Neutrosophic Sets and Systems, Vol. 31, 2020
University of New Mexico

T-Neutrosophic Cubic Set on BF-Algebra

Mohsin Khalid [1], Neha Andaleeb Khalid [2] and Said Broumi [3,*]

1 The University of Lahore, 1Km Raiwind Road, Lahore, 54000, Pakistan, E-mail: mk4605107@gmail.com
2 Department of Mathematics, Lahore Collage for Women University, Lahore, Pakistan, E-mail: nehakhalid97@gmail.com
3 Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, Casablanca, Morocco
E-mail: s.broumi@flbenmsik.ma

* Correspondence; Mohsin Khalid; mk4605107@gmail.com

Abstract: In this paper, the concept of t-neutrosophic cubic set is introduced and investigated the t-neutrosophic cubic set through subalgebra, ideal and closed ideal of BF-algebra. Homomorphic properties of t-neutrosophic cubic subalgebra and ideal are also investigated with some related properties.

Keywords: BF-algebra, t-neutrosophic cubic set, t-neutrosophic cubic subalgebra, t-neutrosophic cubic closed ideal.

---

1 Introduction

Zadeh [33, 34] introduced the concept of fuzzy set. Jun et al. [7] defined interval-valued fuzzy set and discussed its properties. Jun et al. [8] presented the notion of cubic subgroups. Senapati et al. [26] generalized the idea of cubic set to subalgebras, ideals and closed ideals of *B*-algebra. Imai and Iseki [5, 6] introduced the two classes of algebra which were BCK algebra and BCI-algebra. Huang [4] investigated the BCI-algebra. Jun et al. [10, 11] applied the idea of cubic set to subalgebras, ideals and q-ideals in BCK/BCI-algebra. Neggers et al. [13] defined and studied the B-algebra. Cho et al. [3] studied the relations of B-algebra with different topics. Park et al. [15] studied quadratic *B*-algebra on field *X* with a *BCI*-algebra. Saeid [16] was given the idea of interval valued fuzzy subalgebra in *B*-algebra. Walendziak [32] proved the conditions of *B*-algebra. Senapati et al. [21, 22, 23, 24, 31] was introduced the fuzzy dot subalgebra of BG-algebra, fuzzy dot subalgebra, fuzzy dot ideals, interval-valued fuzzy closed ideals and fuzzy subalgebra with respect to t-norm in *B*-algebra. Senapati et. al. [17, 25] was introduced *L*-fuzzy *G*-subalgebra of *G*-algebra and bipolar fuzzy set which was related to *B*-algebra. Khalid et. al. [20] studied the intuitionistic fuzzy translation. Many researchers [12, 27, 28, 29, 30] have done a lot of work on BG-algebra which was a generalization of *B*-algebra. Smarandache [18, 19] introduced the concept of neutrosophic set. Jun et al. [9] introduced neutrosophic cubic set. Barbhuiya [2] studied the t-intuitionistic fuzzy BG-subalgebra. Takallo et al. [37] introduced the MBJ-neutrosophic set, BMBJ-neutrosophic subalgebra, BMBJ-neutrosophic ideal and BMBJ-neutrosophic ∘-subalgebra. G. Muhiuddin et al. [38] studied the neutrosophic quadruple BCK/BCI-number, neutrosophic quadruple BCK/BCI-algebra, neutrosophic quadruple subalgebra and (positive implicative) neutrosophic quadruple ideal. Park [39] introduced the notion of neutrosophic ideal in subtraction algebra and discussed conditions for a neutrosophic set to be a neutrosophic ideal. Borzooei et al. [40] introduced the concept of MBJ-neutrosophic set, BMBJ-neutrosophic ideal and positive implicative BMBJ-neutrosophic ideal. Jun et al. [41] studied the commutative falling neutrosophic ideals in BCK-algebra. Song et al. [42] investigated the interval neutrosophic set and applied to ideals in BCK/BCI-algebra. Khalid et al. [43] interestingly investigated the neutrosophic soft cubic subalgebra through significant results. Muhiuddin et al. [44] was studied neutrosophic quadruple BCK/BCI-number, neutrosophic quadruple BCK/BCI-algebra, (regular) neutrosophic quadruple ideal and neutrosophic quadruple q-ideal. Muhiuddin et al. [45] investigated the (ϵ, ϵ)-neutrosophic subalgebra, (ϵ, ϵ)-neutrosophic ideal. Akinleye et al. [46] defined the neutrosophic quadruple algebraic structures, also studied neutrosophic quadruple rings and presented their elementary properties. Basset et al. [47] studied integrated neutrosophic ANP and VIKOR method for achieving sustainable supplier selection. Basset et al. [48] studied the type 2 neutrosophic number, score and accuracy function, multi attribute decision making TOPSIS and T2NN-TOPSIS.

The purpose of this paper is to introduce the idea of t-neutrosophic cubic set [t-NCS] and to investigate this set through the concepts of subalgebra, ideal and closed ideal of BF-algebra. Homomorphic image and inverse homomorphic image of t-neutrosophic cubic subalgebra [t-NCSU] and t-neutrosophic cubic ideal [t-NCID] are also studied.

2 Preliminaries

In this section, basic definitions are cited that are necessary for this paper.

Definition 2.1 [32] A nonempty set X with a constant 0 and a binary operation $*$ is called BF-algebra when it fulfills these axioms.

1. $t_1 * t_1 = 0$
2. $t_1 * 0 = 0$
3. $0 * (t_1 * t_2) = t_2 * t_1$ for all $t_1, t_2 \in X$.

A BF-algebra is denoted by $(X,*,0)$.

Definition 2.2 [1] A nonempty subset S of G-algebra X is called a subalgebra of X if $t_1 * t_2 \in S$ $\forall$ $t_1, t_2 \in S$.

Definition 2.3 [14] Mapping $f|X \to Y$ of B-algebra is called homomorphism if $f(t_1 * t_2) = f(t_1) * f(t_2)$ $\forall$ $t_1, t_2 \in X$.

Definition 2.4 [23] A nonempty subset I of B-algebra X is called an ideal if for any $t_1, t_2 \in X$, (i) $0 \in I$, (ii) $t_1 * t_2 \in I$ and $t_2 \in I \Rightarrow t_1 \in I$.

An ideal I of B-algebra X is called closed if $0 * t_2 \in I$, $\forall$ $t_2 \in I$.

Definition 2.5 [33] Let X be the set of elements which are denoted generally by t_1. Then a fuzzy set C in X is defined as $C = \{< t_1, \mu_C(t_1) > |t_1 \in X\}$, where $\mu_C(t_1)$ is called the existenceship value of t_1 in C and $\mu_C(t_1) \in [0,1]$.

For a family $C_i = \{< t_1, \mu_{C_i}(t_1) > |t_1 \in X\}$ of fuzzy sets in X, where $i \in k$ and k is index set, we define the join (∨) meet (∧) operations as follows:

$$\bigvee_{i\in k} C_i = (\bigvee_{i\in k} \mu_{C_i})(t_1) = \sup\{\mu_{C_i} | i \in k\}$$

and

$$\bigwedge_{i\in k} C_i = (\bigwedge_{i\in k} \mu_{C_i})(t_1) = \inf\{\mu_{C_i} | i \in k\}$$

respectively, $\forall\ t_1 \in X$.

Definition 2.6 [2] Let two elements $D_1, D_2 \in D[0,1]$. If $D_1 = [(t_1)_1^-, (t_1)_1^+]$ and $D_2 = [(t_1)_2^-, (t_1)_2^+]$, then $\mathrm{rmax}(D_1, D_2) = [\max((t_1)_1^-, (t_1)_2^-), \max((t_1)_1^+, (t_1)_2^+)]$ which is denoted by $D_1 \vee^r D_2$ and $\mathrm{rmin}(D_1, D_2) = [\min((t_1)_1^-, (t_1)_2^-), \min((t_1)_1^+, (t_1)_2^+)]$ which is denoted by $D_1 \wedge^r D_2$. Thus, if $D_i = [((t_1)_1)_i^-, ((t_1)_2)^+] \in D[0,1]$ for $i = 1,2,3,\ldots,$ then we define $\mathrm{rsup}_i(D_i) = [\sup_i(((t_1)_1)_i^-), \sup_i(((t_1)_1)_i^+)]$, i.e., $\vee_i^r D_i = [\vee_i ((t_1)_1)_i^-, \vee_i ((t_1)_1)_i^+]$. In the same way we define $\mathrm{rinf}_i(D_i) = [\inf_i(((t_1)_1)_i^-), \inf_i(((t_1)_1)_i^+)]$, i.e., $\wedge_i^r D_i = [\wedge_i ((t_1)_1)_i^-, \wedge_i ((t_1)_1)_i^+]$. Now we call $D_1 \geq D_2 \Leftarrow (t_1)_1^- \geq (t_1)_2^-$ and $(t_1)_1^+ \geq (t_1)_2^+$. Similarly the relations $D_1 \leq D_2$ and $D_1 = D_2$ are defined.

Definition 2.7 [1,22] A fuzzy set $C = \{< t_1, \mu_C(t_1) > |t_1 \in X\}$ is called a fuzzy subalgebra of X if $\mu_C(t_1 * t_2) \geq \min\{\mu_C(t_1), \mu_C(t_2)\}\ \forall\ t_1, t_2 \in X$. A fuzzy set $C = \{< t_1, \mu_C(t_1) > |t_1 \in X\}$ in X is called a fuzzy ideal of X if it satisfies (i) $\mu_C(0) \geq \mu_C(t_1)$ and (ii) $\mu_C(t_1) \geq \min\{\mu_C(t_1 * t_2), \mu_A(t_2)\}\ \forall\ t_1, t_2 \in X$.

Definition 2.8 [33] An IVFS B over X is an object of the form $B = \{< t_1, \mu_B(t_1) > |t_1 \in X\}$ Where $\mu_B(t_1)$: $X \to D[0{:}1]$, Where D[0,1] is the collection of all subintervals of [0,1]. The interval $\mu_B(t_1)$ shows the interval of the degree of membership of the element t_1 to the set B, Where $\mu_B(t_1) = \{\mu_{LB}(t_1), \mu_{UB}(t_1)\},\ \forall\ t_1 \in X$.

Definition 2.9 [16] A interval valued fuzzy set $C = \{< t_1, \mu_C(t_1) > |t_1 \in X\}$ is called a interval valued fuzzy subalgebra of X if it satisfies $\mu_C(t_1 * t_2) \geq \mathrm{rmin}\{\mu_C(t_1), \mu_C(t_2)\}\ \forall\ t_1, t_2 \in X$.

Definition 2.10 [15] A pair $\tilde{\mathcal{P}}_k = (A, \Lambda)$ is called NCS where $A = \{\langle t_1; A_T(t_1), A_I(t_1), A_F(t_1)\rangle\ |t_1 \in Y\}$ is an INS in Y and $\Lambda = \{\langle t_1; \lambda_T(t_1), \lambda_I(t_1), \lambda_F(t_1)\rangle |\ t_1 \in Y\}$ is a neutrosophic set in Y.

Definition 2.11 [26] Let $C = \{\langle t_1, \kappa(t_1), \sigma(t_1)\rangle\}$ be a cubic set, where $\kappa(t_1)$ is an interval-valued fuzzy set in X, $\sigma(t_1)$ is a fuzzy set in X. Then C is cubic subalgebra under binary operation $*$ if following axioms are satisfied:

C1: $\kappa(t_1 * t_2) \geq \mathrm{rmin}\{\kappa(t_1), \kappa(t_2)\}$,

C2: $\sigma(t_1 * t_2) \leq \max\{\sigma(t_1), \sigma(t_2)\}\ \forall\ t_1, t_2 \in X$.

Definition 2.12 [9] Suppose X be a nonempty set. A neutrosophic cubic set in X is pair $\mathcal{C} = (\kappa, \sigma)$ where $\kappa = \{\langle t_1; \kappa_E(t_1), \kappa_I(t_1), \kappa_N(t_1)\rangle\ |t_1 \in X\}$ is an interval neutrosophic set in X and $\sigma = \{\langle t_1; \sigma_E(t_1), \sigma_I(t_1), \sigma_N(t_1)\rangle\ |t_1 \in X\}$ is a neutrosophic set in X.

Definition 2.13 [9] For any $\mathcal{C}_i = (\kappa_i, \sigma_i)$ where

$\kappa_i = \{\langle t_1; \kappa_{iE}(t_1), \kappa_{iI}(t_1), \kappa_{iN}(t_1)\rangle\ |t_1 \in X\}$,

$\sigma_i = \{\langle t_1; \sigma_{iE}(t_1), \sigma_{iI}(t_1), \sigma_{iN}(t_1)\rangle\ |t_1 \in X\}$ for $i \in k$, P-union, P-inersection, R-un -ion and R-intersection are defined respectively by

P-union $\bigcup_{P\, i\in k} \mathcal{C}_i = (\bigcup_{i\in k} \kappa_i, \bigvee_{i\in k} \sigma_i)$, **P-intersection** $\bigcap_{P\, i\in k} \mathcal{C}_i = (\bigcap_{i\in k} \kappa_i, \bigwedge_{i\in k} \sigma_i)$,

R-union $\bigcup_{R\, i\in k} \mathcal{C}_i = (\bigcup_{i\in k} \kappa_i, \bigwedge_{i\in k} \sigma_i)$, **R-intersection:** $\bigcap_{R\, i\in k} \mathcal{C}_i = (\bigcap_{i\in k} \kappa_i, \bigvee_{i\in k} \sigma_i)$,

where

$$\bigcup_{i\in k} \kappa_i = \{\langle t_1; (\bigcup_{i\in k} \kappa_{iE})(t_1), (\bigcup_{i\in k} \kappa_{iI})(t_1), (\bigcup_{i\in k} \kappa_{iN})(t_1)\rangle | t_1 \in X\},$$

$$\bigvee_{i\in k} \sigma_i = \{\langle t_1; (\bigvee_{i\in k} \sigma_{iE})(t_1), (\bigvee_{i\in k} \sigma_{iI})(t_1), (\bigvee_{i\in k} \sigma_{iN})(t_1)\rangle | t_1 \in X\},$$

$$\bigcap_{i\in k} \kappa_i = \{\langle t_1; (\bigcap_{i\in k} \kappa_{iE})(t_1), (\bigcap_{i\in k} \kappa_{iI})(t_1), (\bigcap_{i\in k} \kappa_{iN})(t_1)\rangle | t_1 \in X\},$$

$$\bigwedge_{i\in k} \sigma_i = \{\langle t_1; (\bigwedge_{i\in k} \sigma_{iE})(t_1), (\bigwedge_{i\in k} \sigma_{iI})(t_1), (\bigwedge_{i\in k} \sigma_{iN})(t_1)\rangle | t_1 \in X\},$$

Definition 2.14 [36] Let $C = (\mu_C, \nu_C)$ be an IFS in BF-algebra X and $t \in [0,1]$, then the IFS C^t is called the t-intuitionistic fuzzy subset of X w.r.t C and is defined as $C^t = \{< t_1, \mu_{C^t}(t_1), \nu_{C^t}(t_1) > | t_1 \in Y\} = < \mu_{C^t}, \nu_{C^t} >$ where $\mu_{C^t}(t_1) = \min\{\mu_C(t_1), t\}$ and $\mu_{C^t}(t_1) = \max\{\nu_C(t_1), 1 - t\}$ $\forall$ $t_1 \in X$.

Definition 2.15 [36] Let $B^t = (\mu_{B^t}, \nu_{B^t})$ be a t-intuitionistic fuzzy subset of BF-algebra X and $t \in [0,1]$ then B^t is called t-intuitionistic fuzzy subalgebra of X if it fulfills these axioms.

(i) $\mu_{B^t}(t_1 * t_2) \geq \min\{\mu_{B^t}(t_1), \mu_{B^t}(t_2)\}$,

(ii) $\nu_{B^t}(t_1 * t_2) \leq \max\{\nu_{B^t}(t_1), \nu_{B^t}(t_2)\}$, $\forall$ $t_1, t_2 \in X$.

