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The Novel Attempt for Finding Minimum Solution in Fuzzy Neutrosophic Relational Geometric Programming (FNRGP) with (max,min) Composition

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Abstract. This article sheds light on the possibility of finding the minimum solution set of neutrosophic relational geometric programming with (max, min) composition. This work examines the privacy enjoyed by both neutrosophic logic and geometric programming, and how it affects the minimum solutions. It is the first attempt to solve this type of problems. Neutrosophic relation equations are important branches of neutrosophic mathematics. At present they have been widely applied in chemical plants, transportation problem, study of bonded labor problem [5] .

Keyword:- Geometric Programming, Neutrosophic Relational Equations, Fuzzy Integral Neutrosophic Matrices, Minimum Solution, Fuzzy Neutrosophic Relational Geometric Programming (FNRGP).

Introduction

The notion of neutrosophic relational equations which are abundant with the concept of indeterminacy, was first introduced by Florentin Smarandache [5]. We call

$$xoA = b \qquad (1)$$

a neutrosophic relational equations, where $A = (a_{ij})_{m\times n}$ is fuzzy integral neutrosophic matrix with entries from $[0,1] \cup I$, $b = (b_1, \dots, b_n), b_j \in [0,1] \cup I$ and $'o'$ is the $(max - min)$ composition operator. The pioneering contribution for the theory of geometric programming (GP) problems goes to Zener , Duffin and Peterson in 1961. A large number of applications for GP and fuzzy relation GP can be found in business administration, technological economic analysis, resource allocation, environmental engineering, engineering optimization designment and modernization of management, therefore it is significant to solve such a programming. B.Y. Cao proposed the fuzzy GP problems in 1987. He was the first to deal with fuzzy relation equations with GP at 2007. In a similar way to fuzzy relational equations, when the solution set of problem (1) is not empty, it's in general a non-convex set that can be completely determined by one maximum solution and a finite number of minimal solutions. H.E. Khalid presented in details and for the first time the structure of maximum solution for FNRGP at 2015. Recently there is not an effective method to confirm whether the solution set has a minimal solution, which makes the solving problem more difficult. In the consideration of the importance of the GP and the neutrosophic relation equation in theory and applications, we propose a fuzzy neutrosophic relation GP, discussed optimal solutions.

1. Fundamentals Concepts

Definition 1.1 [5]

Let $N = [0, 1] \cup I$ where I is the indeterminacy. The $m \times n$ matrices $A = \{(a_{ij}) \mid a_{ij} \in N\}$ are called fuzzy integral neutrosophic matrices. Clearly the class of $m \times n$ matrices is contained in the class of fuzzy integral neutrosophic matrices.

Definition 1.2

The optimization problem

$$\left.\begin{aligned} &\min f(x) = \left(c_1 \wedge x_1^{\gamma_1}\right) \vee, \ldots\ldots, \vee\left(c_m \wedge x_m^{\gamma_m}\right) \\ &\quad s.t. \\ &\quad xoA = b \\ &\quad (x_i \in N)(1 \le i \le m) \end{aligned}\right\} \quad (2)$$

is called (V, Λ) (max-min) fuzzy neutrosophic relational GP. Where $A = (a_{ij})$ $(a_{ij} \in N, 1 \le i \le m, 1 \le j \le n)$ is an (m x n)-dimensional fuzzy integral neutrosophic matrix, $x = (x_1, x_2, \ldots, x_m)$ an m-dimensional variable vector, $c = (c_1, c_2, \ldots, c_m)$ $(c_i \ge 0)$ and $b = (b_1, b_2, \ldots, b_n)$ $(b_j \in N)$ are (m & n) – dimensional constant vectors respectively, γ_i is an arbitrary real number.

Without loss of generality, the elements of b must be rearranged in decreasing order and the elements of the matrix A is correspondingly rearranged.

Definition 1.3 [3]:
The neutrosophic algebraic structures are algebraic structures based on sets of neutrosophic numbers of the form $Z = a + bI$, where a, b are real (or complex) numbers, and a is called the determinate part of Z and b is called the indeterminate part of Z, while $I = indeterminacy$, with $mI + nI = (m + n) I, 0 \cdot I = 0, I^n = I$ for integer $n \ge 1$, and $I / I = undefined$. When a, b are real numbers, then $a + bI$ is called a neutrosophic real number. While if a, b are complex numbers, then $a + bI$ is called a neutrosophic complex number.

