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Topology on Ultra Neutrosophic Set

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Abstract:

In this paper, we introduce the ultra neutrosophic set, and define some operations and establish a few properties of the ultra neutrosophic sets. Using the notion of topology on ultra neutrosophic sets, we introduce the concept of ultra neutrosophic topology. Further, we define the notion of ultra neutrosophic interior and ultra neutrosophic closure via ultra neutrosophic topological space.

Keywords: *Neutrosophic Set*; *Ultra Neutrosophic Crisp Set*; *Ultra Neutrosophic Set*.

1. Introduction:

The fundamental concept of Neutrosophic Set (NS) was introduced by Smarandache [1], which is the generalization of the Fuzzy Set (FS) [2] and the Intuitionistic Fuzzy Set (IFS) [3]. The concept of Neutrosophic Crisp Set (NCS) was grounded by Alblowi et al. [4] in 2014. Afterwards, Neutrosophic Crisp Topological Space (NCTS) was studied by Salama et al. [5]. In 2015, Salama et al. [6] further studied NCS theory. Later on, the Ultra Neutrosophic Crisp Set (UNCS) was presented by El Ghawably and Salama [7] in 2015.

Research Gap: The Ultra Neutrosophic Set (UNS) and the Ultra Neutrosophic Topology (UNT) on UNSs have not yet been introduced in the literature.

Motivation: To address the research gap, we introduce the Ultra Neutrosophic Set (UNS) and present a few basic properties of UNSs. Also, we present the Ultra Neutrosophic Topology (UNT) on UNSs.

The rest of the paper has been divided into the following sections. Section 2 presents the preliminaries and definitions on NS, NCS, UNCS. In section 3, we introduce the concept of ultra neutrosophic set and ultra neutrosophic topology. Besides, we formulate several results on them. Finally, in section 4, we conclude the paper by stating some future direction of research.

2. Preliminaries & Definitions:

In this section, we present some definitions on neutrosophic crisp set, neutrosophic set and ultra neutrosophic crisp set those are relevant for developing the main results of this article.

Definition 2.1. Suppose that Z be a non-empty fixed set. Then B_N, an NCS [4] is a triplet defined by $B_N=(B_1, B_2, B_3)$, where B_i (i = 1, 2, 3) be any subset of Z.

Definition 2.2. Assume that $B_N=(B_1, B_2, B_3)$ is an NCS. Then, the complement [4] of $B_N=(B_1, B_2, B_3)$ is defined by $B_N{}^c=(B_1{}^c, B_2{}^c, B_3{}^c)$.

Definition 2.3. Suppose that $B_N=(B_1, B_2, B_3)$ and $A_N=(A_1, A_2, A_3)$ are any two NCSs [4]. Then,

(i) $B_N \subseteq A_N$ iff $B_1 \subseteq A_1$, $B_2 \subseteq A_2$, $B_3 \supseteq A_3$;

(ii) $B_N \cup A_N = (B_1 \cup A_1, B_2 \cup A_2, B_3 \cap A_3)$;

(iii) $B_N \cap A_N = (B_1 \cap A_1, B_2 \cap A_2, B_3 \cup A_3)$.

Definition 2.4. Assume that Z is a fixed set. Then, an UNCS [7] $\widetilde{B_N}$ is defined as follows:

$\widetilde{B_N}=(B_1, B_2, B_3, M_B)$, where $M_B=(\cup_{i=1}^{3} B_i)^c$.

Definition 2.5. Suppose that $\widetilde{B_N}=(B_1, B_2, B_3, M_B)$ be an UNCS. Then, the complement [7] of $\widetilde{B_N}=(B_1, B_2, B_3, M_B)$ is defined by $\widetilde{B_N}^c=(B_1{}^c, B_2{}^c, B_3{}^c, M_B{}^c)$.

Definition 2.6. Suppose that $\widetilde{B_N}=(B_1, B_2, B_3, M_B)$ and $\widetilde{A_N}=(A_1, A_2, A_3, M_A)$ are two UNCSs. Then [7],

(i) $\widetilde{B_N} \subseteq \widetilde{A_N}$ iff $B_1 \subseteq A_1$, $B_2 \subseteq A_2$, $B_3 \supseteq A_3$, $M_B \supseteq M_A$;

(ii) $\widetilde{B_N} \cup \widetilde{A_N} = (B_1 \cup A_1, B_2 \cup A_2, B_3 \cap A_3, M_B \cap M_A)$;

(iii) $\widetilde{B_N} \cap \widetilde{A_N} = (B_1 \cap A_1, B_2 \cap A_2, B_3 \cup A_3, M_B \cup M_A)$.

Definition 2.7. Assume that Z is a fixed set. Then, an NS [1] U over Z is defined as follows:

$U = \{(\delta, T_U(\delta), I_U(\delta), F_U(\delta)) : \delta \in Z$, and $T_U(\delta), I_U(\delta), F_U(\delta) \in]^{-}0, 1^{+}[\}$, where $^{-}0 \leq T_U(\delta) + I_U(\delta) + F_U(\delta) \leq 3^{+}$.

Definition 2.8. Suppose that $U = \{(\delta, T_U(\delta), I_U(\delta), F_U(\delta)) : \delta \in Z$, and $T_U(\delta), I_U(\delta), F_U(\delta) \in [0, 1]\}$ be an NS over a fixed set Z. Then, complement [1] of U is $U^c = \{(\delta, 1-T_U(\delta), 1-I_U(\delta), 1-F_U(\delta)) : \delta \in Z\}$.