3 t-Neutrosophic Cubic Subalgebra of BF-algebra

Let $\mathcal{C} = (\kappa_{\mathcal{C}}, \sigma_{\mathcal{C}})$ be a neutrosophic cubic set [**NCS**] of BF-algebra X, then the NCS $\mathcal{C}$ is called the t-neutrosophic cubic set (**t-NCS**) of X w.r.t $\mathcal{C}$ and is defined as $\mathcal{C}^t = \{< t_1, \hat{\kappa}^t(t_1), \sigma^t(t_1) > | t_1 \in X\} = < \hat{\kappa}^t, \sigma^t >$ such that $\hat{\kappa}^t(t_1) = \{< \hat{\kappa}^t_E(t_1), \hat{\kappa}^t_I(t_1), \hat{\kappa}^t_N(t_1) > | t_1 \in X\}$ and $\sigma(t_1) = \{< \sigma^t_E(t_1), \sigma^t_I(t_1), \sigma^t_N(t_1) > | t_1 \in X\}$ with two independent components where $\hat{\kappa}^t(t_1) = \{\mathrm{rmin}(\hat{\kappa}_E(t_1), t), \mathrm{rmin}(\hat{\kappa}_I(t_1), t'), \mathrm{rmin}(\hat{\kappa}_N(t_1), 2 - t - t')\}, \sigma^t(t_1) = \{\max(\sigma_E(t_1), t), \max(\sigma_I(t_1), t'), \max(\sigma_N(t_1), 2 - t - t')\}$ and $\forall$ $t, t', 2 - t - t' \in [0,1]$ and now concept of cubic subalgebra can be extended to t-NCSU.

Definition 3.1 Let $\mathcal{C} = (\hat{\kappa}, \sigma)$ be a cubic set, where X is subalgebra. Then $\mathcal{C}$ is t-NCSU under binary operation $*$ if it satisfies the following conditions:

N1:

$\hat{\kappa}^t{}_E(t_1 * t_2) \geq \mathrm{rmin}\{\hat{\kappa}^t_E(t_1), \hat{\kappa}^t_E(t_2)\}$,

$\hat{\kappa}^t{}_I(t_1 * t_2) \geq \mathrm{rmin}\{\hat{\kappa}^t_I(t_1), \hat{\kappa}^t_I(t_2)\}$,

$\hat{\kappa}^t{}_N(t_1 * t_2) \geq \mathrm{rmin}\{\hat{\kappa}^t_N(t_1), \hat{\kappa}^t_N(t_2)\}$,

N2:

$\sigma^t{}_E(t_1 * t_2) \leq \max\{\sigma^t_E(t_1), \sigma^t_E(t_2)\}$

$\sigma^t{}_I(t_1 * t_2) \leq \max\{\sigma^t_I(t_1), \sigma^t_I(t_2)\}$

$\sigma^t{}_N(t_1 * t_2) \leq \max\{\sigma^t_N(t_1), \sigma^t_N(t_2)\}$.

Where **E** means existenceship/membership value, **I** means indeterminacy existenceship/membership value and N means non existenceship/membership value. For our convenience we introduce new notation for t-neutrosophic cubic set as

$$\mathcal{C} = (\hat{\boldsymbol{\kappa}}^{\mathbf{t}}_{\mathbf{E,I,N}}, \boldsymbol{\sigma}^{\mathbf{t}}_{\mathbf{E,I,N}}) = \{\langle \mathbf{t_1}, \hat{\boldsymbol{\kappa}}^{\mathbf{t}}_{\mathbf{E,I,N}}(\mathbf{t_1}), \boldsymbol{\sigma}^{\mathbf{t}}_{\mathbf{E,I,N}}(\mathbf{t_1})\rangle\} = \{\langle \mathbf{t_1}, \hat{\boldsymbol{\kappa}}^{\mathbf{t}}_{\Xi}(\mathbf{t_1}), \boldsymbol{\sigma}^{\mathbf{t}}_{\Xi}(\mathbf{t_1})\rangle\}$$

and for conditions N1, N2 as

N1: $\hat{\kappa}^t_\Xi(t_1 * t_2) \geq \text{rmin}\{\hat{\kappa}^t_\Xi(t_1), \hat{\kappa}^t_\Xi(t_2)\}$,

N2: $\sigma^t_\Xi(t_1 * t_2) \leq \max\{\sigma^t_\Xi(t_1), \sigma^t_\Xi(t_2)\}$.

Example 3.2 Let $X = \{0, t_1, t_2, t_3, t_4, t_5\}$ be a BF-algebra with the following Cayley table.

$*$	0	t_1	t_2	t_3	t_4	t_5
0	0	t_5	t_4	t_3	t_2	t_1
t_1	t_1	0	t_5	t_4	t_3	t_2
t_2	t_2	t_1	0	t_5	t_4	t_3
t_3	t_3	t_2	t_1	0	t_5	t_4
t_4	t_4	t_3	t_2	t_1	0	t_5
t_5	t_5	t_4	t_3	t_2	t_1	0

A t-neutrosophic cubic set $\mathcal{C} = (\hat{\kappa}^t{}_\Xi, \sigma^t_\Xi)$ of X is defined by

	0	t_1	t_2	t_3	t_4	t_5
$\hat{\kappa}^t{}_E$	[0.7,0.9]	[0.6,0.8]	[0.7,0.9]	[0.6,0.8]	[0.7,0.9]	[0.6,0.8]
$\hat{\kappa}^t{}_I$	[0.3,0.2]	[0.2,0.1]	[0.3,0.2]	[0.2,0.1]	[0.3,0.2]	[0.2,0.1]
$\hat{\kappa}^t{}_N$	[0.2,0.4]	[0.1,0.4]	[0.2,0.4]	[0.1,0.4]	[0.2,0.4]	[0.1,0.4]

	0	t_1	t_2	t_3	t_4	t_5
$\sigma^t{}_E$	0.1	0.3	0.1	0.3	0.1	0.3
$\sigma^t{}_I$	0.3	0.5	0.3	0.5	0.3	0.5
$\sigma^t{}_N$	0.5	0.6	0.5	0.6	0.5	0.6

Both the conditions of definition are satisfied by the set $\mathcal{C}$. Thus $\mathcal{C} = (\hat{\kappa}^t{}_\Xi, \sigma^t_\Xi)$ is a t-NCSU of X.

Proposition 3.3 Let $\mathcal{C} = \{\langle t_1, \hat{\kappa}^t_\Xi(t_1), \sigma^t_\Xi(t_1)\rangle\}$ is a t-NCSU of X, then $\forall\ t_1 \in X$, $\hat{\kappa}^t_\Xi(t_1) \geq \hat{\kappa}^t_\Xi(0)$ and $\sigma^t_\Xi(0) \leq \sigma^t_\Xi(t_1)$. Thus, $\hat{\kappa}^t_\Xi(0)$ and $\sigma^t_\Xi(0)$ are the upper bound and lower bound of $\hat{\kappa}^t_\Xi(t_1)$ and $\sigma^t_\Xi(t_1)$ respectively.

Proof. $\forall\ t_1 \in X$, we have $\hat{\kappa}^t_\Xi(0) = \hat{\kappa}^t{}_\Xi(t_1 * t_1) \geq \text{rmin}\{\hat{\kappa}^t_\Xi(t_1), \hat{\kappa}^t{}_\Xi(t_1)\} = \hat{\kappa}^t_\Xi(t_1) \Rightarrow \hat{\kappa}^t_\Xi(0) \geq \hat{\kappa}^t_\Xi(t_1)$ and $\sigma^t_\Xi(0) = \sigma^t_\Xi(t_1 * t_1) \leq \max\{\sigma^t_\Xi(t_1), \sigma^t_\Xi(t_1)\} = \sigma^t_\Xi(t_1) \Rightarrow \sigma^t_\Xi(0) \leq \sigma^t_\Xi(t_1)$.

Theorem 3.4 Let $\mathcal{C}$=$\{\langle t_1, \hat{\kappa}^t_\Xi(t_1), \sigma^t_\Xi(t_1)\rangle\}$ be a t-NCSU of X. If there exists a sequence $\{(t_1)_n\}$ of X such that $\lim_{n\to\infty}\hat{\kappa}^t{}_\Xi((t_1)_n) = [1,1]$ and $\lim_{n\to\infty}\sigma^t_\Xi((t_1)_n) = 0$.Then $\hat{\kappa}^t{}_\Xi(0) = [1,1]$ and $\sigma^t_\Xi(0) = 0$.

Proof. Using above proposition, $\hat{\kappa}^t_\Xi(0) \geq \hat{\kappa}^t_\Xi(t_1)\ \forall\ t_1 \in X$, $\therefore\ \hat{\kappa}^t_\Xi(0) \geq \hat{\kappa}^t_\Xi((t_1)_n)$ for $n \in Z^+$. Consider, $[1,1] \geq \hat{\kappa}^t_\Xi(0)\ \geq \lim_{n\to\infty}\hat{\kappa}^t_\Xi((t_1)_n) = [1,1]$. Hence $\hat{\kappa}^t_\Xi(0) = [1,1]$.

Again, using proposition, $\sigma^t_\Xi(0) \leq \sigma^t_\Xi(t_1)\ \forall\ t_1 \in X$, $\therefore\ \sigma^t_\Xi(0) \leq \sigma^t_\Xi((t_1)_n)$ for $n \in Z^+$. Consider, $0 \leq \sigma^t_\Xi(0)\ \leq \lim_{n\to\infty}\sigma^t_\Xi((t_1)_n) = 0$. Hence $\sigma^t_\Xi(0) = 0$.

Theorem 3.5 The R-intersection of any set of t-NCSU of X is t-NCSU of X.

Proof. Let $\mathcal{C}^t_i = \{\langle t_1, (\hat{\kappa}^t{}_i)_\Xi, (\sigma^t_i)_\Xi\rangle | t_1 \in X\}$ where $i \in k$, is family of sets of t-NCSU of X and $t_1, t_2 \in X$ and $t \in [0,1]$ Then

$$(\cap\ (\hat{\kappa}^t{}_i)_\Xi)(t_1 * t_2) = \mathrm{rinf}(\hat{\kappa}^t{}_i)_\Xi(t_1 * t_2)$$

$$\geq \mathrm{rinf}\{\mathrm{rmin}\{(\hat{\kappa}^t{}_i)_\Xi(t_1), (\hat{\kappa}^t{}_i)_\Xi(t_2)\}\}$$

$$= \mathrm{rmin}\{\mathrm{rinf}(\hat{\kappa}^t{}_i)_\Xi(t_1), \mathrm{rinf}(\hat{\kappa}^t{}_i)_\Xi(t_2)\}$$

$$= \mathrm{rmin}\{(\cap\ (\hat{\kappa}^t{}_i)_\Xi)(t_1), (\cap\ (\hat{\kappa}^t{}_i)_\Xi)(t_2)\}$$

$$\Rightarrow (\cap\ (\hat{\kappa}^t{}_i)_\Xi)(t_1 * t_2) \geq \mathrm{rmin}\{(\cap\ (\hat{\kappa}^t{}_i)_\Xi)(t_1), (\cap\ (\hat{\kappa}^t{}_i)_\Xi)(t_2)\}$$

and

$$(\vee\ (\sigma^t_i)_\Xi)(t_1 * t_2) = \sup(\sigma^t_i)_\Xi(t_1 * t_2)$$

$$\leq \sup\{\max\{(\sigma^t_i)_\Xi(t_1), (\sigma^t_i)_\Xi(t_2)\}\}$$

$$= \max\{\sup(\sigma^t_i)_\Xi(t_1), \sup(\sigma^t_i)_\Xi(t_2)\}$$

$$= \max\{(\vee\ (\sigma^t_i)_\Xi)(t_1), (\vee\ (\sigma^t_i)_\Xi)(t_2)\}$$

$$\Rightarrow (\vee\ (\sigma^t_i)_\Xi)(t_1 * t_2) \leq \max\{(\vee\ (\sigma^t_i)_\Xi)(t_1), (\vee\ (\sigma^t_i)_\Xi)(t_2)\},$$

which show that R-intersection of $\mathcal{C}^t_i$ is t-NCSU of X.

Remark 3.6 The R-union, P-intersection and P-union of t-NCSU need not to be a t-NCSU which is explained through example.

let $X = \{0, t_1, t_2, t_3, t_4, t_5\}$ be a BF-algebra with the following Caley table.

$*$	0	t_1	t_2	t_3	t_4	t_5
0	0	t_2	t_1	t_3	t_4	t_5
t_1	t_1	0	t_2	t_5	t_3	t_4
t_2	t_2	t_1	0	t_4	t_5	t_3
t_3	t_3	t_4	t_5	0	t_1	t_2
t_4	t_4	t_5	t_3	t_2	0	t_1
t_5	t_5	t_3	t_4	t_1	t_2	0

Let $\mathcal{C}^t_1 = ((\hat{\kappa}^t)^1_\Xi, (\sigma^t)^1_\Xi)$ and $\mathcal{C}^t_2 = ((\hat{\kappa}^t)^2_\Xi, (\sigma^t)^2_\Xi)$ are t-neutrosophic cubic sets of X which are defined by

	0	t_1	t_2	t_3	t_4	t_5
$\hat{\kappa}_1^t E$	[0.4,0.5]	[0.2,0.3]	[0.2,0.3]	[0.4,0.5]	[0.2,0.3]	[0.2,0.3]
$\hat{\kappa}_1^t I$	[0.6,0.7]	[0.3,0.4]	[0.3,0.4]	[0.6,0.7]	[0.3,0.4]	[0.3,0.4]
$\hat{\kappa}_1^t N$	[0.7,0.8]	[0.4,0.5]	[0.4,0.5]	[0.7,0.8]	[0.4,0.5]	[0.4,0.5]
$\hat{\kappa}_2^t E$	[0.7,0.8]	[0.3,0.4]	[0.3,0.4]	[0.3,0.4]	[0.7,0.8]	[0.3,0.4]
$\hat{\kappa}_2^t I$	[0.8,0.7]	[0.2,0.3]	[0.2,0.3]	[0.2,0.3]	[0.8,0.7]	[0.2,0.3]
$\hat{\kappa}_2^t N$	[0.7,0.6]	[0.2,0.4]	[0.2,0.4]	[0.2,0.4]	[0.7,0.6]	[0.2,0.4]

	0	t_1	t_2	t_3	t_4	t_5
$\sigma_1^t E$	0.2	0.9	0.9	0.2	0.9	0.9
$\sigma_1^t I$	0.3	0.8	0.8	0.3	0.8	0.8
$\sigma_1^t N$	0.5	0.7	0.7	0.5	0.7	0.7
$\sigma_2^t E$	0.3	0.6	0.6	0.6	0.3	0.6
$\sigma_2^t I$	0.4	0.8	0.8	0.8	0.4	0.8
$\sigma_2^t N$	0.5	0.8	0.8	0.8	0.3	0.8

$(\cup\ (\hat{\kappa}^t)^i_\Xi)(a_3 * a_4) = ([0.3,0.4],[0.3,0.4],[0.4,0.5])_\Xi \ngeq ([0.7,0.8],[0.6,0.7],[0.5,0.6])_\Xi =$ $\mathrm{rmin}\{(\cup\ (\hat{\kappa}^t)^i_\Xi)(a_3),(\cup\ (\hat{\kappa}^t)^i_\Xi)(a_4)\}$ and $(\wedge\,(\sigma^t{}_i)_\Xi)(a_3 * a_4) = (0.5,0.6,0.7)_\Xi \nleq (0.3,0.4,0.5)_\Xi = \max\{(\wedge$ $(\sigma^t{}_i)_\Xi)(a_3),(\wedge\,(\sigma_i^t)_\Xi)(a_4)\}$.

Theorem 3.7. Let $\mathcal{C}_i^t = \{\langle t_1,(\hat{\kappa}^t{}_i)_\Xi,(\sigma_i^t)_\Xi\rangle | t_1 \in X\}$ be a collection of sets of t-NCSU of X, where $i \in k$ and $t \in [0,1]$. If $\inf\{\max\{(\sigma_i^t)_\Xi(t_1),(\sigma_i^t)_\Xi(t_1)\}\} = \max\{\inf(\sigma_i^t)_\Xi(t_1)$ $,\inf(\sigma_i^t)_\Xi(t_1)\}\ \forall\ t_1 \in X$, then the P-intersection of $\mathcal{C}_i^t$ is also a t-NCSU of X.

