VIKOR based MAGDM Strategy with Trapezoidal Neutrosophic Numbers

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Abstract. ViseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) is a popular strategy for multi-attribute decision making (MADM). We extend the VIKOR strategy for MAGDM problems in trapezoidal neutrosophic number environment. In decision making situation, single-valued trapezoidal neutrosophic numbers are employed to express the attribute values. Then we develop an extended VIKOR strategy to deal with MAGDM in single-valued trapezoidal neutrosophic number environment. The influence of decision-making mechanism coefficient is presented. To illustrate and validate the proposed VIKOR strategy, an illustrative numerical example of MAGDM problem is solved in trapezoidal neutrosophic number environment.

Keywords: Neutrosophic set, Trapezoidal neutrosophic fuzzy number, Multi-attribute decision making, VIKOR strategy.

1. Introduction:


VIKOR strategy in trapezoidal neutrosophic number (TrNN) environment is not studied in the literature. To fill up this research gap, we propose a VIKOR strategy to deal with MAGDM problems in TrNN environment. Also, we solve an MAGDM problem based on VIKOR strategy in trapezoidal neutrosophic number.

The rest of the paper is developed as follows. In section 2, we briefly describe definitions of trapezoidal fuzzy number, TrNN, trapezoidal neutrosophic weighted arithmetic averaging (TrNWAA) operator, Hamming distance between two TrNNs. In section 3, we briefly describe extended VIKOR strategy. Thereafter in section 4, we present a VIKOR strategy in TrNN environment. In section 5, we solve an MAGDM problem using the proposed
VIKOR strategy. In section 6, we present the sensitivity analysis. We represent conclusion and scope of future research in section 7.

2. Preliminaries
We present some fundamental definitions of fuzzy sets, neutrosophic set, SVNS, and TrNN.

**Definition 2.1** [58] Let \( \overline{Y} \) be a universal set. Then, a fuzzy set \( F \) is presented as:

\[
F = \{ y, v_F(y) : y \in \overline{Y} \} \tag{1}
\]

where \( v_F(y) \) is the degree of membership which maps \( \overline{Y} \) to \([0,1]\) or we can express by \( v_F : \overline{Y} \rightarrow [0,1] \).

**Definition 2.2**[1] Let \( \overline{Y} \) be an universal set. A neutrosophic set \( N \) can be presented of the form:

\[
N = \{ z, T_N(y), I_N(y), F_N(y) : y \in \overline{Y} \} \tag{2}
\]

where the functions \( T_N, I_N, F_N : [0,1] \rightarrow \overline{Y} \) define respectively the degree of truth membership, the degree of indeterminacy, and the degree of non-membership or falsity of the component \( y \in \overline{Y} \) and satisfy the condition,

\[
0 \leq T_N(y) + I_N(y) + F_N(y) \leq 3
\]

**Definition 2.3**[2] Let \( \overline{Y} \) be a universal set. An SVNS \( N \) in \( \overline{Y} \) is described by

\[
N = \{ z, T_N(y), I_N(y), F_N(y) : y \in \overline{Y} \} \tag{3}
\]

where \( T_N(y), I_N(y), F_N(y) : [0,1] \rightarrow \overline{Y} \) with the condition \( 0 \leq T_N(y) + F_N(y) + I_N(y) \leq 3 \) for all \( y \in \overline{Y} \). The functions \( T_N(y), I_N(y), F_N(y) \) are respectively, the truth membership function, the indeterminacy membership function and the falsity membership function of the element \( y \) to the set \( N \).

**Definition 2.4**[59] A generalized trapezoidal fuzzy number \( \tilde{T} \) denoted by \( \tilde{T} = (b_1, b_2, \hat{b}_1, \hat{b}_2; v) \) is described as a fuzzy subset of the real number \( \mathbb{R} \) with membership function \( \kappa_T(x) \) which is defined by

\[
\kappa_T(x) = \begin{cases} 
\frac{(x - \hat{b}_1)v}{(b_2 - \hat{b}_1)}, & \hat{b}_1 \leq x < b_2 \\
\frac{v}{b_2 - \hat{b}_1}, & b_1 \leq x \leq \hat{b}_1 \\
\frac{(b_2 - x)v}{(b_2 - \hat{b}_1)}, & \hat{b}_2 < x \leq \hat{b}_2 \\
0, & \text{otherwise}
\end{cases} \tag{4}
\]

where \( \hat{b}_1, \hat{b}_2, b_1, b_2 \) are real number satisfying \( \hat{b}_1 \leq \hat{b}_2 \leq b_1 \leq b_2 \) and \( v \) is the membership degree.

