-1}{8}\\ \frac{-3}{8}&\frac{3}{8}&\frac{9}{8}&\frac{-1}{8}\\ \frac{1}{8}&\frac{-1}{8}&\frac{-3}{8}&\frac{3}{8}\end{bmatrix}.$$

Now, we obtain the (α,β,γ)-cut of the right hand side vector. By Definition 4, we get:

$b^{\aleph}_{1\,(\alpha,\beta,\gamma)}=<[0.9(\alpha),0.9(2-\alpha)],[0.4(1-\beta),0.4(1+\beta)],[0.2(1-\gamma),0.2(1+\gamma)]>,$

$b^{\aleph}_{2\,(\alpha,\beta,\gamma)}=<[0.8(4+\alpha),0.8(8-2\alpha)],[0.3(5-\beta),0.3(5+2\beta)],[0.3(5-\gamma),0.3(5+2\gamma)]>.$

Also, $(D_1+D_2)^{-1}=\begin{bmatrix}\frac{3}{2}&\frac{-1}{2}\\ \frac{-1}{2}&\frac{1}{2}\end{bmatrix}.$

So:

$$(D_1+D_2)^{-1}(\underline{b}^T-\overline{b}^T)=\begin{bmatrix}\frac{3}{2}&\frac{-1}{2}\\ \frac{-1}{2}&\frac{1}{2}\end{bmatrix}\begin{bmatrix}\frac{9}{5}(\alpha-1)\\ \frac{12}{5}(\alpha-1)\end{bmatrix}=\begin{bmatrix}\frac{3}{2}(\alpha-1)\\ \frac{3}{10}(\alpha-1)\end{bmatrix}\leq\begin{bmatrix}0\\0\end{bmatrix},$$

$$(D_1+D_2)^{-1}(\underline{b}^I-\overline{b}^I)=\begin{bmatrix}\frac{3}{2}&\frac{-1}{2}\\ \frac{-1}{2}&\frac{1}{2}\end{bmatrix}\begin{bmatrix}-0.8\beta\\ -0.9\beta\end{bmatrix}=\begin{bmatrix}\frac{-3}{4}\beta\\ \frac{-1}{20}\beta\end{bmatrix}\leq\begin{bmatrix}0\\0\end{bmatrix},$$

$$(D_1+D_2)^{-1}(\underline{b}^F-\overline{b}^F)=\begin{bmatrix}\frac{3}{2}&\frac{-1}{2}\\ \frac{-1}{2}&\frac{1}{2}\end{bmatrix}\begin{bmatrix}-0.4\gamma\\ -0.9\gamma\end{bmatrix}=\begin{bmatrix}\frac{-3}{20}\gamma\\ \frac{-1}{4}\gamma\end{bmatrix}\leq\begin{bmatrix}0\\0\end{bmatrix},$$

Therefore, by theorems 3 and 4, The SNLE (16) should has a strong solution. To obtain this solution, form Eq.(11) we have:

$$x^T = \begin{bmatrix} \underline{x}_1^T \\ \underline{x}_2^T \\ \overline{x}_1^T \\ \overline{x}_2^T \end{bmatrix} = D^{-1}B^T = \left[\frac{1}{40}(26\alpha + 41),\quad \frac{1}{40}(2\alpha + 29),\quad \frac{1}{40}(-34\alpha + 101),\quad \frac{1}{40}(-10\alpha - 41)\right]^t,$$

$$x^I = \begin{bmatrix} \underline{x}_1^I \\ \underline{x}_2^I \\ \overline{x}_1^I \\ \overline{x}_2^I \end{bmatrix} = D^{-1}B^I = \left[\frac{-27}{80}(\beta - 2),\quad \frac{1}{80}(\beta + 22),\quad \frac{3}{80}(11\beta + 18),\quad \frac{1}{80}(5\beta - 22)\right]^t,$$

$$x^F = \begin{bmatrix} \underline{x}_1^F \\ \underline{x}_2^F \\ \overline{x}_1^F \\ \overline{x}_2^F \end{bmatrix} = D^{-1}B^F = \left[\frac{-3}{80}(\gamma - 14),\quad \frac{-1}{80}(7\gamma + 26),\quad \frac{3}{80}(3\gamma + 14),\quad \frac{13}{80}(\gamma + 2)\right]^t.$$

$$x_{1\,(\alpha,\beta,\gamma)}^{\aleph} = <[\frac{1}{40}(26\alpha + 41), \frac{1}{40}(-34\alpha + 101)], [\frac{-27}{80}(\beta - 2), \frac{3}{80}(11\beta + 18)], [\frac{-3}{80}(\gamma - 14), \frac{3}{80}(3\gamma + 14)]>,$$

$$x_{2\,(\alpha,\beta,\gamma)}^{\aleph} = <[\frac{1}{40}(2\alpha + 29), \frac{1}{40}(-10\alpha - 41)], [\frac{1}{80}(\beta + 22), \frac{1}{80}(5\beta - 22)], [\frac{-1}{80}(7\gamma + 26), \frac{13}{80}(\gamma + 2)]>.$$