Proof. Suppose that $\mathcal{C}_i^t = \{\langle t_1,(\hat{\kappa}^t{}_i)_\Xi,(\sigma_i^t)_\Xi\rangle | t_1 \in X\}$ where $i \in k$, be a collection of sets of t-NCSU of X such that $\inf\{\max\{(\sigma_i^t)_\Xi(t_1),(\sigma_i^t)_\Xi(t_1)\}\} = \max\{\inf(\sigma_i^t)_\Xi(t_1),\inf(\sigma_i^t)_\Xi(t_1)\}\ \forall\ a \in X$. Then for $t_1, t_2 \in X$ and $t \in [0,1]$. Then

$$(\cap\ (\hat{\kappa}^t{}_i)_\Xi)(t_1 * t_2) = \mathrm{rinf}\{(\hat{\kappa}^t{}_i)_\Xi(t_1 * t_2)\}$$

$$\geq \mathrm{rinf}\{\mathrm{rmin}\{(\hat{\kappa}^t{}_i)_\Xi(t_1),(\hat{\kappa}^t{}_i)_\Xi(t_2)\}\}$$

$$= \mathrm{rmin}\{\mathrm{rinf}(\hat{\kappa}^t{}_i)_\Xi(t_1),\mathrm{rinf}(\hat{\kappa}^t{}_i)_\Xi(t_2)\}$$

$$= \mathrm{rmin}\{(\cap\ (\hat{\kappa}^t{}_i)_\Xi)(t_1),(\cap\ (\hat{\kappa}^t{}_i)_\Xi)(t_2)\}$$

$$\Rightarrow (\cap\ (\hat{\kappa}^t{}_i)_\Xi)(t_1 * t_2) \geq \mathrm{rmin}\{(\cap\ (\hat{\kappa}^t{}_i)_\Xi)(t_1),(\cap\ (\hat{\kappa}^t{}_i)_\Xi)(t_2)\}$$

and

$$(\wedge\,(\sigma_i^t))_\Xi)(t_1 * t_2) = \inf(\sigma_i^t)_\Xi(t_1 * t_2)$$

$$\leq \inf\{\max\{(\sigma_i^t)_\Xi(t_1),(\sigma_i^t))_\Xi(t_2)\}\}$$

$$= \max\{\inf(\sigma_i^t)_\Xi(t_1),\inf(\sigma_i^t)_\Xi(t_2)\}$$

$$= \max\{(\wedge\,(\sigma_i^t)_\Xi)(t_1),(\wedge\,(\sigma_i^t))_\Xi)(t_2)\}$$

$$\Rightarrow (\wedge (\sigma_i^t)_\Xi)(t_1 * t_2) \leq \max\{(\wedge (\sigma_i^t)_\Xi)(t_1), (\wedge (\sigma_i^t))_\Xi)(t_2)\},$$

which show that P-intersection of $\mathcal{C}_i^t$ is t-NCSU of X.

Theorem 3.8. Let $\mathcal{C}_i^t = \{\langle t_1, (\hat{\kappa}^t{}_i)_\Xi, (\sigma_i^t)_\Xi\rangle | t_1 \in X\}$ where $i \in k$, be a collection of sets of t-NCSU of X. If $\sup\{\mathrm{rmin}\{(\hat{\kappa}^t{}_i)_\Xi(t_1), (\hat{\kappa}^t{}_i)_\Xi(t_2)\}\} = \mathrm{rmin}\{\sup(\hat{\kappa}^t{}_i)_\Xi(t_1), \sup(\hat{\kappa}^t{}_i)_\Xi(t_2)\}$ and $\inf\{\max\{(\sigma_i^t)_\Xi(t_1), (\sigma_i^t)_\Xi(t_2)\}\} = \max\{\inf(\sigma_i^t)_\Xi(t_1), \inf(\sigma_i^t)_\Xi(t_2)\}$, $\forall\, t_1 \in X$. Then P-union of $\mathcal{C}_i^t$ is t-NCSU of X.

Proof. Let $\mathcal{C}_i^t = \{\langle t_1, (\hat{\kappa}^t{}_i)_\Xi, (\sigma_i^t)_\Xi\rangle | t_1 \in X\}$ where $i \in k$, be a collection of sets of t-NCSU of X such that $\sup\{\mathrm{rmin}\{(\hat{\kappa}^t{}_i)_\Xi(t_1), (\hat{\kappa}^t{}_i)_\Xi(t_2)\}\} = \mathrm{rmin}\{\sup(\hat{\kappa}^t{}_i)_\Xi(t_1), \sup(\hat{\kappa}^t{}_i)_\Xi(t_2)\}$

$\forall\, t_1 \in X$. Then for $t_1, t_2 \in X$, and $t \in [0,1]$.

$$(\cup\, (\hat{\kappa}^t{}_i)_\Xi)(t_1 * t_2) = \mathrm{rsup}(\hat{\kappa}^t{}_i)_\Xi(t_1 * t_2)$$

$$\geq \mathrm{rsup}\{\mathrm{rmin}\{(\hat{\kappa}^t{}_i)_\Xi(t_1), (\hat{\kappa}^t{}_i)_\Xi(t_2)\}\}$$

$$= \mathrm{rmin}\{\mathrm{rsup}(\hat{\kappa}^t{}_i)_\Xi(t_1), \mathrm{rsup}(\hat{\kappa}^t{}_i)_\Xi(t_2)\}$$

$$= \mathrm{rmin}\{(\cup\, (\hat{\kappa}^t{}_i)_\Xi)(t_1), (\cup\, (\hat{\kappa}^t{}_i)_\Xi)(t_2)\}$$

$$\Rightarrow (\cup\, (\hat{\kappa}^t{}_i)_\Xi)(t_1 * t_2) \geq \mathrm{rmin}\{(\cup\, (\hat{\kappa}^t{}_i)_\Xi)(t_1), (\cup\, (\hat{\kappa}^t{}_i)_\Xi)(t_2)\}$$

and

$$(\vee (\sigma_i^t)_\Xi)(t_1 * t_2) = \sup(\sigma_i^t)_\Xi(t_1 * t_2)$$

$$\leq \sup\{\max\{(\sigma_i^t)_\Xi(t_1), (\sigma_i^t)_\Xi(t_2)\}\}$$

$$= \max\{\sup(\sigma_i^t)_\Xi(t_1), \sup(\sigma_i^t)_\Xi(t_2)\}$$

$$= \max\{(\vee (\sigma_i^t)_\Xi)(t_1), (\vee (\sigma_i^t)_\Xi)(t_2)\}$$

$$\Rightarrow (\vee (\sigma_i^t)_\Xi)(t_1 * t_2) \leq \max\{(\vee (\sigma_i^t)_\Xi)(t_1), (\vee (\sigma_i^t)_\Xi)(t_2)\},$$

which show that P-union of $\mathcal{C}_i^t$ is t-NCSU of X.

Theorem 3.9 Let $\mathcal{C}_i^t = \{\langle t_1, (\hat{\kappa}^t{}_i)_\Xi, (\sigma_i^t)_\Xi\rangle | t_1 \in X\}$ where $i \in k$, be a collection of sets of t-NCSU of X. If $\inf\{\max\{(\sigma_i^t)_\Xi(t_1), (\sigma_i^t)_\Xi(t_2)\}\} = \max\{\inf(\sigma_i^t)_\Xi(t_1), \inf(\sigma_i^t)_\Xi(t_2)\}$ and $\sup\{\mathrm{rmin}\{(\hat{\kappa}^t{}_i)_\Xi(t_1), (\hat{\kappa}^t{}_i)_\Xi(t_2)\}\} = \mathrm{rmin}\{\sup(\hat{\kappa}^t{}_i)_\Xi(t_1), \sup(\hat{\kappa}^t{}_i)_\Xi(t_2)\}$ $\forall\, t_1 \in X$ and $t \in [0,1]$. Then R-union of $\mathcal{C}_i^t$ is a t-NCSU of X.

Proof. Let $\mathcal{C}_i^t = \{\langle t_1, (\hat{\kappa}^t{}_i)_\Xi, (\sigma_i^t)_\Xi\rangle | t_1 \in X\}$ where $i \in k$, and $t \in [0,1]$ be collection of sets of t-NCSU of X such that $\inf\{\max\{(\sigma_i^t)_\Xi(t_1), (\sigma_i^t)_\Xi(t_2)\}\} = \max\{\inf(\sigma_i^t)_\Xi(t_1), \inf(\sigma_i^t)_\Xi(t_2)\}\}$ and $\sup\{\mathrm{rmin}\{(\hat{\kappa}^t{}_i)_\Xi(t_1), (\hat{\kappa}^t{}_i)_\Xi(t_2)\}\} = \mathrm{rmin}$

$\{\sup(\hat{\kappa}^t{}_i)_\Xi(t_1), \sup(\hat{\kappa}^t{}_i)_\Xi(t_2)\}$ $\forall\, t_1 \in X$. Then for $t_1, t_2 \in X$ and $t \in [0,1]$

$$(\cup\, (\hat{\kappa}^t{}_i)_\Xi)(t_1 * t_2) = \mathrm{rsup}(\hat{\kappa}^t{}_i)_\Xi(t_1 * t_2)$$

$$\geq \mathrm{rsup}\{\mathrm{rmin}\{(\hat{\kappa}^t{}_i)_\Xi(t_1), (\hat{\kappa}^t{}_i)_\Xi(t_2)\}\}$$

$$= \mathrm{rmin}\{\mathrm{rsup}(\hat{\kappa}^t{}_i)_\Xi(t_1), \mathrm{rsup}(\hat{\kappa}^t{}_i)_\Xi(t_2)\}$$

$$= \mathrm{rmin}\{(\cup\, \hat{\kappa}^t{}_i)_\Xi)(t_1), (\cup\, \hat{\kappa}^t{}_i)_\Xi)(t_2)\}$$

$$\Rightarrow (\cup\, (\hat{\kappa}^t{}_i)_\Xi)(t_1 * t_2) \geq \mathrm{rmin}\{(\cup\, (\hat{\kappa}^t{}_i)_\Xi)(t_1), (\cup\, (\hat{\kappa}^t{}_i)_\Xi)(t_2)\}$$

and

$$(\wedge (\sigma_i^t)_\Xi)(t_1 * t_2) = \inf(\sigma_i^t)_\Xi(t_1 * t_2)$$

$$\leq \inf\{\max\{(\sigma_i^t)_\Xi(t_1), (\sigma_i^t)_\Xi(t_2)\}\}$$

$= \max\{\inf(\sigma_i^t)_\Xi(t_1), \inf(\sigma_i^t)_\Xi(t_2)\}$

$= \max\{(\wedge (\sigma_i^t)_\Xi)(t_1), (\wedge (\sigma_i^t)_\Xi)(t_2)\}$

$\Rightarrow (\wedge (\sigma_i^t)_\Xi)(t_1 * t_2) \leq \max\{(\wedge (\sigma_i^t)_\Xi)(t_1), (\wedge (\sigma_i^t)_\Xi)(t_2)\}$,

which show that R-union of $\mathcal{C}_i^t$ is t-NCSU of X.

Theorem 3.10 If t-neutrosophic cubic set $\mathcal{C}^t = (\hat{\kappa}^t{}_\Xi, \sigma_\Xi^t)$ of X is subalgebra, then $\forall\ t_1 \in X$, $\hat{\kappa}^t{}_\Xi(0 * t_1) \geq \hat{\kappa}^t{}_\Xi(t_1)$ and $\sigma_\Xi^t(0 * t_1) \leq \sigma_\Xi^t(t_1)$.

Proof. For all $t_1 \in X$, $\hat{\kappa}^t{}_\Xi(0 * t_1) \geq \operatorname{rmin}\{\hat{\kappa}^t{}_\Xi(0), \hat{\kappa}^t{}_\Xi(t_1)\} = \operatorname{rmin}\{\hat{\kappa}^t{}_\Xi(t_1 * t_1), \hat{\kappa}^t{}_\Xi(t_1)\} \geq \operatorname{rmin}\{\operatorname{rmin}\{\hat{\kappa}^t{}_\Xi(t_1), \hat{\kappa}^t{}_\Xi(t_1)\}, \hat{\kappa}^t{}_\Xi(t_1)\} = \hat{\kappa}^t{}_\Xi(t_1)$and similarly $\sigma_\Xi^t(0 * t_1) \leq \max\{\sigma_\Xi^t(0), \sigma_\Xi^t(t_1)\} = \sigma_\Xi^t(t_1)$.

Theorem 3.11 If t-netrosophic cubic set $\mathcal{C}^t = (\hat{\kappa}^t{}_\Xi, \sigma_\Xi^t)$ of X is subalgebra then $\mathcal{C}^t(t_1 * t_2) = \mathcal{C}^t(t_1 * (0 * (0 * t_2)))\ \forall\ t_1, t_2 \in X$.

Proof. Let X be a BF-algebra and $t_1, t_2 \in X$. Then we know by above lemma that $t_2 = 0 * (0 * t_2)$. Hence $\hat{\kappa}^t{}_\Xi(t_1 * t_2) = \hat{\kappa}^t{}_\Xi(t_1 * (0 * (0 * t_2)))$ and $\sigma_\Xi^t(t_1 * t_2) = \sigma_\Xi^t(t_1 * (0 * (0 * t_2)))$. Therefore, $\mathcal{C}_\Xi^t(t_1 * t_2) = \mathcal{C}_\Xi^t(t_1 * (0 * (0 * t_2)))$.

Theorem 3.12 If t-neutrosophic cubic set $\mathcal{C}^t = (\hat{\kappa}_\Xi^t, \sigma_\Xi^t)$ of X is t-NCSU, then $\forall\ t_1, t_2 \in$, $\hat{\kappa}_\Xi^t\big(t_1 * (0 * t_2)\big) \geq \operatorname{rmin}\{\hat{\kappa}_\Xi^t(t_1), \hat{\kappa}^t{}_\Xi(t_2)\}$ and $\sigma_\Xi^t\big(t_1 * (0 * t_2)\big) \leq \max\{\sigma_\Xi^t(t_1),\ \sigma_\Xi^t(t_2)\}$.

Proof. Let $t_1, t_2 \in X$. Then we have $\hat{\kappa}_\Xi^t(t_1 * (0 * t_2)) \geq \operatorname{rmin}\{\hat{\kappa}_\Xi^t(t_1), \hat{\kappa}_\Xi^t(0 * t_2)\} \geq \operatorname{rmin}\{\hat{\kappa}_\Xi^t(t_1), \hat{\kappa}_\Xi^t(t_2)\}$ and $\sigma_\Xi^t(t_1 * (0 * t_2)) \leq \max\{\sigma_\Xi^t(t_1), \sigma_\Xi^t(0 * t_2)\} \leq \max\ \{\sigma_\Xi^t(t_1), \sigma_\Xi^t(t_2)\}$ by definition and proposition.

Theorem 3.13 If a t-neutrosophic cubic set $\mathcal{C}^t = (\hat{\kappa}^t{}_\Xi, \sigma_\Xi^t)$ of X satisfies the following conditions, then $\mathcal{C}^t$ refers to a t-NCSU of X:

1. $\hat{\kappa}_\Xi^t(0 * t_1) \geq \hat{\kappa}_\Xi^t(t_1)$ and $\sigma_\Xi^t(0 * t_1) \leq \sigma_\Xi^t(t_1)\ \forall\ t_1 \in X$
2. $\hat{\kappa}_\Xi^t(t_1 * (0 * t_2)) \geq \operatorname{rmin}\{\hat{\kappa}_\Xi^t(t_1), \hat{\kappa}_\Xi^t(t_2)\}$ and $\sigma_\Xi^t(t_1 * (0 * t_2)) \leq \max\{\sigma_\Xi^t(t_1), \sigma_\Xi^t(t_2)\}$, $\forall\ t_1, t_2 \in X$ and $t \in [0,1]$.

Proof. Assume that the t-neutrosophic cubic set $\mathcal{C}^t = (\hat{\kappa}_\Xi^t, \sigma_\Xi^t)$ of X satisfies the above conditions (1 and 2). Then by lemma, we have $\hat{\kappa}_\Xi^t(t_1 * t_2) = \hat{\kappa}_\Xi^t(t_1 * (0 * (0 * t_2))) \geq \operatorname{rmin}\{\hat{\kappa}_\Xi^t(t_1), \hat{\kappa}_\Xi^t(0 * t_2)\} \geq \operatorname{rmin}\{\hat{\kappa}_\Xi^t(t_1), \hat{\kappa}_\Xi^t(t_2)\}$ and $\sigma_\Xi^t(t_1 * t_2) = \sigma_\Xi^t(t_1 * (0 * (0 * t_2))) \leq \max\{\sigma_\Xi^t(t_1), \sigma_\Xi^t(0 * t_2)\} \leq \max\{\sigma_\Xi^t(t_1), \sigma_\Xi^t(t_2)\}\ \forall\ t_1, t_2 \in X$. Hence $\mathcal{C}^t$ is t-NCSU of X.

Theorem 3.14 A t-neutrosophic cubic set $\mathcal{C}^t = (\hat{\kappa}_\Xi^t, \sigma_\Xi^t)$ of X is t-NCSU of X $\Leftarrow$ $\hat{\kappa}_\Xi^{t-}, \hat{\kappa}_\Xi^{t+}$ and σ_Ξ^t are fuzzy subalgebra of X.

