The Characteristic Function of a Neutrosophic Set

A. A. Salama¹, Florentin Smarandache² and S. A. Alblowi³

1 Department of Mathematics and Computer Science, Faculty of Sciences, Port Said University, 23 December Street, Port Said 42522, Egypt.
Email: drsalama44@gmail.com

2 Department of Mathematics, University of New Mexico 705 Gurley Ave. Gallup, NM 87301, USA.
Email: smarand@unm.edu

3 Department of Mathematics, King Abdulaziz University, Jeddah, Saudi Arabia
Email: salwaalblowi@hotmail.com

Abstract. The purpose of this paper is to introduce and study the characteristic function of a neutrosophic set. After given the fundamental definitions of neutrosophic set operations generated by the characteristic function of a neutrosophic set, we obtain several properties, and discussed the relationship between neutrosophic sets generated by $Ng$ and others. Finally, we introduce the neutrosophic topological spaces generated by $Ng$. Possible application to GIS topology rules are touched upon.

Keywords: Neutrosophic Set; Neutrosophic Topology; Characteristic Function.

1 Introduction

Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts, such as a neutrosophic set theory. After the introduction of the neutrosophic set concepts in [2-13]. In this paper we introduce definitions of neutrosophic sets by characteristic function. After given the fundamental definitions of neutrosophic set operations by $Ng$, we obtain several properties, and discussed the relationship between neutrosophic sets and others. Added to, we introduce the neutrosophic topological spaces generated by $Ng$.

2 Terminologies

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [7-9], Hanafy, Salama et al. [2-13] and Demirci in [1].

3 Neutrosophic Sets generated by $Ng$

We shall now consider some possible definitions for basic concepts of the neutrosophic sets generated by $Ng$ and its operations.

3.1 Definition

Let $X$ is a non-empty fixed set. A neutrosophic set ( NS for short) $A$ is an object having the form $A = \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ represent the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-membership (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set $A$ and let $g_A : X \times [0,1] \rightarrow [0,1] = I$ be reality function, then $N_{g_A}(\lambda) = N_{g_A}(\langle x, \lambda_1, \lambda_2, \lambda_3 \rangle)$ is said to be the characteristic function of a neutrosophic set on $X$ if $N_{g_A}(\lambda) = \begin{cases} 1 & \text{if } \mu_A(x) = \lambda_1, \sigma_A(x) = \lambda_2, \nu_A(x) = \lambda_3 \\ 0 & \text{otherwise} \end{cases}$

Where $\lambda = \langle x, \lambda_1, \lambda_2, \lambda_3 \rangle$. Then the object $G(A) = \langle x, \mu_{G(A)}(x), \sigma_{G(A)}(x), \nu_{G(A)}(x) \rangle$ is a neutrosophic set generated by $Ng$ where $\mu_{G(A)} = \sup \lambda_1 \{ N_{g_A}(\lambda) \wedge \lambda \}$ $\sigma_{G(A)} = \sup \lambda_2 \{ N_{g_A}(\lambda) \wedge \lambda \}$ $\nu_{G(A)} = \sup \lambda_3 \{ N_{g_A}(\lambda) \wedge \lambda \}$

3.1 Proposition

1) $A \subseteq Ng \iff G(A) \subseteq G(B)$.
2) \( A = N^c \) \( B \Leftrightarrow G(A) = G(B) \)

3.2 Definition

Let \( A \) be neutrosophic set of \( X \). Then the neutrosophic complement of \( A \) generated by \( N^c \) denoted by \( A^c \) may be defined as the following:

\[
\begin{align*}
(N^c)^1 & = \{ x, \mu^c_A(x), \nu^c_A(x), \nu^c_A(x) \} \\
(N^c)^2 & = \{ x, v_A(x), \sigma_A(x), \mu_A(x) \} \\
(N^c)^3 & = \{ x, v_A(x), \sigma^c_A(x), \mu^c_A(x) \}
\end{align*}
\]

3.1 Example. Let \( X = \{x\} \), \( A = \{x : 0.5, 0.7, 0.6\} \), \( N^c_A = 1 \), \( N^c_A = 0 \). Then \( G(A) = \{x : 0.5, 0.7, 0.6\} \)

Since our main purpose is to construct the tools for developing neutrosophic set and neutrosophic topology, we must introduce the \( g(0_n) \) and \( g(1_n) \) as follows: \( G(0_n) \) may be defined as:

\[
\begin{align*}
i) & G(0_n) = \{x : 0, 0, 1\} \\
ii) & G(0_n) = \{x : 0, 1, 1\} \\
iii) & G(0_n) = \{x : 0, 1, 0\} \\
v) & G(0_n) = \{x : 0, 0, 0\}
\end{align*}
\]

\( G(1_n) \) may be defined as:

\[
\begin{align*}
i) & G(1_n) = \{x : 1, 0, 0\} \\
ii) & G(1_n) = \{x : 1, 1, 0\} \\
iii) & G(1_n) = \{x : 1, 1, 1\} \\
v) & G(1_n) = \{x : 1, 1, 1\}
\end{align*}
\]

We will define the following operations intersection and union for neutrosophic sets generated by \( N^c \) denoted by \( \cap^c \) and \( \cup^c \) respectively.