Definition 2.9. Suppose that $U = \{(\delta, T_U(\delta), I_U(\delta), F_U(\delta)) : \delta \in Z$, and $T_U(\delta), I_U(\delta), F_U(\delta) \in [0, 1]\}$ and $Y = \{(\delta, T_Y(\delta), I_Y(\delta), F_Y(\delta)) : \delta \in Z$, and $T_Y(\delta), I_Y(\delta), F_Y(\delta) \in [0, 1]\}$ be two NSs [1] over Z. Then,

i. $U \subseteq Y$ iff $T_U(\delta) \leq T_Y(\delta)$, $I_U(\delta) \geq I_Y(\delta)$, $F_U(\delta) \geq F_Y(\delta)$, for each $\delta \in Z$.

ii. $U \cup Y = \{(\delta, T_U(\delta) \vee T_Y(\delta), I_U(\delta) \wedge I_Y(\delta), F_U(\delta) \wedge F_Y(\delta)) : \delta \in Z\}$;

iii. $U \cap Y = \{(\delta, T_U(\delta) \wedge T_Y(\delta), I_U(\delta) \vee I_Y(\delta), F_U(\delta) \vee F_Y(\delta)) : \delta \in Z\}$.

Definition 2.11. The null NS (0_N) [1] and the whole NS (1_N) [1] over a fixed set Z are defined as follows:

(i) $0_N = \{(\delta, 0, 0, 1) : \delta \in Z\}$;

(ii) $1_N = \{(\delta, 1, 0, 0) : \delta \in Z\}$.

Clearly, $0_N \subseteq U \subseteq 1_N$, for any NS U over a fixed set Z.

3. Ultra Neutrosophic Set and Ultra Neutrosophic Topology:

In this section, we procure the notion of ultra neutrosophic set and ultra neutrosophic topology. Besides, we establish several results on them.

Definition 3.1. An ultra neutrosophic set R over a non-empty set Z is defined by $R=\{(\delta, T_R(\delta), I_R(\delta), F_R(\delta), M_R(\delta)) : \delta \in Z\}$, where $M_R(\delta) = T_R(\delta) \wedge I_R(\delta) \wedge F_R(\delta)$, $\forall \delta \in Z$ or $M_R(\delta) = T_R(\delta) \vee I_R(\delta) \vee F_R(\delta)$, $\forall \delta \in Z$.

Example 3.1. Consider a fixed set Z = {p, q, r}. Then, R = {(p, 0.3, 0.2, 0.5, 0.2), (q, 0.5, 0.7, 0.6, 0.5), (r, 0.6, 0.4, 0.2, 0.2)} is an UNS over Z.

Remark 3.1. The notion of ultra neutrosophic set is fully different from the notion of ultra neutrosophic crisp set. In an Ultra Neutrosophic Crisp Set $\widetilde{B_N}=(B_1, B_2, B_3, M_B)$, the crisp set M_B is defined by $M_B = (\cup_{i=1}^{3} B_i)^c$. But in the case of Ultra Neutrosophic Set $R=\{(\delta, T_R(\delta), I_R(\delta), F_R(\delta), M_R(\delta)) : \delta \in Z\}$ over Z, where $T_R(\delta)$, $I_R(\delta)$, and $F_R(\delta)$ denote respectively the truth, indeterminacy and false membership values of each $\delta \in Z$, another membership function $M_R : Z \to [0, 1]$ is defined by $M_R(\delta) = T_R(\delta) \wedge I_R(\delta) \wedge F_R(\delta)$, $\forall \delta \in Z$ or $M_R(\delta) = T_R(\delta) \vee I_R(\delta) \vee F_R(\delta)$, $\forall \delta \in Z$, which is totally different from the point of view of ultra neutrosophic crisp set.

Definition 3.2. The complement of an UNS $R = \{(\delta, T_R(\delta), I_R(\delta), F_R(\delta), M_R(\delta)) : \delta \in Z\}$ is defined by $R^c = \{(\delta, 1-T_R(\delta), 1-I_R(\delta), 1-F_R(\delta), 1-M_R(\delta)) : \delta \in Z\}$.

Definition 3.3. Consider any two UNSs $R = \{(\delta, T_R(\delta), I_R(\delta), F_R(\delta), M_R(\delta)) : \delta \in Z\}$ and $S = \{(\delta, T_S(\delta), I_S(\delta), F_S(\delta), M_S(\delta)) : \delta \in Z\}$ over a fixed set Z. Then,

i. $R \subseteq S \Leftrightarrow T_R(\delta) \leq T_S(\delta), I_R(\delta) \geq I_S(\delta), F_R(\delta) \geq F_S(\delta), M_R(\delta) \leq M_S(\delta)$, for each $\delta \in Z$;

ii. R=S if and if $R \subseteq S$ and $S \subseteq R$;

iii. $R \cup S=\{(\delta, T_R(\delta) \vee T_S(\delta), I_R(\delta) \wedge I_S(\delta), F_R(\delta) \wedge F_S(\delta), M_R(\delta) \vee M_S(\delta)) : \delta \in Z\}$;

iv. $R \cap S=\{(\delta, T_R(\delta) \wedge T_S(\delta), I_R(\delta) \vee I_S(\delta), F_R(\delta) \vee F_S(\delta), M_R(\delta) \wedge M_S(\delta)) : \delta \in Z\}$.

Definition 3.4. The null UNS $\widehat{0_N}$ and whole UNS $\widehat{1_N}$ over a fixed set Z is defined as follows:

i. $\widehat{0_N}=\{(\delta, 0, 1, 1, 0) : \delta \in Z\}$;

ii. $\widehat{1_N}=\{(\delta, 1, 0, 0, 1) : \delta \in Z\}$.

Clearly, $\widehat{0_N} \subseteq U \subseteq \widehat{1_N}$, for any UNS U over Z.

Theorem 3.1. Let N, R and C be three UNSs over a fixed set Z. Then, the following results hold:

i. $N \cup [R \cup C] = [N \cup R] \cup C$;

ii. $N \cap [R \cap C] = [N \cap R] \cap C$;

iii. $N \cup R = R \cup N$ and $N \cap R = R \cap N$;

iv. $N \cup N = N$ and $N \cap N = N$;

v. $N \cup [R \cap C] = [N \cup R] \cap [N \cup C]$;

vi. $N \cap [R \cup M] = [N \cap R] \cup [N \cap C]$;

vii. $[N^c]^c = N$.