Proof. Let $\hat{\kappa}_\Xi^{t-}, \hat{\kappa}_\Xi^{t+}$ and σ_Ξ^t are fuzzy subalgebra of X and $t_1, t_2 \in X$ and $t \in [0,1]$. Then $\hat{\kappa}_\Xi^{t-}(t_1 * t_2) \geq \min\{\hat{\kappa}_\Xi^{t-}(t_1), \hat{\kappa}_\Xi^{t-}(t_2)\}, \hat{\kappa}_\Xi^{t+}(t_1 * t_2) \geq \min\{\hat{\kappa}_\Xi^{t+}(t_1), \hat{\kappa}_\Xi^{t+}(t_2)\}$ and $\sigma_\Xi^t(t_1 * t_2) \leq \max\{\sigma_\Xi^t(t_1), \sigma_\Xi^t(t_2)\}$. Now, $\hat{\kappa}_\Xi^t(t_1 * t_2) = [\hat{\kappa}_\Xi^{t-}(t_1 * t_2), \hat{\kappa}_\Xi^{t+}(t_1 * t_2)] \geq [\min\{\hat{\kappa}_\Xi^{t-}(t_1), \hat{\kappa}_\Xi^{t-}(t_2)\}, \min\{\hat{\kappa}_\Xi^{t+}(t_1), \hat{\kappa}_\Xi^{t+}(t_2)\}] \geq \operatorname{rmin}\{[\hat{\kappa}_\Xi^{t-}(t_1), \hat{\kappa}^{t+}{}_\Xi(t_2)], [\hat{\kappa}_\Xi^{t-}(t_1), \hat{\kappa}_\Xi^{t+}$

$(t_2)]\} = \operatorname{rmin}\{\hat{\kappa}_\Xi^t(t_1), \hat{\kappa}_\Xi^t(t_2)\}$. Therefore, $\mathcal{C}^t$ is t-NCSU of X. Conversely, assume that $\mathcal{C}^t$ is a t-NCSU of X . For any $t_1, t_2 \in X$, $[\hat{\kappa}_\Xi^{t-}(t_1 * t_2), \hat{\kappa}_\Xi^{t+}(t_1 * t_2)] = \hat{\kappa}_\Xi^t(t_1 * t_2) \geq \operatorname{rmin}\{\hat{\kappa}_\Xi^t(t_1), \hat{\kappa}_\Xi^t(t_2)\} = \operatorname{rmin}\{[\hat{\kappa}_\Xi^{t-}(t_1), \hat{\kappa}^{t+}{}_\Xi(t_1)], [\hat{\kappa}_\Xi^{t-}(t_2),\ \hat{\kappa}_\Xi^{t+}(t_2)]\} =[\min\{\hat{\kappa}_\Xi^{t-}(t_1), \hat{\kappa}_\Xi^{t-}$

$(t_2)\}, \min\{\hat{\kappa}_\Xi^{t+}(t_1), \hat{\kappa}_\Xi^{t+}(t_2)\}]$. Thus, $\hat{\kappa}_\Xi^{t-}(t_1 * t_2) \geq \min\{\hat{\kappa}_\Xi^{t-}(t_1), \hat{\kappa}_\Xi^{t-}(t_2)\}$, $\hat{\kappa}_\Xi^{t+}(t_1 * t_2) \geq \min\{\hat{\kappa}_\Xi^{t+}(t_1), \hat{\kappa}_\Xi^{t+}(t_2)\}$ and $\sigma_\Xi^t(t_1 * t_2) \leq \max\{\sigma_\Xi^t(t_1), \sigma_\Xi^t(t_2)\}$. Hence $\hat{\kappa}_\Xi^{t+}, \hat{\kappa}_\Xi^{t-}$ and σ_Ξ^t are fuzzy subalgebra of X.

Theorem 3.15 Let $\mathcal{C}^t = (\hat{\kappa}^t_\Xi, \sigma^t_\Xi)$ be a t-NCSU of X and $n \in \mathbb{Z}^+$(the set of positive integer). Then

1. $\hat{\kappa}^t_\Xi(Л_n t_1 * t_1) \geq \hat{\kappa}^t_\Xi(t_1)$ for $n \in \mathbb{O}$,
2. $\sigma^t_\Xi(Л_n t_1 * t_1) \leq \sigma^t_\Xi(t_1)$ for $n \in \mathbb{O}$,
3. $\hat{\kappa}^t_\Xi(Л_n t_1 * t_1) = \hat{\kappa}^t_\Xi(t_1)$ for $n \in \mathbb{E}$,
4. $\sigma^t_\Xi(Л_n t_1 * t_1) = \sigma^t_\Xi(t_1)$ for $n \in \mathbb{E}$.

Proof. Let $t_1 \in X$ and n is odd. Then $n = 2q - 1$ for some positive integer q. We prove the theorem by induction. Now $\hat{\kappa}^t_\Xi(t_1 * t_1) = \hat{\kappa}^t_\Xi(0) \geq \hat{\kappa}^t_\Xi(t_1)$ and $\sigma^t_\Xi(t_1 * t_1) = \sigma^t_\Xi(0) \leq \sigma^t_\Xi(t_1)$. Suppose that $\hat{\kappa}^t_\Xi(Л_{2q-1} t_1 * t_1) \geq \hat{\kappa}^t_\Xi(t_1)$ and $\sigma^t_\Xi(Л_{2q-1} t_1 * t_1) \leq \sigma^t_\Xi(t_1)$. Then by assumption, $\hat{\kappa}^t_\Xi(Л_{2(q+1)-1} t_1 * t_1) = \hat{\kappa}^t_\Xi(Л_{2q+1} t_1 * t_1) = \hat{\kappa}^t_\Xi(Л_{2q-1} t_1 * (t_1 * (t_1 * t_1))) = \hat{\kappa}^t_\Xi(Л_{2q-1} t_1 * t_1) \geq \hat{\kappa}^t{}_\Xi(t_1)$ and $\sigma^t_\Xi(Л_{2(q+1)-1} t_1 * t_1) = \sigma^t_\Xi(Л_{2q+1} t_1 * t_1) = \sigma^t_\Xi(Л_{2q-1} t_1 * (t_1 * (t_1 * t_1))) = \sigma^t_\Xi(Л_{2q-1} t_1 * t_1) \leq \sigma^t_\Xi(t_1)$, which prove (1) and (2), similarly we can prove the remaining cases (3) and (4).

Theorem 3.16 The sets denoted by $I_{\hat{\kappa}^t_\Xi}$ and $I_{\sigma^t_\Xi}$ are also subalgebras of X, which are defined as: $I_{\hat{\kappa}^t_\Xi} = \{t_1 \in X | \hat{\kappa}^t_\Xi(t_1) = \hat{\kappa}^t_\Xi(0)\}$, $I_{\sigma^t_\Xi} = \{t_1 \in X | \sigma^t_\Xi(t_1) = \sigma^t_\Xi(0)\}$. Let $\mathcal{C}^t = (\hat{\kappa}^t_\Xi, \sigma^t_\Xi)$ be a t-NCSU of X. Then the sets $I_{\hat{\kappa}^t_\Xi}$ and $I_{\sigma^t_\Xi}$ are subalgebras of X.

Proof. Let $t_1, t_2 \in I_{\hat{\kappa}^t_\Xi}$. Then $\hat{\kappa}^t_\Xi(t_1) = \hat{\kappa}^t_\Xi(0) = \hat{\kappa}^t_\Xi(t_2)$ and $\hat{\kappa}^t_\Xi(t_1 * t_2) \geq \mathrm{rmin}\{\hat{\kappa}^t_\Xi(t_1), \hat{\kappa}^t_\Xi(t_2)\} = \hat{\kappa}^t{}_\Xi(0)$. By using Proposition 3.3, we know that $\hat{\kappa}^t_\Xi(t_1 * t_2) = \hat{\kappa}^t{}_\Xi(0)$ or equivalently $t_1 * t_2 \in I_{\hat{\kappa}^t{}_\Xi}$.

Again let $t_1, t_2 \in I_{\hat{\kappa}^t_\Xi}$. Then $\sigma^t_\Xi(t_1) = \sigma^t_\Xi(0) = \sigma^t_\Xi(t_2)$ and $\sigma^t_\Xi(t_1 * t_2) \leq \max\{\sigma^t_\Xi(t_1), \sigma^t_\Xi(t_2)\} = \sigma^t_\Xi(0)$. Again by using Proposition 3.3, we know that $\sigma^t_\Xi(t_1 * t_2) = \sigma^t_\Xi(0)$ or equivalently $t_1 * t_2 \in I_{\hat{\kappa}^t_\Xi}$. Hence the sets $I_{\hat{\kappa}^t_\Xi}$ and $I_{\sigma^t_\Xi}$ are subalgebras of X.

Theorem 3.17 Let A be a nonempty subset of X and $\mathcal{C}^t = (\hat{\kappa}^t_\Xi, \sigma^t_\Xi)$ be a t-neutrosophic cubic set of X defined by

$$\hat{\kappa}^t_\Xi(t_1) = \begin{cases} [\mu_{\Xi_1}, \mu_{\Xi_2}], & \text{if } t_1 \in A \\ [\nu_{\Xi_1}, \nu_{\Xi_2}], & \text{otherwise,} \end{cases} \quad \sigma^t_\Xi(t_1) = \begin{cases} \phi_\Xi, & \text{if } t_1 \in A \\ \delta_\Xi, & \text{otherwise} \end{cases}$$

, $\forall\ [\mu_{\Xi_1}, \mu_{\Xi_2}], [\nu_{\Xi_1}, \nu_{\Xi_2}] \in D[0,1]$ and ϕ_Ξ, $\delta_\Xi \in [0,1]$ with $[\mu_{\Xi_1}, \mu_{\Xi_2}] \geq [\nu_{\Xi_1}, \nu_{\Xi_2}]$ and $\phi_\Xi \leq \delta_\Xi$. Then $\mathcal{C}^t$ is a t-NCSU of X $\Leftrightarrow$ A is a subalgebra of X. Moreover, $I_{\hat{\kappa}^t_\Xi} = A = I_{\sigma^t_\Xi}$

Proof. Let $\mathcal{C}^t$ be a t-NCSU of X and $t_1, t_2 \in X$ such that $t_1, t_2 \in A$. Then $\hat{\kappa}^t_\Xi(t_1 * t_2) \geq \mathrm{rmin}\{\hat{\kappa}^t_\Xi(t_1), \hat{\kappa}^t_\Xi(t_2)\} = \mathrm{rmin}\{[\mu_{\Xi_1}, \mu_{\Xi_2}], [\mu_{\Xi_1}, \mu_{\Xi_2}]\} = [\mu_{\Xi_1}, \mu_{\Xi_2}]$ and $\sigma^t_\Xi(t_1 * t_2) \leq \max\{\sigma^t_\Xi(t_1), \sigma^t_\Xi(t_2)\} = \max\{\phi_\Xi, \phi_\Xi\} = \phi_\Xi$. Therefore $t_1 * t_2 \in A$. Hence A is a subalgebra of X.

Conversely, suppose that A is a subalgebra of X and $t_1, t_2 \in X$. Consider two cases.
Case 1: If $t_1, t_2 \in A$ then $t_1 * t_2 \in A$, thus $\hat{\kappa}^t_\Xi(t_1 * t_2) = [\mu_{\Xi_1}, \mu_{\Xi_2}] = \mathrm{rmin}\{\hat{\kappa}^t_\Xi(t_1), \hat{\kappa}^t_\Xi(t_2)\}$ and $\sigma^t_\Xi(t_1 * t_2) = \phi_\Xi = \max\{\sigma^t_\Xi(t_1), \sigma^t_\Xi(t_2)\}$.
Case 2: If $t_1 \notin A$ or $t_2 \notin A$, then $\hat{\kappa}^t_\Xi(t_1 * t_2) \geq [\nu_{\Xi_1}, \nu_{\Xi_2}] = \mathrm{rmin}\{\hat{\kappa}^t_\Xi(t_1), \hat{\kappa}^t_\Xi(t_2)\}$ and $\sigma^t_\Xi(t_1 * t_2) \leq \delta_\Xi = \max\{\sigma^t_\Xi(t_1), \sigma^t_\Xi(t_2)\}$. Hence $\mathcal{C}^t$ is a t-NCSU of X.

Now, $I_{\hat{\kappa}^t_\Xi} = \{t_1 \in X, \hat{\kappa}^t_\Xi(t_1) = \hat{\kappa}^t_\Xi(0)\} = \{t_1 \in X, \hat{\kappa}^t_\Xi(t_1) = [\alpha_{\Xi_1}, \alpha_{\Xi_2}]\} = A$ and $I_{\sigma^t_\Xi} = \{t_1 \in X, \sigma^t_\Xi(t_1) = \sigma^t_\Xi(0)\} = \{t_1 \in X, \sigma^t_\Xi(t_1) = \gamma_\Xi\} = A$.

Definition 3.18 Let $\mathcal{C}^t = (\hat{\kappa}^t{}_\Xi, \sigma^t_\Xi)$ be a t-neutrosophic cubic set of X. For $[s_{E_1}, s_{E_2}], [s_{I_1}, s_{I_2}], [s_{N_1}, s_{N_2}] \in D[0,1]$ and $t_{E_1}, t_{I_1}, t_{N_1} \in [0,1]$, the set $U(\hat{\kappa}^t{}_\Xi | ([s_{E_1}, s_{E_2}], [s_{I_1}, s_{I_2}], [s_{N_1}, s_{N_2}])) = \{t_1 \in X | \hat{\kappa}^t_E(t_1) \geq [s_{E_1}, s_{E_2}], \hat{\kappa}^t_I(t_1) \geq [s_{I_1}, s_{I_2}], \hat{\kappa}^t_N(t_1) \geq [s_{N_1}, s_{N_2}]\}$ is called upper $([s_{E_1}, s_{E_2}], [s_{I_1}, s_{I_2}], [s_{N_1}, s_{N_2}])$-level of $\mathcal{C}^t$ and $L(\sigma^t_\Xi | (t_{E_1}, t_{I_1}, t_{N_1})) = \{t_1 \in X | \sigma^t{}_E(t_1) \leq t_{E_1}, \sigma^t{}_I(t_1) \leq t_{I_1}, \sigma^t{}_N(t_1) \leq t_{N_1}\}$ is called lower $(t_{E_1}, t_{I_1}, t_{N_1})$-level of $\mathcal{C}^t$.

For comfort, we introduce the new notions for upper level and lower level of $\mathcal{C}^t$ as, $U(\hat{\kappa}^t_\Xi|[s_{\Xi_1}, s_{\Xi_2}]$={$t_1 \in X|\hat{\kappa}^t_\Xi(t_1) \geq [s_{\Xi_1}, s_{\Xi_2}]$} is called upper ($[s_{\Xi_1}, s_{\Xi_2}]$)-level of $\mathcal{C}^t$ and $L(\sigma^t_\Xi|t_{\Xi_1})$={$t_1 \in X|\sigma^t_\Xi(t_1) \leq t_{\Xi_1}$} is called lower t_{Ξ_1}-level of $\mathcal{C}^t$.

Theorem 3.19 If $\mathcal{C}^t = (\hat{\kappa}^t_\Xi, \sigma^t_\Xi)$ is t-NCSU of X, then the upper $[s_{\Xi_1}, s_{\Xi_2}]$-level and lower t_{Ξ_1}-level of $\mathcal{C}^t$ are subalgebras of X.

Proof. Let $t_1, t_2 \in U(\hat{\kappa}^t_\Xi|[s_{\Xi_1}, s_{\Xi_2}])$. Then $\hat{\kappa}^t_\Xi(t_1) \geq [s_{\Xi_1}, s_{\Xi_2}]$ and $\hat{\kappa}^t_\Xi(t_2) \geq [s_{\Xi_1}, s_{\Xi_2}]$. It follows that $\hat{\kappa}^t_\Xi(t_1 * t_2) \geq \text{rmin}\{\hat{\kappa}^t_\Xi(t_1), \hat{\kappa}^t_\Xi(t_2)\} \geq [s_{\Xi_1}, s_{\Xi_2}] \Rightarrow t_1 * t_2 \in U(\hat{\kappa}^t_\Xi|[s_{\Xi_1}, s_{\Xi_2}])$. Hence, $U(\hat{\kappa}^t_\Xi|[s_{\Xi_1}, s_{\Xi_2}]$ is a subalgebra of X. Let $t_1, t_2 \in L(\sigma^t_\Xi|t_{\Xi_1})$. Then $\sigma^t_\Xi(t_1) \leq t_{\Xi_1}$ and $\sigma^t_\Xi(t_2) \leq t_{\Xi_1}$. It follows that $\sigma^t_\Xi(t_1 * t_2) \leq \max\{\sigma^t_\Xi(t_1), \sigma^t_\Xi(t_2)\} \leq t_{\Xi_1} \Rightarrow t_1 * t_2 \in L(\sigma^t_\Xi|t_{\Xi_1})$. Hence $L(\sigma^t_\Xi|t_{\Xi_1})$ is a subalgebra of X.

Corollary 3.20 Let $\mathcal{C}^t = (\hat{\kappa}^t_\Xi, \sigma^t_\Xi)$ is t-NCSU of X. Then $\hat{\kappa}^t_\Xi([s_{\Xi_1}, s_{\Xi_2}]; t_{\Xi_1})$= $U(\hat{\kappa}^t_\Xi|[s_{\Xi_1}, s_{\Xi_2}]) \cap L(\sigma^t_\Xi|t_{\Xi_1})$ = $\{t_1 \in X|\hat{\kappa}^t_\Xi(t_1) \geq [s_{\Xi_1}, s_{\Xi_2}], \sigma^t_\Xi(t_1) \leq t_{\Xi_1}\}$ is a subalgebra of X.