3.3 Definition. Let two neutrosophic sets \( A = \{x, \mu_A(x), \sigma_A(x), v_A(x)\} \) and

\( B = \{x, \mu_B(x), \sigma_B(x), v_B(x)\} \) on \( X \), and

\( G(A) = \{x, \mu_G(A)(x), \sigma_G(A)(x), v_G(A)(x)\} \),

\( G(B) = \{x, \mu_G(B)(x), \sigma_G(B)(x), v_G(B)(x)\} \).

Then \( A \cap^c B \) may be defined as three types:

\[
\begin{align*}
i) & \text{Type I: } G(A \cap B) = \{x, \mu_{G(A)}(x) \wedge \mu_{G(B)}(x), \sigma_{G(A)}(x) \wedge \sigma_{G(B)}(x), v_{G(A)}(x) \lor v_{G(B)}(x)\} \\
ii) & \text{Type II: }
\end{align*}
\]

\( G(A \cup B) = \{x, \mu_{G(A)}(x) \vee \mu_{G(B)}(x), \sigma_{G(A)}(x) \vee \sigma_{G(B)}(x), v_{G(A)}(x) \wedge v_{G(B)}(x)\} \).

\[
\begin{align*}
\end{align*}
\]

\( ii) \text{ Type III: }
\]

\( G(A \cup B) = \{x, \mu_{G(A)}(x) \vee \mu_{G(B)}(x), \sigma_{G(A)}(x) \wedge \sigma_{G(B)}(x), v_{G(A)}(x) \lor v_{G(B)}(x)\} \)

\( A \cap N^c \) \( B \) may be defined as two types:

Type I:

\( G(A \cap B) = \{x, \mu_{G(A)}(x) \wedge \mu_{G(B)}(x), \sigma_{G(A)}(x) \wedge \sigma_{G(B)}(x), v_{G(A)}(x) \lor v_{G(B)}(x)\} \)

Type II:

\( G(A \cup B) = \{x, \mu_{G(A)}(x) \vee \mu_{G(B)}(x), \sigma_{G(A)}(x) \wedge \sigma_{G(B)}(x), v_{G(A)}(x) \lor v_{G(B)}(x)\} \)

\( iii) \text{ Type III: }
\]

\( G(A \cup B) = \{x, \mu_{G(A)}(x) \vee \mu_{G(B)}(x), \sigma_{G(A)}(x) \wedge \sigma_{G(B)}(x), v_{G(A)}(x) \lor v_{G(B)}(x)\} \)

3.4 Definition

Let a neutrosophic set \( A = \{x, \mu_A(x), \sigma_A(x), v_A(x)\} \) and

\( G(A) = \{x, \mu_{G(A)}(x), \sigma_{G(A)}(x), v_{G(A)}(x)\} \). Then

\( i) \) \( J^e A = \{x : \mu_{G(A)}(x), \sigma_{G(A)}(x), v_{G(A)}(x)\} \)

\( ii) \) \( N^e A = \{x : \mu_{G(A)}(x), \sigma_{G(A)}(x), v_{G(A)}(x)\} \)

3.2 Proposition

For all two neutrosophic sets \( A \) and \( B \) on \( X \) generated by \( N^c \), then the following are true

\( i) \) \( (A \cap B) = A \cap B \)

\( ii) \) \( (A \cup B) = A \cup B \)

We can easily generalize the operations of intersection and union in definition 3.2 to arbitrary family of neutrosophic subsets by generated by \( N^c \) as follows:

3.3 Proposition.

Let \( \{A_j : j \in J\} \) be arbitrary family of neutrosophic subsets in \( X \) generated by \( N^c \), then

\( a) \) \( \cap^c A_j \) may be defined as:

\( i) \) \( \text{Type I: } G(\cap A_j) = \{x : \mu_{G(A_j)}(x) \wedge \mu_{G(A_j)}(x), \sigma_{G(A_j)}(x) \wedge \sigma_{G(A_j)}(x), v_{G(A_j)}(x) \lor v_{G(A_j)}(x)\} \)