**Definition 2.5**[43, 44] Let \( x \) be a TrNN. Then, its truth membership, indeterminacy membership, and falsity membership functions are presented respectively as:

\[
T_x(z) = \begin{cases} 
\frac{(z - b_1)u}{(b_2' - b_1')}, & b_1' \leq z < b_2' \\
\frac{u}{b_2' - b_1'}, & b_1 \leq z \leq b_2' \\
\frac{(b_2' - z)u}{(b_2' - b_1')}, & b_1' \leq z \leq b_2' \\
0, & \text{otherwise}
\end{cases} \tag{5}
\]

\[
I_x(z) = \begin{cases} 
\frac{(b_2' - z) + (z - b_1')u}{(b_2' - b_1')}, & b_1' \leq z < b_2' \\
\frac{u}{b_2' - b_1'}, & b_1 \leq z \leq b_2' \\
\frac{z - b_2' + (b_2' - z)u}{b_2' - b_1'}, & b_1' \leq z \leq b_2' \\
0, & \text{otherwise}
\end{cases} \tag{6}
\]
\[
F_{i}(z) = \begin{cases} \frac{b_i^1 - z + (z - b_i^1) f_i}{b_i^2 - b_i^1}, & b_i^1 \leq z < b_i^2 \\ \frac{f_i, b_i^1 \leq z \leq b_i^1}{0}, & \text{otherwise} \end{cases}
\]

Here \(0 \leq T_{i}(z) \leq 1.0 \leq I_{i}(z) \leq 1\) and \(0 \leq F_{i}(z) \leq 1\) and \(0 \leq T_{i}(z) + I_{i}(z) + F_{i}(z) \leq 3; b_i^1, b_i^2, b_i^3, b_i^4 \in R\). Then \(x = ([b_i^1, b_i^2, b_i^3, b_i^4]; t_i, i, f_i)\) is called a TrNN.

**Definition 2.6** [43] Let \(m_i = ([p_{i1}, q_{i1}, r_{i1}, s_{i1}]; t_{i1}, i, f_{i1})\) be a group of TrNNs, then a trapezoidal neutrosophic weighted arithmetic averaging (TrNWAA) operator is defined as follows:

\[
\text{TrNWAA}(m_1, m_2, ..., m_n) = \sum_{i=1}^{n} \bar{w}_i m_i
\]

where, \(\bar{w}_i\) is the weight of \(m_i (i = 1, 2, ..., n)\) such that \(\bar{w}_i > 0\) and \(\sum_{i=1}^{n} \bar{w}_i = 1\). Specially, when \(\bar{w}_i = 1/n\) for i=1,2,..., n' the TrNWAA operator transform into the trapezoidal neutrosophic arithmetic averaging (TrNAA) operator.

**Definition 2.7**[44] Let \(m_1 = ([p_{11}, q_{11}, r_{11}, s_{11}]; t_{11}, i, f_{11})\) and \(m_2 = ([p_{21}, q_{21}, r_{21}, s_{21}]; t_{21}, i, f_{21})\) be any two TrNNs. The normalized Hamming distance between \(m_1\) and \(m_2\) is defined as:

\[
d(m_1, m_2) = \frac{1}{12} \left( \left| \frac{\tilde{p}_1(2 + t_{1n} - i_n - f_n) - \tilde{p}_2(2 + t_{2n} - i_n - f_n)}{\tilde{p}_1(2 + t_{1n} - i_n - f_n) + \tilde{p}_2(2 + t_{2n} - i_n - f_n)} \right| + \left| \frac{\tilde{q}_1(2 + t_{1n} - i_n - f_n) - \tilde{q}_2(2 + t_{2n} - i_n - f_n)}{\tilde{q}_1(2 + t_{1n} - i_n - f_n) + \tilde{q}_2(2 + t_{2n} - i_n - f_n)} \right| \right)
\]

2.8. **Standardize the decision matrix**[44]

Let \(D = (b_{ij})_{p \times s}\) be a neutrosophic matrix, where \(b_{ij} = ([b_{ij}^1, b_{ij}^2, b_{ij}^3, b_{ij}^4]; t_{ij}, i_n, f_{ij})\) is the rating value of the alternative \(x_i\) with respect to attribute \(y_j\). To remove the effect of several physical dimensions, we standardize the decision matrix \((b_{ij})_{p \times s}\) for benefit type and cost type attributes.