Proof. Suppose that $N=\{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\}$, $R=\{(\delta, T_R(\delta), I_R(\delta), F_R(\delta), M_R(\delta)) : \delta\in Z\}$ and $C=\{(\delta, T_C(\delta), I_C(\delta), F_C(\delta), M_C(\delta)) : \delta\in Z\}$ be three UNSs over a fixed set Z.

i. We have, $N\cup[R\cup C]$

$= \{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\} \cup [\{(\delta, T_R(\delta), I_R(\delta), F_R(\delta), M_R(\delta)) : \delta\in Z\} \cup \{(\delta, T_C(\delta), I_C(\delta), F_C(\delta), M_C(\delta)) : \delta\in Z\}]$

$= \{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\} \cup \{(\delta, T_R(\delta)\vee T_C(\delta), I_R(\delta)\wedge I_C(\delta), F_R(\delta)\wedge F_C(\delta), M_R(\delta)\vee M_C(\delta)) : \delta\in Z\}$

$= \{(\delta, T_N(\delta)\vee(T_R(\delta)\vee T_C(\delta)), I_N(\delta)\wedge(I_R(\delta)\wedge I_C(\delta)), F_N(\delta)\wedge(F_R(\delta)\wedge F_C(\delta)), M_N(\delta)\vee(M_R(\delta)\vee M_C(\delta))) : \delta\in Z\}$

$= \{(\delta, (T_N(\delta)\vee T_R(\delta))\vee T_C(\delta), (I_N(\delta)\wedge I_R(\delta))\wedge I_C(\delta), (F_N(\delta)\wedge F_R(\delta))\wedge F_C(\delta), (M_N(\delta)\vee M_R(\delta))\vee M_C(\delta)) : \delta\in Z\}$

$= \{(\delta, T_N(\delta)\vee T_R(\delta), I_N(\delta)\wedge I_R(\delta), F_N(\delta)\wedge F_R(\delta), M_N(\delta)\vee M_R(\delta)) : \delta\in Z\} \cup \{(\delta, T_C(\delta), I_C(\delta), F_C(\delta), M_C(\delta)) : \delta\in Z\}$

$=[\{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\} \cup \{(\delta, T_R(\delta), I_R(\delta), F_R(\delta), M_R(\delta)) : \delta\in Z\}] \cup \{(\delta, T_C(\delta), I_C(\delta), F_C(\delta), M_C(\delta)) : \delta\in Z\}$

$= [N\cup R]\cup C$

Therefore, $N\cup[R\cup C] = [N\cup R]\cup C$.

ii. We have, $N\cap[R\cap C]$

$= \{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\} \cap [\{(\delta, T_R(\delta), I_R(\delta), F_R(\delta), M_R(\delta)) : \delta\in Z\} \cap \{(\delta, T_C(\delta), I_C(\delta), F_C(\delta), M_C(\delta)) : \delta\in Z\}]$

$= \{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\} \cap \{(\delta, T_R(\delta)\wedge T_C(\delta), I_R(\delta)\vee I_C(\delta), F_R(\delta)\vee F_C(\delta), M_R(\delta)\wedge M_C(\delta)) : \delta\in Z\}$

$= \{(\delta, T_N(\delta)\wedge(T_R(\delta)\wedge T_C(\delta)), I_N(\delta)\vee(I_R(\delta)\vee I_C(\delta)), F_N(\delta)\vee(F_R(\delta)\vee F_C(\delta)), M_N(\delta)\wedge(M_R(\delta)\wedge M_C(\delta))) : \delta\in Z\}$

$= \{(\delta, (T_N(\delta)\wedge T_R(\delta))\wedge T_C(\delta), (I_N(\delta)\vee I_R(\delta))\vee I_C(\delta), (F_N(\delta)\vee F_R(\delta))\vee F_C(\delta), (M_N(\delta)\wedge M_R(\delta))\wedge M_C(\delta)) : \delta\in Z\}$

$= \{(\delta, T_N(\delta)\wedge T_R(\delta), I_N(\delta)\vee I_R(\delta), F_N(\delta)\vee F_R(\delta), M_N(\delta)\wedge M_R(\delta)) : \delta\in Z\} \cap \{(\delta, T_C(\delta), I_C(\delta), F_C(\delta), M_C(\delta)) : \delta\in Z\}$

$=[\{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\} \cap \{(\delta, T_R(\delta), I_R(\delta), F_R(\delta), M_R(\delta)) : \delta\in Z\}] \cap \{(\delta, T_C(\delta), I_C(\delta), F_C(\delta), M_C(\delta)) : \delta\in Z\}$

$= [N\cap R]\cap C$

Therefore, $N\cap[R\cap C] = [N\cap R]\cap C$.

iii. We have, $N\cup R$

$= \{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\} \cup \{(\delta, T_R(\delta), I_R(\delta), F_R(\delta), M_R(\delta)) : \delta\in Z\}$

$= \{(\delta, T_N(\delta)\vee T_R(\delta), I_N(\delta)\wedge I_R(\delta), F_N(\delta)\wedge F_R(\delta), M_N(\delta)\vee M_R(\delta)) : \delta\in Z\}$

$= \{(\delta, T_R(\delta)\vee T_N(\delta), I_R(\delta)\wedge I_N(\delta), F_R(\delta)\wedge F_N(\delta), M_R(\delta)\vee M_N(\delta)) : \delta\in Z\}$

$= \{(\delta, T_R(\delta), I_R(\delta), F_R(\delta), M_R(\delta)) : \delta\in Z\} \cup \{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\}$

$= R\cup N$

Therefore, $N\cup R = R\cup N$.

Further, we have $N\cap R$

$= \{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\} \cap \{(\delta, T_R(\delta), I_R(\delta), F_R(\delta), M_R(\delta)) : \delta\in Z\}$

$= \{(\delta, T_N(\delta)\wedge T_R(\delta), I_N(\delta)\vee I_R(\delta), F_N(\delta)\vee F_R(\delta), M_N(\delta)\wedge M_R(\delta)) : \delta\in Z\}$

$= \{(\delta, T_R(\delta)\wedge T_N(\delta), I_R(\delta)\vee I_N(\delta), F_R(\delta)\vee F_N(\delta), M_R(\delta)\wedge M_N(\delta)) : \delta\in Z\}$

$= \{(\delta, T_R(\delta), I_R(\delta), F_R(\delta), M_R(\delta)) : \delta\in Z\} \cap \{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\}$

$= R\cap N$

Therefore, $N\cap R = R\cap N$.

iv. We have, $N\cup N$

$= \{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\} \cup \{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\}$

$= \{(\delta, T_N(\delta)\vee T_N(\delta), I_N(\delta)\wedge I_N(\delta), F_N(\delta)\wedge F_N(\delta), M_N(\delta)\vee M_N(\delta)) : \delta\in Z\}$

$= \{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\}$

$= N$

Therefore, $N\cup N = N$.