\( ii) \) \( \text{Type II: } G(\cup A_j) = \{x : \mu_{G(A_j)}(x) \lor \mu_{G(A_j)}(x), \sigma_{G(A_j)}(x) \lor \sigma_{G(A_j)}(x), v_{G(A_j)}(x) \wedge v_{G(A_j)}(x)\} \)

\( b) \) \( \cup^c A_j \) may be defined as:

\( i) \) \( \text{Type I: } G(\cup A_j) = \{x : \mu_{G(A_j)}(x) \lor \mu_{G(A_j)}(x), \sigma_{G(A_j)}(x) \lor \sigma_{G(A_j)}(x), v_{G(A_j)}(x) \wedge v_{G(A_j)}(x)\} \)

\( ii) \) \( \text{Type II: } G(\cup A_j) = \{x : \mu_{G(A_j)}(x) \lor \mu_{G(A_j)}(x), \sigma_{G(A_j)}(x) \lor \sigma_{G(A_j)}(x), v_{G(A_j)}(x) \wedge v_{G(A_j)}(x)\} \)
2) \( G(\cup A_j) = \left\{ \vee_{G(A_j)}(x), \wedge_{G(A_j)}(x), \wedge_{G(A_j)}(x) \right\} \).

3.4 Definition

Let \( f: X \rightarrow Y \) be a mapping.

(i) The image of a neutrosophic set \( A \) generated by \( Ng \) on \( X \) under \( f \) is a neutrosophic set \( B \) on \( Y \) generated by \( Ng \), denoted by \( f(A) \) whose reality function \( g_b: Y \times I \rightarrow I = [0, 1] \) satisfies the property \( \mu_{G(B)}(x) = \sup_{\lambda_1} \{ Ng_A(\lambda) \wedge \lambda \} \).

\[ \sigma_{G(B)} = \sup_{\lambda_2} \{ Ng_A(\lambda) \wedge \lambda \} \]

\[ \nu_{G(B)} = \sup_{\lambda_3} \{ Ng_A(\lambda) \wedge \lambda \} \]

(ii) The preimage of a neutrosophic set \( B \) on \( Y \) generated by \( Ng \) under \( f \) is a neutrosophic set \( A \) on \( X \) generated by \( Ng \), denoted by \( f^{-1}(B) \), whose reality function \( g_a: X \times I \rightarrow I = [0, 1] \) satisfies the property \( G(A) = G(B) \circ f \).

3.5 Proposition

Let \( f \) be mappings \( f: X \rightarrow Y \), respectively. Then for a mapping \( f: X \rightarrow Y \), the following properties hold:

(i) \( f(A_j) \subseteq Ng \) if \( j \in I \), then \( f(A_j) \subseteq Ng \) if \( A \subseteq Ng \).

(ii) \( f^{-1}(B_j) \subseteq Ng \) for \( j, K \in I \), then \( f^{-1}(B_j) \subseteq Ng \) for \( j \in I \).

(iii) \( f^{-1}( \bigcup_{j \in I} Ng B_j ) = \bigcap_{j \in I} f^{-1}(B_j) \).

3.6 Definition

Let \( (X, \Psi) \) be a ngts, \( A \) a neutrosophic set on \( X \) generated by \( Ng \). Then the neutrosophic interior of \( A \) generated by \( Ng \), denoted by \( ngintA \), is a set characterized by \( G(ngintA) = \bigcap G(A) \), where \( \bigcap G(\Psi) = G(\psi) \) denotes the interior operation in neutrosophic topological spaces generated by \( Ng \). Similarly, the neutrosophic closure of \( A \) generated by \( Ng \), denoted by \( gclA \), is a neutrosophic set characterized by \( G(gclA) = cL G(A) \), where \( cL \) denotes the closure operation in neutrosophic topological spaces generated by \( Ng \).

The neutrosophic interior \( gnint(A) \) and the genuine neutrosophic closure \( gnclA \) generated by \( Ng \) can be characterized by:

\[ gnintA = \bigcap Ng \{ U: U \in \Psi, U \subseteq Ng, A \} \]

\[ gnclA = \bigcap Ng \{ C: C \text{ is neutrosophic closed generated by } Ng, A \subseteq Ng, C \} \]

Since \( G(\text{gnint } A) = \bigcup \{ G(U): G(U) \in G(\Psi), G(U) \subseteq G(A) \} \),

\[ G(\text{gnc } A) = \bigcap \{ G(C): G(C) \in G(\psi), G(A) \subseteq G(C) \}. \]
3.6 Proposition. For any neutrosophic set \( A \) generated by \( N\) on a NTS \((X,\Psi)\), we have

(i) \(\text{cl} A^N = N\, (\text{int} A)^N\)

(ii) \(\text{Int} A^N = N\, (\text{cl} A)^N\)

References


Received: April 23rd, 2014. Accepted: May 4th, 2014.