Definition 1.4: [partial ordered relation of integral fuzzy neutrosophic numbers]

Depending upon the definition of integral neutrosophic lattice [5] , the author propose the following axioms:

a- decreasing partial order

1-The greatest element in $[0,1) \cup I$ is I,

$\max(I, x) = I \quad \forall\ x \in [0,1)$

2- The fuzzy values in a decreasing order will be rearranged as follows :

$1 > x_1 > x_2 > x_3 > \cdots > x_m \ge 0$

3- One is the greatest element in $[0,1] \cup I$,

$\max(I, 1) = 1$

b- Increasing partial order

1- the smallest element in $(0,1] \cup I$ is I ,

$\min(I, x) = I \quad \forall\ x \in (0,1]$

2- The fuzzy values in increasing order will be rearranged as follows :

$0 < x_1 < x_2 < x_3 < \cdots < x_m \le 1$

3- Zero is the smallest element in $[0,1] \cup I$,

$\min(I, 0) = 0$

Example :- To rearrange the following matrices:-
$b = [I \quad .5 \quad I \quad .85]^T$

$c = [1 \quad I \quad 0 \quad I \quad .4 \quad .1 \quad .85]^T$

in

1- decreasing order

$b^T = [I \ \ I \ \ 0.85 \ \ 0.5]$

$c^T = [1 \ \ I \ \ I \ \ 0.85 \ \ 0.4 \ \ 0.1 \ \ 0]$

2- increasing order

$b^T = [I \ \ I \ \ 0.5 \ \ 0.85\ ]$

$c^T = [0 \ \ I \ \ I \ \ 0.1 \ \ 0.4 \ \ 0.85 \ \ 1]$

Definition 1.5

If there exists a solution to Eq.(1) it's called compatible.

Suppose $X(A, b) = \{(x_1, x_2, \ldots, x_m)^T \in [0,1]^n \cup I,\ I^n = I \mid xoA = b, x_i \in N\}$ is a solution set of Eq.(1) we define $x^1 \le x^2 \Leftrightarrow x_i^1 \le x_i^2$ $(1 \le i \le m), \forall\, x^1, x^2 \in X(A, b)$. Where " $\le$ " is a partial order relation on $X(A, b)$.

Definition 1.6 [4]:

If $\exists \hat{x} \in X(A, b)$, such that $x \le \hat{x}, \forall x \in X(A, b)$, then $\hat{x}$ is called the greatest solution to Eq.(1) and

$$\hat{x}_i = \begin{cases} 1 & a_{ij} \leq b_j \text{ or } a_{ij} = b_j = I \\ b_j & a_{ij} > b_j \\ 0 & a_{ij} = I \text{ and } b_j = [0,1] \\ I & b_j = I \text{ and } a_{ij} = (0,1] \\ \text{not comp.} & a_{ij} = 0 \text{ and } b_j = I \end{cases} \quad (3)$$

Corollary 1.7 [2]:

If $X(A,b) \neq \emptyset$. then $\hat{x} \in X(A,b)$.

Similar to fuzzy relation equations , the above corollary works on fuzzy neutrosophic relation equations.

Notes 1.8:

1- Every fuzzy variable is always a neutrosophic variable, but all neutrosophic variables in general are not fuzzy variables. [5]

2- The set of all minimal solutions to Eq.(1) are denoted by $\breve{X}(A,b)$.

3- X (A,b) is non-convex, but it is composed of several n-dimensional rectangular regions with each rectangular region being a closed convex set [2].

2. The theory concept for exponents of variables in the geometric programming via fuzzy neutrosophic relation equations:-

B.Y Cao (2010) [1] had discussed optimization for fuzzy relation GP by considering the following three cases:

1- if $\gamma_i < 0$ $(1 \leq i \leq m)$, then the greatest solution $\hat{x}$ to Eq.(1) is an optimal solution for problem (2).

2- if $\gamma_i \geq 0$, then a minimal solution $\breve{x}$ to Eq.(1) is an optimal solution to (2).

3- the optimal solution to optimization problem (2) must exist in $\breve{X}(A,b)$. Let $f(\breve{x}^*) = \min\{f(\breve{x}) | \breve{x} \in \breve{X}(A,b)\}$, where $\breve{x}^* \in \breve{X}(A,b)$, then $\forall\, x \in X(A,b)$ $f(x) \geq f(\breve{x}^*)$. Therefore , $\breve{x}^*$ is an optimal solution to optimization problem (2).