Further, we have $N\cap N$

$= \{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\} \cap \{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\}$

$= \{(\delta, T_N(\delta)\wedge T_N(\delta), I_N(\delta)\vee I_N(\delta), F_N(\delta)\vee F_N(\delta), M_N(\delta)\wedge M_N(\delta)) : \delta\in Z\}$

$= \{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\}$

$= N$

Therefore, $N\cap N = N$.

v. We have $N\cup[R\cap C]$

$= \{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\} \cup [\{(\delta, T_R(\delta), I_R(\delta), F_R(\delta), M_R(\delta)) : \delta\in Z\} \cap \{(\delta, T_C(\delta), I_C(\delta), F_C(\delta), M_C(\delta)) : \delta\in Z\}]$

$= \{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\} \cup [\{(\delta, T_R(\delta)\wedge T_C(\delta), I_R(\delta)\vee I_C(\delta), F_R(\delta)\vee F_C(\delta), M_R(\delta)\wedge M_C(\delta)) : \delta\in Z\}]$

$= [\{(\delta, T_N(\delta)\vee(T_R(\delta)\wedge T_C(\delta)), I_N(\delta)\wedge(I_R(\delta)\vee I_C(\delta)), F_N(\delta)\wedge(F_R(\delta)\vee F_C(\delta)), M_N(\delta)\vee(M_R(\delta)\wedge M_C(\delta))) : \delta\in Z\}]$

Now, we have $[N\cup R]\cap[N\cup C]$

$= [\{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\} \cup \{(\delta, T_R(\delta), I_R(\delta), F_R(\delta), M_R(\delta)) : \delta\in Z\}] \cap [\{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\} \cup \{(\delta, T_C(\delta), I_C(\delta), F_C(\delta), M_C(\delta)) : \delta\in Z\}]$

$= \{(\delta, T_N(\delta)\vee T_R(\delta), I_N(\delta)\wedge I_R(\delta), F_N(\delta)\wedge F_R(\delta), M_N(\delta)\vee M_R(\delta)) : \delta\in Z\} \cap \{(\delta, T_N(\delta)\vee T_C(\delta), I_N(\delta)\wedge I_C(\delta), F_N(\delta)\wedge F_C(\delta), M_N(\delta)\vee M_C(\delta)) : \delta\in Z\}$

$= \{(\delta, (T_N(\delta)\vee T_R(\delta)) \wedge (T_N(\delta)\vee T_C(\delta)), (I_N(\delta)\wedge I_R(\delta)) \vee (I_N(\delta)\wedge I_C(\delta)), (F_N(\delta)\wedge F_R(\delta)) \vee (F_N(\delta)\wedge F_C(\delta)), (M_N(\delta)\vee M_R(\delta)) \wedge (M_N(\delta)\vee M_C(\delta))) : \delta\in Z\}$

$= \{(\delta, T_N(\delta)\vee(T_R(\delta)\wedge T_C(\delta)), I_N(\delta)\wedge(I_R(\delta)\vee I_C(\delta)), F_N(\delta)\wedge(F_R(\delta)\vee F_C(\delta)), M_N(\delta)\vee(M_R(\delta)\wedge M_C(\delta))) : \delta\in Z\}$

$= N\cup[R\cap C]$

Therefore, $N\cup[R\cap C] = [N\cup R]\cap[N\cup C]$.

vi. We have $N\cap[R\cup C]$

$= \{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\} \cap [\{(\delta, T_R(\delta), I_R(\delta), F_R(\delta), M_R(\delta)) : \delta\in Z\} \cup \{(\delta, T_C(\delta), I_C(\delta), F_C(\delta), M_C(\delta)) : \delta\in Z\}]$

$= \{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\} \cap [\{(\delta, T_R(\delta)\vee T_C(\delta), I_R(\delta)\wedge I_C(\delta), F_R(\delta)\wedge F_C(\delta), M_R(\delta)\vee M_C(\delta)) : \delta\in Z\}]$

$= [\{(\delta, T_N(\delta)\wedge(T_R(\delta)\vee T_C(\delta)), I_N(\delta)\vee(I_R(\delta)\wedge I_C(\delta)), F_N(\delta)\vee(F_R(\delta)\wedge F_C(\delta)), M_N(\delta)\wedge(M_R(\delta)\vee M_C(\delta))) : \delta\in Z\}]$

Now, we have $[N\cap R]\cup[N\cap C]$

$= [\{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\} \cap \{(\delta, T_R(\delta), I_R(\delta), F_R(\delta), M_R(\delta)) : \delta\in Z\}] \cup [\{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\} \cap \{(\delta, T_C(\delta), I_C(\delta), F_C(\delta), M_C(\delta)) : \delta\in Z\}]$

$= \{(\delta, T_N(\delta)\wedge T_R(\delta), I_N(\delta)\vee I_R(\delta), F_N(\delta)\vee F_R(\delta), M_N(\delta)\wedge M_R(\delta)) : \delta\in Z\} \cup \{(\delta, T_N(\delta)\wedge T_C(\delta), I_N(\delta)\vee I_C(\delta), F_N(\delta)\vee F_C(\delta), M_N(\delta)\wedge M_C(\delta)) : \delta\in Z\}$

$= \{(\delta, (T_N(\delta)\wedge T_R(\delta))\vee (T_N(\delta)\wedge T_C(\delta)), (I_N(\delta)\vee I_R(\delta))\wedge (I_N(\delta)\vee I_C(\delta)), (F_N(\delta)\vee F_R(\delta))\wedge (F_N(\delta)\vee F_C(\delta)), (M_N(\delta)\wedge M_R(\delta))\vee (M_N(\delta)\wedge M_C(\delta))) : \delta\in Z\}$

$= \{(\delta, T_N(\delta)\wedge(T_R(\delta)\vee T_C(\delta)), I_N(\delta)\vee(I_R(\delta)\wedge I_C(\delta)), F_N(\delta)\vee(F_R(\delta)\wedge F_C(\delta)), M_N(\delta)\wedge(M_R(\delta)\vee M_C(\delta))) : \delta\in Z\}$

$= N\cap[R\cup C]$

Therefore, $N\cap[R\cup C] = [N\cap R]\cup[N\cap C]$.

vii. We have, $N^c = \{(\delta, 1-T_N(\delta), 1-I_N(\delta), 1-F_N(\delta), 1-M_N(\delta)) : \delta\in Z\}$.