Proof. We can prove it by using above proved Theorem. The converse of above corollary is not valid.

Theorem 3.21 Every subalgebra of X can be realized as both the upper $[s_{\Xi_1}, s_{\Xi_2}]$-level and lower t_{Ξ_1}-level of some t-NCSU of X.

Proof. Let $\mathcal{A}^t$ be a t-NCSU of X, and t-neutrosophic cubic set $\mathcal{C}^t$ on X is defined by

$$\hat{\kappa}^t_\Xi = \begin{cases}[\mu_{\Xi_1}, \mu_{\Xi_2}] & \text{if } t_1 \in \mathcal{A}^t \\ [0,0] & \text{otherwise}.\end{cases}, \sigma^t_\Xi = \begin{cases}\nu_{\Xi_1} & \text{if } t_1 \in \mathcal{A}^t \\ 0 & \text{otherwise}.\end{cases}$$

$\forall$ $[\mu_{\Xi_1}, \mu_{\Xi_2}] \in D[0,1]$ and $\nu_{\Xi_1} \in [0,1]$. We investigate the following cases.

Case 1 If $\forall$ $t_1, t_2 \in \mathcal{A}^t$ then $\hat{\kappa}^t_\Xi(t_1) = [\mu_{\Xi_1}, \mu_{\Xi_2}]$, $\sigma^t_\Xi(t_1) = \nu_{\Xi_1}$ and $\hat{\kappa}^t_\Xi(t_2) = [\mu_{\Xi_1}, \mu_{\Xi_2}]$, $\sigma^t_\Xi(t_2) = \nu_{\Xi_1}$.Thus $\hat{\kappa}^t_\Xi(t_1 * t_2) = [\mu_{\Xi_1}, \mu_{\Xi_2}] = \text{rmin}\{[\mu_{\Xi_1}, \mu_{\Xi_2}], [\mu_{\Xi_1}, \mu_{\Xi_2}]\} = \text{rmin}\{\hat{\kappa}^t_\Xi(t_1), \hat{\kappa}^t_\Xi(t_2)\}$ and $\sigma^t_\Xi(t_1 * t_2) = \nu_{\Xi_1} = \max\{\nu_{\Xi_1}, \nu_{\Xi_1}\} = \max\{\sigma^t_\Xi(t_1), \sigma^t_\Xi(t_2)\}$.

Case 2 If $t_1 \in \mathcal{A}^t$ and $t_2 \notin \mathcal{A}^t$, then $\hat{\kappa}^t_\Xi(t_1) = [\mu_{\Xi_1}, \mu_{\Xi_2}]$, $\sigma^t_\Xi(t_1) = \nu_{\Xi_1}$ and $\hat{\kappa}^t_\Xi(t_2) = [0,0]$, $\sigma^t_\Xi(t_2) = 1$. Thus $\hat{\kappa}^t_\Xi(t_1 * t_2) \geq [0,0] = \text{rmin}\{[\mu_{\Xi_1}, \mu_{\Xi_2}], [0,0]\} = \text{rmin}\{\hat{\kappa}^t_\Xi(t_1), \hat{\kappa}^t_\Xi(t_2)\}$ and $\sigma^t_\Xi(t_1 * t_2) \leq 1 = \max\{\nu_{\Xi_1}, 1\} = \max\{\sigma^t_\Xi(t_1), \sigma^t_\Xi(t_2)\}$.

Case 3 If $t_1 \notin \mathcal{A}^t$ and $t_2 \in \mathcal{A}^t$, then $\hat{\kappa}^t_\Xi(t_1) = [0,0]$, $\sigma^t_\Xi(t_1) = 1$ and $\hat{\kappa}^t_\Xi(t_2) = [\mu_{\Xi_1}, \mu_{\Xi_2}]$, $\sigma^t_\Xi(t_2) = \nu_{\Xi_1}$. Thus $\hat{\kappa}^t_\Xi(t_1 * t_2) \geq [0,0] = \text{rmin}\{[0,0], [\mu_{\Xi_1}, \nu_{\Xi_2}]\} = \text{rmin}\{\hat{\kappa}^t_\Xi(t_1), \hat{\kappa}^t_\Xi(t_2)\}$ and $\sigma^t_\Xi(t_1 * t_2) \leq 1 = \max\{1, \nu_{\Xi_1}\} = \max\{\sigma^t_\Xi(t_1), \sigma^t_\Xi(t_2)\}$.

Case 4 If $t_1 \notin \mathcal{A}^t$ and $t_2 \notin \mathcal{A}^t$, then $\hat{\kappa}^t_\Xi(t_1) = [0,0]$, $\sigma^t_\Xi(t_1) = 1$ and $\hat{\kappa}^t_\Xi(t_2) = [0,0]$, $\sigma^t_\Xi(t_2) = 1$. Thus $\hat{\kappa}^t_\Xi(t_1 * t_2) \geq [0,0] = \text{rmin}\{[0,0], [0,0]\} = \text{rmin}\{\hat{\kappa}^t_\Xi(t_1), \hat{\kappa}^t_\Xi(t_2)\}$ and $\sigma^t_\Xi(t_1 * t_2) \leq 1 = \max\{1,1\} = \max\{\sigma^t_\Xi(t_1), \sigma^t_\Xi(t_2)\}$. Therefore, $\mathcal{C}^t$ is a t-NCSU of X.

Theorem 3.22 Let $\mathcal{A}^t$ be a subset of X and $\mathcal{C}^t$ be a t-neutrosophic cubic set on X which is given in the proof of above theorem. If $\mathcal{C}^t$ is realized as lower level subalgebra and upper level subalgebra of some t-NCSU of X, then $\mathcal{B}^t$ is a t-neutrosophic cubic one of X.

Proof. Let $\mathcal{C}^t$ be a t-NCSU of X, and $t_1, t_2 \in \mathcal{C}^t$. Then $\hat{\kappa}^t_\Xi(t_1) = \hat{\kappa}^t_\Xi(t_2) = [\alpha_{\Xi_1}, \alpha_{\Xi_2}]$ and $\sigma^t_\Xi(t_1) = \sigma^t_\Xi(t_2) = \beta_{\Xi_1}$. Thus $\hat{\kappa}^t_\Xi(t_1 * t_2) \geq \text{rmin}\{\hat{\kappa}^t_\Xi(t_1), \hat{\kappa}^t_\Xi(t_2)\} = \text{rmin}\{[\alpha_{\Xi_1}, \alpha_{\Xi_2}], [\alpha_{\Xi_1}, \alpha_{\Xi_2}]\} = [\alpha_{\Xi_1}, \alpha_{\Xi_2}]$ and $\sigma^t_\Xi(t_1 * t_2) \leq \max\{\sigma^t_\Xi(t_1), \sigma^t_\Xi(t_2)\} = \max\{\beta_{\Xi_1}, \beta_{\Xi_1}\} = \beta_{\Xi_1} \Rightarrow t_1 * t_2 \in \mathcal{A}^t$. Hence proof is completed.

4 Image and Pre-image of t-Neutrosophic Cubic Subalgebra

In this section, homomorphism of t-neutrosophic cubic subalgebra is defined and some results are studied.

Suppose Γ be a mapping from X into Y and $\mathcal{C}^t = (\hat{\kappa}^t_\Xi, \sigma^t_\Xi)$ be a t-neutrosophic cubic set in X. Then the inverse-image of $\mathcal{C}^t$is defined as $\Gamma^{-1}(\mathcal{C}^t) = \{\langle t_1, \Gamma^{-1}(\hat{\kappa}^t_\Xi), \Gamma^{-1}(\sigma^t_\Xi)\rangle | t_1 \in X\}$ and $\Gamma^{-1}(\hat{\kappa}^t_\Xi)(t_1) = \hat{\kappa}^t_\Xi(\Gamma(t_1))$and $\Gamma^{-1}(\sigma^t_\Xi)(t_1) = \sigma^t_\Xi(\Gamma(t_1))$. It can be shown that $\Gamma^{-1}(\mathcal{C}^t)$ is a t-neutrosophic cubic set.

Theorem 4.1 Suppose that $\Gamma | X \to Y$ be a homomorphism of BF-algebra. If $\mathcal{C}^t = (\hat{\kappa}^t_\Xi, \sigma^t_\Xi)$ is a t-NCSU of Y, then the pre-image $\Gamma^{-1}(\mathcal{C}^t)$=$\{\langle t_1, \Gamma^{-1}(\hat{\kappa}^t_\Xi), \Gamma^{-1}(\sigma^t_\Xi)\rangle | t_1 \in X\}$ of $\mathcal{C}^t$ under Γ is a t-NCSU of X.

Proof. Assume that $\mathcal{C}^t = (\hat{\kappa}^t_\Xi, \sigma^t_\Xi)$ is a t-NCSU of Y and $t_1, t_2 \in X$. Then $\Gamma^{-1}(\hat{\kappa}^t_\Xi)(t_1 * t_2) = \hat{\kappa}^t_\Xi(\Gamma(t_1 * t_2)) = \hat{\kappa}^t_\Xi(\Gamma(t_1) * \Gamma(t_2)) \geq \text{rmin}\{\hat{\kappa}^t_\Xi(\Gamma(t_1)), \hat{\kappa}^t_\Xi(\Gamma(t_2))\} = \text{rmin}\{\Gamma^{-1}(\hat{\kappa}^t_\Xi)(t_1), \Gamma^{-1}(\hat{\kappa}^t_\Xi)(t_2)\}$ and $\Gamma^{-1}(\sigma^t_\Xi)(t_1 * t_2) = \sigma^t_\Xi(\Gamma(t_1 * t_2)) = \sigma^t_\Xi(\Gamma(t_1) * \Gamma(t_2)) \leq \max\{\sigma^t_\Xi(\Gamma(t_1)), \sigma^t_\Xi(\Gamma(t_2))\} = \max\{\Gamma^{-1}(\sigma^t_\Xi)(t_1), \Gamma^{-1}(\sigma^t_\Xi)(t_2)\}$. $\therefore \Gamma^{-1}(\mathcal{C}^t) = \{\langle t_1, \Gamma^{-1}(\hat{\kappa}^t{}_\Xi), \Gamma^{-1}(\sigma^t_\Xi)\rangle | t_1 \in X\}$ is t-NCSU of X.

Theorem 4.2 Consider $\Gamma | X \to Y$ be a homomorphism of BF-algebra and $\mathcal{C}^t_j = ((\hat{\kappa}^t_j)_\Xi, (\sigma^t_j)_\Xi)$ be a t-NCSU of Y, where $j \in k$. If $\inf\{\max\{(\sigma^t_j)_\Xi(t_2), (\sigma^t_j)_\Xi(t_2)\}\} = \max\{\inf(\sigma^t_j)_\Xi(t_2), \inf(\sigma^t_j)_\Xi(t_2)\}$, $\forall$ $t_2 \in Y$. Then $\Gamma^{-1}(\bigcap_{R\, j\in k} \mathcal{C}^t_j)$ is t-NCSU of X.

Proof. Let $\mathcal{C}^t_j = ((\kappa^t_j)_\Xi, (\sigma^t_j)_\Xi)$ be a t-NCSU of Y where $j \in k$satisfying $\inf\{\max\{(\sigma^t_j)_\Xi(t_2), (\sigma^t_j)_\Xi(t_2)\}\}$ = $\max\{\inf(\sigma^t_j)_\Xi(t_2), \inf(\sigma^t_j)_\Xi(t_2)\}$, $\forall$ $t_2 \in Y$. Then by Theorem 3.7 we know, $\bigcap_{R\, j\in k} \mathcal{C}^t_j$ is a t-NCSU of Y. Hence $\Gamma^{-1}(\bigcap_{R\, j\in k} \mathcal{C}^t_j)$ is t-NCSU of X.

Theorem 4.3 Let $\Gamma | X \to Y$ be a homomorphism of BF-algebra. Assume that $\mathcal{C}^t_j = ((\hat{\kappa}^t_j)_\Xi, (\sigma^t_j)_\Xi)$ be a collection of sets of t-NCSU of Y where $j \in k$. If $\text{rsup}\{\text{rmin}\{(\hat{\kappa}^t_j)_\Xi(t_2), (\hat{\kappa}^t_j)_\Xi(t_2)\}\} = \text{rmin}\{\text{rsup}(\hat{\kappa}^t_j)_\Xi(t_2), \text{rsup}(\hat{\kappa}^t_j)_\Xi(t_2)\}$, $\forall$ $(t_2), (t_2)' \in Y$. Then $\Gamma^{-1}(\bigcup_{R\, j\in k} \mathcal{C}^t_j)$ is t-NCSU of X.

Proof. Let $\mathcal{C}^t_j = ((\hat{\kappa}^t_j)_\Xi, (\sigma^t_j)_\Xi)$ be a t-NCSU of Y where $j \in k$ satisfying $\text{rsup}\{\text{rmin}\{(\hat{\kappa}^t_j)_\Xi(t_2), (\hat{\kappa}^t_j)_\Xi(t_2')\} = \text{rmin}\{\text{rsup}(\hat{\kappa}^t_j)_\Xi(t_2), \text{rsup}(\hat{\kappa}^t_j)_\Xi(t_2')\}$ $\forall$ $t_2, t_2' \in Y$. Then by Theorem 3.8 we know, $\bigcup_{R\, j\in k} \mathcal{C}^t_j$ is a t-NCSU of Y. Hence $\Gamma^{-1}(\bigcup_{R\, j\in k} \mathcal{C}^t_j)$ is t-NCSU of X.

Definition 4.4 A t-neutrosophic cubic set $\mathcal{C}^t = (\hat{\kappa}^t_\Xi, \sigma^t_\Xi)$ in BF-algebra X is said to have rsup-property and inf-property for any subset P of X, $\exists$ $p_0 \in T$ such that $\hat{\kappa}^t_\Xi(p_0) = \underset{p_0 \in S}{\text{rsup}} \hat{\kappa}^t_\Xi(p_0)$ and $\sigma^t_\Xi(s_0) = \inf_{t_0 \in T} \sigma^t_\Xi(t_0)$ respectively.

Definition 4.5 Let Γ be mapping from X to Y. If $\mathcal{C}^t = (\hat{\kappa}^t_\Xi, \sigma^t_\Xi)$ is neutrosphic cubic set of X, then the image of $\mathcal{C}^t$ under Γ is denoted by $\Gamma(\mathcal{C}^t)$ and is defined as $\Gamma(\mathcal{C}^t)$=$\{\langle t_1, \Gamma_{rsup}(\hat{\kappa}^t_\Xi), \Gamma_{inf}(\hat{\kappa}^t_\Xi)\rangle | t_1 \in X\}$, where

$$\Gamma_{rsup}(\hat{\kappa}^t_\Xi)(t_2) = \begin{cases} \underset{t_1 \in \Gamma^{-1}(t_2)}{\text{rsup}} (\hat{\kappa}^t_\Xi)(t_1), & \text{if } \Gamma^{-1}(t_2) \neq \phi \\ [0,0], & \text{otherwise}, \end{cases}$$

and

$$\Gamma_{inf}(\sigma^t_\Xi)(t_2) = \begin{cases} \inf_{t_1 \in \Gamma^{-1}(t_2)} \sigma^t_\Xi(t_1), & \text{if } \Gamma^{-1}(t_2) \neq \phi \\ 1, & \text{otherwise}. \end{cases}$$

Theorem 4.6 Suppose $\Gamma | X \to Y$ be a homomorphism from a BF-algebra X onto a BF-algebra Y. If $\mathcal{C}^t = (\hat{\kappa}^t_\Xi, \sigma^t_\Xi)$ is a t-NCSU of X, then the image $\Gamma(\mathcal{C}^t)$ $=\{\langle t_1, \Gamma_{rsup}(\hat{\kappa}^t_\Xi), \Gamma_{inf}(\sigma^t_\Xi)\rangle | t_1 \in X\}$ of $\mathcal{A}$ under Γ is t-NCSU of Y.