Note that, in a more general case $\breve{x}^*$ may not be unique.

As for the general situation, the exponent γ_i of x_i is either a positive number or a negative one . B.Y.Cao proposed

$R_1 = \{i | \gamma_i < 0, 1 \leq i \leq m\}$,

$R_2 = \{i | \gamma_i \geq 0, 1 \leq i \leq m\}$.

Then $R_1 \cap R_2 = \emptyset$, $R_1 \cup R_2 = \imath$, where $\imath = \{1,2,\dots,m\}$.

Let $f_1(x) = \prod_{i \in R_1} x_i^{\gamma_i}$, $f_2(x) = \prod_{i \in R_2} x_i^{\gamma_i}$.

Then $f(x) = f_1(x) f_2(x)$. Therefore, if some exponent γ_i of x_i are positive numbers while others are negative, then x^* is an optimal solution to optimization problem (2) where

$$x_i^* = \begin{cases} \hat{x}_i, & i \in R_1 \\ \breve{x}_i^* & i \in R_2 \end{cases} \quad (4)$$

Really the above work can be coincided for our fuzzy neutrosophic relation in GP because the variables exponents (γ_i) are still real numbers in problem (2), note that there is trouble in case of $\gamma_i < 0$ and corollary 3.3 handled it.

3. An adaptive procedure to find the minimal solution for fuzzy neutrosophic geometric programming with $(\boldsymbol{max}, \boldsymbol{min})$ relation composition.

Definition 3.1 [5]:

Matrix $M = (m_{ij})_{m \times n}$ is called "matrix pattern" where $m_{ij} = (\hat{x}_i, a_{ij})$, this matrix is important element in the process of finding minimal solutions.

3.2 Algorithm:

Step 1- Rank the elements of b with decreasing order (definition 1.4) and find the maximum solution $\hat{x}$ (see Eq.(3)).

Step 2- If $\hat{x}$ is not a solution to Eq.(1), then go to step 15, otherwise go to step 3.

Step 3- Find the "matrix pattern" (definition 3.1).

Step 4- Mark m_{ij}, which satisfies $\min(\hat{x}_i, a_{ij}) = b_j$.

Step 5- Let the marked m_{ij} be denoted by $\widetilde{m}_{ij}$

Step 6- If $j_1$is the smallest j in all marked $\widetilde{m}_{ij}$, then set $\breve{x}_{i_1}{}^*$ to be the smaller one of the two elements in $\widetilde{m}_{i_1 j_1}$.

Step 7- Delete the $i_1 th$ row and the $j_1 th$ column of M and then delete all the columns that contain marked $\widetilde{m}_{i_1 j}$, where $j \neq j_1$.

Step 8- In all remained and marked $\widetilde{m}_{ij}$, find the smallest j and set it to be j_2 , then let $\breve{x}_{i_2}{}^*$ be the smaller of the two elements in $\widetilde{m}_{i_2 j_2}$.

Step 9- Delete the $i_2 th$ row and the $j_2 th$ column of M and then delete all the columns that contain marked $\widetilde{m}_{i_2 j}$, where $j \neq j_2$.

Step 10- Repeat step 7 and 8 until no marked $\widetilde{m}_{ij}$ is remained .

Step 11- The other $\breve{x}_i{}^*$, which are not set in 5-9 , are set to be zero.

Step 12- Let $\breve{x}^* = (\breve{x}_1{}^*, \breve{x}_2{}^*, \dots, \breve{x}_m{}^*)$ be the quasi minimum for problem (2).

Step 13- Check the sign of γ_i if $\gamma_i < 0$, then put $\hat{x}_i$ instead of $\breve{x}_i{}^*$ unless $\hat{x}_i = I$ (see Eq.(5))

Step 14- Print $x^* = \breve{x}^*$, $f(x^*)$ and stop.

Step 15- Print "have no solution" and stop.

Corollary 3.3:

If $\gamma_i < 0$ and the component $(\hat{x}_i = I) \in \hat{x}$, then the component $\breve{x}_i^* \in \breve{x}^*$ will be optimal for problem (2).