Therefore, $(N^c)^c$

$= \{(\delta, 1-(1-T_N(\delta)), 1-(1-I_N(\delta)), 1-(1-F_N(\delta)), 1-(1-M_N(\delta))) : \delta\in Z\}$

$= \{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta\in Z\}$

$= N$

Hence, $(N^c)^c = N$.

Definition 3.5. Consider a universe of discourse Z and τ be any collection of UNSs over Z. Then, τ is called an Ultra Neutrosophic Topology (Ultra-N-T) on Z if it satisfies the following three axioms:

i. $\widehat{0_N}, \widehat{1_N} \in\tau$;

ii. $C_1, C_2 \in\tau \Rightarrow C_1\cap C_2 \in\tau$;

iii. $\{C_i : i \in \Delta\} \subseteq \tau \Rightarrow \cup C_i \in\tau$.

In that case, (Z, τ) is called an Ultra Neutrosophic Topological Space (Ultra-N-T-S). If K is an element of τ, then K is said to be an Ultra Neutrosophic Open Set (Ultra-N-O-S) and the complement of K is said to be an Ultra Neutrosophic Closed Set (Ultra-N-C-S).

Example 3.2. Assume that Z={c, d} and L={(c, 0.5, 0.3, 0.7, 0.7), (d, 0.4, 0.5, 0.3, 0.5)}, M={(c, 0.4, 0.5, 0.7, 0.4), (d, 0.2, 0.6, 0.5, 0.2)}, N={(c, 0.8, 0.2, 0.5, 0.8), (d, 0.5, 0.4, 0.3, 0.5)} are three UNSs over Z. Then, the collection τ={$\widehat{0_N}$, $\widehat{1_N}$, L, M, N} is an Ultra-N-T on Z.

Definition 3.6. Assume that (Z, τ) is an Ultra-N-T-S and K be an UNS over Z. Then, Ultra Neutrosophic Interior (Ultra-N_{int}) and Ultra Neutrosophic Closure (Ultra-N_{cl}) of U are defined by

Ultra-N_{int} (K) = ∪{L : L is an Ultra-N-O-S in Z and L⊆K},

Ultra-N_{cl} (K) = ∩{H : H is an Ultra-N-C-S in Z and K⊆H}.

It is clear that, Ultra-N_{int}(K) is the largest Ultra-N-O-S over Z which is contained in K and Ultra-N_{cl}(K) is the smallest Ultra-N-C-S over Z which contains K.

Theorem 3.2. Let (Z, τ) be an Ultra-N-T-S. Assume that A and B are any two UNSs over Z. Then, the following properties hold:

i. Ultra-N_{int}(A) ⊆ A ⊆ Ultra-N_{cl}(A);
ii. A ⊆ B ⟹ Ultra-N_{int}(A) ⊆ Ultra-N_{int}(B);
iii. A ⊆ B ⟹ Ultra-N_{cl}(A) ⊆ Ultra-N_{cl}(B);
iv. Ultra-N_{cl}(0_N) = 0_N & Ultra-N_{cl}(1_N) = 1_N;
v. Ultra-N_{int}(0_N) = 0_N & Ultra-N_{int}(1_N) = 1_N;
vi. Ultra-N_{cl}(A ∪ B) = Ultra-N_{cl}(A) ∪ Ultra-N_{cl}(B);
vii. Ultra-N_{int}(A) ∪ Ultra-N_{int}(B) ⊂ Ultra-N_{int}(A∪B);
viii. Ultra-N_{int}(A ∩ B) = Ultra-N_{int}(A) ∩ Ultra-N_{int}(B);
ix. Ultra-N_{cl}(A ∩ B) ⊂ Ultra-N_{cl}(A) ∩ Ultra-N_{cl}(B).

Proof.

i. By Definition 3.6, we have Ultra-N_{int}(A) = ∪{R : R is an Ultra-N-O-S in (Z, τ) and R⊆A}. Since, each R⊆A, so ∪{R : R is an Ultra-N-O-S in (Z, τ) and R⊆A} ⊆ A, i.e., Ultra-N_{int}(A)⊆A.

   Again, Ultra-N_{cl}(A) = ∩{W : W is an Ultra-N-C-S in (Z, τ) and A⊆W}. Since, each W⊇A, so ∩{W : W is an Ultra-N-C-S in (Z, τ) and A⊆W} ⊇ A, i.e., Ultra-N_{cl}(A)⊇A**.** Therefore, Ultra-N_{int}(A)⊆A⊆Ultra-N_{cl}(A).

   Hence, Ultra-N_{int}(A) ⊆ A ⊆ Ultra-N_{cl}(A), for any ultra neutrosophic set A over Z.

ii. Suppose that (Z, τ) is an Ultra-N-T-S. Let A and B be any two UNSs over Z such that A⊆B. Now, we have

   Ultra-N_{int}(A) = ∪{W : W is an Ultra-N-O-S in (Z, τ) and W⊆A}

   ⊆ ∪{W : W is an Ultra-N-O-S in (Z, τ) and W⊆B} [Since A⊆B]

   = Ultra-N_{int}(B)

   ⇒ Ultra-N_{int}(A) ⊆ Ultra-N_{int}(B).