We denote the standardized decision matrix by \(D' = (s_{ij})_{p \times s'}\)

1. For benefit type attribute

\[
b_{ij}' = \left( \frac{v_{ij}^1}{v_{ij}^2}, \frac{v_{ij}^3}{v_{ij}^4}; t_{ij}, i_n, f_{ij} \right)
\]

2. For cost type attribute:

\[
b_{ij}' = \left( \frac{v_{ij}^1}{v_{ij}^2}, \frac{v_{ij}^3}{v_{ij}^4}; t_{ij}, i_n, f_{ij} \right)
\]

Here \(v_{ij}^1 = \max \{b_{ij}^i: i = 1, 2, ..., p'\}\) and \(v_{ij}^2 = \min \{b_{ij}^i: i = 1, 2, ..., p'\}\) for \(j = 1, 2, ..., n'\)

Hence, we obtain standardized matrix \(D'\) as:

\[
D' = (s_{ij})_{p \times s'}
\]

3.VIKOR Strategy for MADM

Assume that \(B_1, B_2, ..., B_s\) are the s alternatives. For the alternative \(B_i\), assume that the rating of the \(j\)'th criterion is \(h_{ij}\), i.e. \(h_{ij}\) is the value of \(j\)'th criterion for the alternative \(B_i\); the number of criteria is assumed to be \(r\). Development of the extended VIKOR strategy is started with the following form of \(L_p\) metric:
To formulate ranking measure, \( L_{i'q} \) (as \( S_{i'} \)) and \( L_{i'q}^{'} \) (as \( R_{i'} \)) are employed. The solution obtained by \( \min S_{i'} \) reflects a maximum group utility ("majority" rule), and the solution obtained by \( \min R_{i'} \) reflects a minimum individual regret of the "opponent".

VIKOR strategy is presented using the following steps:

(a) Evaluate the best \( h_{j+}^{i'} \) and the worst \( h_{j-}^{i'} \) values of all criteria \( j' = 1, 2, \ldots, n' \).

\[
h_{j+}^{i'} = \max_{i} h_{j+}^{i}, \quad h_{j-}^{i'} = \min_{i} h_{j+}^{i}, \text{for benefit criterion,}
\]

\[
h_{j+}^{i'} = \min_{i} h_{j-}^{i'}, \quad h_{j-}^{i'} = \max_{i} h_{j-}^{i'}, \text{for cost criterion}
\]

(b) Calculate the values \( F_{i'} \) and \( G_{i'} \); \( i' = 1, 2, \ldots, m' \), by these relations:

\[
F_{i'} = \sum_{j'=1}^{n'} w_{j'} \frac{(h_{j+}^{i'} - h_{j-}^{i'})}{(h_{j+}^{i'} - h_{j-}^{i'})}, \quad 1 \leq q \leq \alpha, i' = 1, 2, \ldots, s
\]

\[
G_{i'} = \max_{j'} \left( \frac{w_{j'} (h_{j+}^{i'} - h_{j-}^{i'})}{(h_{j+}^{i'} - h_{j-}^{i'})} \right)
\]

where \( w_{j'} \) (\( j'= 1, 2, \ldots, r \)) represent the weights of criteria.

(c) Sorting by the values \( F \), \( G \) and \( K \) in decreasing order, we rank the alternatives.

(d) Propose the alternative \( B_1^j \) as a compromise solution that is ranked the best by the measure \( K \) (minimal) subject to the conditions A1 and A2:

A1. Acceptable advantage:

\[
K(B_2^j) - K(B_1^j) \geq DK
\]

where \( B_2^j \) is second alternative in the ranking list by \( K \); \( DK = 1/(s-1) \); \( s \) = the number of alternatives.

A2. Acceptable stability in decision making:

Using \( F \) or/and \( G \), we must have alternative \( B_1^j \) as the best ranked. We say the compromise solution as stable subject to

i. "voting by majority rule" (when \( v > 0.5 \) is needed),

ii. or "by consensus" \( v = 0.5 \),

iii. or "with veto" (\( v < 0.5 \)).

\( v \) reflects the weight of the decision making strategy of "the majority of criteria" (or "the maximum group utility").

- \( B_1^j \) and \( B_2^j \) are compromise solutions if A2 is not satisfied, Or

- \( B_1^j, B_2^j, \ldots, B_M^j \) are compromise solutions if A1 is not satisfied.

Evaluate \( K(B_2^j) - K(B_1^j) \geq DK \) to determine \( B_M^j \) for maximum \( M \) (the positions of these alternatives are "in closeness").