Proof. Let $\mathcal{C}^t = (\hat{\kappa}^t_\Xi, \sigma^t_\Xi)$ be a t-NCSU of X and $t_2, t_2' \in Y$. We know that $\{t_1 * t_1' | t_1 \in \Gamma^{-1}(t_2)$ and $t_1' \in \Gamma^{-1}t_2'\} \subseteq \{t_1 \in X | t_1 \in \Gamma^{-1}(t_2 * t_2')\}$. Now $\Gamma_{rsup}(\hat{\kappa}^t_\Xi)(t_2 * t_2')$=rsup$\{\hat{\kappa}^t_\Xi(t_1) | t_1 \in \Gamma^{-1}(t_2 * t_2')\} \geq$ rsup$\{\hat{\kappa}^t_\Xi(t_1 * t_1') | t_1 \in \Gamma^{-1}(t_2)$ and $t_1' \in \Gamma^{-1}(t_2')\} \geq$ rsup$\{$rmin$\{\hat{\kappa}^t_\Xi(t_1), \hat{\kappa}^t_\Xi(t_1')\} | t_1 \in \Gamma^{-1}(t_2)$ and $t_1' \in \Gamma^{-1}(t_2')\} =$ rmin$\{$rsup$\{\hat{\kappa}^t_\Xi(t_1) | t_1 \in \Gamma^{-1}(t_2)\}$, rsup$\{\hat{\kappa}^t_\Xi(t_1') | t_1' \in \Gamma^{-1}(t_2')\}\} =$ rmin$\{\Gamma_{rsup}(\hat{\kappa}^t_\Xi)(t_2)$,

$\Gamma_{rsup}(\hat{\kappa}^t_\Xi)(t_2')\}$ and $\Gamma_{inf}(\sigma^t_\Xi)(t_2 * t_2') = \inf\{\sigma^t_\Xi(t_1) | t_1 \in \Gamma^{-1}(t_2 * t_2')\} \leq \inf\{\sigma^t_\Xi(t_1 * t_1') | t_1 \in \Gamma^{-1}(t_2)$ and $t_1' \in \Gamma^{-1}(t_2')\} \leq \inf\{\max\{\sigma^t_\Xi(t_1), \sigma^t_\Xi(t_1')\} | t_1 \in \Gamma^{-1}(t_2)$ and $t_1' \in \Gamma^{-1}(t_2')\} = \max\{\inf\{\sigma^t_\Xi(t_1) | t_1 \in \Gamma^{-1}(t_2)\}, \inf\{\sigma^t_\Xi(t_1') | t_1' \in \Gamma^{-1}(t_2')\}\} = \max\{\Gamma_{inf}(\sigma^t_\Xi)(t_2), \Gamma_{inf}(\sigma^t_\Xi)(t_2')\}$. Hence $\Gamma(\mathcal{C}^t)=\{\langle t_1, \Gamma_{rsup}(\hat{\kappa}^t{}_\Xi), \Gamma_{inf}(\sigma^t{}_\Xi) \rangle | t_1 \in X\}$

is a t-NCSU of Y.

Theorem 4.7 Assume that $\Gamma | X \to Y$ is a homomorphism of BF-algebra and $\mathcal{C}^t_i = \{(\hat{\kappa}^t_i)_\Xi, (\sigma^t_i)_\Xi\}$ is a t-NCSU of X, where $i \in k$. If $\inf\{\max\{(\sigma^t_i)_\Xi(t_1), (\sigma^t_i)_\Xi(t_1)\}\} = \max\{\inf(\sigma^t_i)_\Xi(t_1), \inf(\sigma^t_i)_\Xi(t_1)\}$, $\forall\, t_1 \in X$. Then $\Gamma(\bigcap_{P\, i\in k} \mathcal{C}^t_i)$ is a t-NCSU of Y.

Proof. Let $\mathcal{C}^t_i = \{(\hat{\kappa}^t_i)_\Xi, (\sigma^t_i)_\Xi\}$ be a collection of sets of t-NCSU of X, where $i \in k$ satisfies $\inf\{\max\{(\sigma^t_i)_\Xi(t_1), (\sigma^t_i)_\Xi(t_1)\}\} = \max\{\inf(\sigma^t_i)_\Xi(t_1), \inf(\sigma^t_i)_\Xi(t_1)\}$ $\forall$ $t_1 \in X$. Then by above stated theorem, $\bigcap_{P\, i\in k} \mathcal{C}^t_i$ is a t-NCSU of X. Hence $\Gamma(\bigcap_{P\, i\in k} \mathcal{C}^t_j)$ is t-NCSU of Y.

Theorem 4.8 Suppose $\Gamma | X \to Y$ be a homomorphism of BF-algebra and $\mathcal{C}^t_i = \{(\hat{\kappa}^t_i)_\Xi, (\sigma^t_i)_\Xi\}$ be a t-NCSU of X where $i \in k$.If rsup$\{$rmin$\{(\kappa^t_i)_\Xi(t_1), (\hat{\kappa}^t_i)_\Xi(t_1)\}\} =$ rmin$\{$rsup $(\hat{\kappa}^t_i)_\Xi(t_1)$, rsup$(\hat{\kappa}^t_i)_\Xi(t_1')\}$, $\forall$ $t_1, t_1' \in Y$. Then $\Gamma(\bigcup_{P\, i\in k} \mathcal{C}^t_i)$ is also a t-NCSU of Y.

Proof. Let $\mathcal{C}^t_i = \{(\hat{\kappa}^t_i)_\Xi, (\sigma^t_i)_\Xi\}$ be a collection of sets of t-NCSU of X where $i \in k$ satisfies rsup$\{$rmin$\{(\hat{\kappa}^t_i)_\Xi(t_1), (\hat{\kappa}^t_i)_\Xi(t_1')\}\}$ = rmin$\{$rsup$(\hat{\kappa}^t_i)_\Xi(t_1)$, rsup$(\hat{\kappa}^t_i)_\Xi(t_1')\}$, $\forall\, t_1, t_1' \in X$. Then by above stated theorem we know that $\bigcup_{P\, i\in k} \mathcal{C}^t_i$ is a t-NCSU of X. Hence $\Gamma(\bigcup_{P\, i\in k} \mathcal{C}^t_i)$ is t-NCSU of Y.

Theorem 4.9 For a homomorphism $\Gamma | X \to Y$ of BF-algebra, the following results hold:

1. If $\forall$ $i \in k$, $\mathcal{C}^t_i$ is t-NCSU of X, then $\Gamma(\bigcap_{R\, i\in k} \mathcal{C}^t_i)$ is t-NCSU of Y,
2. If $\forall\, i \in k$, $\mathcal{D}^t_i$ is t-NCSU of Y, then $\Gamma^{-1}(\bigcap_{R\, i\in k} \mathcal{D}^t_i)$ is t-NCSU of X.

Proof. Straightforward.

Theorem 4.10 Let Γ be an isomorphism from a BF-algebra X onto a BF-algebra Y. If $\mathcal{C}^t$ is a t-NCSU of X. Then $\Gamma^{-1}(\Gamma(\mathcal{C}^t)) = \mathcal{C}^t$.

Proof. For any $t_1 \in X$, let $\Gamma(t_1) = t_2$. Since Γ is an isomorphism, $\Gamma^{-1}(t_2) = \{t_1\}$. Thus $\Gamma(\mathcal{C}^t)(\Gamma(t_1)) = \Gamma(\mathcal{C}^t)(t_2) = \bigcup_{t_1 \in \Gamma^{-1}(t_2)} \mathcal{C}^t(t_1) = \mathcal{C}^t(t_1)$.For any $t_2 \in Y, \Gamma$ is an isomorphism, $\Gamma^{-1}(t_2) = \{t_1\}$ so that $\Gamma(t_1) = t_2$. Thus $\Gamma^{-1}(\mathcal{C}^t)(t_1) = \mathcal{C}^t(\Gamma(t_1)) = \mathcal{C}^t(t_2)$. Hence, $\Gamma^{-1}(\Gamma(\mathcal{C}^t)) = \mathcal{C}^t$.

Corollary 4.11 Consider Γ is an Isomorphism from a BF-algebra X onto a BF-algebra Y. If $\mathcal{C}^t$ is a t-NCSU of Y. Then $\Gamma(\Gamma^{-1}(\mathcal{C}^t)) = \mathcal{C}^t$.

Proof. Straightforward.

Corollary 4.12 Let $\Gamma | X \to X$ be an automorphism. If $\mathcal{C}^t$ is a t-NCSU of X. Then $\Gamma(\mathcal{C}^t) = \mathcal{C}^t \Leftarrow \Gamma^{-1}(\mathcal{C}^t) = \mathcal{C}^t$.

5 t-Neutrosophic Cubic Closed Ideal of BF-algebra

In this section, t-neutrosophic cubic ideal and t-neutrosophic cubic closed ideal of BF-algebra are defined and investigated through related results.

Definition 5.1 A t-neutrosophic cubic set $\mathcal{C}^t = (\hat{\kappa}^t_\Xi, \sigma^t_\Xi)$ of X is called a t-NCID of X if it satisfies following axoims:

N3. $\hat{\kappa}^t{}_\Xi(0) \geq \hat{\kappa}^t_\Xi(t_1)$ and $\sigma^t_\Xi(0) \leq \sigma^t_\Xi(t_1)$,

N4. $\hat{\kappa}^t_\Xi(t_1) \geq \mathrm{rmin}\{\hat{\kappa}^t_\Xi(t_1 * t_2), \hat{\kappa}^t_\Xi(t_2)\}$,

N5. $\sigma^t_\Xi(t_1) \leq \max\{\sigma^t_\Xi(t_1 * t_2), \sigma^t_\Xi(t_2)\}$, $\forall\, t_1, t_2 \in X$.

Example 5.2 Consider a BF-algebra $X = \{0, t_1, t_2, t_3\}$ and binary operation * is defined on X as

*	0	t_1	t_2	t_3
	0	t_1	t_2	t_3
t_1	t_1	0	t_3	t_2
t_2	t_2	t_3	0	t_1
t_3	t_3	t_2	t_1	0

Let $\mathcal{C}^t = \{\hat{\kappa}^t{}_\Xi, \sigma^t_\Xi\}$ be a t-neutrosophic cubic set in X is defined as,

	0	t_1	t_2	t_3
$\hat{\kappa}^t{}_E$	[1,1]	[0.8,0.7]	[1,1]	[0.4,0.6]
$\hat{\kappa}^t{}_I$	[0.8,0.8]	[0.5,0.7]	[0.8,0.8]	[0.6,0.4]
$\hat{\kappa}^t{}_N$	[0.7,0.8]	[0.4,0.5]	[0.7,0.8]	[0.8,0.4]

and

	0	t_1	t_2	t_3
$\sigma^t{}_E$	0	0.7	0	0.6
$\sigma^t{}_I$	0.1	0.5	0.1	0.6
$\sigma^t{}_N$	0.2	0.3	0.2	0.4

Then it can be easy verify that $\mathcal{C}^t$ satisfies the conditions N3, N4 and N5. Hence $\mathcal{C}^t$ is t-NCID of X.

Definition 5.3 Let $\mathcal{C}^t = \{\hat{\kappa}^t_\Xi, \sigma^t_\Xi\}$ be a t-neutrosophic cubic set X then it is called t-neutrosophic cubic closed ideal of X if it satisfies N4, N5 and N6. $\hat{\kappa}^t_\Xi(0 * t_1) \geq \hat{\kappa}^t_\Xi(t_1)$ and $\sigma^t_\Xi(0 * t_1) \leq \sigma^t_\Xi(t_1)$, $\forall\, t_1 \in X$.

Example 5.4 Let $X = \{0, t_1, t_2, t_3, t_4, t_5\}$ be a BF-algebra as in Example 3.2 and $\mathcal{C}^t = \{\hat{\kappa}^t_\Xi, \sigma^t_\Xi\}$ be a t-neutrosophic cubic set in X is defined as

	0	t_1	t_2	t_3	t_4	t_5
$\hat{\kappa}^t{}_E$	[0.4,0.7]	[0.3,0.6]	[0.3,0.6]	[0.2,0.4]	[0.2,0.4]	[0.2,0.4]
$\hat{\kappa}^t{}_I$	[0.5,0.8]	[0.4,0.7]	[0.4,0.7]	[0.3,0.6]	[0.3,0.6]	[0.3,0.6]
$\hat{\kappa}^t{}_N$	[0.6,0.9]	[0.5,0.8]	[0.5,0.8]	[0.3,0.4]	[0.3,0.4]	[0.3,0.4]

	0	t_1	t_2	t_3	t_4	t_5
$\sigma^t{}_E$	0.3	0.6	0.6	0.8	0.8	0.8
$\sigma^t{}_I$	0.4	0.5	0.5	0.7	0.7	0.7
$\sigma^t{}_N$	0.5	0.6	0.6	0.9	0.9	0.9

By calculations it is clear that $\mathcal{C}^t$ is a t-neutrosophic cubic closed ideal of X.

Proposition 5.5 Every t-neutrosophic cubic closed ideal is a t-NCID.

Proof The converse of proposition 5.5 is not true in general as shown in the given example.

Example 5.6 Let $X = \{0, t_1, t_2, t_3, t_4, t_5\}$ be a BF-algebra as in Example 3.2 and $\mathcal{C}^t = \{\hat{\kappa}^t_\Xi, \sigma^t_\Xi\}$ be a t-neutrosophic cubic set in X is defined as

	0	t_1	t_2	t_3	t_4	t_5
$\hat{\kappa}^t{}_E$	[0.5,0.7]	[0.4,0.6]	[0.4,0.6]	[0.3,0.4]	[0.3,0.4]	[0.3,0.4]
$\hat{\kappa}^t{}_I$	[0.6,0.8]	[0.5,0.7]	[0.5,0.7]	[0.4,0.6]	[0.4,0.6]	[0.4,0.6]
$\hat{\kappa}^t{}_N$	[0.7,0.9]	[0.6,0.8]	[0.6,0.8]	[0.5,0.4]	[0.5,0.4]	[0.5,0.4]

	0	t_1	t_2	t_3	t_4	t_5
$\sigma^t{}_E$	0.2	0.5	0.5	0.6	0.6	0.6
$\sigma^t{}_I$	0.3	0.4	0.4	0.7	0.7	0.7
$\sigma^t{}_N$	0.3	0.5	0.5	0.8	0.8	0.8

By calculations verify that $\mathcal{C}^t$ is a t-NCID of X. But it is not a t-neutrosophic cubic closed ideal of X since $\hat{\kappa}^t{}_\Xi(0 * t_1) \ngeq \hat{\kappa}^t_\Xi(t_1)$ and $\sigma^t_\Xi(0 * t_1) \nleq \sigma^t_\Xi(t_1)$, $\forall\ t_1 \in X$.

Corollary 5.7 Every t-NCSU which satisfies N4 and N5 becomes a t-neutrosophic cubic closed ideal.

Theorem 5.8 Every t-neutrosophic cubic closed ideal of a BF-algebra X is also a t-NCSU of X.

Proof. Suppose $\mathcal{C}^t = \{\hat{\kappa}^t_\Xi, \sigma^t_\Xi\}$ be a t-neutrosophic cubic closed ideal of X, then for any $t_1 \in X$ we have $\hat{\kappa}^t_\Xi(0 * t_1) \geq \hat{\kappa}^t_\Xi(t_1)$ and $\sigma^t_\Xi(0 * t_1) \leq \sigma^t_\Xi(t_1)$. Now by N4, N6, Proposition 3.3, we know that $\hat{\kappa}^t_\Xi(t_1 * t_2) \geq \text{rmin}\{\hat{\kappa}^t_\Xi((t_1 * t_2) * (0 * t_2)), \hat{\kappa}^t_\Xi(0 * t_2)\} = \text{rmin}\{\hat{\kappa}^t_\Xi(t_1), \hat{\kappa}^t_\Xi(0 * t_2)\} \geq \text{rmin}\{\hat{\kappa}^t_\Xi(t_1), \hat{\kappa}^t_\Xi(t_2)\}$ and $\sigma^t_\Xi(t_1 * t_2) \leq \max\{\sigma^t_\Xi((t_1 * t_2) * (0 * t_2)), \sigma^t_\Xi(0 * t_2)\} = \max\{\sigma^t_\Xi(t_1), \sigma^t_\Xi(0 * t_2)\} \leq \max\{\sigma^t_\Xi(t_1), \sigma^t_\Xi(t_2)\}$. Hence $\mathcal{C}^t$ is a t-neutrosophic cubic subalgeba of X.

Theorem 5.9 The R-intersection of any set of t-NCIDs of X is a t-NCID of X.