So the Eq.(4) must be improved to appropriate problem (2) as follow:-

$$x_i^* = \begin{cases} \hat{x}_i, & i \in R_1 \ and \ \hat{x}_i \neq I \\ \breve{x}_i^* & i \in R_2 \ or \ (i \in R_1 \ and \ \hat{x}_i = I) \end{cases} \quad (5)$$

□

4. Numerical Example:-

Consider the following fuzzy neutrosophic relation GP problem :-

$$\text{Min} f(x) = (1.5 \Lambda x_1^{.5}) V (I \Lambda x_2) V (.8 \Lambda x_3^{.5}) V(.9 \Lambda x_4^{-2}) V(.7 \Lambda x_5^{-4}) V(\text{I} \Lambda x_6^{-1})$$

s. t.

$xoA = b$ where

$$A = \begin{bmatrix} I & .2 & .8 & .1 \\ .8 & .2 & .8 & .1 \\ .9 & .1 & .4 & .1 \\ .3 & .95 & .1 & .1 \\ .85 & I & .1 & .1 \\ .4 & .8 & .1 & 0 \end{bmatrix}_{64}$$

$b = (.85, .6, .5, .1)$

It is clear that b is arranged in decreasing order.

The maximum solution is

$\hat{x} = (0, .5, .5, .6, 0, .6)$

$$M = \begin{bmatrix} (0, I) & (0, .2) & (0, .8) & (0, .1) \\ (.5, .8) & (.5, .2) & (.5, .8) & (.5, .1) \\ (.5, .9) & (.5, .1) & (.5, .4) & (.5, .1) \\ (.6, .3) & (.6, .95) & (.6, .1) & (.6, .1) \\ (0, .85) & (0, I) & (0, .1) & (0, .1) \\ (.6, .4) & (.6, .8) & (.6, .1) & (.6, 0) \end{bmatrix}_{64}$$

The elements satisfying $\min(\hat{x}_i, a_{ij}) = b_i$ are:

$m_{42}, m_{62}, m_{23}, m_{24}, m_{34}, m_{44}$

$\widetilde{m}_{42}, \widetilde{m}_{62}, \widetilde{m}_{23}, \widetilde{m}_{24}, \widetilde{m}_{34}, \widetilde{m}_{44}$

The element $\widetilde{m}_{42}$ is of least column number, therefore $\breve{x}_{i_1} = \min(.6, .95) = .6$

At the same time, the fourth row and the second column will be deleted.

As well as, the column included the element $\widetilde{m}_{44}$ must be deleted.

All remained elements of the matrix M are

$$\begin{bmatrix} (0, I) & (0, .8) \\ (.5, .8) & (.5, .8) \\ (.5, .9) & (.5, .4) \\ (0.85) & (0, .1) \\ (.6, .5) & (.6, .1) \end{bmatrix}$$

$\breve{x}_{i_2} = \min(.5, .8) = .5$

$\therefore \breve{x}_{i_2} = .5$

Set $\breve{x}_{i_3} = \breve{x}_{i_4} = \breve{x}_{i_5} = \breve{x}_{i_6} = 0$

So the quasi minimum is

$\breve{x}^* = (.6, .5,0,0,0,0)$

The exponents of x_4, x_5, x_6 in objective function $f(x)$ are negative, therefore

$$x^* = (\underbrace{.6, .5,0}_{from\ quasi}\ ,\ \underbrace{.6,0, .6}_{from\ maximum})$$

$$f(x^*) = (1.5\ \Lambda\ 0.6^{.5}) V(I\ \Lambda\ 0.5) V(.8\ \Lambda\ 0^{.5})$$

$$V(0.9\ \Lambda\ 0.6^{-2})\ V(.7\Lambda\ 0^{-4}) V(\mathrm{I}\ \Lambda\ 0.6^{-1})$$

$$f(x^*) = I.$$

5 Open problems:-

1- It will be a good project to search the optimal solution for fuzzy neutrosophic relation GP when the variables exponents (γ_i) in the objective function contain indeterminacy value.

2- More specifically if the variables exponents are negative and containing indeterminacy value.

3- Search for optimal solution in case of fuzzy neutrosophic relation GP if the composition $'o'$ be (max-product).

Conclusion

This essay, contains novel work to find optimal solution for an important branch of nonlinear programming named GP subject to a system of fuzzy neutrosophic relational equation with $(max - min)$ composition. In 1976, Sanchez gave the formula of the maximal solution for fuzzy relation equation concept and described in details its structure. H.E. Khalid introduced the structure of maximal solution for fuzzy neutrosophic relation GP problems at 2015. There is a debuted numerical example which shows that the proposed method is an effective to search for an optimum solution .

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