The minimal value of \( K \) determines the best alternative.
4. VIKOR strategy for solving MAGDM problem in TrNN environment:

Consider an MADGM problem consisting of r alternatives and t attributes. The alternatives and attributes are presented by \( \alpha' = \{\alpha'_1, \alpha'_2, \ldots, \alpha'_r\} \) and \( \beta' = \{\beta'_1, \beta'_2, \ldots, \beta'_t\} \) respectively. Assume that \( \tau = \{\tau_1, \tau_2, \ldots, \tau_t\} \) is the set of weights of the attributes, where \( \tau_j \geq 0 \) and \( \sum_{j=1}^{t} \tau_j = 1 \). Assume that \( B = \{B_1, B_2, \ldots, B_K\} \) be the set of K decision makers and \( \sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_K\} \) be the set of weights of the decision makers, where \( \sigma_k \geq 0 \) and \( \sum_{k=1}^{K} \sigma_k = 1 \). The rating values offered by the experts are presented in terms of TrNN.

The MAGDM strategy is described as follows:

Step-1: Let \( D(p^i_j) \) be the \( N' \)-th decision matrix where \( \alpha'_j \) is alternative with respect to attribute \( \beta'_j \). The \( N' \)-th decision matrix denoted by \( D^{N'} \) is presented as:

\[
D^{N'} = \begin{pmatrix}
\beta'_1 & \beta'_2 & \ldots & \beta'_t \\
\alpha'_1 & p^{N'}_{11} & p^{N'}_{12} & \ldots & p^{N'}_{1t} \\
\alpha'_2 & p^{N'}_{21} & p^{N'}_{22} & \ldots & p^{N'}_{2t} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
\alpha'_t & p^{N'}_{t1} & p^{N'}_{t2} & \ldots & p^{N'}_{tt} \\
\end{pmatrix} \tag{19}
\]

where \( N' = 1, 2, \ldots, s \); \( i' = 1, 2, \ldots, r \); \( j' = 1, 2, \ldots, t \).

Step-2: To standardize the benefit criterion, we use the equation (10) and for cost criterion, we use (11). After standardizing, the decision matrix reduces to

\[
D' = \begin{pmatrix}
\beta'_1 & \beta'_2 & \ldots & \beta'_t \\
\alpha'_1 & p^{N'}_{11} & p^{N'}_{12} & \ldots & p^{N'}_{1t} \\
\alpha'_2 & p^{N'}_{21} & p^{N'}_{22} & \ldots & p^{N'}_{2t} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
\alpha'_t & p^{N'}_{t1} & p^{N'}_{t2} & \ldots & p^{N'}_{tt} \\
\end{pmatrix}
\]

Step-3: To obtain aggregate decision matrix, we use trapezoidal neutrosophic weighted arithmetic operator(TrNWAA) which is presented below:

\[
p^{ij}_q = \sum_{q=1}^{M} \sigma_q p^{ij}_q \tag{20}
\]

Therefore, we obtain the aggregated decision matrix as:

\[
D^{N'} = \begin{pmatrix}
\beta'_1 & \beta'_2 & \ldots & \beta'_t \\
\alpha'_1 & p^{N'}_{11} & p^{N'}_{12} & \ldots & p^{N'}_{1t} \\
\alpha'_2 & p^{N'}_{21} & p^{N'}_{22} & \ldots & p^{N'}_{2t} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
\alpha'_t & p^{N'}_{t1} & p^{N'}_{t2} & \ldots & p^{N'}_{tt} \\
\end{pmatrix}
\]

Step-4: Define the positive ideal solution (PIS) \( S^+ \) and negative ideal solution (NIS) \( S^- \)

\[
S^+ = (|b^1_1, b^2_1, b^3_1, b^4_1|; t^1_1, i^1_1, f^1_1) = (|b^1_1, b^2_1, b^3_1, b^4_1|; \max t_1, \min i_1, \min f_1) \tag{21}
\]

\[
S^- = (|b^1_1, b^2_1, b^3_1, b^4_1|; t^1_1, i^1_1, f^1_1) = (|b^1_1, b^2_1, b^3_1, b^4_1|; \max t_1, \min i_1, \min f_1) \tag{22}
\]

Step 5: Compute

\[
\Gamma_m = \sum_{j=1}^{t} \tau_j \times d((|b^1_j, b^2_j, b^3_j, b^4_j|; t^1_j, i^1_j, f^1_j), (|b^1_j, b^2_j, b^3_j, b^4_j|; t^2_j, i^2_j, f^2_j)) \tag{23}
\]
\[ Z_m = \max \sum \tau_m \times d((b^1_m, b^2_m, b^3_m, b^4_m; t^1_m, i^1_m, f^1_m), (b^1_j, b^1_j, b^1_j, b^1_j; t^1_j, i^1_j, f^1_j), (b^1_l, b^1_l, b^1_l, b^1_l; t^1_l, i^1_l, f^1_l)) \]

where \( \tau_m \) is the weight of \( b^m \).