Proof. Let $\mathcal{C}^t_i = \{(\hat{\kappa}^t_i)_\Xi, (\sigma^t_i)_\Xi\}$ where $i \in k$, be a collection of sets of t-NCID of X and $t_1, t_2 \in X$. Then

$$(\cap\ (\hat{\kappa}^t_i)_\Xi)(0) = \text{rinf}(\hat{\kappa}^t_i)_\Xi(0)$$

$$\geq \text{rinf}(\hat{\kappa}^t_i)_\Xi(t_1)$$

$$= (\cap\ (\hat{\kappa}^t_i)_\Xi)(t_1)$$

$$\Rightarrow (\cap\ (\hat{\kappa}^t_i)_\Xi)(0) \geq (\cap\ (\hat{\kappa}^t_i)_\Xi)(t_1),$$

$(\vee\,(\sigma_i^t)_\Xi)(0) = \sup(\sigma_i^t)_\Xi(0)$

$\leq (\sigma_i^t)_\Xi(t_1)$

$= (\vee\,(\sigma_i^t)_\Xi)(t_1)$

$\Rightarrow (\vee\,(\sigma_i^t)_\Xi)(0) \leq (\vee\,(\sigma_i^t)_\Xi)(t_1),$

$(\cap\,(\hat{\kappa}_i^t)_\Xi)(t_1) = \mathrm{rinf}(\hat{\kappa}_i^t)_\Xi(t_1)$

$\geq \mathrm{rinf}\{\mathrm{rmin}\{(\hat{\kappa}_i^t)_\Xi(t_1 * t_2), (\hat{\kappa}_i^t)_\Xi(t_2)\}\}$

$= \mathrm{rmin}\{\mathrm{rinf}(\hat{\kappa}_i^t)_\Xi(t_1 * t_2), \mathrm{rinf}(\hat{\kappa}_i^t)_\Xi(t_2)\}$

$= \mathrm{rmin}\{(\cap\,(\hat{\kappa}_i^t)_\Xi)(t_1 * t_2), (\cap\,(\hat{\kappa}_i^t)_\Xi)(t_2)\}$

$\Rightarrow (\cap\,(\hat{\kappa}_i^t)_\Xi)(t_1) \geq \mathrm{rmin}\{(\cap\,(\hat{\kappa}_i^t)_\Xi)(t_1 * t_2), (\cap\,(\hat{\kappa}_i^t)_\Xi)(t_2)\}$

and

$(\vee\,(\sigma_i^t)_\Xi)(t_1) = \sup(\sigma_i^t)_\Xi(t_1)$

$\leq \sup\{\max\{(\sigma_i^t)_\Xi(t_1 * t_2), (\sigma_i^t)_\Xi(t_2)\}\}$

$= \max\{\sup(\sigma_i^t)_\Xi(t_1 * t_2), \sup(\sigma_i^t)_\Xi(t_2)\}$

$= \max\{(\vee\,(\sigma_i^t)_\Xi)(t_1 * t_2), (\vee\,(\sigma_i^t)_\Xi)(t_2)\}$

$\Rightarrow (\vee\,(\sigma_i^t)_\Xi)(t_1) \leq \max\{(\vee\,(\sigma_i^t)_\Xi)(t_1 * t_2), (\vee\,(\sigma_i^t)_\Xi)(t_2)\},$

which show that R-intersection is a t-NCID of X.

Theorem 5.10 The R-intersection of any set of t-neutrosophic cubic closed ideals of X is also a t-neutrosophic cubic closed ideal of X.

Proof. It is similar to the proof of Theorem 5.9.

Theorem 5.11 For a t-neutrosophic cubic ideal $\mathcal{C}^t = \{\hat{\kappa}_\Xi^t, \sigma_\Xi^t\}$ of X, the following assertions are valid:

1. if $t_1 * t_2 \leq z$, then $\hat{\kappa}_\Xi^t(t_1) \geq \mathrm{rmin}\{\hat{\kappa}_\Xi^t(t_2), \hat{\kappa}_\Xi^t(t_3)\}$ and $\sigma_\Xi^t(t_1) \leq \max\{\sigma_\Xi^t(t_2), \sigma_\Xi^t(t_3)\}$,
2. if $t_1 \leq t_2$, then $\hat{\kappa}_\Xi^t(t_1) \geq \hat{\kappa}_\Xi^t(t_2)$ and $\sigma_\Xi^t(t_1) \leq \sigma_\Xi^t(t_2)$, $\forall\ t_1, t_2, t_3 \in X$.

Proof. 1. Assume that $t_1, t_2, t_3 \in X$ such that $t_1 * t_2 \leq t_3$. Then $(t_1 * t_2) * t_3 = 0$ and thus $\hat{\kappa}_\Xi^t(t_1) \geq \mathrm{rmin}\{\hat{\kappa}_\Xi^t(t_1 * t_2), \hat{\kappa}_\Xi^t(t_2)\} \geq \mathrm{rmin}\{\mathrm{rmin}\{\hat{\kappa}_\Xi^t((t_1 * t_2) * t_3), \hat{\kappa}_\Xi^t(t_3)\}, \hat{\kappa}_\Xi^t(t_2)\} = \mathrm{rmin}\{\mathrm{rmin}\{\hat{\kappa}_\Xi^t(0), \hat{\kappa}_\Xi^t(t_3)\}, \hat{\kappa}_\Xi^t(t_2)\} = \mathrm{rmin}\{\hat{\kappa}_\Xi^t(t_2), \hat{\kappa}_\Xi^t(t_3)\}$ and $\sigma_\Xi^t(t_1) \leq \max\{\sigma_\Xi^t(t_1 * t_2), \sigma_\Xi^t(t_2)\} \leq \max\{\max\{\sigma_\Xi^t((t_1 * t_2) * t_3), \sigma_\Xi^t(t_3)\}, \sigma_\Xi^t(t_2)\} = \max\{\max\{\sigma_\Xi^t(0), \sigma_\Xi^t(t_3)\},$

$\sigma_\Xi^t(t_2)\} = \max\{\sigma_\Xi^t(b), \sigma_\Xi^t(t_3)\}$.

2. Again, take $t_1, t_2 \in X$ such that $t_1 \leq t_2$. Then $t_1 * t_2 = 0$ and thus $\hat{\kappa}_\Xi^t(t_1) \geq \mathrm{rmin}\{\hat{\kappa}_\Xi^t(t_1 * t_2), \hat{\kappa}_\Xi^t(t_2)\} = \mathrm{rmin}\{\hat{\kappa}_\Xi^t(0), \hat{\kappa}_\Xi^t(t_2)\} = \hat{\kappa}_\Xi^t(t_2)$ and $\sigma_\Xi^t(t_1) \leq \mathrm{rmin}\{\sigma_\Xi^t(t_1 * t_2), \sigma_\Xi^t(t_2)\} = \mathrm{rmin}\{\sigma_\Xi^t(0), \sigma_\Xi^t(t_2)\} = \sigma_\Xi^t(t_2)$.

Theorem 5.12 Let $\mathcal{C}^t = \{\hat{\kappa}_\Xi^t, \sigma_\Xi^t\}$ is a neutrosophic cubic ideal of X. If $t_1 * t_2 \leq t_1, \forall\ t_1, t_2 \in X$. Then $\mathcal{C}^t$ is a t-NCSU of X.

Proof. Assume that $\mathcal{C}^t = \{\hat{\kappa}_\Xi^t, \sigma_\Xi^t\}$ is a t-neutrosophic cubic ideal of X. Suppose that $t_1 * t_2 \leq t_1\ \forall\ t_1, t_2 \in X$. Then

$\hat{\kappa}_\Xi^t(t_1 * t_2) \geq \hat{\kappa}_\Xi^t(t_1)$ ($\because$ By Theorem 5.11)

$\geq rmin\{\hat{\kappa}^t_\Xi(t_1 * t_2), \hat{\kappa}^t_\Xi(t_2)\}$ ($\because$ By N4)

$\geq rmin\{\hat{\kappa}^t_\Xi(t_1), \hat{\kappa}^t_\Xi(t_2)\}$ ($\because$ By Theorem 5.11)

$\Rightarrow \hat{\kappa}^t_\Xi(t_1 * t_2) \geq rmin\{\hat{\kappa}^t_\Xi(t_1), \hat{\kappa}^t_\Xi(t_2)\}$

and

$\sigma^t_\Xi(t_1 * t_2) \leq \sigma^t_\Xi(t_1)$ ($\because$ By Theorem 5.11)

$\leq \max\{\sigma^t_\Xi(t_1 * t_2), \sigma^t_\Xi(t_2)\}$ ($\because$ By N5)

$\leq \max\{\sigma^t_\Xi(t_1), \sigma^t_\Xi(t_2)\}$ ($\because$ By Theorem 5.11)

$\Rightarrow \sigma^t_\Xi(t_1 * t_2) \leq \max\{\sigma^t_\Xi(t_1), \sigma^t_\Xi(t_2)\}$.

Hence $\mathcal{C}^t = \{\hat{\kappa}^t_\Xi, \sigma^t_\Xi\}$ is a t-NCSU of X.

Theorem 5.13 If $\mathcal{C}^t = \{\hat{\kappa}^t_\Xi, \sigma^t_\Xi\}$ is a t-neutrosophic cubic ideal of X, then $(\ldots((t_1 * x_1) * x_2) * \ldots) * x_n = 0$ for any $t_1, x_1, x_2, \ldots, x_n \in X \Rightarrow \hat{\kappa}^t{}_\Xi(t_1) \geq rmin\{\hat{\kappa}^t_\Xi(x_1), \hat{\kappa}^t_\Xi(x_2), \ldots,$ $\hat{\kappa}^t_\Xi(x_n)\}$ and $\sigma^t_\Xi(t_1) \leq \max\{\sigma^t_\Xi(x_1), \sigma^t_\Xi(x_2), \ldots, \sigma^t_\Xi(x_n)\}$.

Proof. We can prove this theorem by using induction on n and Theorem 5.11.

Theorem 5.14 A t-neutrosophic cubic set $\mathcal{C}^t = (\hat{\kappa}^t_\Xi, \sigma^t_\Xi)$ is a t-neutrosophic cubic closed ideal of $X \Leftarrow U(\hat{\kappa}^t_\Xi|[s_{\Xi_1}, s_{\Xi_2}])$ and $L(\sigma^t_\Xi|t_{\Xi_1})$ are closed ideals of X for every $[s_{\Xi_1}, s_{\Xi_2}] \in D[0,1]$ and $t_{\Xi_1} \in [0,1]$.

Proof. Assume that $\mathcal{C}^t = (\hat{\kappa}^t_\Xi, \sigma^t_\Xi)$ is a t-neutrosophic cubic closed ideal of X. For $[s_{\Xi_1}, s_{\Xi_2}] \in D[0,1]$, clearly, $0 * t_1 \in U(\hat{\kappa}^t_\Xi|[s_{\Xi_1}, s_{\Xi_2}])$, where $t_1 \in X$. Let $t_1, t_2 \in X$ be such that $t_1 * t_2 \in U(\hat{\kappa}^t_\Xi|[s_{\Xi_1}, s_{\Xi_2}])$ and $t_2 \in U(\hat{\kappa}^t_\Xi|[s_{\Xi_1}, s_{\Xi_2}])$. Then $\hat{\kappa}^t_\Xi(t_1) \geq rmin\{\hat{\kappa}^t_\Xi(t_1 * t_2), \hat{\kappa}^t_\Xi(t_2)\} \geq [s_{\Xi_1}, s_{\Xi_2}] \Rightarrow t_1 \in U(\hat{\kappa}^t_\Xi|[s_{\Xi_1}, s_{\Xi_2}]$. Hence $U(\hat{\kappa}^t_\Xi|[s_{\Xi_1}, s_{\Xi_2}])$ is a closed ideal of X.

For $t_{\Xi_1} \in [0,1]$. Clearly, $0 * t_1 \in L(\sigma^t_\Xi|t_{\Xi_1})$. Let $t_1, t_2 \in X$ be such that $t_1 * t_2 \in L(\sigma^t_\Xi|t_{\Xi_1})$ and $t_2 \in L(\sigma^t_\Xi|t_{\Xi_1})$. Then $\sigma^t_\Xi(t_1) \leq \max\{\sigma^t_\Xi(t_1 * t_2), \sigma^t_\Xi(t_2)\} \leq t_{\Xi_1} \Rightarrow t_1 \in L(\sigma^t_\Xi|t_{\Xi_1})$. Hence $L(\sigma^t_\Xi|t_{\Xi_1})$ is a t-neutrosophic cubic closed ideal of X.

Conversely, suppose that each nonempty level subset $U(\hat{\kappa}^t_\Xi|[s_{\Xi_1}, s_{\Xi_2}])$ and $L(\sigma^t_\Xi|t_{\Xi_1})$ are closed ideals of X. For any $t_1 \in X$, let $\hat{\kappa}^t_\Xi(t_1) = [s_{\Xi_1}, s_{\Xi_2}]$ and $\sigma^t_\Xi(t_1) = t_{\Xi_1}$. Then $t_1 \in U(\hat{\kappa}^t_\Xi|[s_{\Xi_1}, s_{\Xi_2}])$ and $t_1 \in L(\sigma^t_\Xi|t_{\Xi_1})$. Since $0 * t_1 \in U(\hat{\kappa}^t_\Xi|[s_{\Xi_1}, s_{\Xi_2}]) \cap L(\sigma^t_\Xi|t_{\Xi_1})$, it follows that $\hat{\kappa}^t_\Xi(0 * t_1) \geq [s_{\Xi_1}, s_{\Xi_2}] = \hat{\kappa}^t_\Xi(t_1)$ and $\sigma^t_\Xi(0 * t_1) \leq t_{\Xi_1} = \sigma^t_\Xi(t_1)$ $\forall$ $t_1 \in X$. If there exists $\alpha_{\Xi_1}, \beta_{\Xi_1} \in X$ such that $\hat{\kappa}^t_\Xi(\alpha_{\Xi_1}) \leq rmin\{\hat{\kappa}^t_\Xi(\alpha_{\Xi_1} * \beta_{\Xi_1}), \beta_{\Xi_1}\}$, then by taking $[s'_{\Xi_1}, s'_{\Xi_2}] = \frac{1}{2}[\hat{\kappa}^t_\Xi(\alpha_{\Xi_1} * \beta_{\Xi_1}) + rmin\{\hat{\kappa}^t_\Xi(\alpha_{\Xi_1}), \hat{\kappa}^t_\Xi(\beta_{\Xi_1})\}]$.

It follows that $\alpha_{\Xi_1} * \beta_{\Xi_1} \in U(\hat{\kappa}^t_\Xi|[s'_{\Xi_1}, s'_{\Xi_2}])$ and $\beta_{\Xi_1} \in U(\hat{\kappa}^t_\Xi|[s'_{\Xi_1}, s'_{\Xi_2}])$, but $\alpha_{\Xi_1} \notin U(\hat{\kappa}^t_\Xi|[s'_{\Xi_1}, s'_{\Xi_2}])$, which is contradiction. Hence, $U(\hat{\kappa}^t_\Xi|[s'_{\Xi_1}, s'_{\Xi_2}])$ is not closed ideal of X.

Again, if there exists $\alpha_{\Xi_1}, \beta_{\Xi_1} \in X$ such that $\sigma^t_\Xi(\alpha_{\Xi_1}) \geq \max\{\sigma^t_\Xi(\alpha_{\Xi_1} * \beta_{\Xi_1}), \sigma^t_\Xi(\beta_{\Xi_1})\}$, then by taking $t'_{\Xi_1} = \frac{1}{2}[\sigma^t_\Xi(\alpha_{\Xi_1} * \beta_{\Xi_1}) + \max\{\sigma^t_\Xi(\alpha_{\Xi_1}), \sigma^t_\Xi(\beta_{\Xi_1})\}]$.

It follows that $\alpha_{\Xi_1} * \beta_{\Xi_1} \in L(\sigma^t_\Xi|t'_{\Xi_1})$ and $\beta_{\Xi_1} \in L(\sigma^t_\Xi|t'_{\Xi_1})$, but $\alpha_{\Xi_1} \notin L(\sigma^t_\Xi|t'_{\Xi_1})$, which is contradiction. So $L(\sigma^t_\Xi|t'_{\Xi_1})$ is not closed ideal of X. Hence $\mathcal{C}^t = (\hat{\kappa}^t_\Xi, \sigma^t_\Xi)$ is a t-neutrosophic cubic ideal of X because it satisfies N3 and N4.

6 Neutrosophic Cubic Ideals under Homomorphism

In this section, t-neutrosophic cubic ideals are investigated under homomorphism through some results.

Theorem 6.1 Suppose that $\Gamma|X \to Y$ is a homomorphism of BF-algebra. If $\mathcal{C}^t = (\hat{\kappa}^t_\Xi, \sigma^t_\Xi)$ is a t-NCID of Y. Then pre-image $\Gamma^{-1}(\mathcal{C}^t) = (\Gamma^{-1}(\hat{\kappa}^t_\Xi), \Gamma^{-1}(\sigma^t_\Xi))$ of $\mathcal{C}^t$ under Γ of X is a t-NCID of X.