Therefore, $A \subseteq B \Rightarrow$ Ultra-$N_{int}(A) \subseteq$ Ultra-$N_{int}(B)$.

iii. Assume that (Z, τ) is an Ultra-N-T-S. Let A and B be any two UNSs over Z such that $A \subseteq B$. Now, we have

Ultra-$N_{cl}(A) = \cap\{W : W$ is an Ultra-N-C-S in (Z, τ) and $A \subseteq W\}$

$\subseteq \cap\{W : W$ is an Ultra-N-C-S in (Z, τ) and $B \subseteq W\}$ [Since $A \subseteq B$]

= Ultra-$N_{cl}(B)$

$\Rightarrow$ Ultra-$N_{cl}(A) \subseteq$ Ultra-$N_{cl}(B)$.

Therefore, $A \subseteq B \Rightarrow$ Ultra-$N_{cl}(A) \subseteq$ Ultra-$N_{cl}(B)$.

iv. It is known that, Ultra-$N_{cl}(A) = \cap \{W : W$ is an Ultra-N-C-S in (Z, τ) and $A \subseteq W\}$.

We have, Ultra-$N_{cl}(0_N)$

$= \cap \{W : W$ is an Ultra-N-C-S in (Z, τ) and $0_N \subseteq W\}$

$= 0_N \cap \{W : W$ is an Ultra-N-C-S in (Z, τ) and $0_N \subset W\}$

$= 0_N \cap M$, where $M = \cap\{W$: W is an Ultra-N-C-S in (Z, τ) and $0_N \subset W\}$ is a neutrosophic sub-set of (Z, τ).

$= 0_N$

Therefore, Ultra-$N_{cl}(0_N) = 0_N$.

Further, we have

Ultra-$N_{cl}(1_N)$

$= \cap \{W : W$ is an Ultra-N-C-S in (Z, τ) and $1_N \subseteq W\}$

$= 1_N$ [since there exists no Ultra-N-C-S W in (Z, τ) such that $1_N \subset W$]

Therefore, Ultra-$N_{cl}(1_N) = 1_N$.

v. It is known that, Ultra-$N_{int}(A) = \cup \{W : W$ is an Ultra-N-O-S in (Z, τ) and $W \subseteq A\}$.

We have,

Ultra-$N_{int}(0_N)$

$= \cup \{W : W$ is an Ultra-N-O-S in (Z, τ) and $W \subseteq 0_N\}$

$= 0_N$ [since there exists no Ultra-N-O-S W in (Z, τ) such that $W \subset 0_N$]

Therefore, Ultra-$N_{int}(0_N) = 0_N$.

Further, we have Ultra-$N_{int}(1_N)$

$= \cup \{W : W$ is an Ultra-N-O-S in (Z, τ) and $W \subseteq 1_N\}$

$= \cup \{W : W$ is an Ultra-N-O-S in (Z, τ) and $W \subseteq 1_N\} \cup 1_N$

$= M \cup 1_N$, where $M = \cup\{W : W$ is an Ultra-N-O-S in (Z, τ) and $W \subseteq 1_N\}$ is a neutrosophic subset of (Z, τ).

$= 1_N$

Therefore, Ultra-$N_{int}(1_N) = 1_N$.

vi. Let A and B be any two ultra neutrosophic sub-sets of an Ultra-N-T-S (Z, τ). It is known that, $A \subseteq A \cup B$ and $B \subseteq A \cup B$.

Now, $A \subseteq A \cup B$

$\Rightarrow$ Ultra-$N_{cl}(A) \subseteq$ Ultra-$N_{cl}(A \cup B)$;

and $B \subseteq A \cup B$

$\Rightarrow$ Ultra-$N_{cl}(B) \subseteq$ Ultra-$N_{cl}(A \cup B)$.

Therefore, Ultra-$N_{cl}(A) \cup$Ultra-$N_{cl}(B) \subseteq$ Ultra-$N_{cl}(A \cup B)$ (1)

We have, $A \subseteq$Ultra-$N_{cl}(A)$, $B \subseteq$Ultra-$N_{cl}(B)$. Therefore, $A \cup B \subseteq$ Ultra-$N_{cl}(A) \cup$Ultra-$N_{cl}(B)$. Further, it is known that Ultra-$N_{cl}(A) \cup$Ultra-$N_{cl}(B)$ is an Ultra-N-C-S in (Z, τ). It is clear that, Ultra-$N_{cl}(A) \cup$Ultra-$N_{cl}(B)$ is an Ultra-N-C-S in (Z, τ), which contains $A \cup B$. But it is known that Ultra-$N_{cl}(A \cup B)$ is the smallest Ultra-N-C-S in (Z, τ), which contains $A \cup B$. Therefore, Ultra-$N_{cl}(A \cup B) \subseteq$ Ultra-$N_{cl}(A) \cup$Ultra-$N_{cl}(B)$ (2)

From eqs. (1) and (2), we have Ultra-$N_{cl}(A \cup B)$ = Ultra-$N_{cl}(A) \cup$Ultra-$N_{cl}(B)$.

vii. Suppose that A and B are two ultra neutrosophic sub-sets of an Ultra-N-T-S (Z, τ). It is known that $A \subset A \cup B$ and $B \subset A \cup B$.

Thus, we obtain

$A \subset A \cup B$

$\Rightarrow$ Ultra-$N_{int}(A) \subset$ Ultra-$N_{int}(A \cup B)$;

and $B \subset A \cup B$

$\Rightarrow$ Ultra-$N_{int}(B) \subset$ Ultra-$N_{int}(A \cup B)$.

Therefore, Ultra-$N_{int}(A) \cup$Ultra-$N_{int}(B) \subset$ Ultra-$N_{int}(A \cup B)$.

viii. Assume that A and B are any two ultra neutrosophic sub-sets of an Ultra-N-T-S (Z, τ). It is known that $A \cap B \subseteq A$ and $A \cap B \subseteq B$.

Now, we have

$A \cap B \subseteq A$

$\Rightarrow$ Ultra-$N_{int}(A \cap B) \subseteq$ Ultra-$N_{int}(A)$;

and $A \cap B \subseteq B$

$\Rightarrow$ Ultra-$N_{int}(A \cap B) \subseteq$ Ultra-$N_{int}(B)$.