Using equation (9), we obtain
\[ d((b^1_m, b^2_m, b^3_m, b^4_m; t^1_m, i^1_m, f^1_m), (b^1_j, b^1_j, b^1_j, b^1_j; t^1_j, i^1_j, f^1_j)) \]
\[ = \frac{1}{12} \left[ b^1_j (2 + t^1_j - i^1_j - f^1_j) - b^1_j (2 + t^1_j - i^1_j - f^1_j) \right] \]
\[ + b^2_j (2 + t^1_j - i^1_j - f^1_j) - b^2_j (2 + t^1_j - i^1_j - f^1_j) \]
\[ + b^3_j (2 + t^1_j - i^1_j - f^1_j) - b^3_j (2 + t^1_j - i^1_j - f^1_j) \]
\[ + b^4_j (2 + t^1_j - i^1_j - f^1_j) - b^4_j (2 + t^1_j - i^1_j - f^1_j) \]

Step 6: Compute the \( \Theta \) by the following formula:
\[ \Theta_m = \Psi \left( \frac{\Gamma_m - \Gamma^*_m}{\Gamma_m - \Gamma^*_m} \right) \frac{(Z_m - Z^*_m)}{(Z_m - Z^*_m)} \]

where \( \Gamma^*_m = \min \Gamma_m \), \( \Gamma^*_m = \max \Gamma_m \)
\[ Z^*_m = \min Z_m \), \( Z^*_m = \max Z_m \)

Here, \( \Psi \) denotes “decision-making mechanism coefficient”.

i. \( \Theta \) is the minimal if \( \Psi \leq 0.5 \)

ii. \( \Theta \) is the “maximum group utility” if \( \Psi = 0.5 \)

iii. \( \Theta \) is both the minimal and the “maximum group utility” if \( \Psi = 0.5 \).

Step 7: Ranking the alternative by \( \Gamma_m, Z_m, \) and \( \Theta_m \).

Step 8: Determine the compromise solution

Obtain alternative \( \alpha^1 \) as a compromise solution, that is ranked as the best by the measure \( \alpha^1 \) (minimal) if the A1 and A2 are satisfied:

A1. Acceptable stability:
\[ \Theta(\alpha^2) - \Theta(\alpha^1) \geq \frac{1}{r-1} \]

where \( \alpha^1, \alpha^2 \) are the alternatives with 1st and 2nd positions in the ranking by \( \Theta \); \( r = \) the number of alternatives.

A2. Acceptable stability in decision making:

Alternative \( \alpha^1 \) must also be the best ranked by \( \Gamma \) or/and \( Z \). This compromise solution is stable within whole decision making process.

- \( \alpha^1 \) and \( \alpha^2 \) are compromise solutions if A2 is not satisfied, or
- \( \alpha^1, \alpha^2, \ldots, \alpha^r \) are compromise solutions if A1 is not satisfied and \( \alpha^r \) is decided by
\[ \Theta(\alpha^2) - \Theta(\alpha^1) \leq \frac{1}{r-1} \]

for maximum \( r \).

The minimal value of \( \Theta \) determines the best alternative.
5. Numerical example
To illustrate the developed VIKOR strategy, we consider an MAGDM problem adapted from [57]. The considered MAGDM problem is described as follows:
An investment company constitutes a decision making board with three experts to invest certain amount of money in the best alternative. The experts evaluate the four alternatives and three attributes which are described below:

**Alternatives:**
1. Car company ($\alpha_1'$)
2. Food company ($\alpha_2'$)
3. Computer company ($\alpha_3'$)
4. Arms company ($\alpha_4'$)

**Attributes:**
1. Risk factor ($\beta_1'$)
2. Growth factor ($\beta_2'$)
3. Environment impact ($\beta_3'$)

Suppose, $\tau = (0.30,0.42,0.28)$ be the set of weights of the decision makers and $\sigma = (0.33,0.39,0.28)$ be the set of weights of the attributes.