For different values of $0 \le \alpha, \beta, \gamma \le 1$,the graphical interpretation of the above results is shown in figures 1and 2.

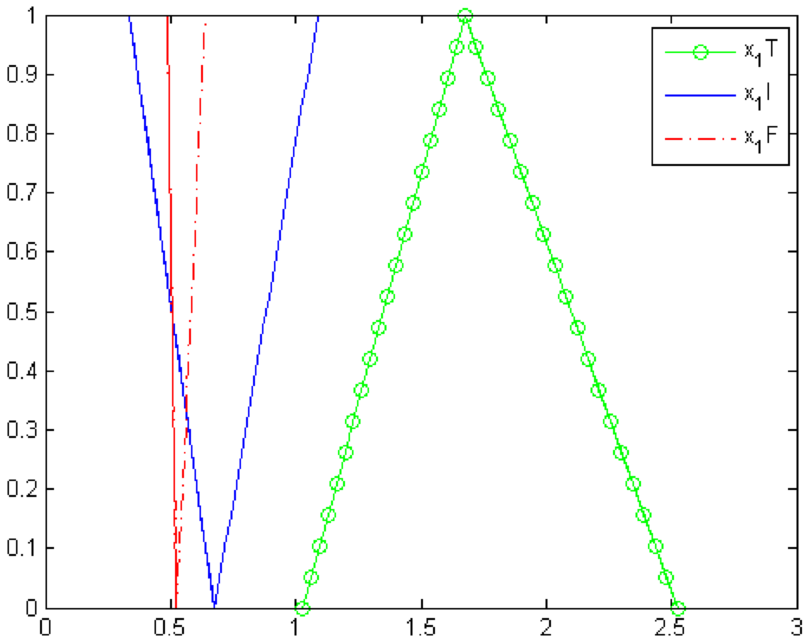

Figure 1. The value of $x_1^{\aleph}$.

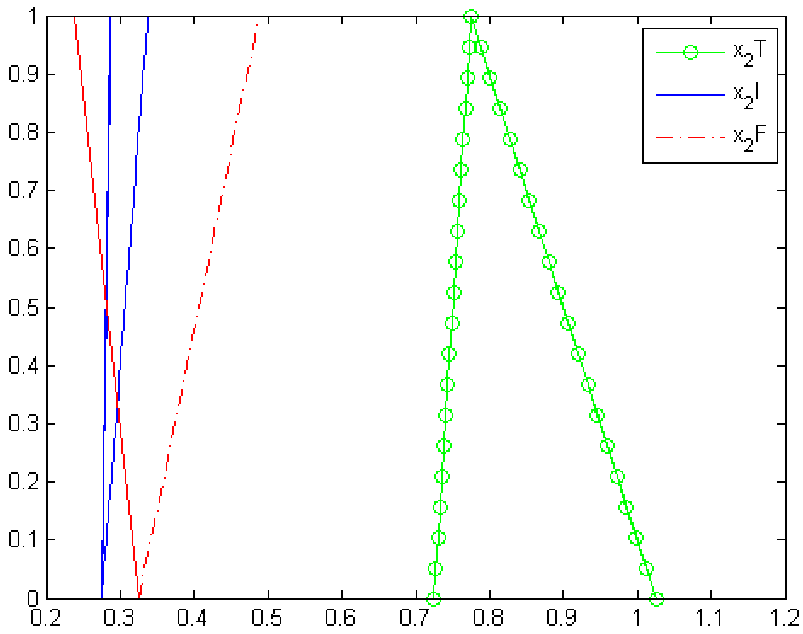

Figure 2. The value of $x_2^{\aleph}$.

5. Conclusions

In this study, we present for first time, the system of Neutrosophic Linear Equations (SNLE) and establish a general model to solve it. Some theorems about SNLE are investigated and the conditions of a strong neutrosophic solution to n x n system of linear equations is proved in details. Finally, from numerical and theoretical studies it can be concluded that the model is efficient and convenient.

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