Therefore, Ultra-$N_{int}(A \cap B) \subseteq$ Ultra-$N_{int}(A) \cap$Ultra-$N_{int}(B)$.

For any two ultra neutrosophic sets A and B, we have Ultra-$N_{int}(A) \subseteq A$ & Ultra-$N_{int}(B) \subseteq B$. This implies, Ultra-$N_{int}(A) \cap$ Ultra-$N_{int}(B) \subseteq A \cap B$. It is known that, Ultra-$N_{int}(A) \cap$Ultra-N_{int} (B) is an Ultra-N-O-S in (Z, τ). Therefore, Ultra-$N_{int}(A) \cap$Ultra-$N_{int}(B)$ is an Ultra-N-O-S in (Z, τ), which is contained in $A \cap B$. We know that Ultra-$N_{int}(A \cap B)$ is the largest Ultra-N-O-S in (Z, τ), which is contained in $A \cap B$. Therefore, Ultra-$N_{int}(A) \cap$Ultra-$N_{int}(B) \subseteq A \cap B$.

Hence, Ultra-$N_{int}(A \cap B) \subseteq$ Ultra-$N_{int}(A) \cap$Ultra-$N_{int}(B)$ and Ultra-$N_{int}(A) \cap$Ultra-$N_{int}(B) \subseteq A \cap B$.

Therefore, Ultra-$N_{int}(A \cap B)$ = Ultra-$N_{int}(A) \cap$Ultra-$N_{int}(B)$.

ix. Suppose that A and B be any two ultra neutrosophic sub-sets of an Ultra-N-T-S (Z, τ). It is known that $A \cap B \subseteq A$, $A \cap B \subseteq B$.

Now, $A \cap B \subseteq A$

$\Rightarrow$ Ultra-$N_{cl}(A \cap B) \subseteq$ Ultra-$N_{cl}(A)$;

and $A \cap B \subseteq B$

$\Rightarrow$ Ultra-$N_{cl}(A \cap B) \subseteq$ Ultra-$N_{cl}(B)$.

Therefore, Ultra-$N_{cl}(A \cap B) \subseteq$ Ultra-$N_{cl}(A) \cap$Ultra-$N_{cl}(B)$.

Theorem 3.3. Let (Z, τ) be an Ultra-N-T-S.

i. If K is an Ultra-N-C-S in (Z, τ), then Ultra-$N_{cl}(K)$=K;

ii. If K is an Ultra-N-O-S in (Z, τ), then Ultra-$N_{int}(K)$=K.

Proof.

i. Let (Z, τ) be an Ultra-N-T-S , and S be an Ultra-N-C-S in (Z, τ).

Now, Ultra-$N_{cl}(S) = \cap\{W$: W is an Ultra-N-C-S in (Z, τ) and $S \subseteq W\}$. Since, S is an Ultra-N-C-S in (Z, τ), so S is the smallest Ultra-N-C-S, which contains S. This implies, $\cap\{W$: W is an Ultra-N-C-S in (Z, τ) and $S \subseteq W\}$ = S. Therefore, Ultra-$N_{cl}(S)$ = S.

ii. Ley (Z, τ) be an Ultra-N-T-S , and S be an Ultra-N-C-S in (Z, τ).

Now, Ultra-$N_{int}(S) = \cup\{W$: W is an Ultra-N-O-S in (Z, τ) and $W \subseteq S\}$. Since, S is an Ultra-N-O-S in (Z, τ), so S is the largest Ultra-N-O-S, which is contained in S. This implies, $\cup\{W$: W is an Ultra-N-O-S in (Z, τ) and $W \subseteq S\}$ = S. Therefore, Ultra-$N_{int}(S)$ = S.

Theorem 3.4. Assume that N is an Ultra neutrosophic sub-set of an Ultra-N-T-S (Z, τ). Then,

i. (Ultra-$N_{int}(N))^c$ = Ultra-$N_{cl}(N^c)$;

ii. (Ultra-$N_{cl}(N))^c$ = Ultra-$N_{int}(N^c)$.

Proof.

i. Let (Z, τ) be an Ultra-N-T-S, and $N=\{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta \in Z\}$ be an Ultra neutrosophic sub-set of (Z, τ).

Now, we have

Ultra-$N_{int}(N)$

$= \cup\{W_i : i \in \Delta$ and W_i is an Ultra-N-O-S in (Z, τ) such that $W_i \subseteq N\}$

$= \{(\delta, \vee T_{W_i}(\delta), \vee I_{W_i}(\delta), \wedge F_{W_i}(\delta), \vee M_{W_i}(\delta)) : \delta \in Z\}$, where for all $i \in \Delta$ and W_i is an Ultra-N-O-S in (Z, τ) such that $W_i \subseteq N$.

$\Rightarrow$ (Ultra-$N_{int}(N))^c = \{(\delta, \wedge T_{W_i}(\delta), \vee I_{W_i}(\delta), \vee F_{W_i}(\delta), \wedge M_{W_i}(\delta)) : \delta \in Z\}$.

Since, $\wedge T_{W_i}(\delta) \leq T_N(\delta)$, $\vee I_{W_i}(\delta) \leq I_N(\delta)$, $\vee F_{W_i}(\delta) \geq F_N(\delta)$, $\wedge M_{W_i}(\delta) \geq M_N(\delta)$, for each $i \in \Delta$ and $\delta \in Z$, so Ultra-$N_{cl}(N^c) = \{(\delta, \wedge T_{W_i}(\delta), \vee I_{W_i}(\delta), \vee F_{W_i}(\delta), \wedge M_{W_i}(\delta)) : \delta \in Z\}\} = \cap\{W_i : i \in \Delta$ and W_i is an Ultra-N-C-S in (Z, τ) such that $N^c \subseteq W_i\}$.

Therefore, (Ultra-$N_{int}(N))^c$ = Ultra-$N_{cl}(N^c)$.

ii. Assume that (Z, τ) be an Ultra-N-T-S, and $N=\{(\delta, T_N(\delta), I_N(\delta), F_N(\delta), M_N(\delta)) : \delta \in Z\}$ be an Ultra neutrosophic sub-set of (Z, τ).