**Step-1:** In this step, we construct the decision matrix in TrNNs form.

**Decision matrix $D^1$**

\[
\begin{bmatrix}
\alpha'_1 & \beta'_1 & \gamma'_1 \\
(0.5,0.6,0.7,0.8);0.1,0.4,0.7 & (0.1,0.1,0.2,0.3);0.6,0.7,0.5 & (0.1,0.2,0.2,0.3);0.7,0.2,0.4 \\
\vdots & \vdots & \vdots \\
\alpha'_4 & \beta'_4 & \gamma'_4 \\
(0.7,0.8,0.9);0.3,0.3,0.2 & (0.1,0.2,0.3,0.3);0.6,0.5,0.2 & (0.2,0.2,0.2,0.2);0.5,0.2,0.2 \\
\end{bmatrix}
\]  

**Decision matrix $D^2$**

\[
\begin{bmatrix}
\alpha'_1 & \beta'_1 & \gamma'_1 \\
(0.1,0.1,0.2,0.3);0.2,0.5,0.1 & (0.2,0.2,0.3,0.4);0.2,0.5,0.1 & (0.2,0.2,0.3,0.4);0.4,0.5,0.2 \\
\vdots & \vdots & \vdots \\
\alpha'_4 & \beta'_4 & \gamma'_4 \\
(0.5,0.6,0.7,0.7);0.5,0.2,0.1 & (0.2,0.2,0.2,0.2);0.3,0.4,0.5 & (0.1,0.1,0.2,0.2);0.3,0.7,0.4 \\
\end{bmatrix}
\]

**Decision matrix $D^3$**

\[
\begin{bmatrix}
\alpha'_1 & \beta'_1 & \gamma'_1 \\
(0.3,0.4,0.4,0.5);0.5,0.1,0.1 & (0.1,0.2,0.2,0.3);0.5,0.1,0.1 & (0.2,0.2,0.3,0.4);0.6,0.2,0.1 \\
\vdots & \vdots & \vdots \\
\alpha'_4 & \beta'_4 & \gamma'_4 \\
(0.3,0.4,0.4,0.5);0.4,0.5,0.3 & (0.2,0.3,0.3,0.4);0.5,0.4,0.3 & (0.1,0.2,0.2,0.3);0.5,0.1,0.1 \\
\end{bmatrix}
\]

**Step-2:** We do not need to standardize the defining matrix as all the criteria are profit type.

**Step-3:** Using TrNWAA operator of equation (20), we get aggregate decision matrix of (29), (30), and (31) which is presented below:

\[
\begin{bmatrix}
\beta'_1 & \beta'_2 & \beta'_3 \\
(0.276,0.334,0.406,0.506);0.273,0.298,0.179 & (0.142,0.17,0.242,0.342);0.431,0.352,0.162 & (0.254,0.326,0.396,0.496);0.633,0.294,0.203 \\
\vdots & \vdots & \vdots \\
\beta'_4 & \beta'_5 & \beta'_6 \\
(0.504,0.604,0.646,0.704);0.447,0.226,0.123 & (0.142,0.20,0.258,0.358);0.461,0.290,0.242 & (0.13,0.158,0.228,0.256);0.343,0.338,0.346 \\
\end{bmatrix}
\]

**Step-4:** Here we define positive ideal solution and negative solution by employing equations (21) and (22).

The positive ideal solution $R^+$ is presented as:

\[
\beta'_1 = (0.504,0.604,0.646,0.704);0.447,0.226,0.123 \\
\beta'_2 = (0.2,0.3,0.33,0.372);0.537,0.242,0.162 \\
\beta'_3 = (0.13,0.158,0.228,0.256);0.343,0.338,0.346
\]

The negative ideal solution $R^-$ is presented as:

\[
\beta'_1 = (0.188,0.258,0.286,0.356);0.235,0.380,0.401 \\
\beta'_2 = (0.1,0.13,0.172,0.274);0.277,0.354,0.249 \\
\beta'_3 = (0.432,0.532,0.604,0.662);0.640,0.169,0.136 \\
\]