Proof. For all $t_1 \in X$, $\Gamma^{-1}(\hat{\kappa}^t_\Xi)(t_1) = \hat{\kappa}^t_\Xi(\Gamma(t_1)) \leq \hat{\kappa}^t_\Xi(0) = \hat{\kappa}^t_\Xi(\Gamma(0)) = \Gamma^{-1}(\hat{\kappa}^t_\Xi)(0)$ and $\Gamma^{-1}(\sigma^t_\Xi)(t_1) = \sigma^t_\Xi(\Gamma(t_1)) \geq \sigma^t_\Xi(0) = \sigma^t_\Xi(\Gamma(0)) = \Gamma^{-1}(\sigma^t_\Xi)(0)$. Let $t_1, t_2 \in X, \Gamma^{-1}(\hat{\kappa}^t_\Xi)(t_1) = \hat{\kappa}^t_\Xi(\Gamma(t_1)) \geq \text{rmin}\{\hat{\kappa}^t_\Xi(\Gamma(t_1) * \Gamma(t_2)), \hat{\kappa}^t_\Xi(\Gamma(t_2))\} = \text{rmin}\{\hat{\kappa}^t_\Xi(\Gamma(t_1 * t_2)), \hat{\kappa}^t_\Xi(\Gamma(t_2))\} = \text{rmin}\{\Gamma^{-1}(\hat{\kappa}^t_\Xi)(t_1 * t_2), \Gamma^{-1}(\hat{\kappa}^t_\Xi)(t_2)\}$ and $\Gamma^{-1}(\sigma^t_\Xi)(a) = \sigma^t_\Xi(\Gamma(t_1)) \leq \max\{\sigma^t_\Xi(\Gamma(t_1) * \Gamma(t_2)), \sigma^t_\Xi(\Gamma(t_2))\} = \max\{\sigma^t_\Xi(\Gamma(t_1 * t_2)), \sigma^t_\Xi(\Gamma(t_2))\} = \max\{\Gamma^{-1}(\sigma^t_\Xi)(t_1 * t_2), \Gamma^{-1}(\sigma^t_\Xi)(t_2)\}$. Hence $\Gamma^{-1}(\mathcal{C}^t) = (\Gamma^{-1}(\hat{\kappa}^t_\Xi), \Gamma^{-1}(\sigma^t_\Xi))$ is a t-NCID of X.

Corollary 6.2 A homomorphic pre-image of a t-neutrosophic cubic closed ideal is a t-NCID.

Proof. Using Proposition 5.5 and Theorem 6.1, we can prove this corollary .

Corollary 6.3 A homomorphic preimage of a t-neutrosophic cubic closed ideal is also a t-NCSU.

Proof. Using Theorem 5.8 and Theorem 6.1, we can prove this corollary.

Corollary 6.4 Let $\Gamma|X \to Y$ be a homomorphism of BF-algebra. If $\mathcal{C}^t_i = ((\hat{\kappa}^t_i)_\Xi, (\sigma^t_i)_\Xi)$ is a t-NCID of Y where $i \in k$ then the pre image $\Gamma^{-1}(\bigcap_{i\in k_R} (\mathcal{C}^t_i)_\Xi) = (\Gamma^{-1}(\bigcap_{i\in k_R} (\hat{\kappa}^t_i)_\Xi)$, $\Gamma^{-1}(\bigcap_{i\in k_R} (\sigma^t_i)_\Xi))$ is a t-NCID of X.

Proof. Using Theorem 5.9 and Theorem 6.1, we can prove this corollary.

Corollary 6.5 Let $\Gamma|X \to Y$ be a homomorphism of BF-algebra. If $\mathcal{C}^t_i = ((\hat{\kappa}^t_i)_\Xi, (\sigma^t_i)_\Xi)$ is a t-neutrosophic cubic closed ideals of Y where $i \in k$ then the pre-image $\Gamma^{-1}(\bigcap_{i\in k_R} (\mathcal{C}^t_i)_\Xi) = (\Gamma^{-1}(\bigcap_{i\in k_R} (\hat{\kappa}^t_i)_\Xi), \Gamma^{-1}(\bigcap_{i\in k_R} (\sigma^t_i)_\Xi))$ is a t-neutrosophic cubic closed ideal of X.

Proof. Straightforward, using Theorem 5.10 and Theorem 6.1.

Theorem 6.6 Suppose that $\Gamma|X \to Y$ is an epimorphism of BF-algebra. Then $\mathcal{C}^t = (\hat{\kappa}^t_\Xi, \sigma^t_\Xi)$ is a t-NCID of Y, if $\Gamma^{-1}(\mathcal{C}^t) = (\Gamma^{-1}(\hat{\kappa}^t_\Xi), \Gamma^{-1}(\sigma^t_\Xi))$ of $\mathcal{C}^t$ under Γ of X is a t-NCID of X.

Proof. For any $t_2 \in Y, \exists\ t_1 \in X$ such that $t_2 = \Gamma(t_1)$. Then $\hat{\kappa}^t_\Xi(t_2) = \hat{\kappa}^t_\Xi(\Gamma(t_1)) = \Gamma^{-1}(\hat{\kappa}^t_\Xi)(t_1) \leq \Gamma^{-1}(\hat{\kappa}^t_\Xi)(0) = \hat{\kappa}^t_\Xi(\Gamma(0)) = \hat{\kappa}^t{}_\Xi(0)$ and $\sigma^t_\Xi(t_2) = \sigma^t_\Xi(\Gamma(t_1)) = \Gamma^{-1}(\sigma^t_\Xi)$

$(t_1) \geq \Gamma^{-1}(\sigma^t_\Xi)(0) = \sigma^t_\Xi(\Gamma(0)) = \sigma^t_\Xi(0)$.

Suppose $(t_2)_1, (t_2)_2 \in Y$. Then $\Gamma((t_1)_1) = (t_2)_1$ and $\Gamma((t_1)_2) = (t_2)_2$ for some $(t_1)_1, (t_1)_2 \in X$. Thus $\hat{\kappa}^t_\Xi((t_2)_1) = \hat{\kappa}^t_\Xi(\Gamma((t_1)_1)) = \Gamma^{-1}(\hat{\kappa}^t_\Xi)((t_1)_1) \geq \text{rmin}\{\Gamma^{-1}(\hat{\kappa}^t_\Xi)$

$((t_1)_1 * (t_1)_2),\ \Gamma^{-1}(\hat{\kappa}^t_\Xi)((t_1)_2)\} = \text{rmin}\{\hat{\kappa}^t_\Xi(\Gamma((t_1)_1 * (t_1)_2)), \hat{\kappa}^t_\Xi(\Gamma((t_1)_2))\} = \text{rmin}\{\hat{\kappa}^t_\Xi$

$(\Gamma((t_1)_1) * \Gamma((t_1)_2)), \hat{\kappa}^t_\Xi(\Gamma((t_1)_2))\} = \text{rmin}\{\hat{\kappa}^t_\Xi((t_2)_1 * (t_2)_2), \hat{\kappa}^t_\Xi((t_2)_2)\}$ and

$$\begin{aligned}\sigma^t_\Xi((t_2)_1) &= \sigma^t_\Xi(\Gamma((t_1)_1)) = \Gamma^{-1}(\sigma^t_\Xi)((t_1)_1) \leq \max\{\Gamma^{-1}(\sigma^t_\Xi)((t_1)_1 * (t_1)_2), \Gamma^{-1}(\sigma^t_\Xi)((t_1)_2)\}\\ &= \max\{\sigma^t_\Xi(\Gamma((t_1)_1 * (t_1)_2)), \sigma^t_\Xi(\Gamma((t_1)_2))\} = \max\{\sigma^t_\Xi(\Gamma((t_1)_1) * \Gamma((t_1)_2)), \sigma^t_\Xi(\Gamma((t_1)_2))\}\\ &= \max\{\sigma^t_\Xi((t_2)_1 * (t_2)_2), \sigma^t_\Xi((t_2)_2)\}.\end{aligned}$$

Hence $\mathcal{C}^t = (\hat{\kappa}^t{}_\Xi, \sigma^t_\Xi)$ is a t-NCID of Y.

7 Conclusion

In this paper, the concept of t-neutrosophic cubic set was defined and investigated it on BF-algebra through several useful results. For future work this study will provide base for t-neutrosophic soft cubic set, t-neutrosophic soft cubic (M-subalgebra, normal ideals) and different algebras like G-algebra and B-algebra.

Acknowledgments: The authors express their sincere thanks to the referees for valuable comments and suggestions which improve the paper a lot.

Conflicts of Interest

The authors declare no conflict of interest.

References

1. Ahn, S. S. Bang, K. On fuzzy subalgebras in B-algebra, Communications of the Korean Mathematical Society 18 (2003) 429-437.
2. Biswas, R. Rosenfeld's fuzzy subgroup with interval valued membership function, Fuzzy Sets and Systems, 63 (1994) 87-90.
3. Cho, J. R. Kim, H.S. On B-algebras and quasigroups, Quasigroups and Related System 8 (2001) 1-6.
4. Huang, Y. BCI-algebra, Science Press Beijing, 2006.
5. Imai, Y. Iseki, K. On Axiom systems of Propositional calculi XIV, Proc, Japan Academy, 42 (1966) 19-22.
6. Iseki, K. An algebra related with a propositional calculus, Proc. Japan Academy, 42 (1966) 26-29.
7. Jun, Y. B. Kim, C. S. Yang, K. O. Cubic sets, Annuals of Fuzzy Mathematics and Informatics, 4 (2012) 83-98.
8. Jun, Y. B. Jung, S. T. Kim, M. S. Cubic subgroup, Annals of Fuzzy Mathematics and Infirmatics, 2 (2011) 9-15.
9. Jun, Y. B. Smarandache, F. Kim, C. S. Neutrosophic Cubic Sets, New Math. and Natural Computation, (2015) 8-41.
10. Jun, Y. B. Kim, C. S. Kang, M. S. Cubic Subalgebras and ideals of BCK/BCI-algebra, Far East Journal of Mathematical Sciences 44 (2010) 239-250.
11. Jun, Y. B. Kim, C. S. Kang, J. G. Cubic q -Ideal of *BCI*-algebras, Annals of Fuzzy Mathematics and Informatics 1 (2011) 25-31.
12. Kim, C. B. Kim, H.S. On *BG*-algebra, Demonstration Mathematica 41 (2008) 497-505.
13. Neggers, J. Kim, H. S. On *B*-algebras, Mathematichki Vensnik, 54 (2002) 21-29.
14. Neggers, J. Kim, H. S. A fundamental theorem of *B*-homomorphism for *B*-algebras, International Mathematical Journal 2 (2002) 215-219.
15. Park, H. K. Kim, H. S On quadratic *B*-algebras, Qausigroups and Related System 7 (2001) 67-72.
16. Saeid, A. B. Interval-valued fuzzy *B*-algebras, Iranian Journal of Fuzzy System 3 (2006) 63-73.
17. Senapati, T. Bipolar fuzzy structure of *BG*-algebras, The Journal of Fuzzy Mathematics 23 (2015) 209-220.
18. Smarandache, F. Neutrosophic set a generalization of the intuitionistic fuzzy set, Int. J. Pure Appl. Math. 24 (3) (2005) 287-297.
19. Smarandache, F. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability, (American Reserch Press, Rehoboth, NM, 1999).
20. Khalid, M. Iqbal, R. Zafar , S. Khalid, H. Intuitionistic Fuzzy Translation and Multipl-ication of G-algebra, The Journal of Fuzzy Mathematics 27 (3) 17 (2019).
21. Senapati, T. Bhowmik, M. Pal, M. Fuzzy dot subalgebras and fuzzy dot ideals of B-algebra, Journal of Uncertain System 8 (2014) 22-30.
22. Senapati, T. Bhowmik, M. Pal, M. Fuzzy closed ideals of B-algebras, International Journal of Computer Science, Engineering and Technology 1 (2011) 669-673

23. Senapati, T. Bhowmik, M. Pal, M. Fuzzy closed ideals of B-algebras with interval-valued membership function, International Journal of Fuzzy Mathematical Archive 1 (2013) 79-91.
24. Senapati, T. Bhowmik, M. Pal, M. Fuzzy B-subalgebras of B-algebra with resepect to t-norm, Journal of Fuzzy Set Valued Analysis 2012 (2012) 11 pages, doi: 10.5899/2012/jfsva-00111.
25. Senapati, T. Jana, C. Bhowmik, M. Pal, M. L-fuzzy G-subalgebra of G-algebras, Journal of the Egyptian Mathematical Society (2014) http://dx.doi.org/10. 1016 /j.joems .2014.05.010.
26. Senapati, T. Kim, C. H. Bhowmik, M. Pal, M. Cubic subalgebras and cubic closed ideals of B-algebras, Fuzzy. Inform. Eng. 7 (2015) 129-149.
27. Senapati, T. Bhowmik, M. Pal, M. Intuitionistic L-fuzzy ideals of BG-algebras, Afrika Matematika 25 (2014) 577-590.
28. Senapati, T. Bhowmik, M. Pal, M. Interval-valued intuitionistic fuzzy BG-subalgebras, The Journal of Fuzzy Mathematics 20 (2012) 707-720.
29. Senapati, T. Bhowmik, M. Pal, M. Interval-valued intuitionistic fuzzy closed ideals BG-algebras and their products, International Journal of Fuzzy Logic Systems 2 (2012) 27-44.
30. T. Bhowmik, M. Pal, M. Intuitionistic fuzzifications of ideals in BG-algebra, Mathematica Aeterna 2 (2012) 761-778.
31. Senapati, T. Bhowmik, M. Pal, M. Fuzzy dot structure of BG-algrbras, Fuzzy Informa-tion and Engineering 6 (2014) 315-329.
32. Walendziak, A. Some axiomation of B-algebras, Mathematics Slovaca 56 (2006) 301 -306.
33. Zadeh, L. A. Fuzzy sets, Information and control 8 (1965) 338-353.
34. Zadeh, L. A. The concept of a linguistic variable and its application to approximate reasoning, Information science 8 (1975) 199-249.
35. Barbhuiya, S. R. t-intuitionistic Fuzzy Subalgebra of BG-Algebras, Advanced Trends in Mathematics 06-01, Vol. 3 (2015) pp16-24.
36. Sharma, P. K. t-intuitionistic Fuzzy Quotient Group, Advances in Fuzzy Mathematics, 7 (1) (2012) 1-9.
37. Takallo, M. M. Bordbar, H. Borzooei, R. A. Jun, Y. B. BMBJ-neutrosophic ideals in BCK/BCI-algebras, Neutrosophic Sets and Systems, vol. 27 (2019) pp. 1-16, DOI: 10.5281/zenodo.3275167.
38. Muhiuddin, G. Smarandache, F. Jun, Y. B. Neutrosophic Quadruple Ideals in Neutrosophic Quadruple BCI-algebras, Neutrosophic Sets and Systems, vol. 25 (2019) pp. 161-173, DOI: 10.5281/zenodo.2631518.
39. Park, C. H. Neutrosophic ideal of Subtraction Algebras, Neutrosophic Sets and Systems, vol. 24 (2019) pp. 36-45, DOI:10.5281/zenodo.2593913.
40. Borzooei, R. A. Takallo, M. M. Smarandache, F. Jun, Y. B. Positive implicative BMBJ -neutrosophic ideals in BCK-algebras, Neutrosophic Sets and Systems, vol. 23 (2018) pp. 126-141, DOI: 10.5281/zenodo.2158370.
41. Jun, Y. B. Smarandache, F. Ozturk, M. A. Commutative falling neutrosophic ideals in BCK-algebras, Neutrosophic Sets and Systems, vol. 20 (2018) pp. 44-53, http://doi.org/ 10.5281/zenodo.1235351.
42. Song, S. Z. Khan, M. Smarandache, F. Jun, Y. B. Interval neutrosophic sets applied to ideals in BCK/BCI-algebras, Neutrosophic Sets and Systems, vol. 18 (2017) pp. 16-26, http://doi.org/10.5281/zenodo.1175164.
43. Khalid, M. Iqbal, R. Broumi, S. Neutrosophic soft cubic Subalgebras of G-algebras. 28, (2019), 259-272. 10.5281/zenodo.3382552.
44. Muhiuddin, G. Jun, Y. B. Smarandache, F. Neutrosophic quadruple ideals in neutrosophic quadruple BCI-algebras, Neutrosophic Sets and Systems, Vol. 25, (2019).

45. G. Muhiuddin, H. Bordbar, F. Smarandache, Y.B. Jun, Further results on (ϵ, ϵ)-neutrosophic subalgebras and ideals in BCK/BCI-algebras, Neutrosophic Sets and Systems, Vol. 20, (2018).
46. Akinleye, S.A. Smarandache, F. Agboola, A.A.A. On neutrosophic quadruple algebraic structures, Neutrosophic Sets and Systems 12 (2016) 122–126.
47. Basset, M. A. Chang, V. Gamal, A., Smarandache, F. An integrated neutrosophic ANP and VIKOR method for achieving sustainable supplier selection: A case study in importing field, Computers in Industry 106, 94-110, 2019.
48. Basset, M. A. Saleh, M. Gamal, A. Smarandache, F. An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. Applied Soft Computing, 77 (2019) 438-452.

Received: Sep 30, 2019. Accepted: Jan 28, 2020