Now, we have

Ultra-N_{cl}(N)

= $\cap$\{W_i : i$\in\Delta$ and W_i is an Ultra-N-C-S in (Z, τ) such that $W_i \supseteq$ N\}

= \{(δ, $\wedge T_{W_i}(\delta)$, $\vee I_{W_i}(\delta)$, $\vee F_{W_i}(\delta)$, $\wedge M_{W_i}(\delta)$) : $\delta\in$Z\}, where for all i$\in\Delta$ and W_i is an Ultra-N-C-S in (Z, τ) such that $W_i \supseteq$ N.

$\Rightarrow$ (Ultra-N_{cl}(N))^c = \{(δ, $\vee T_{W_i}(\delta)$, $\wedge I_{W_i}(\delta)$, $\wedge F_{W_i}(\delta)$, $\vee M_{W_i}(\delta)$) : $\delta\in\Omega$\}.

Since, $\vee T_{W_i}(\delta) \geq T_N(\delta)$, $\wedge I_{W_i}(\delta) \geq I_N(\delta)$, $\wedge F_{W_i}(\delta) \leq F_N(\delta)$, $\vee M_{W_i}(\delta) \leq M_N(\delta)$, for each i$\in\Delta$ and $\delta\in$Z, so Ultra-N_{int}(N^c) = \{(δ, $\vee T_{W_i}(\delta)$, $\wedge I_{W_i}(\delta)$, $\wedge F_{W_i}(\delta)$, $\vee M_{W_i}(\delta)$) : $\delta\in$Z\} = $\cup$\{W_i : i$\in\Delta$ and W_i is an Ultra-N-O-S in (Z, τ) such that $W_i \subseteq N^c$\}. Therefore, (Ultra-N_{cl}(N))^c = Ultra-N_{int}(N^c).

Definition 3.7. Assume that (Z,τ) be an Ultra-N-T-S and K be an UNS over Z. Then K is said to be an Ultra Neutrosophic Semi Open (Ultra-N-S-O) Set if and only if

K$\subseteq$Ultra-N_{cl}(Ultra-N_{int}(K)).

Definition 3.8. Assume that (Z,τ) be an Ultra-N-T-S and K be an UNS over Z. Then K is said to be an Ultra Neutrosophic Pre Open (Ultra-N-P-O) Set if and only if K$\subseteq$ Ultra-N_{int}(Ultra-N_{cl}(K)).

Definition 3.9. Assume that (Z,τ) be an Ultra-N-T-S and K be an UNS over Z. Then K is said to be an Ultra Neutrosophic b-Open (Ultra-N-b-O) Set if and only if

K $\subseteq$ Ultra-N_{cl}(Ultra-N_{int}(K)) $\cup$ Ultra-N_{int}(Ultra-N_{cl}(K)).

An UNS H is said to be an Ultra Neutrosophic b-Closed (Ultra-N-b-C) Set iff H^c is an Ultra Neutrosophic b-Open Set.

Theorem 3.5. Assume that (Z, τ) be an Ultra-N-T-S. Then

i. Every Ultra-N-O-S is an Ultra-N-S-O set;
ii. Every Ultra-N-O-S is an Ultra-N-P-O set;
iii. Every Ultra-N-S-O set is an Ultra-N-b-O set;
iv. Every Ultra-N-P-O set is an Ultra-N-b-O set.

Proof.

i. Assume that (Z, τ) is an Ultra-N-T-S, and K is an Ultra-N-O-S in (Z,τ). So Ultra-N_{int}(K)=K. It is known that K$\subseteq$Ultra-N_{cl}(K). This implies, K$\subseteq$Ultra-N_{cl}(Ultra-N_{int}(K)). Therefore, K is an Ultra-N-S-O set in (Z, τ).

ii. Suppose that (Z, τ) is an Ultra-N-T-S. Assume that K is an Ultra-N-O-S in (Z,τ). Therefore, Ultra-N_{int}(K)=K. It is known that, K$\subseteq$ Ultra-N_{cl}(K).

Now, K $\subseteq$ Ultra-N_{cl}(K)

$\Rightarrow$ Ultra-N_{int}(K) $\subseteq$ Ultra-N_{int}(Ultra-N_{cl}(K))

$\Rightarrow$ K = Ultra-N_{int}(K) $\subseteq$ Ultra-N_{int}(Ultra-N_{cl}(K))

$\Rightarrow$ K $\subseteq$ Ultra-N_{int}(Ultra-N_{cl}(K))

Therefore, K is an Ultra-N-P-O set in (Z, τ).

iii. Suppose that (Z, τ) is an Ultra-N-T-S, and K is an Ultra-N-S-O set in (Z, τ). Therefore, K$\subseteq$ Ultra-N_{cl}(Ultra-N_{int}(K)). This implies, K $\subseteq$ Ultra-N_{cl}(Ultra-N_{int}(K)) $\cup$ Ultra-N_{int}(Ultra-N_{cl}(K)). Therefore, K is an Ultra-N-b-O set in (Z, τ).

iv. Assume that (Z, τ) is an Ultra-N-T-S. Suppose that K is an Ultra-N-P-O set in (Z, τ). So K$\subseteq$ Ultra-N_{int}(Ultra-N_{cl}(K)). This implies, K $\subseteq$ Ultra-N_{cl}(Ultra-N_{int}(K)) $\cup$ Ultra-N_{int}(Ultra-N_{cl}(K)). Therefore, K is an Ultra-N-b-O set in (Z, τ).

4. Conclusions:

In this paper, we introduce the ultra neutrosophic sets and investigate some of their basic properties. Also, we introduce the ultra neutrosophic topology, Ultra Neutrosophic interior, Ultra Neutrosophic closure. By defining Ultra Neutrosophic Set, Ultra Neutrosophic Topology, Ultra Neutrosophic interior and Ultra Neutrosophic closure, we formulate and prove some theorems on Ultra-N-T-Ss and few illustrative examples are provided. We hope that based on these notions in Ultra-N-T-Ss, many new investigations can be carried out in future.

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