**Step-5:** Using equations (23) and (24), we compute $\Gamma_m$ and $Z_m$ which are presented as:
\[\Gamma_1 = \left(\frac{0.33 \times 0.202}{0.294}\right) + \left(\frac{0.39 \times 0.070}{0.121}\right) + \left(\frac{0.28 \times 0.172}{0.327}\right) = 0.601,\]
\[\Gamma_2 = \left(\frac{0.33 \times 0.238}{0.294}\right) + \left(\frac{0.39 \times 0.119}{0.121}\right) + \left(\frac{0.28 \times 0.181}{0.327}\right) = 0.805,\]
\[\Gamma_3 = \left(\frac{0.33 \times 0.298}{0.294}\right) + \left(\frac{0.39 \times 0}{0.121}\right) + \left(\frac{0.28 \times 0}{0.327}\right) = 0.334,\]
\[\Gamma_4 = \left(\frac{0.33 \times 0}{0.294}\right) + \left(\frac{0.39 \times 0.080}{0.121}\right) + \left(\frac{0.28 \times 0.284}{0.327}\right) = 0.501.\]

Here we use Hamming distance to measure the distance between two TrNN.

\[Z_i = \max \left\{ \left(\frac{0.33 \times 0.202}{0.294}\right), \left(\frac{0.39 \times 0.070}{0.121}\right), \left(\frac{0.28 \times 0.172}{0.327}\right) \right\} = 0.228,\]
\[Z_i = \max \left\{ \left(\frac{0.33 \times 0.238}{0.294}\right), \left(\frac{0.39 \times 0.119}{0.121}\right), \left(\frac{0.28 \times 0.181}{0.327}\right) \right\} = 0.383,\]
\[Z_i = \max \left\{ \left(\frac{0.33 \times 0.298}{0.294}\right), \left(\frac{0.39 \times 0}{0.121}\right), \left(\frac{0.28 \times 0}{0.327}\right) \right\} = 0.334,\]
\[Z_i = \max \left\{ \left(\frac{0.33 \times 0}{0.294}\right), \left(\frac{0.39 \times 0.080}{0.121}\right), \left(\frac{0.28 \times 0.284}{0.327}\right) \right\} = 0.258.\]

Step 6: Using (25), (26), and (27) we calculate \(\Theta_i\)
\[\Theta_1 = 0.283, \Theta_2 = 1, \Theta_3 = 0.342, \Theta_4 = 0.274\]

Step 7: The ranking order of alternatives is
\[\Theta_3 \leq \Theta_1 \leq \Theta_2 \leq \Theta_4\]

**Table 1.** Preference ranking order and compromise solution based on \(\Gamma\), \(Z\) and \(\Theta\)

<table>
<thead>
<tr>
<th>(\alpha'_1)</th>
<th>(\alpha'_2)</th>
<th>(\alpha'_3)</th>
<th>(\alpha'_4)</th>
<th>Ranking</th>
<th>Compromise solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Gamma)</td>
<td>0.6</td>
<td>0.805</td>
<td>0.334</td>
<td>0.501</td>
<td>(\alpha'_1 &lt; \alpha'_2 &lt; \alpha'_3 &lt; \alpha'_4)</td>
</tr>
<tr>
<td>(Z)</td>
<td>0.228</td>
<td>0.383</td>
<td>0.334</td>
<td>0.258</td>
<td>(\alpha'_1 &lt; \alpha'_2 &lt; \alpha'_3 &lt; \alpha'_4)</td>
</tr>
<tr>
<td>(\Theta(\psi=0.5))</td>
<td>0.282</td>
<td>1</td>
<td>0.342</td>
<td>0.274</td>
<td>(\alpha'_1 &lt; \alpha'_2 &lt; \alpha'_3 &lt; \alpha'_4)</td>
</tr>
</tbody>
</table>

**Step 8: Determine the compromise solution**

If we rank \(\Theta\) in decreasing order, the best position alternative is \(\alpha'_1\) with \(\Theta(\alpha'_1) = 0.274\), and the 2nd best position \(\alpha''\) with \(\Theta(\alpha'') = 0.283\). Therefore, \(\Theta(\alpha'_2) - \Theta(\alpha'_1) > 0.008 > 0.33\) (since \(\frac{1}{0.274 - 0.283} = 33\)), which does not satisfy the condition 1\((\Theta(\alpha'_2) - \Theta(\alpha'_1) > 0.33)\).

Here \(\alpha'_1\) is ranked best by \(\Gamma\) and \(Z\) and satisfies the condition 2.

So, the compromise solution as follows:
\[\Theta(\alpha'_1) - \Theta(\alpha'_4) > 0.008 > 0.33,\]
\[\Theta(\alpha'_2) - \Theta(\alpha'_4) > 0.726 > 0.33,\]
\[ \Theta (\alpha'_3) - \Theta (\alpha'_4) = 0.05 < 0.33, \]

Therefore, \( \alpha'_1, \alpha'_2 \) and \( \alpha'_4 \) are compromise solutions.

### 6.1 The impact of parameter \( \Psi \)

Table 2 demonstrates how the different values of \( \Psi \) impact the ranking order of the alternatives \( \alpha'_i \).

**Table 2.** For different values of \( \Psi \), ranking the value of \( \alpha'_i \) \( (i' = 1, 2, 3, 4) \).

<table>
<thead>
<tr>
<th>Values of ( \Psi )</th>
<th>Values of ( \alpha'_i )</th>
<th>Preference order</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Psi = 0.1 )</td>
<td>( \alpha'_1 = 0.057, \alpha'_2 = 1, \alpha'_3 = 0.615, \alpha'_4 = 0.209 )</td>
<td>( \alpha'_1 &lt; \alpha'_4 &lt; \alpha'_3 &lt; \alpha'_2 )</td>
</tr>
<tr>
<td>( \Psi = 0.2 )</td>
<td>( \alpha'_1 = 0.113, \alpha'_2 = 1, \alpha'_3 = 0.547, \alpha'_4 = 0.225 )</td>
<td>( \alpha'_1 &lt; \alpha'_4 &lt; \alpha'_3 &lt; \alpha'_2 )</td>
</tr>
<tr>
<td>( \Psi = 0.3 )</td>
<td>( \alpha'_1 = 0.170, \alpha'_2 = 1, \alpha'_3 = 0.479, \alpha'_4 = 0.241 )</td>
<td>( \alpha'_1 &lt; \alpha'_4 &lt; \alpha'_3 &lt; \alpha'_2 )</td>
</tr>
<tr>
<td>( \Psi = 0.4 )</td>
<td>( \alpha'_1 = 0.227, \alpha'_2 = 1, \alpha'_3 = 0.410, \alpha'_4 = 0.257 )</td>
<td>( \alpha'_1 &lt; \alpha'_4 &lt; \alpha'_3 &lt; \alpha'_2 )</td>
</tr>
<tr>
<td>( \Psi = 0.5 )</td>
<td>( \alpha'_1 = 0.282, \alpha'_2 = 1, \alpha'_3 = 0.342, \alpha'_4 = 0.274 )</td>
<td>( \alpha'_1 &lt; \alpha'_4 &lt; \alpha'_3 &lt; \alpha'_2 )</td>
</tr>
<tr>
<td>( \Psi = 0.6 )</td>
<td>( \alpha'_1 = 0.340, \alpha'_2 = 1, \alpha'_3 = 0.274, \alpha'_4 = 0.290 )</td>
<td>( \alpha'_1 &lt; \alpha'_4 &lt; \alpha'_3 &lt; \alpha'_2 )</td>
</tr>
<tr>
<td>( \Psi = 0.7 )</td>
<td>( \alpha'_1 = 0.370, \alpha'_2 = 1, \alpha'_3 = 0.205, \alpha'_4 = 0.306 )</td>
<td>( \alpha'_1 &lt; \alpha'_4 &lt; \alpha'_3 &lt; \alpha'_2 )</td>
</tr>
<tr>
<td>( \Psi = 0.8 )</td>
<td>( \alpha'_1 = 0.454, \alpha'_2 = 1, \alpha'_3 = 0.137, \alpha'_4 = 0.399 )</td>
<td>( \alpha'_1 &lt; \alpha'_4 &lt; \alpha'_3 &lt; \alpha'_2 )</td>
</tr>
<tr>
<td>( \Psi = 0.9 )</td>
<td>( \alpha'_1 = 0.510, \alpha'_2 = 1, \alpha'_3 = 0.068, \alpha'_4 = 0.338 )</td>
<td>( \alpha'_1 &lt; \alpha'_4 &lt; \alpha'_3 &lt; \alpha'_2 )</td>
</tr>
</tbody>
</table>

### 7. Conclusions

Extended VIKOR strategy for MAGDM in trapezoidal neutrosophic number environment is presented in the paper. TrNWAA operator and Hamming distance are employed to develop the VIKOR strategy for MAGDM. Finally, an MAGDM problem is solved to demonstrate the proposed VIKOR strategy. Here, a sensitivity analysis is performed to demonstrate the impact of different values of the “decision making mechanism coefficient” on ranking system. The proposed extended VIKOR strategy for MAGDM problems can be used to deal with decision making problems such as brick selection [60, 61], logistics center selection [62], teacher selection [63], weaver selection [64], etc.

### References


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